# Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

## 10th Lecture / 10. Vorlesung

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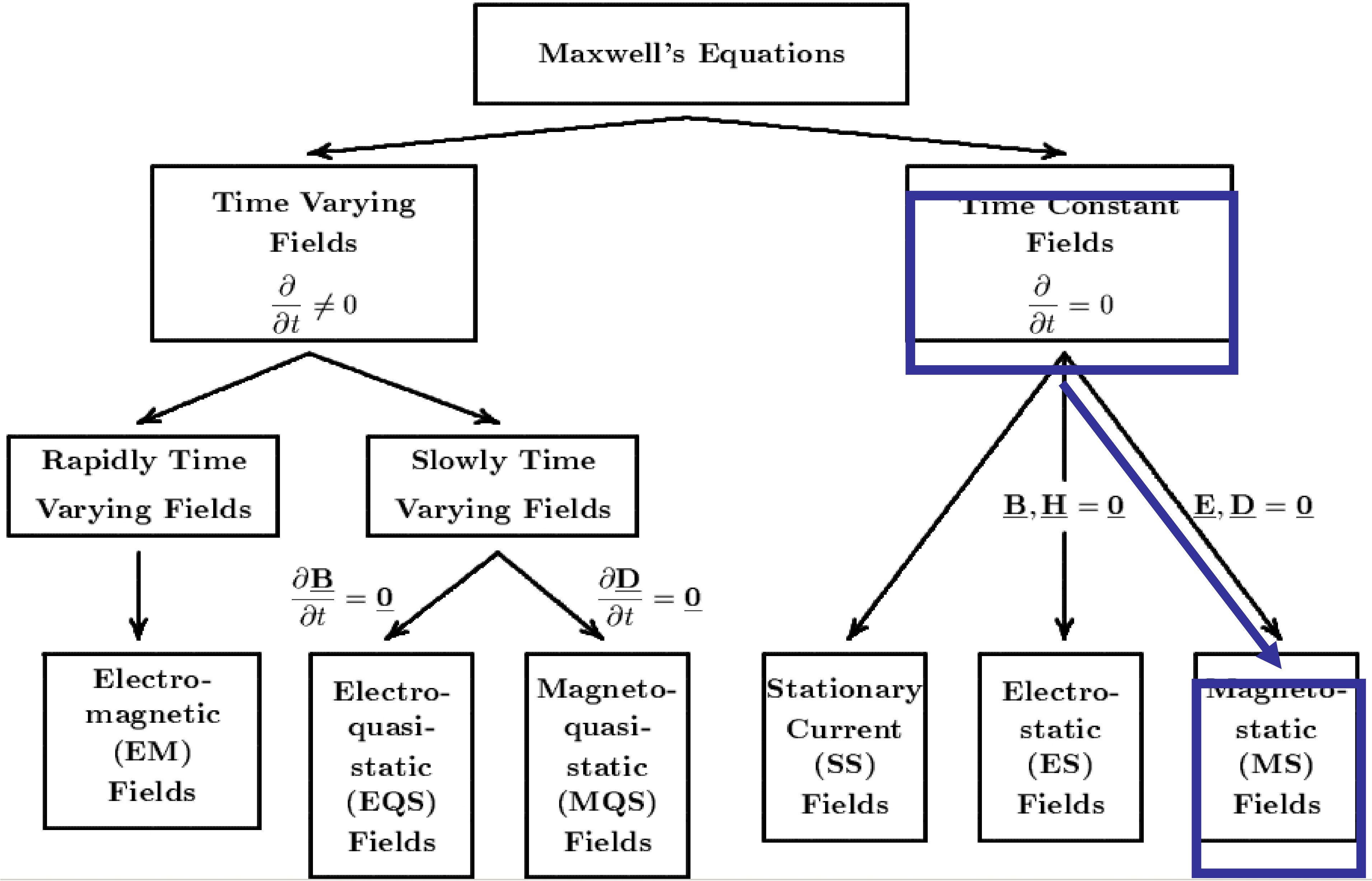
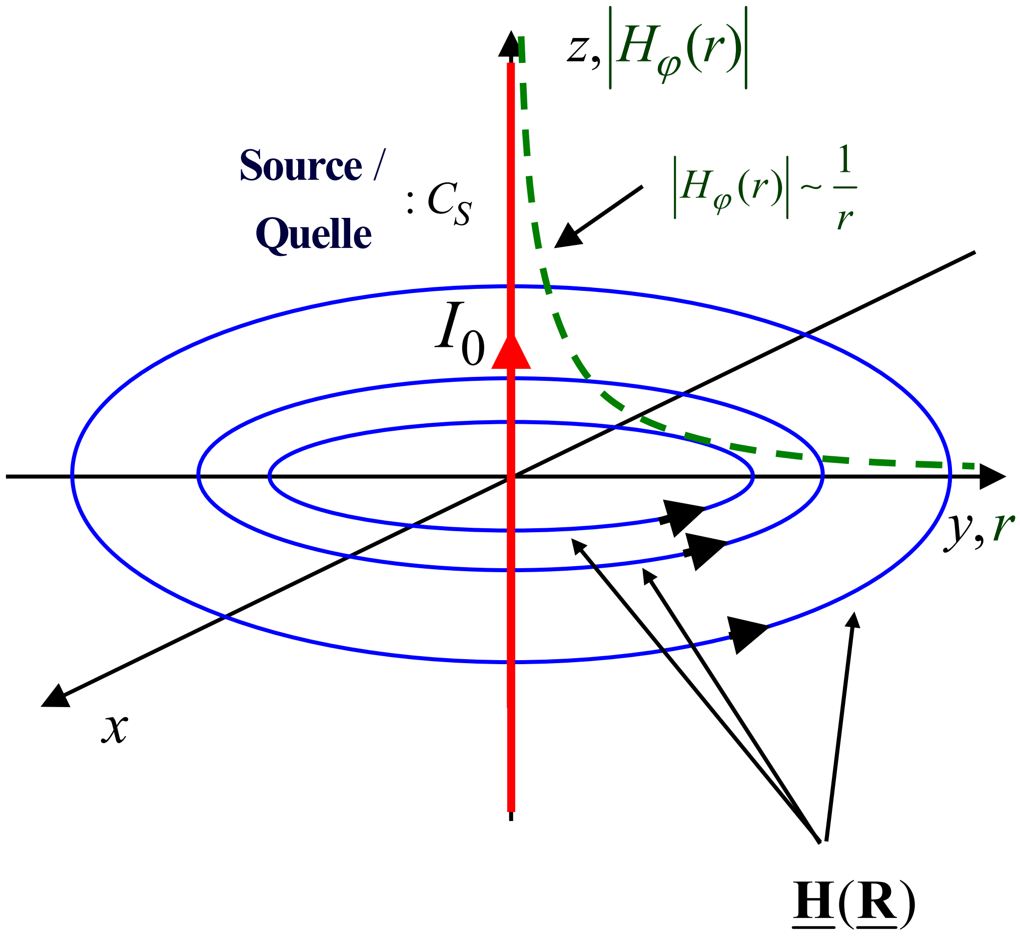
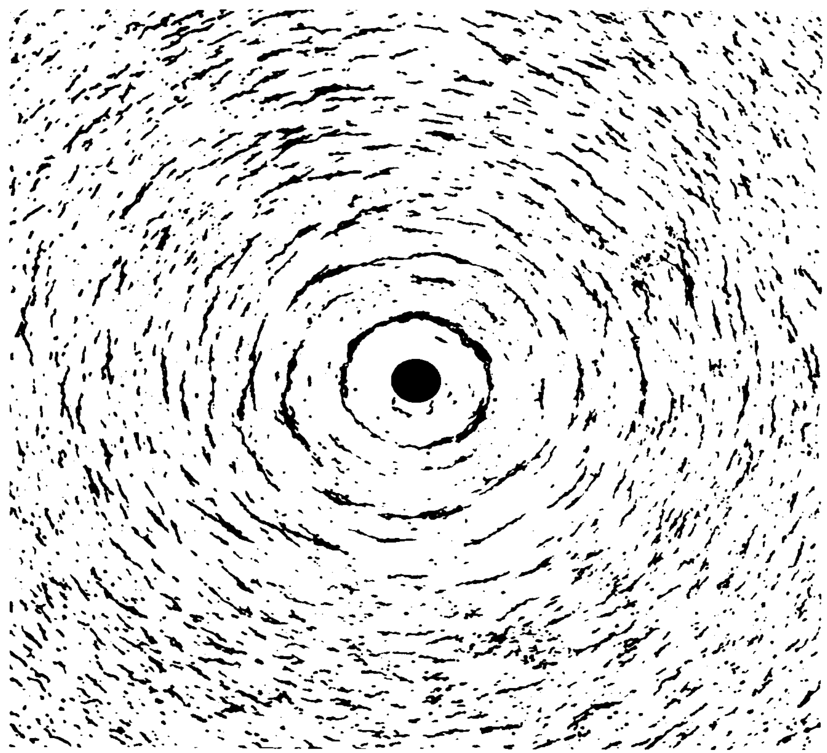
# Magnetostatic (MS) Fields – Classification / Magnetostatische (MS) Felder – Klassifikation



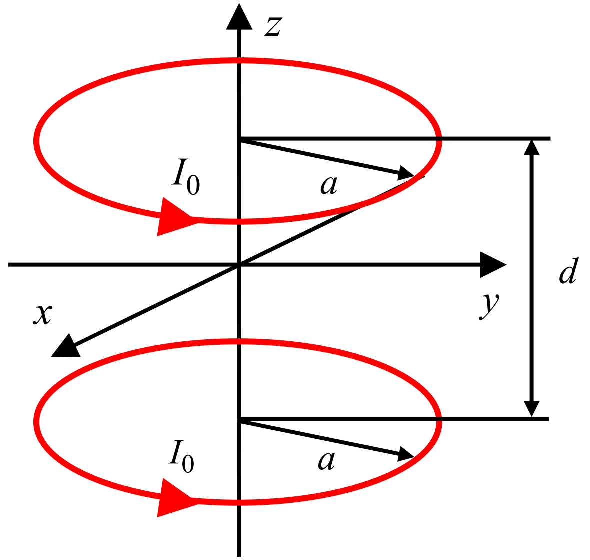
# MS Fields – Magnetic Field of a Line Current / MS-Felder – Magnetfeld eines Linienstromes



