Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

12th Lecture / 12. Vorlesung

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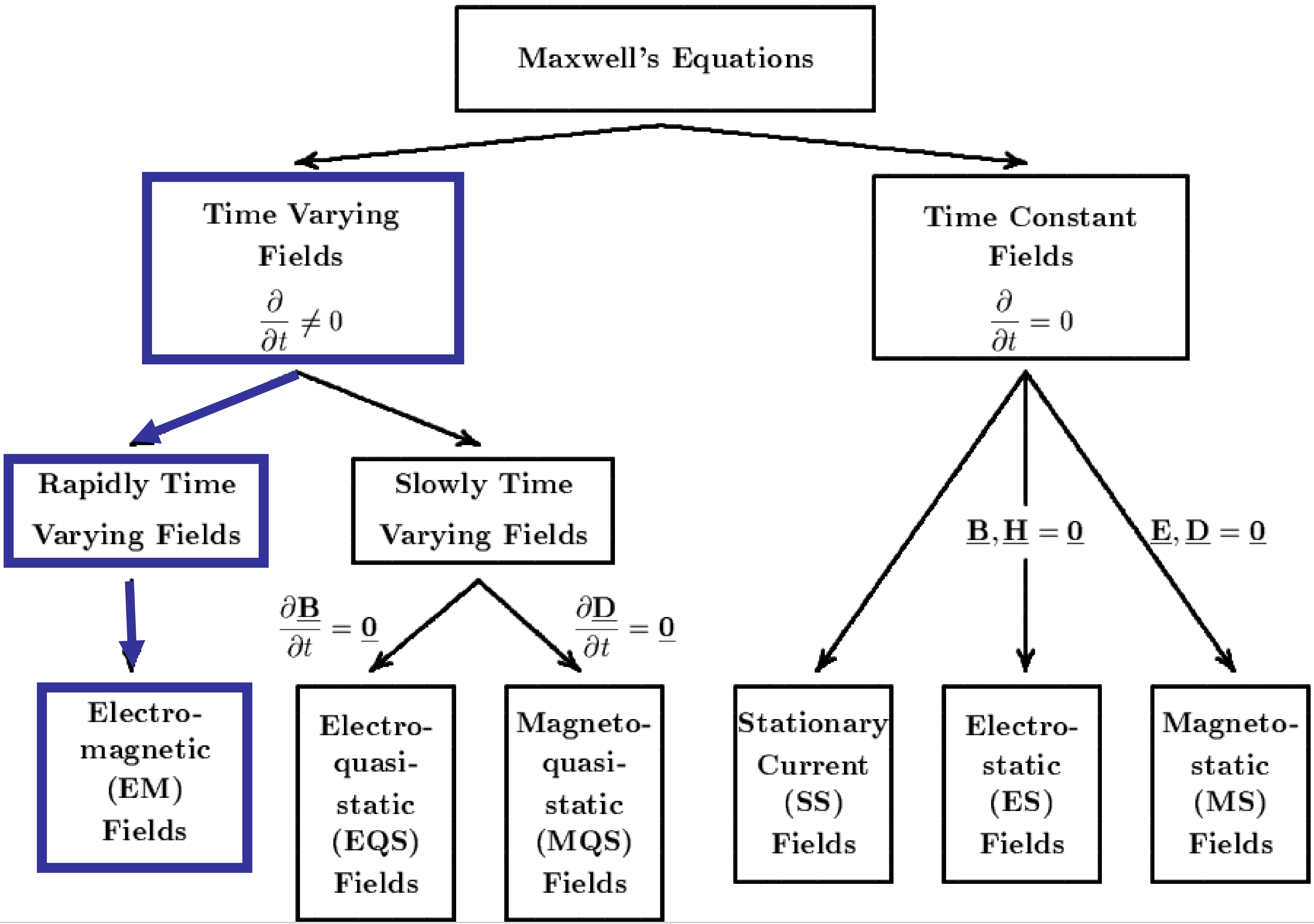
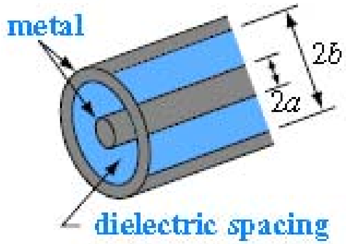
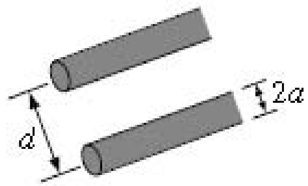
Electromagnetic (EM) Fields – Classification / Elektromagnetische (EM) Felder – Klassifikation



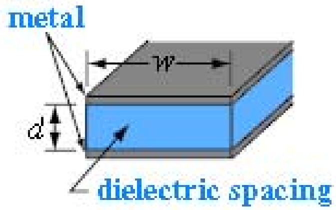
EM Fields – EM Waves – Guided EM Waves /
EM Felder – EM–Wellen – Geführte EM–Wellen



