Aggregate Productivity, Human–Capital Investments, and Trade

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Abstract

We develop an alternative mechanism to explain the distribution of productivities across firms and their self–selection with respect to export activities. The mechanism is based on the decisions about acquiring education made by heterogeneous individuals. Our framework thus integrates standard approaches developed in heterogeneous–firms trade theory, and extends it by applying different specifications of preferences. Most importantly, the impact of trade liberalization on the self–selection of firms is fully preserved by our model. However, welfare results in heterogeneous–firm trade models are not robust with respect to the modeling of preferences and firm heterogeneities.

Keywords: Heterogeneous firms, education, gains from trade

JEL–Code: F12, I21

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1 Introduction

Recent literature on the effects of trade liberalization emphasizes aggregate productivity gains caused by endogenous self–selection effects of heterogeneous firms. These heterogeneous–firm trade theories are guided by results from empirical research revealing (i) persistent productivity differences among firms within narrowly defined industries and (ii) that firms engaging in international trade are (on average) more productive than non–exporters.\(^1\) Based on the monopolistic–competition approach of the new trade theory (cf. Helpman and Krugman, 1985), Manasse and Turrini (2001) and Melitz (2003) have developed models with firm–specific productivities that provide an explanation of these findings.\(^2\) Due to a fixed–cost component in trade costs only the most productive firms find it profitable to export. As a result, large highly productive firms acting on large world markets coexist with smaller less productive firms that constrain their activities to national markets. In such a framework, the opening up to trade or trade liberalization due to declining trade costs generates further expansion of high–productive firms relative to low–productive ones—in Melitz’s approach some low–productive firms even exit the market—thus raising an economy’s aggregate productivity. Although trade is costly, the compositional change within the pool of firms generated by trade liberalization is shown to raise aggregate social welfare by its positive effect on aggregate productivity. That positive welfare impact from productivity growth typically is enhanced by access to a greater spectrum of product varieties induced by trade liberalization.\(^3\)

Although drawing on similar mechanisms of how trade liberalization affects the composition of firms and thus aggregate productivity and social welfare, the papers mentioned above substantially differ with respect to how differences in firm productivity are introduced. Manasse and Turrini (2001) attribute firm heterogeneity to exogenously given differences in the abilities of entrepreneurs that

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\(^1\)Cf. Bernard et al. (2007) for a recent survey on the theoretical and empirical studies in that field.

\(^2\)An alternative framework to analyze the relationship between firm heterogeneity and trade is the Ricardian model with stochastic firm productivities developed by Bernard et al. (2003).

\(^3\)Since some domestic firms exit the market while foreign firms start to export, the spectrum of available product varieties may actually decline in Melitz’s model. However, any negative impact on aggregate welfare from reduced product differentiation is always dominated by the welfare gain from productivity growth.
determine firms’ productivities. With the number of entrepreneurs/firms given exogenously, trade liberalization draws resources to the high–productive exporting firms thus raising aggregate productivity. Melitz (2003) assumes that \textit{ex ante} identical firms have to spend a fixed investment in order to discover their productivity. Specifically, firms draw their productivities from an exogenously given pool of productivities in the process of developing their differentiated products. Consequently, firms become heterogeneous \textit{ex post} and self–select into exporters, nonexporters, and those firms exiting the market. Again, the outcome of this selection process depends on the extent to which an economy is exposed to trade.\footnote{A related paper is Yeaple (2005) who allows firms to employ different technologies and different types of workers. Highly skilled labor is assumed to have a comparative advantage in technologies with low unit costs but high fixed costs. Depending on the availability of skills and on the costs of trade, firms select into the group of exporters by employing highly skilled labor together with low–unit–cost technologies or into the group of nonexporters by employing less skilled labor together with high–unit–cost technologies. Trade liberalization then causes more firms to employ low–unit–cost technologies. However, Yeaple does not emphasize productivity differentials between firms and aggregate productivity effects. Furthermore, he does not analyze effects on aggregate welfare.}

The present paper integrates both approaches. Alike Manasse and Turrini (2001) we attribute differences in the productivities of firms to differences in the skills of manager owners (entrepreneurs) operating that firms. Rather than taking the distribution of skills as given, we extend their analysis by allowing individuals to react to changes in the rentals of acquiring education thus endogenizing both the equilibrium mass of active firms and the supply of workers. As a result, we arrive at basically the same endogenous selection mechanism as Melitz (2003) where both the mass of active firms and the mass of firms engaging in exporting is endogenous. Although preserving the endogenous self–selection process of heterogeneous firms and its impact on aggregate productivity, however, our model argues about the robustness of the welfare results derived so far. As we will show, the welfare effects of market integration are ambiguous in general. More specifically, welfare effects are shown to be sensitive with respect to consumers’ valuation of having different product varieties (love of varieties).

The paper is organized as follows. Section 2 presents the basic model. In section 3 we characterize the closed–economy equilibrium. In section 4 we characterize the open–economy equilibrium, and we derive the implications of trade liberalization. Section 5 then concludes.
2 The Model

2.1 Demand

We assume that consumers have preferences over product varieties according to the following generalized CES function (cf. Benassy, 1996)

\[ U = M^{\frac{\eta}{\rho}} \left[ \int_{j \in J} c(j)^\rho \, dj \right]^{1/\rho}, \quad \rho \in (0, 1), \quad \sigma \equiv \frac{1}{1-\rho} > 1, \]  

where \( M \) is the mass of available goods, \( c(j) \) denotes consumption of product variety \( j \), and \( J \) represents the set of available goods; \( \sigma \) denotes the constant elasticity of substitution between any two goods, and \( \eta \) is a measure of the love of variety that is increasing in \( \eta \). Two widely applied special cases arise from this general specification: (i) With \( \eta = 0 \) we have the case of the standard Dixit-Stiglitz specification (Dixit and Stiglitz, 1977) that is widely applied in trade theory, esp. by Manasse and Turrini (2001) and Melitz (2003). (ii) With \( \eta = \rho - 1 < 0 \) we are in the case proposed by Blanchard and Giavazzi (2003) that neutralizes for any love of variety; this specification has been applied to heterogeneous–firm models in context of labor–market imperfections by, e.g., Egger and Kreickemeier (2008).