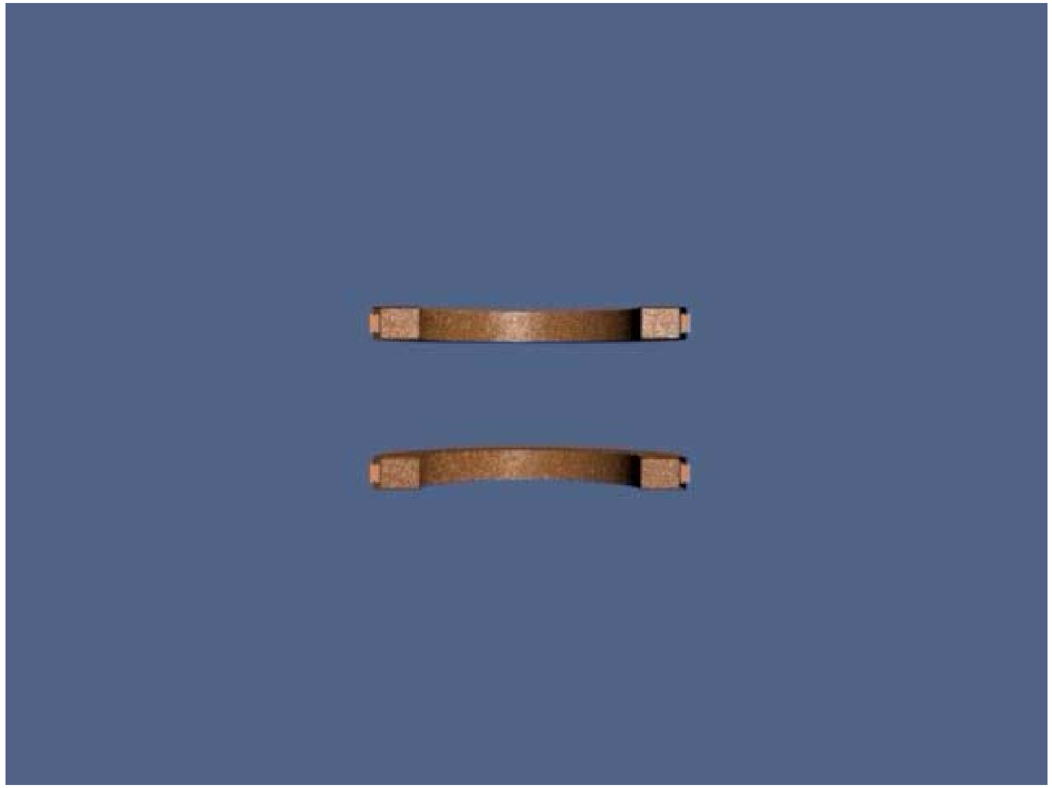



# MS Fields – Magnetic Field of a Helmholtz Coil / MS–Felder – Magnetfeld einer Helmholtz–Spule

## Helmholtz Coil / Helmholtz-Spule

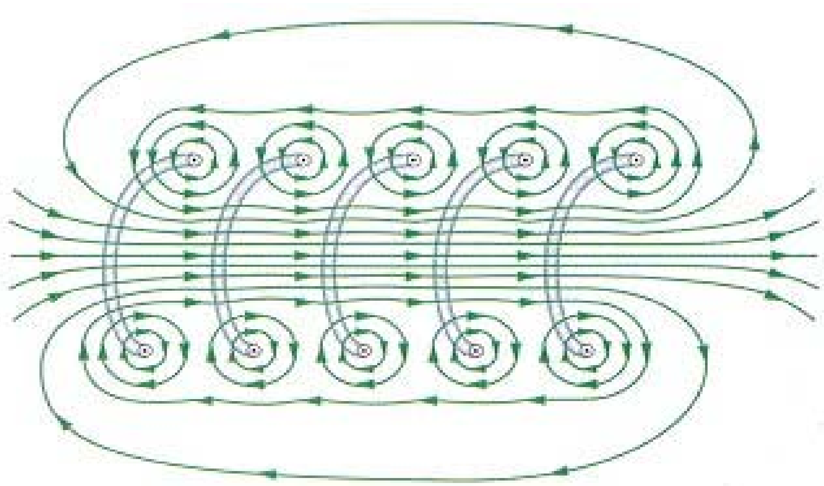


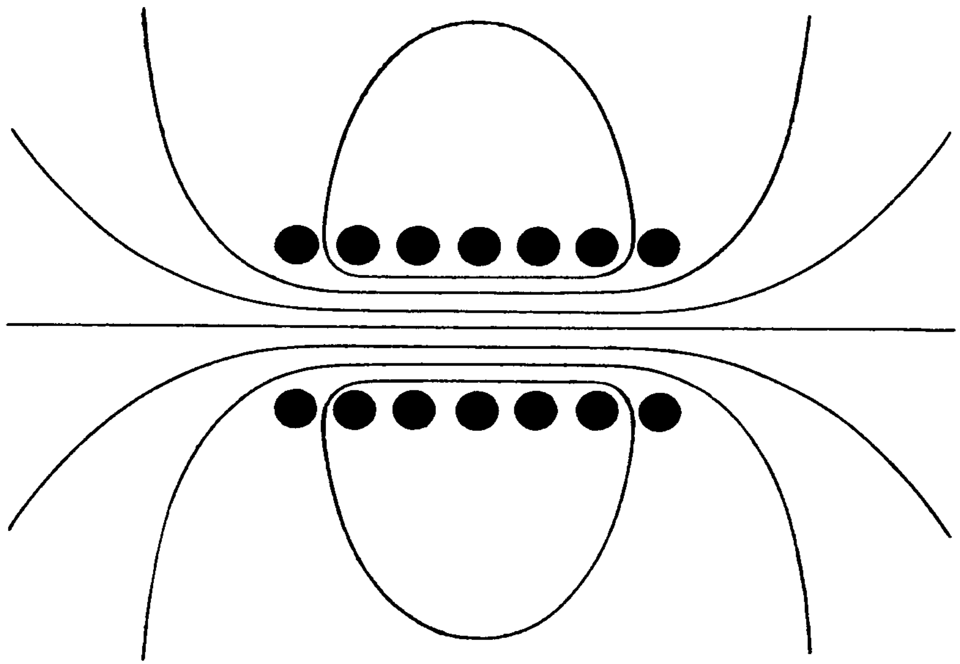

$$B_z(\underline{\mathbf{R}} = z\underline{\mathbf{e}}_z) = \frac{\mu_0 I_0 a^2}{2\left[a^2 + (d/2 - z)^2\right]^{3/2}} + \frac{\mu_0 I_0 a^2}{2\left[a^2 + (d/2 + z)^2\right]^{3/2}}$$

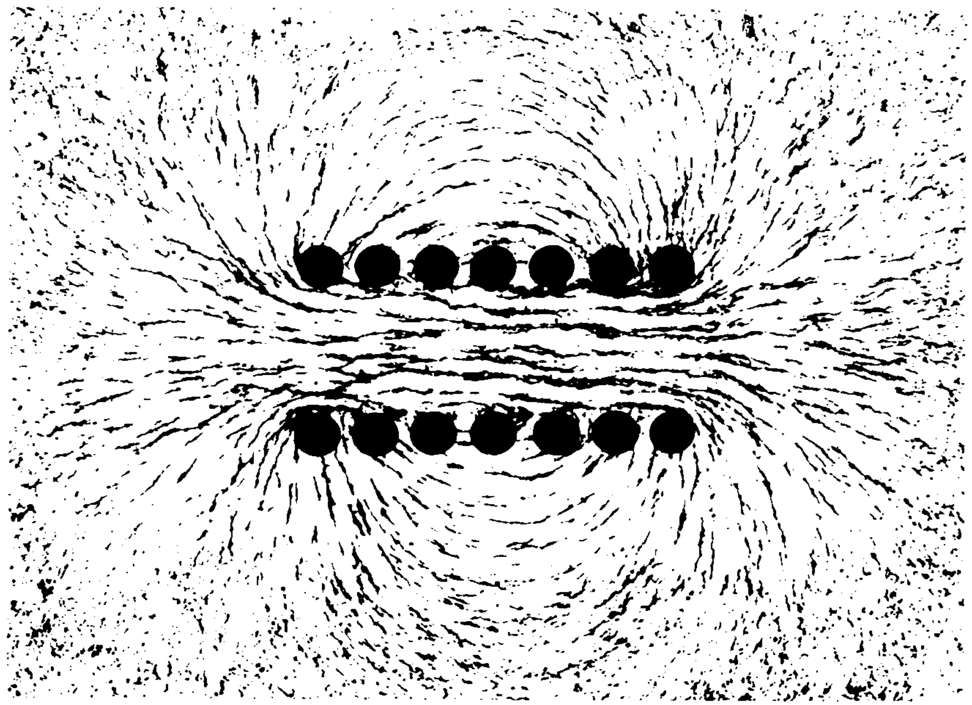


http://web.mit.edu/8.02t/www/802TEAL3D/visualizations/magnetostatics/coilsaligned/coilsaligned.htm

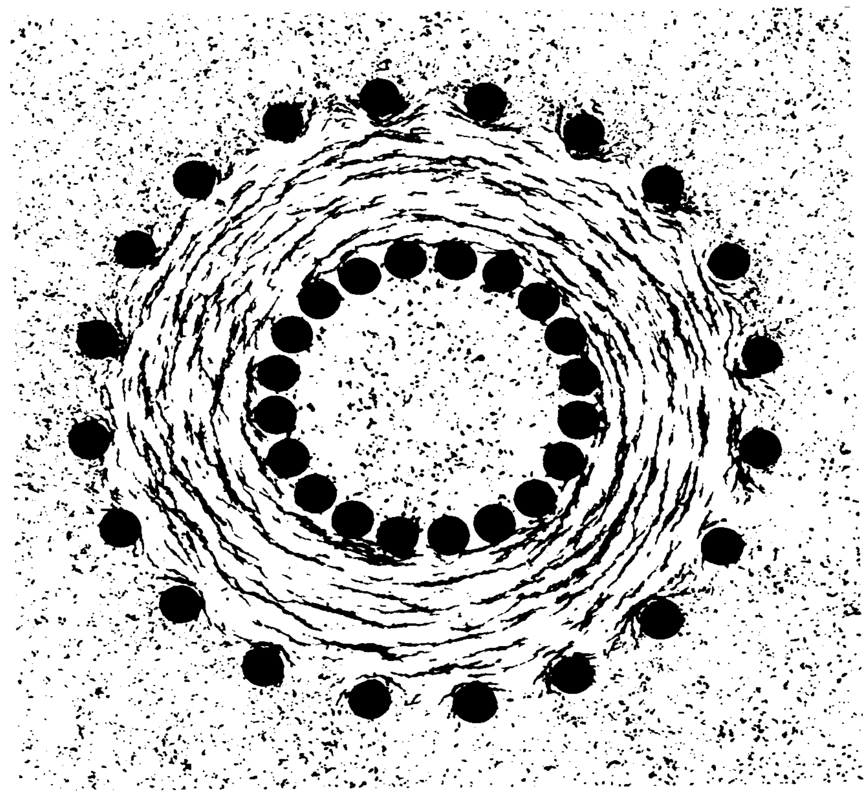
# MS Fields – Magnetic Field of a Solenoid / MS-Felder – Magnetfeld eines Zylinderspule



