QM II: Übungszettel 2
(Abgabetermin: 29.04.2015)

Problem 1 - Propagator for the harmonic oscillator (20 pts)

Show that the propagator for the one-dimensional harmonic oscillator is given by

$$K(x,t;x',t_0) = \sqrt{\frac{m\omega}{2\pi i\hbar \sin[\omega(t-t_0)]}} \exp\left[\frac{im\omega}{2\hbar \sin[\omega(t-t_0)]}\left\{(x+x')^2\cos[\omega(t-t_0)] - 2xx'\right\}\right]$$

Hint: $K(x,t;x',t_0) = \langle x|e^{-\frac{i}{\hbar}\hat{\mathbf{H}}(t-t_0)}|x'\rangle$ *in general and* $\hat{\mathbf{H}} = \hat{\mathbf{p}}^2/2m + m\omega\hat{\mathbf{x}}^2/2$ *for the harmonic oscillator.*

Problem 2 - Electron in a uniform magnetic field (18 pts)

An electron is subject to a uniform time-independent magnetic field of strength B_0, pointing in the positive z-direction. At $t = 0$, the electron is known to be in an eigenstate of $\hat{\mathbf{S}} \cdot \mathbf{n}$, with eigenvalue $\hbar/2$, where $\mathbf{n}$ is a unit vector, lying in the xz-plane, that makes an angle β with the z-axis.

(a) Obtain the probability for finding the electron in the $s_x = \hbar/2$-state as a function of time.

(b) Find the expectation value of $\hat{\mathbf{S}}_x$ as a function of time.

(c) Show that your answers make sense in the extreme cases (i) $\beta \to 0$ and (ii) $\beta \to \pi/2$.

(d) Sketch the dynamics on the Bloch sphere. *(You may use matlab, mathematica or a similar program for that.)*

Problem 3 - Heisenberg's equations of motion (12 pts)

The interaction between an electron and a given magnetic field is due to the magnetic moment of the electron,

$$\hat{\boldsymbol{\mu}}_e = -\frac{2e}{m_e c}\hat{\mathbf{S}}\,,$$

and the external magnetic field, $\mathbf{B}$. The interaction Hamiltonian reads

$$\hat{\mathbf{H}} = -\hat{\boldsymbol{\mu}}_e \cdot \mathbf{B}\,.$$

We consider a polarized electron, with spin polarization (+) in the z-direction, entering a region of constant magnetic field $\mathbf{B} = B_0\,\hat{x}$. The electron moves in the y-direction.

(a) Write the interaction Hamiltonian for this particular problem.

(b) Solve Heisenberg's equations of motion for the time dependent operators $\hat{\mathbf{S}}_x(t)$, $\hat{\mathbf{S}}_y(t)$ and $\hat{\mathbf{S}}_z(t)$.

(c) Discuss the results on the Bloch sphere.