Due to the homotheticity of the utility function, we can derive aggregate demand from the problem of a representative consumer. With \( E \) denoting aggregate consumer income, demand for variety \( i \) is then given by

\[ c(i) = E p(i)^{-\sigma} P^{\sigma-1}, \]  

where \( p(i) \) is the price of variety \( i \), and \( P \) is the price index defined over prices of varieties

\[ P := \left[ \int_{j \in J} p(j)^{1-\sigma} \, dj \right]^{\frac{1}{1-\sigma}}. \]  

2.2 Production

Production requires two factors of production: managerial skills and raw labor. We assume that each manager can employ her skills in the production of at most one variety. The size of the firm is normalized such that one firm employs the skills of one manager. We follow Manasse and Turrini (2001) and assume that
the skills of the manager determines the productivity of the firm.\textsuperscript{5} Technology
is represented by a cost function with constant marginal costs. The demand for
raw labor $l$ is linear in output $x$: $l = x/q$. Firms differ in their productivity
levels $q$. Following Melitz (2003), higher productivity is modelled as producing a
symmetric product variety at lower variable cost.

Each firm faces a residual demand curve with constant elasticity $\sigma$, and a
wage rate $w$. Profit maximization yields the markup–pricing rule

$$ p = \frac{w}{\rho q}.$$ 

### 2.3 Human–Capital Investment and Labor Supply

The economy is populated by a continuum of individuals of mass $L$. Individuals
are heterogeneous with respect to their innate abilities $a$. Abilities are distributed
according to some continuous and differentiable density function $g(a)$ with sup-
port $[0, \infty)$; the respective distribution function is denoted by $G(a)$.

An individual with ability $a$ can choose to enter the labor force and supply
one unit of raw labor at the wage $w$. Alternatively, an individual can choose to
acquire education and become a skilled manager operating a firm. In that case
her income is her firm’s profit. To simplify, we abstract from any direct cost of
education. Of course, the wage income of unskilled labor is the opportunity cost
of education.

Education is assumed to raise individual abilities according to some function
$h(a)$ with $h'(a) > 0$. We assume that the firms productivity and a manager’s
human capital are related by

$$ q = h(a), \quad h'(a) > 0.$$

The self selection of individuals endogenizes the economy’s supply of raw
labor. In case of $a \in [t, \infty)$ individuals acquiring education, aggregate labor
supply is given by

$$ L^S(t) = G(t)L.$$

\textsuperscript{5}In contrast to Manasse and Turrini we prefer to term the skilled individuals operating
firms managers instead of entrepreneurs, because to become a manager typically requires some
higher education whereas entrepreneurs may not have acquired specific skills but have some
advantage in innate abilities.

\textsuperscript{6}This means that the support can be quite large but is finite at some upper bound.
Consequently, the mass of active firms is given by

\[ M(t) = [1 - G(t)]L. \]

3 The Closed–Economy Equilibrium

We begin the analysis of the closed–economy equilibrium by deriving the threshold ability \( t \). Individuals with abilities \( a \geq t \) will invest in education and establish a firm. Therefore, we solve the individuals’ decisions about acquiring education. Those with abilities \( a < t \) will not educate but supply raw labor.

In order to prove this assertion, we look at the profit of a firm with productivity \( q = h(a) \). With markup pricing according to

\[ p(a) = \frac{w}{\rho h(a)} \quad (4) \]

profits are given by

\[ \pi = (1 - \rho) \frac{w x(a)}{\rho h(a)}. \quad (5) \]

An individual with ability \( a \) invests in education as long as her profits from entrepreneurial activities exceed the wage rate. This requires

\[ \frac{x(a)}{h(a)} \geq \sigma - 1. \quad (6) \]

Making use of the clearing of product markets

\[ x(a) = E[p(a)]^{-\sigma} P^{\sigma - 1}, \quad (7) \]

and applying the markup pricing according to (4), condition (6) can be written as

\[ h(a) \geq \left( \frac{\sigma - 1}{E} \right)^{\frac{1}{\sigma - 1}} \left( \frac{w}{\rho} \right)^{\frac{\sigma}{\sigma - 1}} P^{-1}. \]

Since \( h'(a) > 0 \), there exists a unique threshold \( t = a(w, E, P) \) given by

\[ h(t) = \left( \frac{\sigma - 1}{E} \right)^{\frac{1}{\sigma - 1}} \left( \frac{w}{\rho} \right)^{\frac{\sigma}{\sigma - 1}} P^{-1}. \quad (8) \]