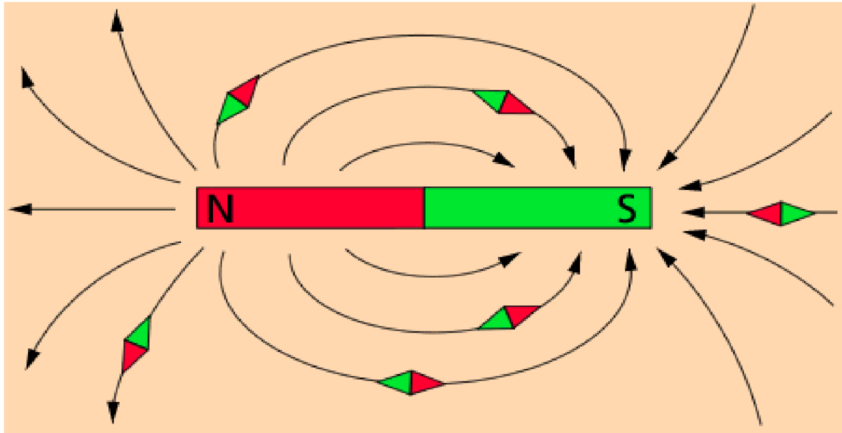


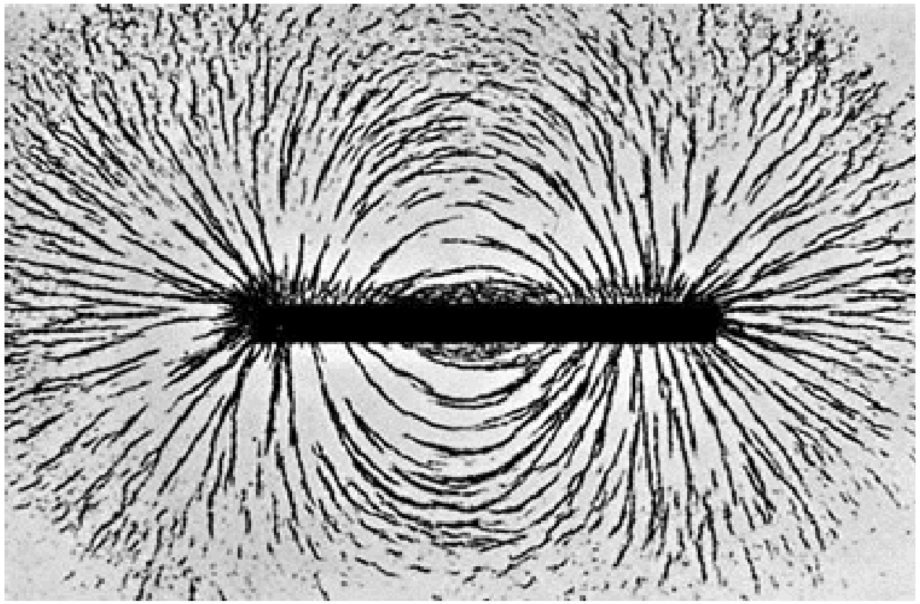


# MS Fields – Magnetic Field of a Toroidal Coil / MS-Felder – Magnetisches Feld einer Ringspule



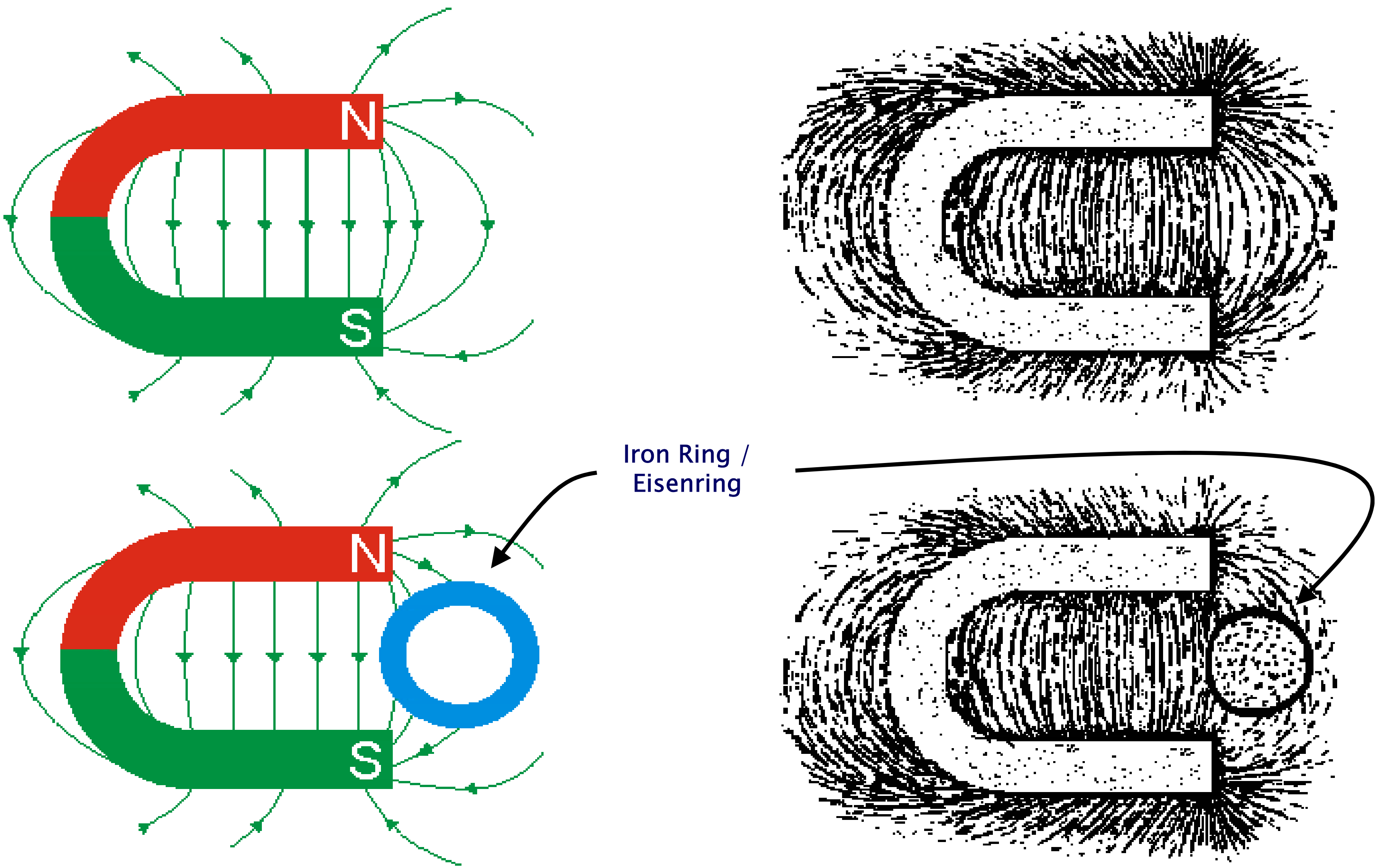
# MS Fields – Magnetic Field of a Bar Magnet / MS-Felder – Magnetfeld eines Stabmagneten



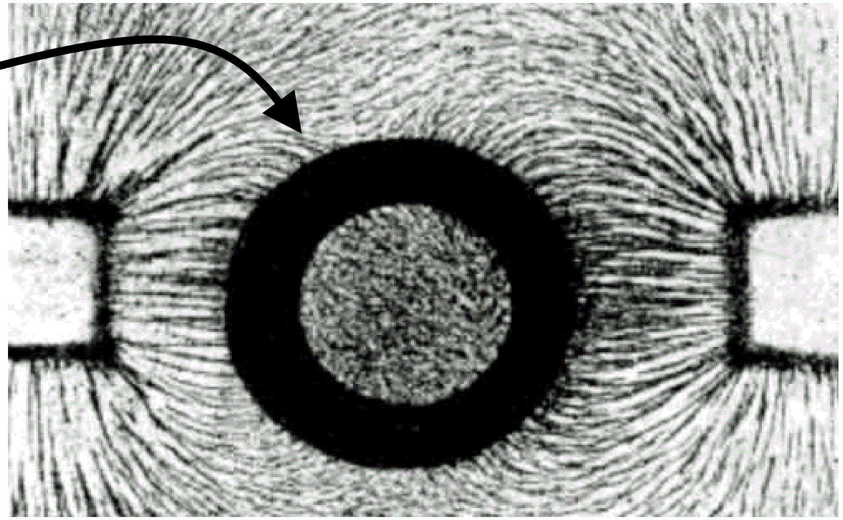
Magnetic Field Lines of a Bar Magnet – Visualized with Iron Filings /
Magnetfeldlinien eines Stabmagneten – Visualisiert mit Eisenfeilspäne

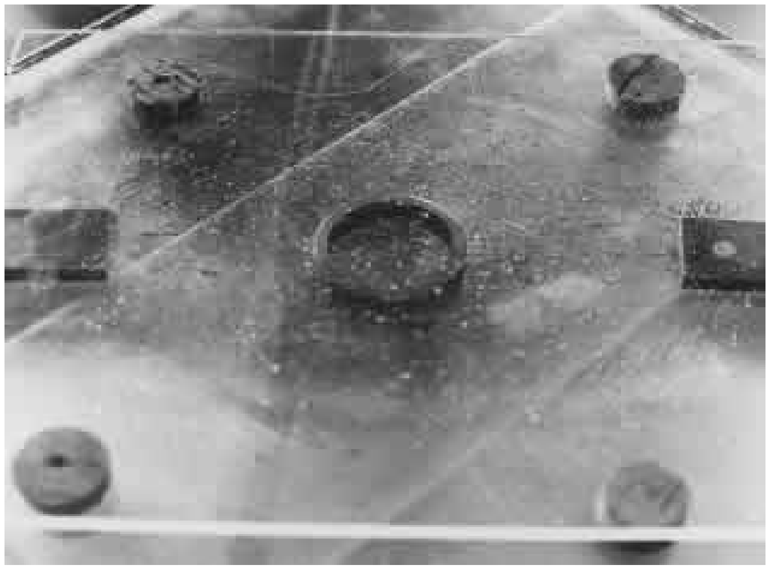
# MS Fields – Magnetic Fields of a Horseshoe Magnet (Permanent Magnet) / MS–Felder – Magnetfeld eines Hufeinsenmagneten (Permanentmagnet)



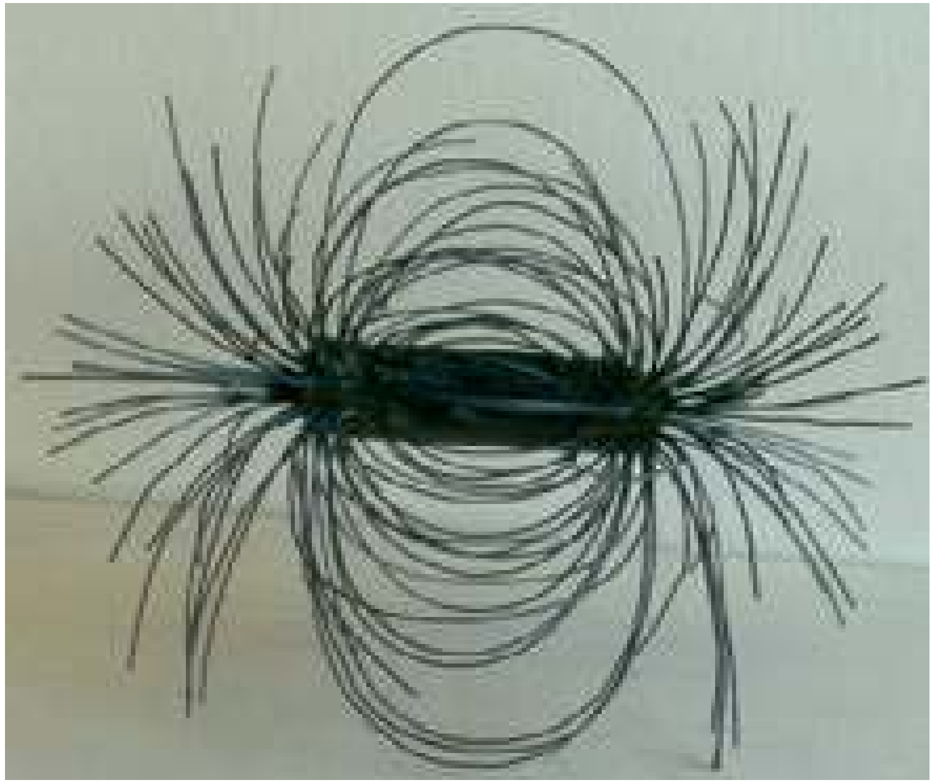
# MS Fields – (... ) Magnetic Shielding / MS–Felder – (...) Magnetische Abschirmung

