

QM II: Übungszettel 6 (Abgabetermin: 28.05.2014)

Problem 1 – Parity-odd and parity-even observables (10pts)

- (a) A state vector $|\Psi\rangle$ is a simultaneous eigenstate of two Hermitian operators $\hat{\mathbf{A}}, \hat{\mathbf{B}}$ which anticommute,

$$[\hat{\mathbf{A}}, \hat{\mathbf{B}}]_+ = 0.$$

What can you say about the eigenvalues of $\hat{\mathbf{A}}, \hat{\mathbf{B}}$ for the state $|\Psi\rangle$?

- (b) Find all possible simultaneous eigenvalues of $|\vec{p}\rangle \otimes |\pi\rangle$, where $|\pi\rangle$ and $|\vec{p}\rangle$ refer to the eigenvectors of the parity and momentum operators, $\hat{\Pi}$ and $\hat{\mathbf{p}}$, respectively.
- (c) Show that the orbital angular momentum eigenfunctions $Y_{lm}(\theta, \phi)$ are also eigenfunctions of the parity operator.

Hint: $\vec{x} \rightarrow -\vec{x}$ implies for spherical coordinates $r \rightarrow r, \theta \rightarrow \pi - \theta, \phi \rightarrow \phi + \pi$.

Problem 2 – Parity violation in atoms (25pts)

Because of weak interactions there is a parity-violating potential between the atomic electron and the nucleus,

$$\hat{\mathbf{H}}_w = \lambda \hat{\mathbf{S}} \cdot [\hat{\mathbf{p}} \delta(\hat{\mathbf{x}}) + \delta(\hat{\mathbf{x}}) \hat{\mathbf{p}}] \quad \text{with} \quad \lambda = \frac{G_F Q_w}{2\sqrt{2}mch},$$

where $G_F \approx 3 \times 10^{-12} mc^2 (\hbar/mc)^3$ is Fermi's constant and $Q_w = -N + (1 - 4 \sin^2 \theta_w)Z$ the dimensionless weak nuclear charge (N is the number of neutrons and θ_w the Weinberg mixing angle with $\sin^2 \theta_w \approx 0.23$). $\hat{\mathbf{S}}$ is the electron spin, $\hat{\mathbf{p}}$ the electron momentum and $\hat{\mathbf{x}}$ the distance between electron and nucleus.

- (a) Show that $\hat{\mathbf{H}}_w$ violates parity, i.e., show that $\hat{\mathbf{H}}_w$ and $\hat{\Pi}$ do not commute.
- (b) Is the product $\hat{\mathbf{S}} \cdot \hat{\mathbf{p}}$ a scalar or pseudoscalar quantity? Justify your answer.
- (c) Due to $\hat{\mathbf{H}}_w$, the ground state of an alkali atom, usually characterized by $|n, l, j, m\rangle$ actually contains tiny contributions from other eigenstates:

$$|n, l, j, m\rangle \rightarrow |n, l, j, m\rangle + \sum_{n', l', j', m'} C_{n', l', j', m'} |n', l', j', m'\rangle$$

On the basis of symmetry considerations alone, what can you say about the (n', l', j', m') that give rise to nonvanishing contributions?

- (d) Suppose the radial wavefunctions and energy levels are all known. Indicate how you may calculate $C_{n', l', j', m'}$. Do you get any further restrictions on (n', l', j', m') ?
- Hint: You can use your answer from question (b) and the Wigner-Eckart theorem.*

- (e) The mixing amplitudes between states $|n, l, j, m\rangle$ and (n', l', j', m') turn out to be imaginary. Why?
- Hint: Consider the behavior of $\hat{\mathbf{H}}_w$ under time-reversal symmetry.*

Problem 3 – Time-reversal symmetry (15pts)

- (a) Let $\varphi(\vec{p}')$ be the momentum-space wavefunction of state $|\varphi\rangle$, $\varphi(\vec{p}') = \langle \vec{p}' | \varphi \rangle$. Is the momentum-space wavefunction for the time-reversed state $\hat{\Theta}|\varphi\rangle$ given by $\varphi(\vec{p}')$, $\varphi(-\vec{p}')$, $\varphi^*(\vec{p}')$, or $\varphi^*(-\vec{p}')$? Why?
- (b) What is the time-reversed state corresponding to $\hat{\mathbf{D}}(R)|j, m\rangle$?
- (c) Using the properties of time reversal and rotation, show that

$$D_{m',m}^{(j)}(R) = (-1)^{m-m'} D_{-m',-m}^{(j)}(R).$$

- (d) Prove that $\hat{\Theta}|jm\rangle = i^{2m}|j, -m\rangle$.