As a result, the individuals’ decision problem about acquiring education has a unique solution for given macroeconomic variables \((w, E, P)\).
In order to determine the general equilibrium we solve for the equilibrium values of the macroeconomic variables. For any threshold \( t = a(w, E, P) \), the price index \( P \) is\(^7\)

\[
P(t) = \left[ L \int_t^\infty p(a)^{1-\sigma} dG(a) \right]^{\frac{1}{1-\sigma}} = \frac{w L^{1-\sigma}}{\rho Q(t)},
\]

where

\[
Q(t) := \left[ \int_t^\infty h(a)^{\sigma-1} dG(a) \right]^{\frac{1}{\sigma-1}}, \quad Q'(t) < 0
\]

is a measure of the per–capita aggregate stock of human capital. Note that \( Q \) is negatively correlated to aggregate productivity measured by

\[
\tilde{Q}(t) := \left[ \frac{1}{1-G(t)} \int_t^\infty h(a)^{\sigma-1} dG(a) \right]^{\frac{1}{\sigma-1}} = \left[ 1 - G(t) \right]^{\frac{1}{1-\sigma}} Q(t).
\]

Substituting in (8) for \( P(t) \), we arrive at the following indifference condition:

\[
h(t) = \left( \frac{\sigma - 1}{\rho} \frac{wL}{E} \right)^{\frac{1}{\sigma-1}} Q(t).
\]

We finally have to solve for \( w/E \). Aggregate income \( E \) is made up from aggregate profits and aggregate wages. Profits of a firm \( a \) can be calculated by substituting in (5) for \( x(a) \) by (7) making use of our solution for \( P(t) \):

\[
\pi(a, t, w, E, L) = (1 - \rho) \frac{E}{L} \left[ h(a) \right]^{\sigma-1} Q(t).
\]

Aggregate profits then are

\[
\Pi(t, w, E, L) = L \int_t^\infty \pi(a, t, E, w, L) dG(a) = (1 - \rho) E.
\]

Adding aggregate wage income \( wG(t)L \), we get

\[
E = \Pi(t, w, E, L) + wG(t)L = (1 - \rho) E + wG(t)L
\]

\(^7\)In contrast to Melitz (2003), we do not formulate the model in terms of the conditional distribution of productivities in equilibrium, but in terms of the distribution of productivities resp. abilities in general. As a result, the price level is written as depending on aggregate human capital \( Q \) rather than on aggregate productivity \( \tilde{Q} \). Alternatively, we could write the price index as \( P(t) = wM(t)^{1-\sigma} / \rho \tilde{Q}(t) \).
or
\[ \frac{E}{wL} = \frac{G(t)}{\rho} \]
By substituting for \( E/wL \) in (10), the threshold is determined as the solution of
\[ h(t) = \phi(t) := (\sigma - 1)\frac{1}{\sigma} G(t)^{1-\sigma} Q(t). \] (11)
From \( h'(t) > 0, \phi'(t) < 0, \) and \( \lim_{t\to0} \phi(t) = \infty, \lim_{t\to\infty} \phi(t) = 0, \) (11) determines a unique solution \( t \in (0, \infty) \). Note that the equilibrium value of \( t \) is independent from the size of the economy. As a result, a country’s aggregate productivity is independent from its size.

Having solved for the equilibrium threshold, all endogenous variables can be calculated as functions of that value. We denote the equilibrium threshold in the closed economy by \( t_a \). For normative analysis, we calculate the equilibrium welfare per capita \( W = U/L \). Substituting for the demand values \( c(j) \) and using our definition of the price index, equilibrium welfare per capita can be written as
\[ W_a = G(t_a)\bar{Q}(t_a) \left\{ [1 - G(t_a)] L \right\}^{\frac{1+\eta}{\sigma+1}}. \] (12)
Welfare is made up of two components: the aggregate quantity of production per capita and the number of differentiated products. With respect to the first component, note that the equilibrium labor employment is given by \( G(t_a)L \) while \( \bar{Q}(t_a) \) denotes the average firm productivity (average output per unit of labor input). It is intuitively clear that aggregate production—and hence welfare per capita—is the higher, the higher either the average productivity of firms given the number of workers using this productivity or the higher the number of workers employed given that productivity. The term \( \left\{ [1 - G(t_a)] L \right\}^{\frac{1+\eta}{\sigma+1}} \) measures the impact of product variety on welfare: in equilibrium, \( [1 - G(t_a)] L \) different varieties are available to consumers. This is to be expected from related models of product differentiation (cf. Krugman, 1980; Melitz, 2003), where per-capita welfare depends on aggregate employment measured by country size \( L \) in exactly the same way. In the special case of \( \eta = \rho - 1 = -1/\sigma \) (Blanchard–Giavazzi specification), the measure of product variety does not not affect welfare and hence per-capita welfare is independent of country size.
4 The Open–Economy Equilibrium

We analyze the interdependence of trade and education in a two–country model. For the self–selection of firms to occur, we assume that trade is associated with fixed costs \( f_X > 0 \) of exporting. These costs measure exporters’ costs to set up and maintain distributional channels in the foreign market. They take the form of output that has to be produced but cannot be sold. Furthermore, we assume that export incurs additional variable costs taking the form of iceberg transportation costs: for one unit of an export good to arrive, \( \tau > 1 \) units have to be produced and shipped. To simplify the analysis we concentrate on the case of two symmetrical countries. Symmetry of both countries allows us to consider the equilibrium allocation and prices in one country. Note that symmetry does not imply that countries are completely identical. Since some firms will only serve their home market while each firm of both countries produces a different variety of the differentiated good, the varieties available to consumers differ between countries. Aggregate variables, however, will be identical in both countries in equilibrium.

4.1 Trade Equilibrium

Allowing for trade increases the set of possibilities for all active firms. Because of the existence of sufficiently high fixed costs of exporting,\(^8\) however, only a subset of firms will find it profitable to export. Thus, we establish a second threshold \( s > t \) such that only firms with \( a \geq s \) engage in trade.