Iron Ring / Eisenring

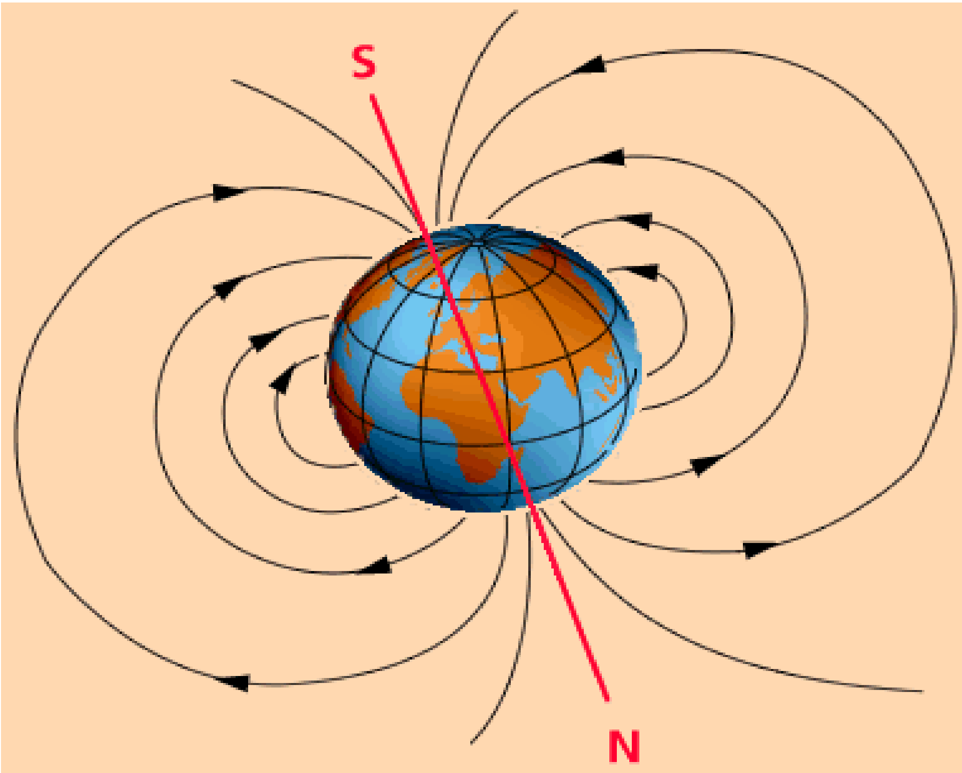


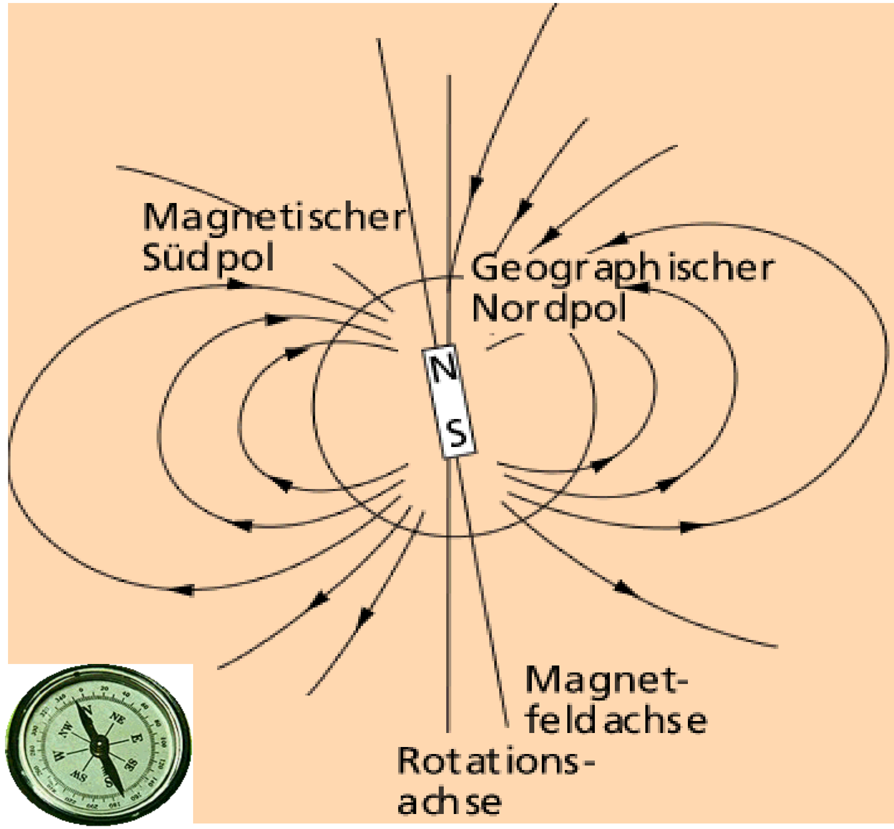


# MS Fields – Magnetic Fields of Permanent Magnets / MS-Felder – Magnetfelder von Permanentmagneten

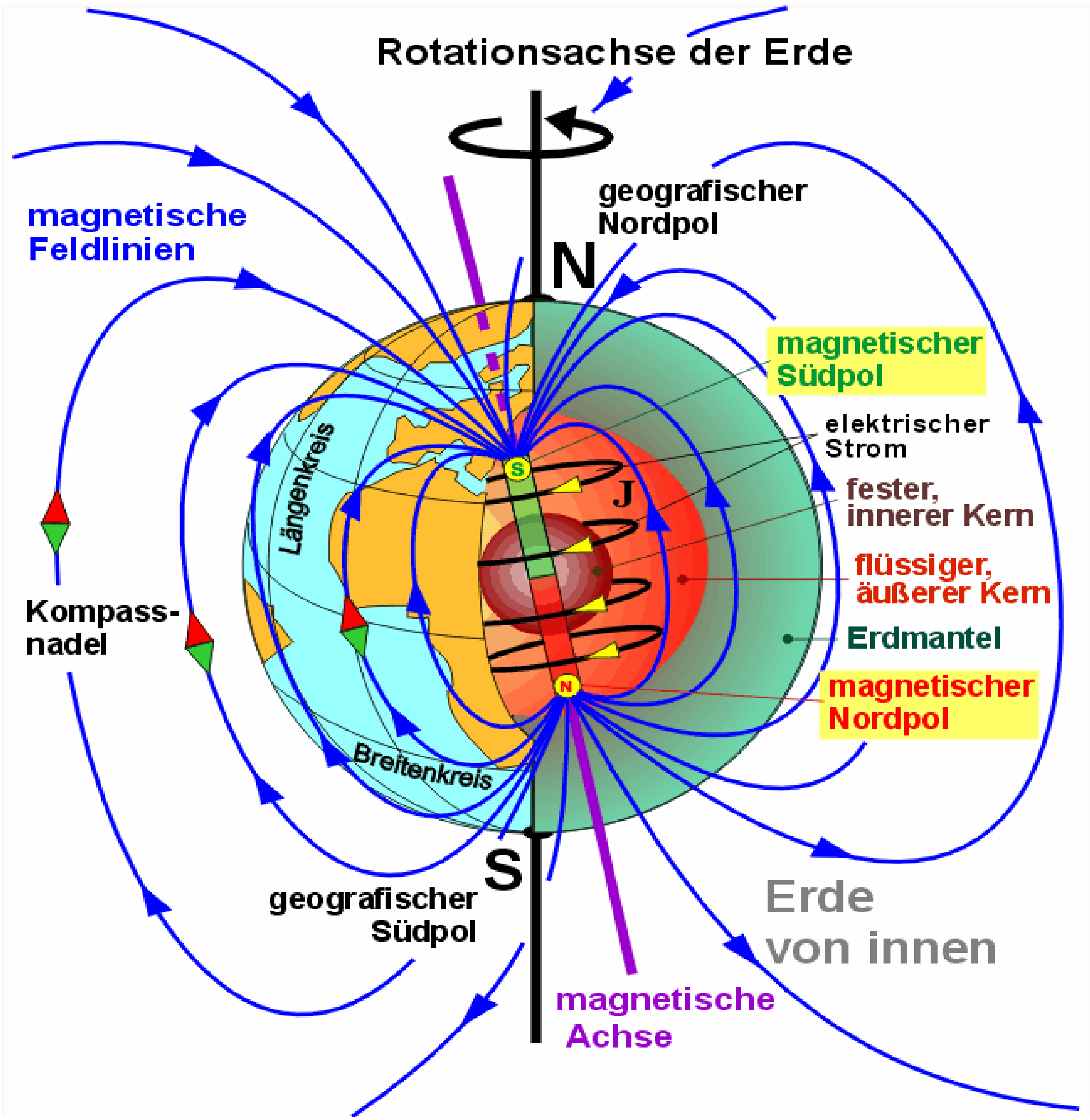


# MS Fields – Magnetic Field of the Earth / MS–Felder – Magnetfeld der Erde

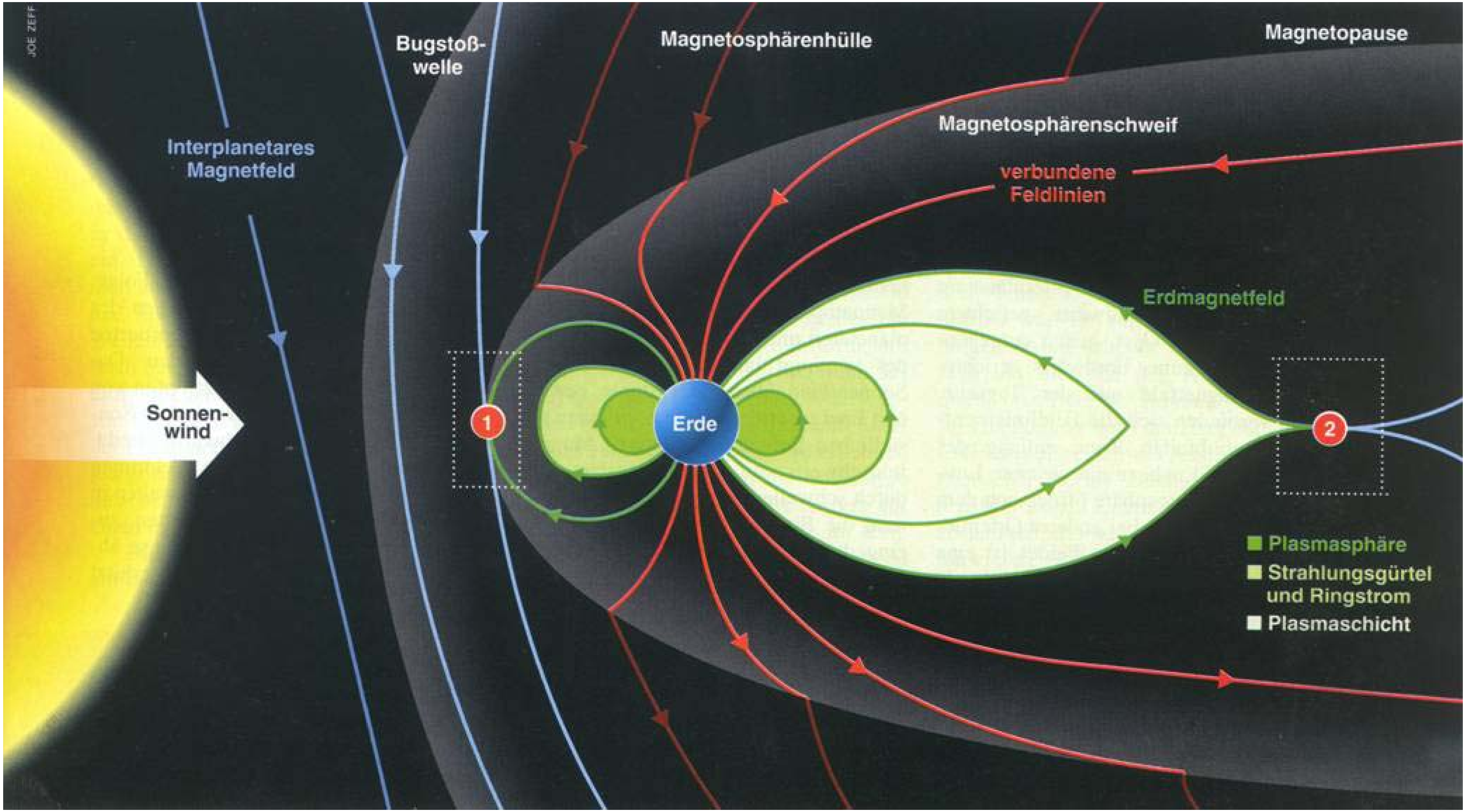


# MS Fields – Magnetic Field of the Earth / MS–Felder – Magnetfeld der Erde



# MS Fields – Magnetic Field of the Earth – Sun Wind / MS–Felder – Magnetfeld der Erde – Sonnenwind



# MS Fields – Governing Equations / MS–Felder – Grundgleichungen

**Integral Form / Integralform**

$$\oint_{C=\partial S} \underline{\mathbf{H}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dR}} = \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}}$$

$$\oiint_S \underline{\mathbf{B}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = 0$$

**Differential Form / Differentialform**

$$\nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}) = \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$\nabla \bullet \underline{\mathbf{B}}(\underline{\mathbf{R}}) = 0$$

$\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})$

is a Known Prescribed Electric Current Density: For Example a Electric Current Density in a Wire /
ist eine bekannte vorgegebene elektrische Stromdichte: Zum Beispiel eine elektrische Stromdichte in einem Draht

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Because of / Weil $\nabla \bullet \underline{\mathbf{B}}(\underline{\mathbf{R}}) = 0$

$\underline{\mathbf{B}}(\underline{\mathbf{R}})$ Can be Represented by / kann dargestellt werden über $\underline{\mathbf{B}}(\underline{\mathbf{R}}) = \nabla \times \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}})$

In Vacuum we have / Im Vakuum gilt

$$\underline{\mathbf{B}}(\underline{\mathbf{R}}) = \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}})$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{1}{\mu_0} \underline{\mathbf{B}}(\underline{\mathbf{R}}) = \frac{1}{\mu_0} \nabla \times \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}})$$

# MS Fields – Vector Potential / MS Felder – Vektorpotential

$$\nabla \bullet \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) = 0$$

Coulomb Gauge / Coulomb–Eichung

It follows / Es folgt

$$\mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}}) = \nabla \times \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}})$$

Applying the Curl Operator Gives / Die Anwendung des Rotationsoperators ergibt

$$\mu_0 \nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}) = \nabla \times \nabla \times \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) = \mu_0 \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$\begin{aligned}\nabla \times \nabla \times \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) &= \nabla \underbrace{\nabla \bullet \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}})}_{=0} - \nabla \bullet \nabla \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) \\ &= -\nabla \bullet \nabla \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) \\ &= -\Delta \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}})\end{aligned}$$

# MS Fields – Vector Potential – Poisson and Laplace Equation / MS Felder – Vektorpotential – Poisson- und Laplace-Gleichung

$$\Delta\, \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) = \begin{cases} -\mu_0\, \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) & \text{for / für} \quad \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) \neq \underline{\mathbf{0}} \quad \text{Vectorial Poisson Equation / Vektorielle Poisson-Gleichung} \\ 0 & \text{for / für} \quad \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}} \quad \text{Vectorial Laplace Equation / Vektorielle Laplace-Gleichung} \end{cases}$$

# MS Fields – Vector Potential – (...) Special Solution / MS Felder – Vector Potential – (...) Spezielle Lösung

$$\Delta\, \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) = -\mu_0\, \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})$$