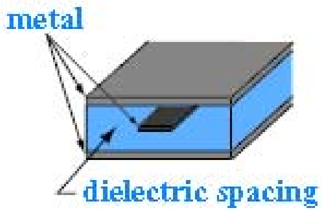
(a) Coaxial Line /
Koaxiale Leitung



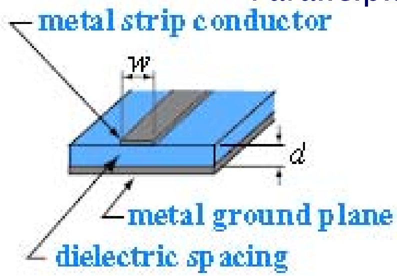
(b) Two–Wire Line /
Zweidrahtleitung



(c) Parallel–Plate Line
/
Parallelplattenleitung

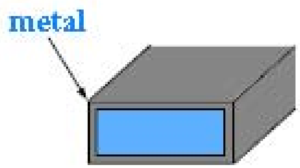


(d) Strip Line /
Streifenleitung



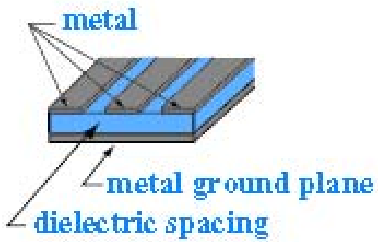
(e) Microstrip Line
/

TEM Transmission Lines / TEM Leitungen (Übertragungsleitungen)



(f) Rectangular Waveguide /
Recheckförmiger
Wellenleiter bzw. Hohlleitung

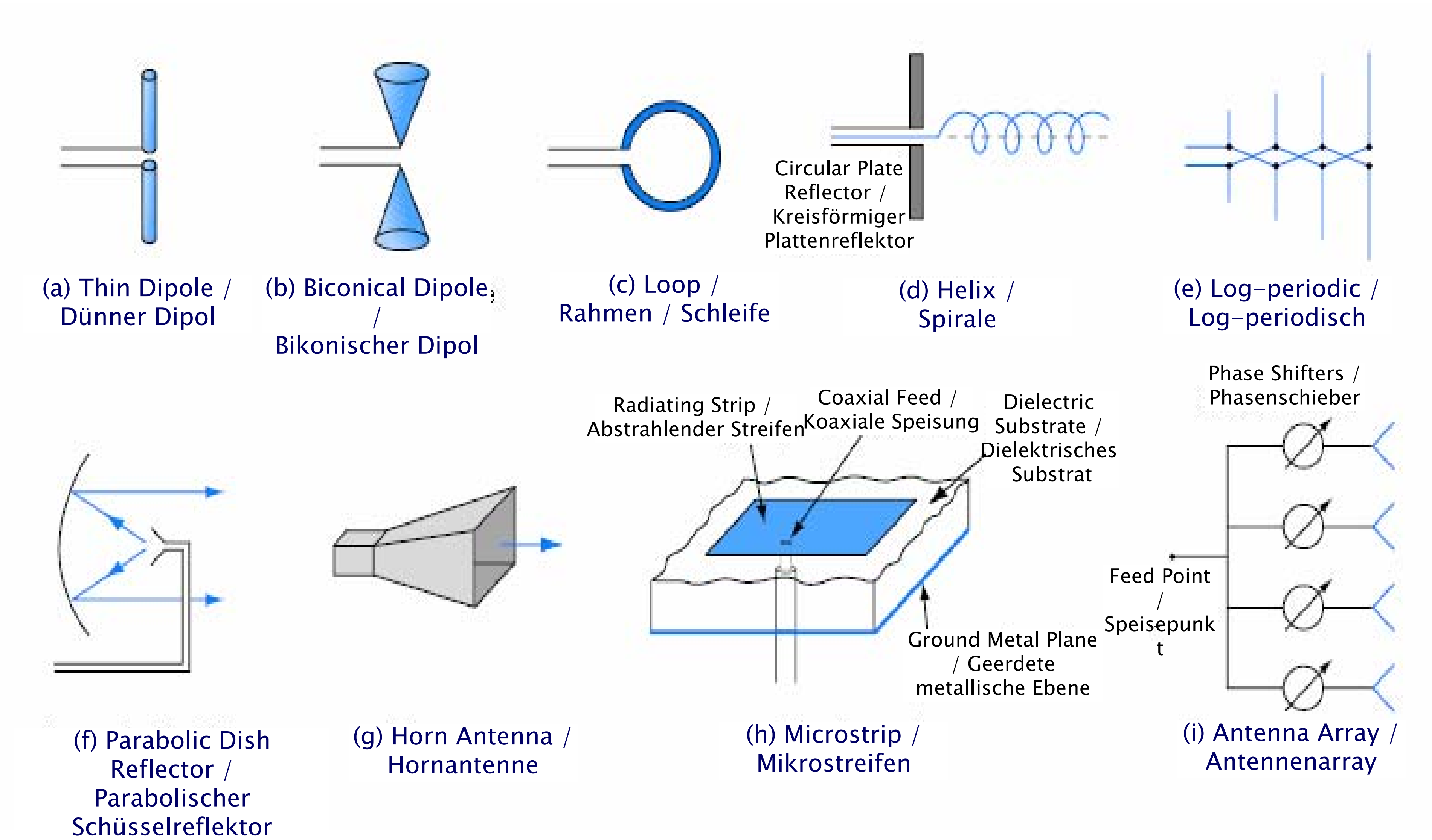
(g) Optical Fiber /
Optische Faser bzw.
Glasfaser