The indifference condition (8) for acquiring education is not affected by the additional options provided by trade; however, the equilibrium values of the macroeconomic variables \((w, E, P)\) are affected by trade. To determine the corresponding indifference condition for exporting, we calculate the profits from exports. Let \( x_X(a) \) denote the export quantity of firm \( a \) sold at export price \( p_X(a) \). Profit maximization for the export activity yields the markup–pricing rule for exports as

\[
p_X(a) = \tau p(a). \tag{13}
\]

\(^{8}\)The corresponding condition for \( f_X \) will be derived below.
The corresponding export profits can be written as:

$$\pi_X = \tau (1 - \rho) \frac{wx_X(a)}{h(a)} - w f_X.$$  \hspace{1cm} (14)

A firm will now engage in trade if the respective profit is non-negative:

$$\frac{x_X(a)}{h(a)} \geq \frac{(\sigma - 1) f_X}{\tau}.$$  

Applying the market–clearing condition \(c(a) = x_X(a)\) and the pricing rule (13), we get

$$h(a) \geq \left( \frac{(\sigma - 1) f_X}{E} \right)^{\frac{1}{\sigma - 1}} \left( \frac{w}{\rho} \right)^{\frac{\sigma - 1}{\sigma - 1}} \tau \frac{\tau}{P}.$$  

Since \(h'(a) > 0\), there exists a unique threshold for exporting firms \(s = a(\tau, w, E, P)\) according to

$$h(s) = \left( \frac{(\sigma - 1) f_X}{E} \right)^{\frac{1}{\sigma - 1}} \left( \frac{w}{\rho} \right)^{\frac{\sigma - 1}{\sigma - 1}} \tau \frac{\tau}{P}.$$  \hspace{1cm} (15)

Given the macroeconomic variables \((w, E, P)\), the firms’ decision problem about exporting has a unique solution. Comparing the two threshold conditions (8) and (15) indicates that \(s > t\) requires \(\tau^{\sigma - 1} f_X > 1\). We assume this condition to hold since otherwise all active firms would engage in export activities in equilibrium.  

In order to solve for the equilibrium, we determine the macroeconomic variables \((w, E, P)\). The mass of domestically produced varieties is given by \(M(t) = [1 - G(t)]L\). Due to symmetry of the countries, the mass of imported varieties amounts to \(M(s) = [1 - G(s)]L\). Thus, the total number of varieties available to consumers is given by \([2 - G(t) - G(s)]L\). For given threshold values \((s, t)\), the price index \(P\) containing domestic varieties as well as imported varieties is

$$P(s, t, \tau) = \frac{w}{\rho} \frac{L^{\frac{1}{1-\sigma}}}{Z(s, t, \tau)},$$  

where

$$Z(s, t, \tau) := \left[ \int_t^\infty h(a)^{\sigma - 1} dG(a) + \tau^{1-\sigma} \int_t^\infty h(a)^{\sigma - 1} dG(a) \right]^{\frac{1}{\sigma - 1}}.$$  

Melitz applies a similar regularity condition to ensure self–selection of firms. If the above condition is not satisfied, all firms export and the threshold for acquiring education is determined by \([p(a) - w/h(a)](x(a) + \tau x_X(a)) = w(1 + f_X)\).
now measures the aggregate stock of human capital, with $\partial Z(\cdot)/\partial s < 0$, $\partial Z(\cdot)/\partial t < 0$ and $\partial Z(\cdot)/\partial \tau < 0$ (cf. the appendix). Note that $\tau$ negatively affects aggregate human capital because part of the productivity must be foregone to cover transport costs in case of exporting.

Substituting in (8) and (15) by (16) gives

$$h(t) = \left(\frac{\sigma - 1}{\rho} \frac{wL}{E}\right)^{\frac{1}{\sigma - 1}} Z(s, t, \tau)$$

(17)

$$h(s) = \tau \left(\frac{\sigma - 1}{\rho} f_X wL}{E}\right)^{\frac{1}{\sigma - 1}} Z(s, t, \tau).$$

(18)

To derive the $wL/E$ we again calculate aggregate profits and wage income as in the closed–economy model. Aggregate profits are now comprised by profits from domestic sales and from exports. Applying our definitions of the different profits, aggregate profits are given by

$$\Pi(a, w, \tau, E, P) = (1 - \rho)E - w f_X [1 - G(s)] L$$

Adding the wage income of workers $wG(t)L$ and rearranging terms we get $wL/E$ as

$$\frac{E}{wL} = \frac{G(t) - [1 - G(s)] f_X}{\rho}.$$  

(19)

Substitution for this term in (17) bzw. (18) gives

$$Y_1(s, t, \tau) := h(t) - (\sigma - 1)^{\frac{1}{\sigma - 1}} [G(t) - [1 - G(s)] f_X]^{\frac{1}{\sigma - 1}} Z(s, t, \tau) = 0$$

(20)

$$Y_2(s, t, \tau) := h(s) - \tau [(\sigma - 1)^{\frac{1}{\sigma - 1}} [G(t) - [1 - G(s)] f_X]^{\frac{1}{\sigma - 1}} Z(s, t, \tau) = 0.$$  

(21)

These two conditions determine both thresholds $(s, t)$ as functions of the parameters $(\tau, f_X, \sigma)$—again, country size is irrelevant for the threshold values—as well as of the underlying density function $g(a)$, and the human–capital function $h(a)$.\(^{10}\)

As shown in the appendix and illustrated in figure 1, both (20) and (21) define a declining curve in $(s, t)$–space. Since at any point of intersection the slope of the $Y_1 = 0$–curve must be less than the slope of the $Y_2 = 0$–curve, the solution for the threshold values is unique. Furthermore, since the $Y_2 = 0$ locus converges to infinity as $t$ approaches to the equilibrium threshold in autarky $t_a$ from above, the equilibrium threshold for acquiring education with trade, $t_t$, must exceed $t_a$.