Vector Possion's Equation / Vektorielle Possion-Gleichung

Special Solution / Spezielle Lösung

$$\underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) = \mu_0 \iiint_{V_S} G(\underline{\mathbf{R}} - \underline{\mathbf{R}}')\, \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\, \mathrm{d}^3\underline{\mathbf{R}}'$$

With the Three-Dimensional Static Green's Function / Mit der dreidimensionalen statischen Greenschen Funktion

$$G(\underline{\mathbf{R}} - \underline{\mathbf{R}}') = \frac{1}{4\pi} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|}$$

$$\begin{aligned}
\underline{\mathbf{H}}(\underline{\mathbf{R}}) &= \frac{1}{\mu_0} \nabla \times \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) \\
&= \frac{1}{\mu_0} \nabla \times \left[ \mu_0 \iiint_{V_S} G(\underline{\mathbf{R}} - \underline{\mathbf{R}}')\, \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\, \mathrm{d}^3\underline{\mathbf{R}}' \right] \\
&= \iiint_{V_S} \nabla \times \left[ G(\underline{\mathbf{R}} - \underline{\mathbf{R}}')\, \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}') \right] \mathrm{d}^3\mathbf{R}'
\end{aligned}$$

$$\nabla \times (\Phi\, \underline{\mathbf{A}}) = \Phi \nabla \times \underline{\mathbf{A}} + \nabla \Phi \times \underline{\mathbf{A}}$$

$$\nabla \times [G(\underline{\mathbf{R}} - \underline{\mathbf{R}}')\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')] = G(\underline{\mathbf{R}} - \underline{\mathbf{R}}') \underbrace{\nabla \times \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')}_{=\underline{\mathbf{0}}} + \nabla G(\underline{\mathbf{R}} - \underline{\mathbf{R}}') \times \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')$$

# MS Fields – Vector Potential – Biot–Savart's Law / MS Felder – Vector Potential – Biot–Savartsche Gesetz

$$\nabla G(\underline{\mathbf{R}}-\underline{\mathbf{R}}') = \nabla \frac{1}{4\pi}\frac{1}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|} = \frac{1}{4\pi}\nabla\frac{1}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|} = -\frac{1}{4\pi}\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}'}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}$$

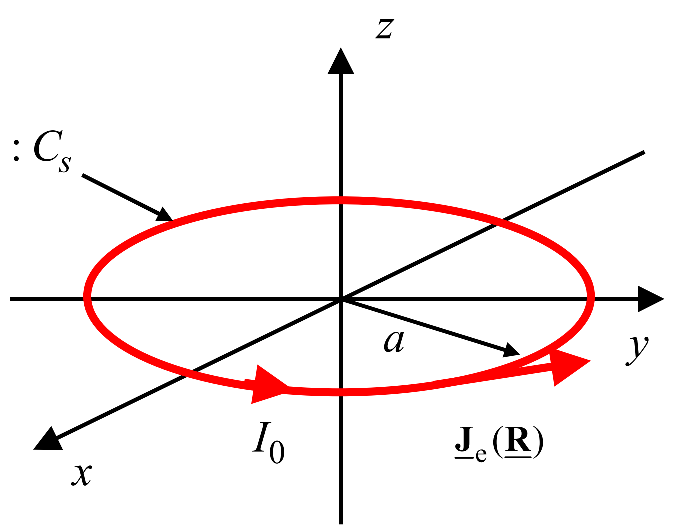
$$\begin{aligned}\underline{\mathbf{H}}(\underline{\mathbf{R}}) &= -\iiint_{V_S} \frac{1}{4\pi}\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}'}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\times \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\,\mathrm{d}^3\underline{\mathbf{R}}' \\ &= \iiint_{V_S} \frac{1}{4\pi}\frac{\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\times(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\,\mathrm{d}^3\underline{\mathbf{R}}' \\ &= \frac{1}{4\pi}\iiint_{V_S} \frac{\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\times(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\,\mathrm{d}^3\underline{\mathbf{R}}'\end{aligned}$$

## Biot–Savart's Law (for a Given Volume Source) / Biot–Savartsches Gesetz (für eine Volumenquelle)

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{1}{4\pi}\iiint_{V_S} \frac{\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\times(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\,\mathrm{d}^3\underline{\mathbf{R}}'$$

# MS Fields – Biot-Savart's Law for a Line Source / MS Felder – Biot-Savartsches Gesetz für eine Linienquelle

**Source / Quelle** $: C_s$




$$\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) = I_0\,\delta(r - a)\,\delta(z)\,\underline{\mathbf{e}}_{\varphi}, \qquad a > 0$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{1}{4\pi}\iiint_{V_s} \frac{\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\times(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\,\mathrm{d}^3\underline{\mathbf{R}}'$$

$$= \frac{1}{4\pi}\int\limits_{z'-\infty}^{\infty}\int\limits_{\varphi'=0}^{2\pi}\int\limits_{r'=0}^{\infty} \frac{I_0\delta(r'-a)\delta(z')\underline{\mathbf{e}}_{\varphi'}(\varphi')\times(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\,r'\mathrm{d}r'\mathrm{d}\varphi'\mathrm{d}z'$$

$$= \frac{I_0}{4\pi}\int\limits_{\varphi'=0}^{2\pi}\int\limits_{r'=0}^{\infty} \frac{\delta(r'-a)\underline{\mathbf{e}}_{\varphi'}(\varphi')\times(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\,r'\mathrm{d}r'\mathrm{d}\varphi'\Bigg|_{\underline{\mathbf{R}}':z'=0}$$

# MS Fields – Biot-Savart's Law for a Line Source / MS-Felder – Biot-Savartsche Gesetz für eine Linienquelle

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0}{4\pi} \int_{\varphi'=0}^{2\pi} \left. \frac{\overbrace{\underline{\mathbf{e}}_{\varphi'}(\varphi')\, a\, \mathrm{d}\varphi'}^{=\underline{\mathbf{dR}}'} \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3} \right|_{\underline{\mathbf{R}}':r'=a;z'=0}$$

$$= \frac{I_0}{4\pi} \int_{\varphi'=0}^{2\pi} \left. \frac{\underline{\mathbf{dR}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3} \right|_{\underline{\mathbf{R}}':r'=a;z'=0}$$

$$= \frac{I_0}{4\pi} \int_{C_s} \left. \frac{\underline{\mathbf{dR}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3} \right|_{C_s:\underline{\mathbf{R}}':r'=a;0\le\varphi'<2\pi;z'=0}$$

Line Source with Contour $C_S$ /
Linienquelle mit der Kontur $C_S$

$$C_S : \underline{\mathbf{R}}' : r' = a; 0 \le \varphi' < 2\pi; z' = 0$$

Biot-Savart Law for a Line Source with Contour $C_S$ /
Biot-Savartsches Gesetz für eine Linienquelle mit der Kontur $C_S$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0}{4\pi} \int_{C_s} \frac{\underline{\mathbf{dR}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

$\Rightarrow$ Arbitrary Contour $C_S$ /
Beliebige Kontur $C_S$

# MS Fields – Biot-Savart‘s Law for a Line Source / MS-Felder – Biot-Savartsche Gesetz für eine Linienquelle

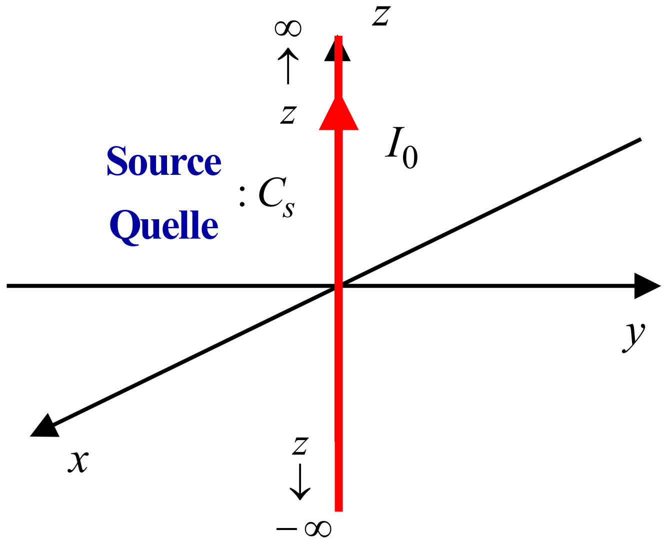
Biot-Savart’s Law for a Volume Source $V_s$ /
Biot-Savartsches Gesetz für eine Volumenquelle $V_s$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{1}{4\pi} \iiint_{V_s} \frac{\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}') \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3} \mathrm{d}^3 \underline{\mathbf{R}}'$$

Biot-Savart Law for a Line Source with Contour $C_s$ /
Biot-Savartsches Gesetz für eine Linienquelle mit der Kontur $C_s$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0}{4\pi} \int_{C_s} \frac{\underline{\underline{\mathbf{dR}}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

# MS Fields – Biot–Savart – Example: Infinite Thin and Infinite Long Wire Carrying a Constant Electric Current / MS–Felder – Biot–Savart – Beispiel: unendlich dünner, unendlich langer Draht, der einen konstanten elektrischen Strom führt




Biot–Savart Law for a
Line Source with Contour $C_S$ /
Biot–Savartsches Gesetz
für eine Linienquelle mit der Kontur $C_S$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0}{4\pi} \int_{C_S} \frac{\underline{\mathbf{dR}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

$$\begin{aligned}
\underline{\mathbf{R}} &= r\underline{\mathbf{e}}_r(\varphi) + z\underline{\mathbf{e}}_z \\
\underline{\mathbf{R}}' &= r'\underline{\mathbf{e}}_{r'}(\varphi') + z'\underline{\mathbf{e}}_z \Big|_{r'=0} \\
&= z'\underline{\mathbf{e}}_z, \qquad -\infty \le z' \le \infty \\
\underline{\mathbf{R}} - \underline{\mathbf{R}}' &= r\underline{\mathbf{e}}_r + (z - z')\underline{\mathbf{e}}_z \\
\underline{\mathbf{dR}}' &= \frac{\mathrm{d}}{\mathrm{d}z'}\underline{\mathbf{R}}' dz' \\
&= \underline{\mathbf{e}}_z \mathrm{d}z'
\end{aligned}$$

$$\underline{\mathbf{dR}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}') = \begin{vmatrix} \underline{\mathbf{e}}_r & \underline{\mathbf{e}}_\varphi(\varphi) & \underline{\mathbf{e}}_z \\ 0 & 0 & dz' \\ r & 0 & z - z' \end{vmatrix}$$

$$= r\,\underline{\mathbf{e}}_\varphi\, dz'$$

$$\left|\underline{\mathbf{R}} - \underline{\mathbf{R}}'\right|^3 = \left[r^2 + (z - z')^2\right]^{3/2}$$

## MS Fields – Biot-Savart – Example: Infinite Thin and Infinite Long Wire Carrying a Constant Electric Current / MS-Felder – Biot-Savart – Beispiel: unendlich dünner, unendlich langer Draht, der einen konstanten elektrischen Strom führt (...)

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0}{4\pi} \int\limits_{z'=-\infty}^{\infty} \frac{r\underline{\mathbf{e}}_{\varphi}(\varphi)}{\left[r^2 + (z-z')^2\right]^{3/2}} \mathrm{d}z'$$

$$= -\frac{I_0}{4\pi} r\underline{\mathbf{e}}_{\varphi}(\varphi) \int\limits_{\alpha=\infty}^{-\infty} \frac{\mathrm{d}\alpha}{\left[r^2 + \alpha^2\right]^{3/2}}$$

$$= \frac{I_0}{4\pi} r\underline{\mathbf{e}}_{\varphi}(\varphi) \int\limits_{\alpha=-\infty}^{\infty} \frac{\mathrm{d}\alpha}{\left[r^2 + \alpha^2\right]^{3/2}}$$

$$= \frac{I_0}{4\pi} r\underline{\mathbf{e}}_{\varphi}(\varphi) \int\limits_{\alpha=-\infty}^{\infty} \frac{\mathrm{d}\alpha}{\left[r^2 + \alpha^2\right]^{3/2}}$$

$$= \frac{I_0}{4\pi} r\underline{\mathbf{e}}_{\varphi}(\varphi) \left. \frac{\alpha}{r^2\sqrt{r^2+\alpha^2}} \right|_{\alpha=-\infty}^{\infty}$$

With the Substitution / Mit der Substitution

$$\alpha = z - z'$$

$$\mathrm{d}z' = -\mathrm{d}\alpha$$

$$z' = -\infty : \quad \alpha = z + \infty = \infty$$

$$z' = \infty : \quad \alpha = z - \infty = -\infty$$

$$\int \frac{\mathrm{d}x}{\left[ax^2 + b\right]^{3/2}} = \frac{x}{b\sqrt{ax^2+b}}$$

**with**
**mit** $a = 1,\ b = r^2$

# MS Fields – Biot–Savart – Example: Infinite Thin and Infinite Long Wire Carrying a Constant Electric Current / MS–Felder – Biot–Savart – Beispiel: unendlich dünner, unendlich langer Draht, der einen konstanten elektrischen Strom führt (...)