\(^{10}\)Manasse and Turrini (2001) can be interpreted as a special case of our model where $t$ is given exogenously. Put formally: (20) is substituted by the condition $t = t_a$.  

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Eventually, defining aggregate productivity of the economy in the presence of trade by
\[ \tilde{Z}(s, t, \tau) := \left(2 - G(t) - G(s)\right)^{\frac{1}{1-\sigma}} Z(s, t, \tau), \]
the equilibrium value of per–capita welfare in the trade equilibrium is given by
\[ W_t = \left[ G(t_t) - \left(1 - G(t_x)\right) f_X \right] \tilde{Z}(t_x, t_t, \tau) \left\{ \left[2 - G(t_t) - G(t_x)\right] L \right\}^{\frac{1+\eta}{1-\sigma}}, \tag{22} \]
where \( t_x \) denotes the equilibrium value of the export threshold \( s \). This again allows a decomposition of per–capita welfare into aggregate production quantities and the number of available products. In equilibrium, \( G(t_t)L \) workers produce at average productivity \( \tilde{Z} \). Consequently, \( G(t_t)\tilde{Z} \) can be interpreted as gross aggregate production of the economy. However, \( \left[1 - G(t_x)\right] f_X \) units of production are used to cover the fixed costs of exporting goods. This amount of production does not generate welfare, only net aggregate production enters the welfare index.

Comparing the welfare levels of the autarky equilibrium and the trade equilibrium gives ambiguous results in general. The opening up of trade can have either a positive or a negative impact on per–capita welfare. This ambiguity of the welfare impact of trade is due to different counteracting partial effects. First, there is no guarantee that aggregate productivity is higher in the open economy than in the closed economy. Trade has two effects on aggregate productivity. Since the least productive firms exit the market (i.e. less individuals acquire in education), and only the most productive firms engage in trade and expand their production, trade gives higher weights to relative productive firms in the measure of aggregate productivity. But due to transportation costs, part of the export production melts away thus reducing aggregate productivity. Consequently, the net effect on aggregate productivity is not uniquely determined. Second, the fact that part of production (resp. part of labor supply) is absorbed to cover the fixed costs of exporting reinforces negative welfare effects. In sum, the reaction of aggregate net production is ambiguous. Finally, the decline in the mass of domestically produced varieties may not be compensated by additional product varieties available from foreign producers. The welfare impact of that effect depends on consumers preferences for varieties measured by \( \eta \). As a result of these multiple counteracting effects, the change in aggregate welfare is ambiguous in general.\footnote{This ambiguity is preserved for the case of the Blanchard–Giavazzi specification of the CES utility index, although the measure of product differentiation does not affect welfare at all.} In order
to proof that trade may indeed have a negative impact on aggregate welfare, we provide a specific numerical example of our model in the appendix.

Let us relate our results to the welfare implications of trade derived by Melitz (2003). In his model, the expansion of highly-productive firms and the shrinking resp. vanishing of low-productive firms does also cause opposite partial welfare impacts that show up in an ambiguous reaction of aggregate productivity due to costs of trade. Additionally, the number of available product varieties may decline as an economy opens up to trade. However, in his model aggregate welfare can be shown to depend exclusively on the productivity of the least-productive firms, implying that the total welfare effect must always be positive. This result depends crucially on the fact that the modelling of the market-entry game ensures average profits of all firms participating in the game (i.e., expected profits before the uncertainty about productivity is resolved) to be zero, and that aggregate employment does not change. As a result of these specific features, any negative welfare effect from a decline in the number of available varieties is dominated by a positive effect on aggregate productivity and \textit{vice versa}.

4.2 Impact of Trade Liberalization

Trade liberalization is interpreted as increased exposure to trade (symmetrical for both countries) and will be modeled as a decline in transportation costs $\tau$. As shown by the comparative-static analysis of the model in the appendix, a decline in $\tau$ raises the equilibrium value of $t$ and reduces that of $s$. As with trade liberalization in the Melitz model, the number of active firms decreases while the number of exporters rises. However, the interpretation of results differ slightly from Melitz. In the present model, fewer individuals invest in education thereby reducing the number of firms while at the same time aggregate labor employment rises. The intuition for this result is straightforward. As transportation costs decline, the demand for exports rises due to the change in relative prices of the differentiated products. Hence exporting becomes profitable for some firms that only served home markets before, and these firms expand their production. Additionally, incumbent exporters face higher demand and also expand their

\footnote{Melitz (2003: 1713, fn. 26, and 1721) emphasizes that aggregate productivity at the factory gate goes up. However, it is aggregate productivity corrected by the loss in product transit that is welfare relevant in the end.}
export production. The resulting additional demand for labor is in parts met by the increase in labor supply as fewer individuals invest in education. Additionally, firms serving only the home markets reduce their production and also compensate for the rise in labor demand.

With respect to welfare effects of trade liberalization we get similar ambiguities as for the opening up of trade. The possibility of negative welfare effects is again established by a specific numerical example in the appendix.

5 Conclusions

This paper has developed an alternative mechanism of explaining the distribution of productivities across firms and their self–selection with respect to export activities that is based on the decisions about acquiring education made by heterogeneous individuals. Thus, our framework integrates the models developed by Manasse and Turrini (2001) and by Melitz (2003). Most importantly, our analysis has shown that the impact of trade liberalization on the self–selection of firms is robust with respect to the source of firms’ productivity differentials. The welfare impact of trade, however, has shown to be ambiguous in general; thus, welfare results in heterogeneous–firm trade models are sensitive with respect to the modeling of firm heterogeneities.

Our model also provides an alternative channel to explain the observed distribution of productivities. Empirical studies (cf., e.g., Del Gatto, Mion and Ottaviano, 2006) find that the distribution of firms’ productivities can be reasonably well approximated by a Pareto distribution. Therefore, many extensions or applications of the Melitz model (cf. Helpman, 2006, for an overview) simply postulate productivities to be distributed that way. In our model, a reasonable approximation of the distribution of productivities can be traced back to the distribution of the individuals’ innate abilities. The literature on psychology established that the distribution of inherent abilities can be well approximated by a normal distribution (cf. Wechsler, 1936). Applying that argument to the present model implies the following. As long as the threshold $t$ is sufficiently high, and provided that education transforms abilities into productivities according to a monotonically increasing function, then the normal distribution of abilities generates a reasonably well approximation to the observed distribution of produc-
tivities. Thus, our approach allows to substitute for the ad-hoc assumption of Pareto-distributed productivities in the market-entry game by an empirically well approved assumption about the distribution of abilities.

Another interesting feature of our approach is that we substitute for Melitz’s abstract lottery with firms drawing their productivities randomly from an arbitrarily specified distribution of productivities by an economically intuitive explanation of the emergence of persistent productivity differentials. It is the human-capital investments of heterogeneous individuals that determines the distribution of productivities in an economy. Consequently, our approach allows for an analysis of policies manipulating the distribution of productivities by educational policies, such as educational subsidies or improvements in the educational technology. Eventually, we can trace back the differences in productivities—and hence comparative advantage (cf. Bernard, Redding and Schott, 2007)—to differences in the quality of labor inputs and thus on institutional differences in national educational systems. As indicated by empirical analyses of augmented neoclassical trade models (cf. Treffer, 1995), such quality differences are important in understanding trade patterns.

References


Wechsler, David (1936), Measurement of Adult Intelligence, Williams & Wilkins: Baltimore.

Appendix

The appendix contains details of the analysis of the two–country trade model.

Open Economy Equilibrium

The conditions for the threshold values for acquiring education and exporting are

\[ Y_1(s, t, \tau) := h(t) - (\sigma - 1)\frac{1}{\sigma} A(s, t) Z(s, t, \tau) = 0 \]  \hspace{1cm} (A.1)
\[ Y_2(s, t, \tau) := h(s) - \tau [(\sigma - 1) f_X]^{\frac{1}{\sigma}} A(s, t) Z(s, t, \tau) = 0, \]  \hspace{1cm} (A.2)

where

\[ A(s, t) := [G(t) - (1 - G(s)) f_X]^{\frac{1}{1-\sigma}} \]

and

\[ Z(s, t, \tau) := \left[ \int_{t}^{\infty} h(a)^{\sigma-1} dG(a) + \tau^{1-\sigma} \int_{s}^{\infty} h(a)^{\sigma-1} dG(a) \right]^{\frac{1}{\sigma}}. \]

The partial derivatives of the functions \( A \) and \( Z \) are given by:

\[ \frac{\partial A}{\partial t} = \frac{A^\sigma}{1-\sigma} g(t) < 0 \]
\[ \frac{\partial A}{\partial s} = \frac{A^\sigma}{1-\sigma} g(s) f_X < 0 \]
\[ \frac{\partial Z}{\partial t} = -\frac{Z^{2-\sigma}}{\sigma-1} h(t)^{\sigma-1} g(t) < 0 \]
\[ \frac{\partial Z}{\partial s} = -\frac{Z^{2-\sigma}}{\sigma-1} h(s)^{\sigma-1} g(s) < 0 \]
\[ \frac{\partial Z}{\partial \tau} = -Z^{2-\sigma} \tau^{-\sigma} \int_{s}^{\infty} h(a)^{\sigma-1} dG(a) < 0. \]

Since all partial derivatives of \( A \) and \( Z \) with respect to the thresholds \((s, t)\) are negative, the derivatives of the product of both functions must be negative as well. This implies

\[ \frac{\partial Y_1}{\partial s} = -h(t) \left[ \frac{\partial A}{\partial s} \frac{1}{A} + \frac{\partial Z}{\partial s} \frac{1}{Z} \right] > 0 \]
\[ \frac{\partial Y_1}{\partial t} = h'(t) - h(t) \left[ \frac{\partial A}{\partial t} \frac{1}{A} + \frac{\partial Z}{\partial t} \frac{1}{Z} \right] > 0 \]
\[ \frac{\partial Y_2}{\partial s} = h'(s) - h(s) \left[ \frac{\partial A}{\partial s} \frac{1}{A} + \frac{\partial Z}{\partial s} \frac{1}{Z} \right] > 0 \]
\[ \frac{\partial Y_2}{\partial t} = -h(s) \left[ \frac{\partial A}{\partial t} \frac{1}{A} + \frac{\partial Z}{\partial t} \frac{1}{Z} \right] > 0. \]
We can now derive the slopes of the equilibrium curves \( Y_1 \) and \( Y_2 \) as follows:

\[
\frac{ds}{dt} \bigg|_{Y_1=0} = -\frac{h'(t)/h(t) - \left( \frac{\partial A}{\partial s} \frac{1}{A} + \frac{\partial Z}{\partial s} \| Z \| \right)}{h'(s)/h(s) - \left( \frac{\partial A}{\partial s} \frac{1}{A} + \frac{\partial Z}{\partial s} \| Z \| \right)} < 0 \quad (A.3)
\]

\[
\frac{ds}{dt} \bigg|_{Y_2=0} = -\frac{\left( \frac{\partial A}{\partial s} \frac{1}{A} + \frac{\partial Z}{\partial s} \| Z \| \right)}{h'(s)/h(s) - \left( \frac{\partial A}{\partial s} \frac{1}{A} + \frac{\partial Z}{\partial s} \| Z \| \right)} < 0. \quad (A.4)
\]