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0}{4\pi}\frac{1}{r}\left.\frac{\operatorname{sgn}(\alpha)\,|\alpha|}{\sqrt{r^2+\alpha^2}}\right|_{\alpha=-\infty}^{\infty}\underline{\mathbf{e}}_{\varphi}(\varphi)$$

$$= \frac{I_0}{4\pi}\frac{1}{r}\left.\frac{\operatorname{sgn}(\alpha)\,|\alpha|}{|\alpha|\sqrt{1+r^2/\alpha^2}}\right|_{\alpha=-\infty}^{\infty}\underline{\mathbf{e}}_{\varphi}(\varphi)$$

$$= \frac{I_0}{4\pi}\frac{1}{r}\left[\underbrace{\lim_{\alpha\to\infty}\frac{\operatorname{sgn}(\alpha)}{\sqrt{1+r^2/\alpha^2}}}_{=1} - \underbrace{\lim_{\alpha\to-\infty}\frac{\operatorname{sgn}(\alpha)}{\sqrt{1+r^2/\alpha^2}}}_{=-1}\right]\underline{\mathbf{e}}_{\varphi}(\varphi)$$

$$= \frac{I_0}{4\pi}\frac{1}{r}\underbrace{\left[1-(-1)\right]}_{2}\underline{\mathbf{e}}_{\varphi}(\varphi)$$

$$= \frac{I_0}{2\pi}\frac{1}{r}\underline{\mathbf{e}}_{\varphi}(\varphi)$$

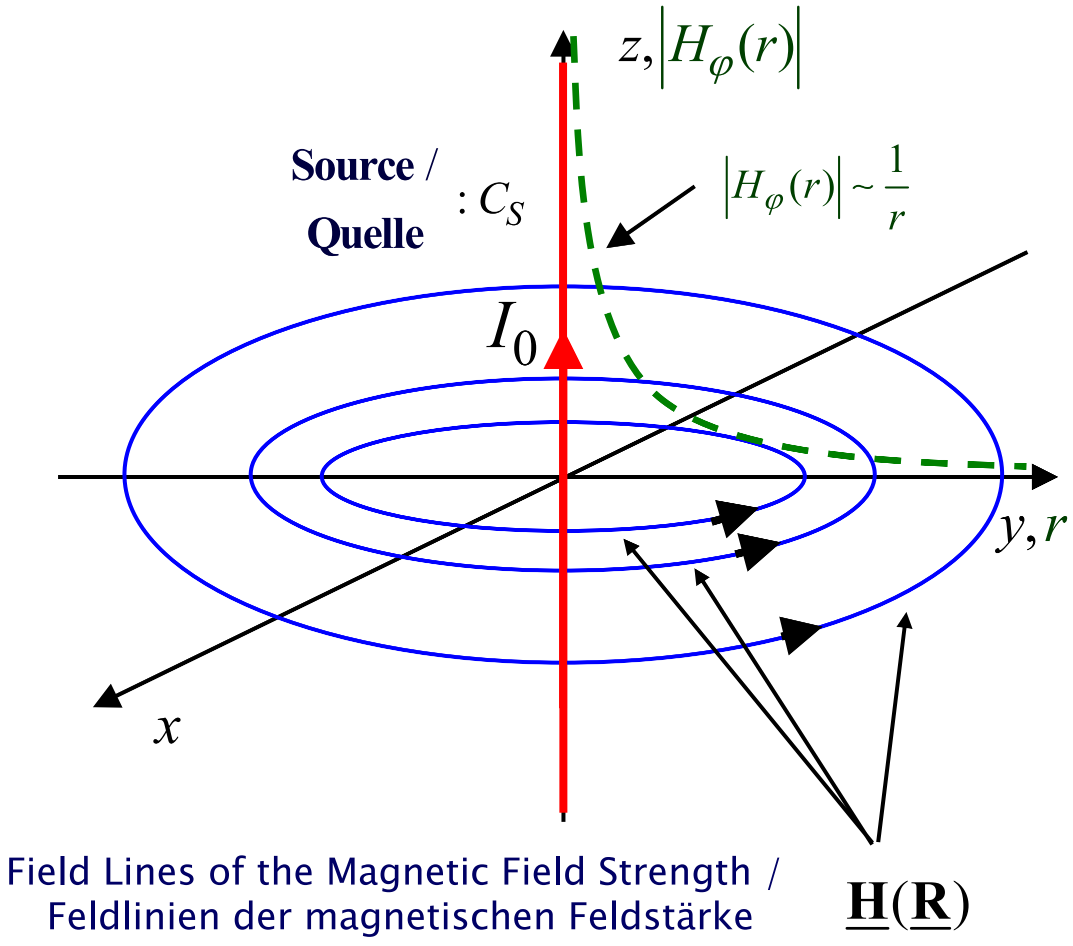
With the Signum Function /
Mit der Signum–Funktion

$$\operatorname{sgn}(\alpha) = \begin{cases} -1 & \alpha < 0 \\ 1 & \alpha > 0 \end{cases}$$

$$\alpha = \operatorname{sgn}(\alpha)\ |\alpha|$$

With /
Mit $\sqrt{\alpha^2} = |\alpha|$

# MS Fields – Biot-Savart – Example: Infinite Thin and Infinite Long Wire Carrying a Constant Electric Current / MS-Felder – Biot-Savart – Beispiel: unendlich dünner, unendlich langer Draht, der einen konstanten elektrischen Strom führt (...)

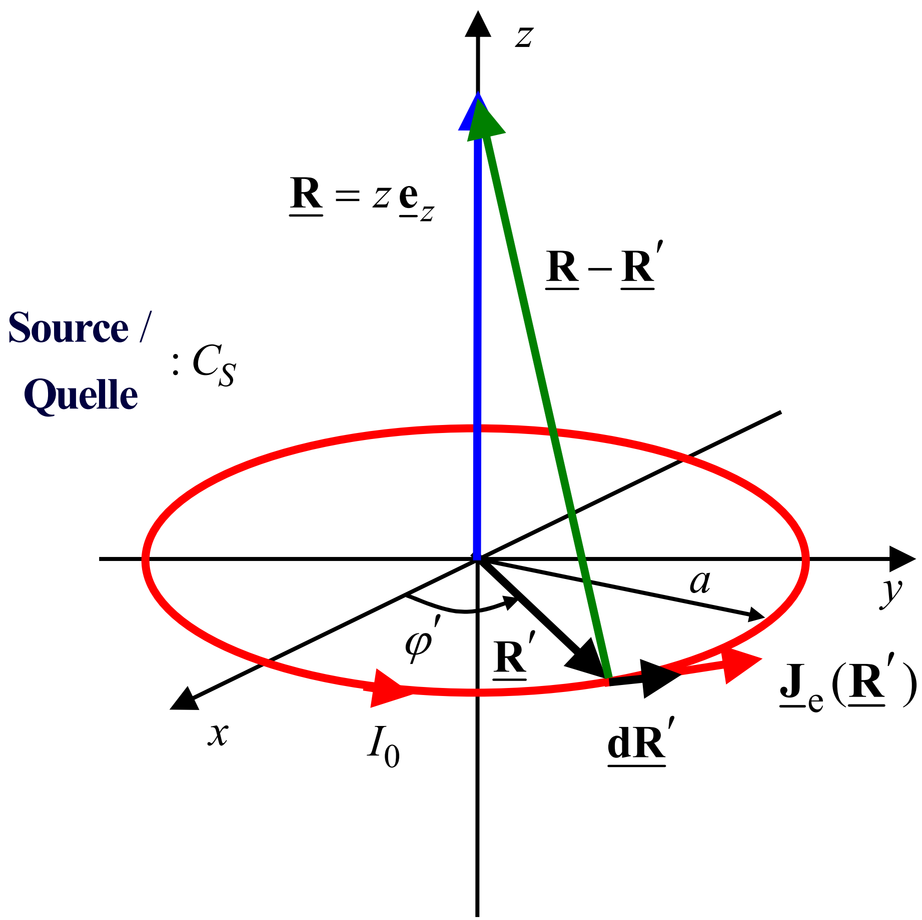



$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \underbrace{\frac{I_0}{2\pi}\frac{1}{r}}_{=H_\varphi(r)} \underline{\mathbf{e}}_\varphi(\varphi)$$

$$= H_\varphi(r)\underline{\mathbf{e}}_\varphi(\varphi)$$

$$\left|\underline{\mathbf{H}}(\underline{\mathbf{R}})\right| = \left|H_\varphi(r)\right| \sim \frac{1}{r}$$

# MS Fields – Biot–Savart's Law – Wire Loop Carrying a Constant Electric Current / MS–Felder – Biot–Savartsches Gesetz – Drahtschleife, die einen konstanten elektrischen Strom führt (...)




$$\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}}) = I_0\,\delta(r-a)\,\delta(z)\,\underline{\mathbf{e}}_\varphi(\varphi), \qquad a > 0$$

Biot–Savart's Law /
Biot–Savartsches Gesetz

$$\underline{\mathbf{H}}(\underline{\mathbf{R}} = z\,\underline{\mathbf{e}}_z) = \frac{I_0}{4\pi} \int_{C_S} \frac{\underline{\mathbf{dR}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

Magnetic Field Strength on the *z* Axis /
Magnetische Feldstärke auf der *z*-Achse

$$\underline{\mathbf{H}}(\underline{\mathbf{R}} = z\,\underline{\mathbf{e}}_z) = \frac{I_0}{4\pi} \int_{C_S} \frac{\underline{\mathbf{dR}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

$$\underline{\mathbf{R}} = z\,\underline{\mathbf{e}}_z \qquad -\infty < z < \infty$$

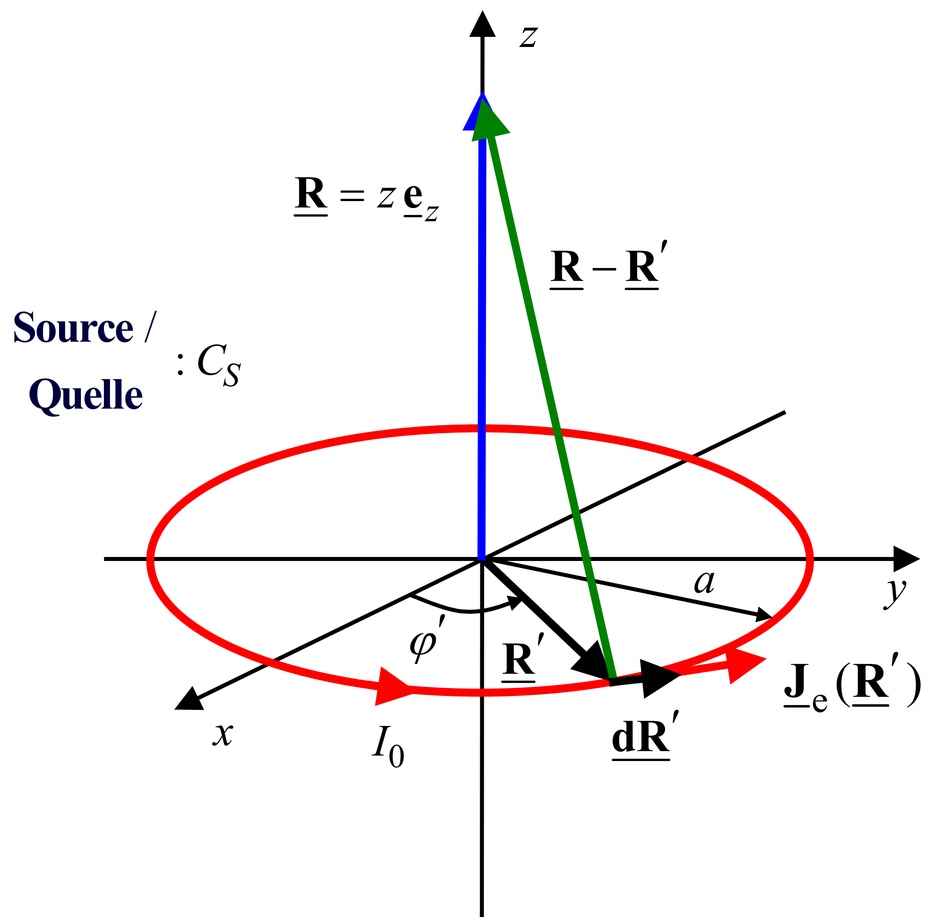
$$\underline{\mathbf{R}}' = a\,\underline{\mathbf{e}}_{r'}(\varphi') \qquad C_s: 0 \le \varphi \le 2\pi$$

$$= a\cos\varphi'\,\underline{\mathbf{e}}_x + a\sin\varphi'\,\underline{\mathbf{e}}_y$$

$$\underline{\mathbf{R}} - \underline{\mathbf{R}}' = z\,\underline{\mathbf{e}}_z - a\cos\varphi'\,\underline{\mathbf{e}}_x - a\sin\varphi'\,\underline{\mathbf{e}}_y$$

$$\underline{\mathbf{dR}}' = \left[\frac{\mathrm{d}}{\mathrm{d}\varphi'}\underline{\mathbf{R}}'(\varphi')\right]\mathrm{d}\varphi'$$