Since the numerator of (A.3) is less than the numerator of (A.4) while the denominator of (A.3) exceeds that of (A.4), we get for the slopes at points of intersection of both curves:

\[
\frac{ds}{dt} \bigg|_{Y_1=0} < \frac{ds}{dt} \bigg|_{Y_2=0}.
\]

With both curves having negative slopes, this implies that the \( Y_1 = 0 \)-curve is steeper at any common point than the \( Y_2 = 0 \)-curve. As a result, the equilibrium is unique.

**Comparative–Static Effects**

The partial derivatives of the functions \( Y_1 \) and \( Y_2 \) with respect to \( \tau \) are:

\[
\frac{\partial Y_1}{\partial \tau} = -\left( \sigma - 1 \right) \frac{1}{\tau} A \frac{\partial Z}{\partial \tau} > 0 \quad (A.5)
\]

\[
\frac{\partial Y_2}{\partial \tau} = \frac{h(s)}{\tau} \left[ \tau^{1-\sigma} Z^{1-\sigma} \int_0^\infty h(a)\sigma^{-1}dG(a) - 1 \right] < 0. \quad (A.6)
\]

Note that the sign of (A.6) follows from

\[
\tau^{1-\sigma} \int_0^\infty h(a)^{-1}dG(a) < Z^{\sigma-1}.
\]

The Jacobian \( J \) of the system (A.1) and (A.2) is given by:

\[
J = \begin{bmatrix}
\frac{\partial Y_1}{\partial s} & \frac{\partial Y_1}{\partial t} \\
\frac{\partial Y_2}{\partial s} & \frac{\partial Y_2}{\partial t}
\end{bmatrix}.
\]

Calculating the determinant of the Jacobian yields:

\[
|J| = \frac{\partial Y_1}{\partial s} \frac{\partial Y_2}{\partial t} - \frac{\partial Y_2}{\partial s} \frac{\partial Y_1}{\partial t} < 0,
\]

where the sign is determined by the slope condition at the point of intersection.
The impact of a change in $\tau$ on the equilibrium values of $t$ and $s$ can now be calculated as

\[
\frac{ds}{d\tau}\bigg|_{Y_1=Y_2=0} = -\frac{\partial Y_1}{\partial \tau} \frac{\partial Y_2}{\partial t} - \frac{\partial Y_2}{\partial \tau} \frac{\partial Y_1}{\partial t} > 0
\]

\[
\frac{dt}{d\tau}\bigg|_{Y_1=Y_2=0} = -\frac{\partial Y_1}{\partial s} \frac{\partial Y_2}{\partial \tau} - \frac{\partial Y_2}{\partial s} \frac{\partial Y_1}{\partial \tau} < 0.
\]

As indicated by (A.5) and (A.6), an increase in $\tau$ shifts down the $Y_1 = 0$–curve while shifting up the $Y_2 = 0$–curve. Since the $Y_1 = 0$–curve is steeper than the $Y_2 = 0$–curve at the initial equilibrium, $s$ rises and $t$ falls.

### A Numerical Example of Negative Welfare Effects

#### Autarky vs. Trade Equilibrium

In this appendix we provide a numerical example of our model to show that two identical countries may experience negative welfare effects in the course of opening up to bilateral trade. In order to solve for the equilibrium values of the model explicitly, we assume individual abilities to be distributed according to a Pareto distribution with distribution function $G(a) = 1 - a^{-k}$. Additionally, we assume the education technology to be the identity function implying productivities $q = a$. In that special case, the indifference condition in autarky (11) reads

\[
t^{\sigma-1}G(t) = (\sigma - 1)Q(t)^{\sigma-1}.
\]

Calculating the various terms under the assumption of $k > \sigma - 1$ gives

\[
Q(t)^{\sigma-1} = \frac{k}{1 + k - \sigma} t^{\sigma-1-k},
\]

and

\[
G(t) = 1 - t^{-k}.
\]

The indifference condition can then be solved:

\[
t_a = \left[\frac{\sigma(k - 1) + 1}{1 + k - \sigma}\right]^{\frac{1}{\sigma}}. \tag{A.7}
\]

Finally, we derive the equilibrium value of our welfare measure in autarky $W = G(t)Q(t)L^{\frac{1+\sigma}{\sigma-1}}$ as

\[
W_a = \left[1 - t_a^{-k}\right] \left[\frac{k}{1 + k - \sigma}\right]^{\frac{1}{\sigma-1}} t_a^{\frac{\sigma-1-k(1+\sigma)}{\sigma-1}} L^{\frac{1+\sigma}{\sigma-1}} \tag{A.8}
\]

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The indifference condition for acquiring education in the trade equilibrium reads

\[ t^{\sigma - 1} [G(t) - (1 - G(s))f_X] = (\sigma - 1)Z(s, t, \tau)^{\sigma - 1}. \]

For our special case, we obtain

\[ Z(s, t, \tau)^{\sigma - 1} = \frac{k}{1 + k - \sigma} t^{\sigma - 1 - k}[1 + Xf_X], \]

where \( X = (t/s)^{k} = (\tau^{1-\sigma}/f_X)^{k/(\sigma-1)}, \) and

\[ G(t) = 1 - t^{-k}; \quad 1 - G(s) = s^{-k} = Xt^{-k}. \]

The indifference condition then reads

\[ t^{\sigma - 1} [1 - t^{-k}(1 + Xf_X)] = (\sigma - 1)\frac{k}{1 + k - \sigma} t^{\sigma - 1 - k}[1 + Xf_X] \]

giving the solution

\[ t_t = \left[(1 + Xf_X)\frac{\sigma(k - 1) + 1}{1 + k - \sigma}\right]^{\frac{1}{\sigma - 1}}. \quad \text{(A.9)} \]

Finally, we derive the equilibrium value of our welfare measure

\[ W = [G(t_t) - (1 - G(t_x))f_X] Z(t_x, t_t, \tau)L^{\frac{1 + \sigma q}{\sigma - 1}} \]

as

\[ W_t = [1 - t_t^{-k}(1 + Xf_X)] \left[\frac{k}{1 + k - \sigma}\right]^{\frac{1}{\sigma - 1}} t_t^{\frac{\sigma - 1 - k(1 + \sigma q)}{\sigma - 1}} (1 + Xf_X)^{\frac{1}{\sigma - 1}} (1 + X)^{\frac{\sigma q}{\sigma - 1}} L^{\frac{1 + \sigma q}{\sigma - 1}}. \quad \text{(A.10)} \]

We can now compare the welfare levels of both equilibria. From (A.8) and (A.10) we get

\[ \frac{W_t}{W_a} = \frac{[G(t_t) - (1 - G(t_x))f_X] Z(t_x, t_t, \tau)}{G(t_x)Q(t_x)} \]

\[ = \frac{[1 - t_t^{-k}(1 + Xf_X)] t_t^{\frac{\sigma - 1 - k(1 + \sigma q)}{\sigma - 1}} (1 + Xf_X)^{\frac{1}{\sigma - 1}} (1 + X)^{\frac{\sigma q}{\sigma - 1}}}{[1 - t_a^{-k}] t_a^{\frac{\sigma - 1 - k(1 + \sigma q)}{\sigma - 1}}}. \]

From (A.7) and (A.9) we get

\[ t_t = t_a(1 + Xf_X)^{\frac{1}{k}}. \]
and hence
\[ W_t = W_a \frac{\sigma - 1 - k\sigma \eta}{\sigma - 1} \left(1 + X\right)^{\frac{\sigma\eta}{\sigma - 1}}. \]

(A.11)

Obviously, negative welfare effects cannot be excluded at the outset. In order to prove that, we discuss two special cases. With \( \eta = 0 \) (Dixit–Stiglitz case), (A.11) reduces to
\[ W_t = W_a \frac{1}{1} > 1. \]

As a result, welfare can never decline as an economy opens up to trade. On the other hand, welfare may well decline in the Blanchard–Giavazzi case with \( \eta = -1/\sigma \). We then get
\[ W_t = W_a \frac{1 + \frac{\sigma - 1}{\sigma - 1}}{1 + \frac{1}{1 - \sigma}}. \]

For sufficiently small \( f_X \), this term may be less than unity implying welfare losses from trade. Taking, e.g., the calibration values from Demidova (2008) and setting \( \sigma = 3.8, k = 3.3, \tau = 1.3 \) and \( X = 0.21 \), we get a negative welfare impact of trade for \( f_X < 0.51727 \). In this setting, the regularity condition \( \tau^{\sigma-1} f_X > 1 \) is fulfilled for all \( f_X > 0.47969 \). Hence, we get negative welfare effects for \( f_X \in (0.47969, 0.51727) \).

**Negative Welfare Impacts from a Decline in Trade Costs**

We derive the welfare impact of a decline in \( \tau \) implying a rise in \( X \) from (A.10). From (A.9) we find that the term in the first bracket on the r.h.s. of (A.10) is a constant. Taking logs, we can rewrite (A.10) as
\[ \ln W_t(X) = \ln K + \frac{\sigma - 1 - k\sigma \eta}{k(\sigma - 1)} \ln(1 + X f_X) + \frac{\sigma\eta}{\sigma - 1} \ln(1 + X). \]

Differentiation w.r.t. \( X \) gives
\[ \frac{W_t'(X)}{W_t(X)} = \frac{\sigma - 1 - k\sigma \eta}{k(\sigma - 1)} \frac{f_X}{1 + X f_X} + \frac{\sigma\eta}{\sigma - 1} \frac{1}{1 + X}. \]

A negative welfare impact of a decline in trade costs requires \( W_t'(X) < 0 \), or
\[ \frac{\sigma - 1 - k\sigma \eta}{k(\sigma - 1)} \frac{f_X}{1 + X f_X} < -\frac{\sigma\eta}{\sigma - 1} \frac{1}{1 + X} \iff \sigma - 1 < k\sigma \eta \frac{f_X - 1}{f_X(1 + X)}. \]
Again, this condition cannot be fulfilled for the Dixit–Stiglitz case $\eta = 0$. However, negative welfare effects are again possible for the Blanchard–Giavazzi case $\eta = -1/\sigma$. Using the same calibration values for the parameters as above, negative welfare effects arise for $f_X < 0.49342$. Additionally, negative welfare effects are possible in both $\eta$ and $f_X$ are sufficiently high.
Figure 1: Determination of equilibrium threshold values with trade