# MS Fields – Biot–Savart's Law – Wire Loop Carrying a Constant Electric Current / MS–Felder – Biot–Savartsches Gesetz – Drahtschleife, die einen konstanten elektrischen Strom führt (…)




Magnetic Field Strength on the $z$ Axis / Magnetische Feldstärke auf der $z$–Achse

$$\underline{\mathbf{H}}(\underline{\mathbf{R}} = z\underline{\mathbf{e}}_z) = \frac{I_0}{4\pi} \int_{C_S} \frac{\mathbf{d}\underline{\mathbf{R}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

$$\underline{\mathbf{R}}' = a\cos\varphi'\,\underline{\mathbf{e}}_x + a\sin\varphi'\,\underline{\mathbf{e}}_y$$

$$\begin{aligned}
\mathbf{d}\underline{\mathbf{R}}' &= \left[\frac{\mathrm{d}}{\mathrm{d}\varphi'}\underline{\mathbf{R}}'(\varphi')\right]\mathrm{d}\varphi' \\
&= \left[\frac{\mathrm{d}}{\mathrm{d}\varphi'}\left(a\cos\varphi'\underline{\mathbf{e}}_x + a\sin\varphi'\underline{\mathbf{e}}_y\right)\right]\mathrm{d}\varphi' \\
&= a\left(-\sin\varphi'\underline{\mathbf{e}}_x + \cos\varphi'\underline{\mathbf{e}}_y\right)\mathrm{d}\varphi'
\end{aligned}$$

# MS Fields – Biot–Savart's Law – Wire Loop Carrying a Constant Electric Current / MS–Felder – Biot–Savartsches Gesetz – Drahtschleife, die einen konstanten elektrischen Strom führt (...)

$$\underline{\mathbf{H}}(\underline{\mathbf{R}} = z\underline{\mathbf{e}}_z) = \frac{I_0}{4\pi} \int\limits_{C_S} \frac{\underline{\underline{\mathbf{dR}}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

$$\underline{\mathbf{R}} - \underline{\mathbf{R}}' = z\underline{\mathbf{e}}_z - a\cos\varphi'\underline{\mathbf{e}}_x - a\sin\varphi'\underline{\mathbf{e}}_y, \qquad 0 \le \varphi' \le 2\pi$$

$$\underline{\underline{\mathbf{dR}}}' = a\left(-\sin\varphi'\underline{\mathbf{e}}_x + \cos\varphi'\underline{\mathbf{e}}_y\right)\mathrm{d}\varphi'$$

$$\underline{\underline{\mathbf{dR}}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}') = \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ -a\sin\varphi'\mathrm{d}\varphi' & a\cos\varphi'\mathrm{d}\varphi' & 0 \\ -a\cos\varphi' & -a\sin\varphi' & z \end{vmatrix}$$

$$= az\cos\varphi'\,\underline{\mathbf{e}}_x\mathrm{d}\varphi' + a^2\sin^2\varphi'\,\underline{\mathbf{e}}_z\mathrm{d}\varphi' + a^2\cos^2\varphi'\,\underline{\mathbf{e}}_z\mathrm{d}\varphi' + az\sin\varphi'\,\underline{\mathbf{e}}_y\mathrm{d}\varphi'$$

$$= az\left(\cos\varphi'\,\underline{\mathbf{e}}_x + \sin\varphi'\,\underline{\mathbf{e}}_y\right)\mathrm{d}\varphi' + a^2\underbrace{\left(\sin^2\varphi' + \cos^2\varphi'\right)}_{=1}\underline{\mathbf{e}}_z\mathrm{d}\varphi'$$

$$= az\left(\cos\varphi'\,\underline{\mathbf{e}}_x + \sin\varphi'\,\underline{\mathbf{e}}_y\right)\mathrm{d}\varphi' + a^2\,\underline{\mathbf{e}}_z\mathrm{d}\varphi'$$

# MS Fields – Biot-Savart's Law – Wire Loop Carrying a Constant Electric Current / MS-Felder – Biot-Savartsches Gesetz – Drahtschleife, die einen konstanten elektrischen Strom führt (...)

$$\underline{\mathbf{H}}(\underline{\mathbf{R}} = z\underline{\mathbf{e}}_z) = \frac{I_0}{4\pi} \int_{C_S} \frac{\underline{\underline{\mathbf{dR}}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

$$\underline{\mathbf{R}} - \underline{\mathbf{R}}' = z\underline{\mathbf{e}}_z - a\cos\varphi'\underline{\mathbf{e}}_x - a\sin\varphi'\underline{\mathbf{e}}_y, \qquad 0 \le \varphi' \le 2\pi$$

$$\left|\underline{\mathbf{R}} - \underline{\mathbf{R}}'\right|^3 = \left[\underbrace{a^2\cos^2\varphi' + a^2\sin^2\varphi'}_{=a^2\underbrace{\left(\cos^2\varphi' + \sin^2\varphi'\right)}_{=1}} + z^2\right]^{3/2}$$

$$= \left[a^2 + z^2\right]^{3/2}$$

$$\underline{\underline{\mathbf{dR}}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}') = az\left(\cos\varphi'\underline{\mathbf{e}}_x + \sin\varphi'\underline{\mathbf{e}}_y\right)\mathrm{d}\varphi' + a^2\,\underline{\mathbf{e}}_z\mathrm{d}\varphi'$$

# MS Fields – Biot-Savart's Law – Wire Loop Carrying a Constant Electric Current / MS-Felder – Biot-Savartsches Gesetz – Drahtschleife, die einen konstanten elektrischen Strom führt (...)

$$\underline{\mathbf{H}}(\underline{\mathbf{R}} = z\underline{\mathbf{e}}_z) = \frac{I_0}{4\pi} \int_{C_S} \frac{\underline{\underline{\mathbf{dR}}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

$$\underline{\mathbf{dR}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}') = za\left(\cos\varphi'\,\underline{\mathbf{e}}_x + \sin\varphi'\,\underline{\mathbf{e}}_y\right) - a^2\,\underline{\mathbf{e}}_z$$

$$\left|\underline{\mathbf{R}} - \underline{\mathbf{R}}'\right|^3 = \left[a^2 + z^2\right]^{3/2}$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}} = z\underline{\mathbf{e}}_z) = \frac{I_0}{4\pi} \int_{C_S} \frac{\underline{\underline{\mathbf{dR}}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

$$= \frac{I_0}{4\pi} \int_{\varphi'=0}^{2\pi} \frac{za\left(\cos\varphi'\,\underline{\mathbf{e}}_x + \sin\varphi'\,\underline{\mathbf{e}}_y\right) + a^2\,\underline{\mathbf{e}}_z}{\left[a^2 + z^2\right]^{3/2}}\mathrm{d}\varphi'$$

$$= \frac{I_0}{4\pi} \int_{\varphi'=0}^{2\pi} \frac{za\left(\cos\varphi'\,\underline{\mathbf{e}}_x + \sin\varphi'\,\underline{\mathbf{e}}_y\right)}{\left[a^2 + z^2\right]^{3/2}}\mathrm{d}\varphi' - \frac{I_0}{4\pi} \int_{\varphi'=0}^{2\pi} \frac{a^2\,\underline{\mathbf{e}}_z}{\left[a^2 + z^2\right]^{3/2}}\mathrm{d}\varphi'$$

# MS Fields – Biot-Savart's Law – Wire Loop Carrying a Constant Electric Current / MS-Felder – Biot-Savartsches Gesetz – Drahtschleife, die einen konstanten elektrischen Strom führt (...)

$$\underline{\mathbf{H}}(\underline{\mathbf{R}} = z\underline{\mathbf{e}}_z) = \frac{I_0}{4\pi} \int_{C_S} \frac{\underline{\mathbf{dR}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

$$\underline{\mathbf{dR}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}') = az\left(\cos\varphi'\,\underline{\mathbf{e}}_x + \sin\varphi'\,\underline{\mathbf{e}}_y\right) - a^2\,\underline{\mathbf{e}}_z$$

$$\left|\underline{\mathbf{R}} - \underline{\mathbf{R}}'\right|^3 = \left[a^2 + z^2\right]^{3/2}$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}} = z\underline{\mathbf{e}}_z) = \frac{I_0}{4\pi} \frac{za}{\left[a^2 + z^2\right]^{3/2}} \underbrace{\int_{\varphi'=0}^{2\pi} \left(\cos\varphi'\,\underline{\mathbf{e}}_x + \sin\varphi'\,\underline{\mathbf{e}}_y\right) \mathrm{d}\varphi'}_{=0} - \frac{I_0}{4\pi} \frac{a\,\underline{\mathbf{e}}_z}{\left[a^2 + z^2\right]^{3/2}} \int_{\varphi'=0}^{2\pi} \mathrm{d}\varphi'$$

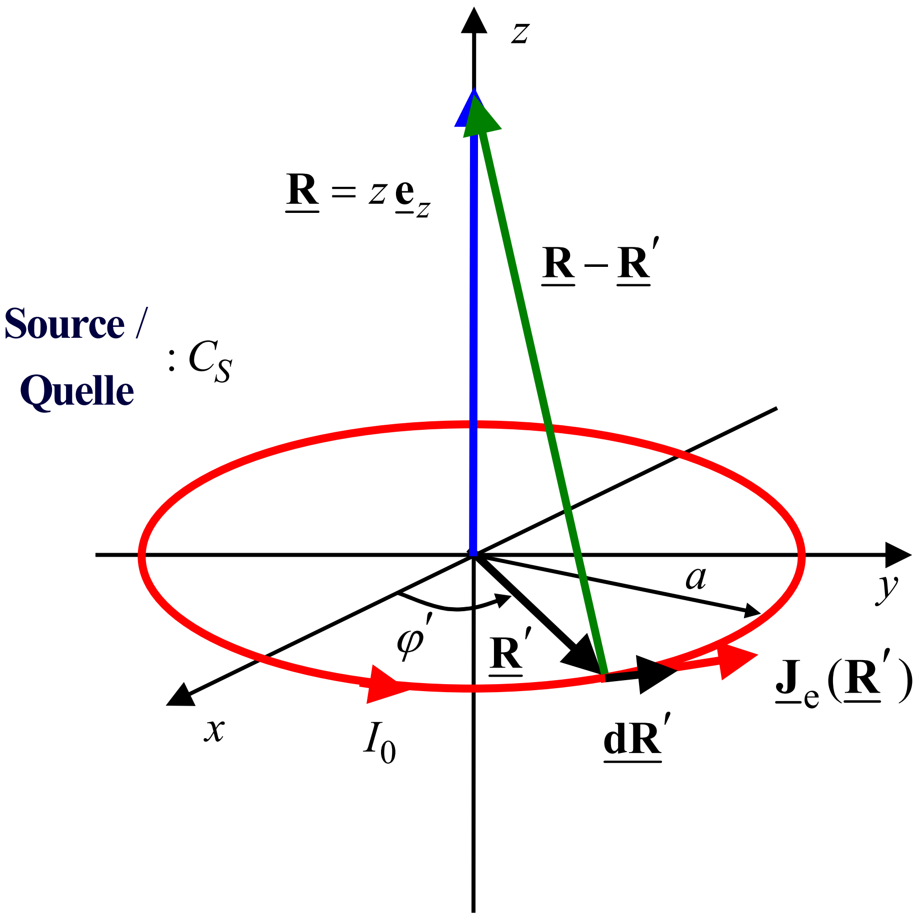
$$= \frac{I_0}{4\pi} \frac{a^2\,\underline{\mathbf{e}}_z}{\left[a^2 + z^2\right]^{3/2}} \underbrace{\int_{\varphi'=0}^{2\pi} \mathrm{d}\varphi'}_{=2\pi}$$

$$= \frac{I_0}{2} \frac{a^2}{\left[a^2 + z^2\right]^{3/2}} \underline{\mathbf{e}}_z$$

$$\int_{\varphi'=0}^{2\pi} \cos\varphi'\,\mathrm{d}\varphi' = 0$$

$$\int_{\varphi'=0}^{2\pi} \sin\varphi'\,\mathrm{d}\varphi' = 0$$

# MS Fields – Biot-Savart's Law – Wire Loop Carrying a Constant Electric Current / MS-Felder – Biot-Savartsches Gesetz – Drahtschleife, die einen konstanten elektrischen Strom führt (...)




Biot-Savart's Law /
Biot-Savartsches Gesetz

$$\underline{\mathbf{H}}(\underline{\mathbf{R}} = z\underline{\mathbf{e}}_z) = \frac{I_0}{4\pi} \int_{C_S} \frac{\mathbf{d}\underline{\mathbf{R}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

Magnetic Field Strength on the $z$ Axis/
Magnetische Feldstärke auf der $z$-Achse

$$\underline{\mathbf{H}}(\underline{\mathbf{R}} = z\underline{\mathbf{e}}_z) = \frac{I_0}{2} \frac{a^2}{\left[a^2 + z^2\right]^{3/2}} \underline{\mathbf{e}}_z$$

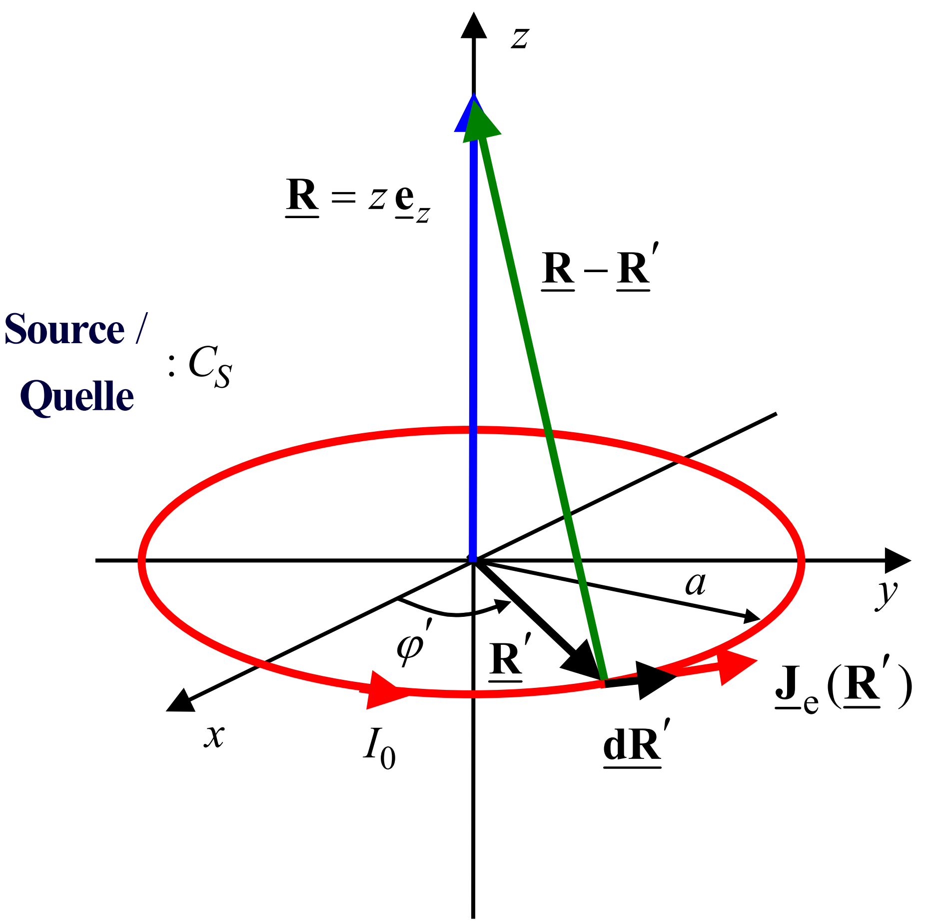
Constitutive Equation for Vacuum /
Materialgleichung für Vakuum

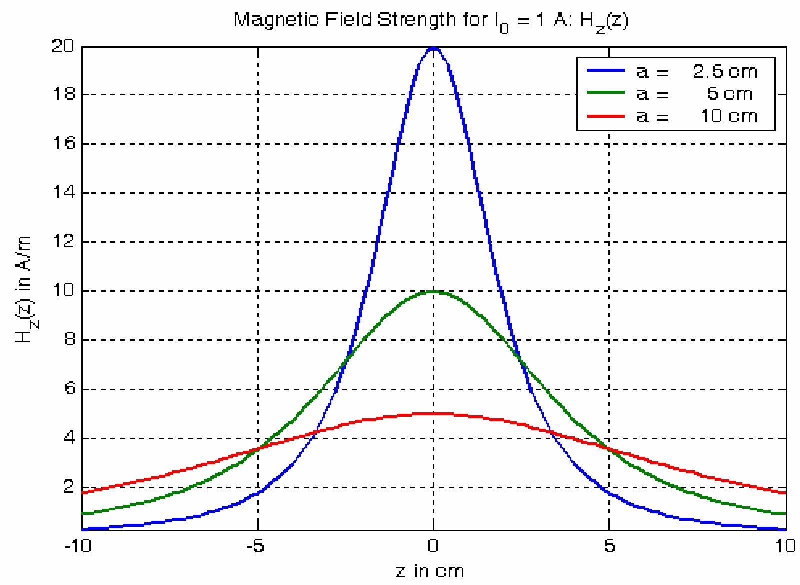
$$\underline{\mathbf{B}}(\underline{\mathbf{R}}) = \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}})$$

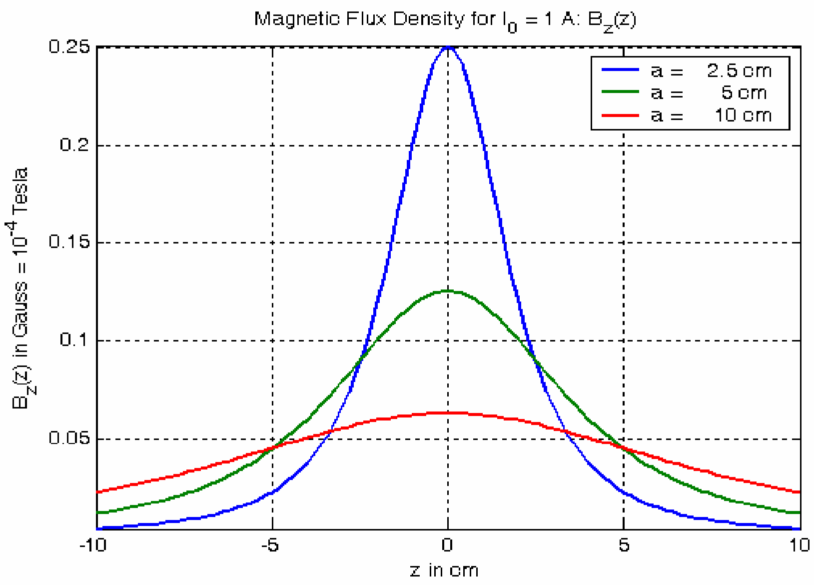
Magnetic Flux Density on the $z$ Axis/
Magnetische Flussdichte auf der $z$-Achse

$$\underline{\mathbf{B}}(\underline{\mathbf{R}} = z\underline{\mathbf{e}}_z) = \frac{\mu_0 I_0}{2} \frac{a^2}{\left[a^2 + z^2\right]^{3/2}} \underline{\mathbf{e}}_z$$

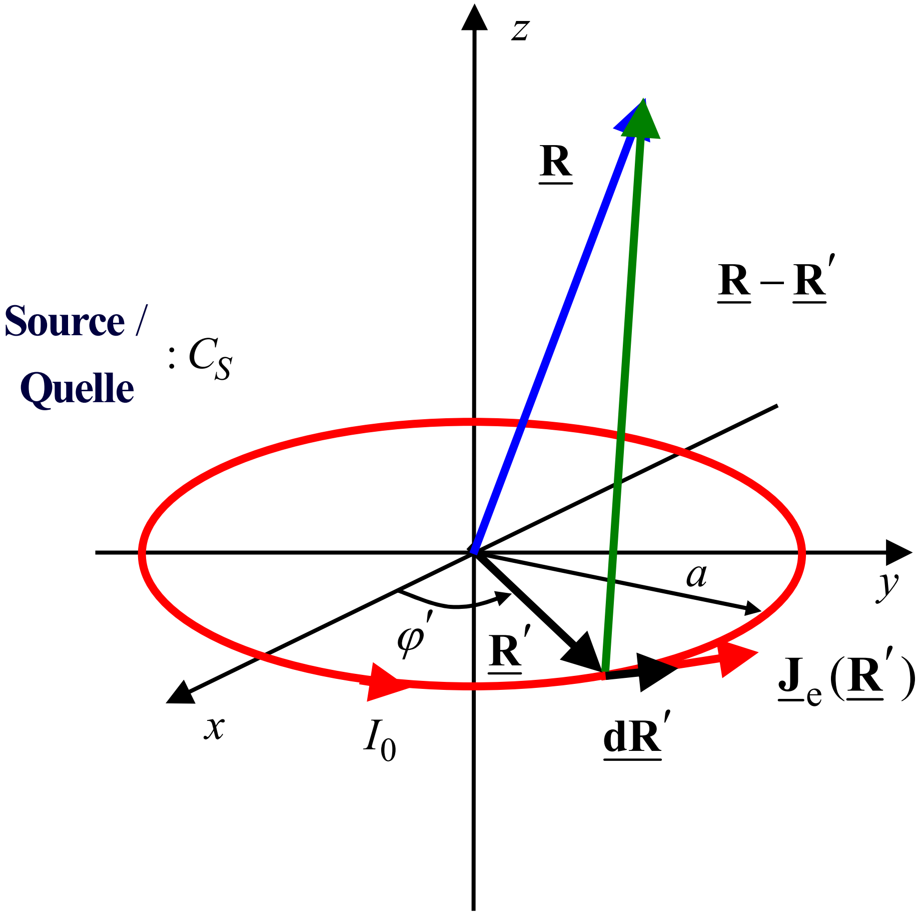
# MS Fields – Biot-Savart's Law – Wire Loop Carrying a Constant Electric Current / MS-Felder – Biot-Savartsches Gesetz – Drahtschleife, die einen konstanten elektrischen Strom führt (...)



# MS Fields – Biot-Savart's Law – Wire Loop Carrying a Constant Electric Current / MS-Felder – Biot-Savartsches Gesetz – Drahtschleife, die einen konstanten elektrischen Strom führt (...)




Magnetic Flux Density – Arbitrary Observation Point /
Magnetische Flussdichte – Beliebiger Beobachtungspunkt

$$\underline{\mathbf{B}}(\underline{\mathbf{R}}) = B_r(\underline{\mathbf{R}})\underline{\mathbf{e}}_r(\varphi) + B_z(\underline{\mathbf{R}})\underline{\mathbf{e}}_z$$

$$B_r(\underline{\mathbf{R}}) = \frac{\mu_0 I_0 k}{4\pi\sqrt{ar}}\left[-K(k) + \frac{a^2 + r^2 + z^2}{(a-r)^2 + z^2}E(k)\right]$$

$$B_z(\underline{\mathbf{R}}) = \frac{\mu_0 I_0 k}{4\pi\sqrt{ar}}\left[K(k) + \frac{a^2 - r^2 - z^2}{(a-r)^2 + z^2}E(k)\right]$$

with / mit

$$k = \sqrt{\frac{4ar}{(a+r)^2 + z^2}}$$

Complete Elliptic Integrals of 1st and 2nd Kind /
Kompletten elliptischen Integrale 1. und 2. Art

$$K(k) = \int_{\theta=0}^{\pi/2} \frac{1}{\sqrt{1 - k^2 \sin^2\theta}}\,\mathrm{d}\theta \qquad E(k) = \int_{\theta=0}^{\pi/2} \sqrt{1 - k^2\sin^2\theta}\,\mathrm{d}\theta$$

# End of Lecture 10 /
# Ende der 10. Vorlesung