(h) Coplanar Waveguide /
Koplanarer Wellenleiter

Higher Order Transmission Lines / Leitungen höherer Ordnung (Übertragungsleitungen)

EM Fields – EM Waves – Antennas / EM Felder – EM Wellen – Antennen

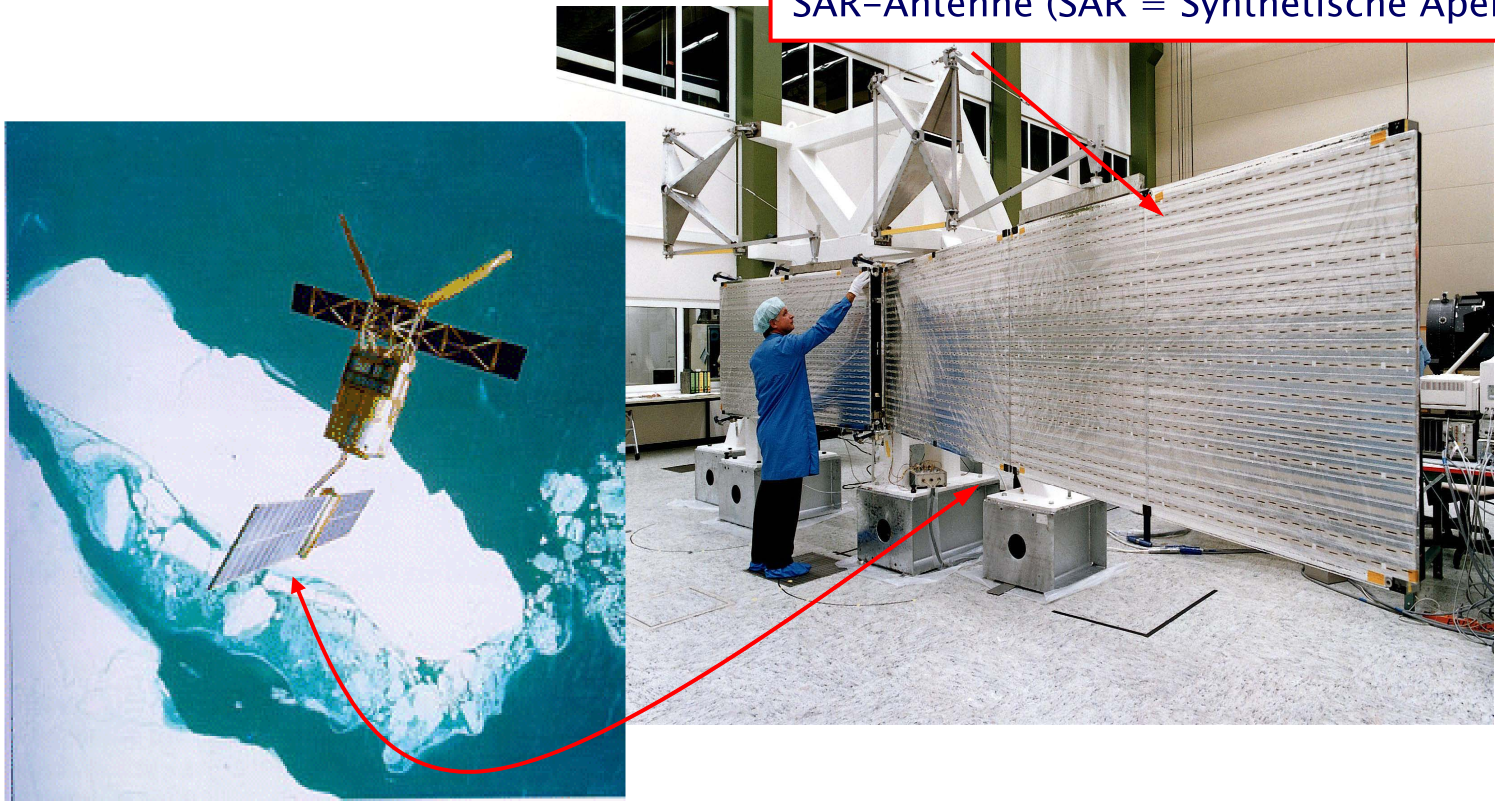


Leitungen – Antennen: Hohlleiterschlitzantennen (ERS-1)

ERS-1 – Satellit (ERS = European Remote Sensing)

(C-Band SAR – C-Band at 5 GHz)

SAR-Antenne (SAR = Synthetische Aperturradar-Antenne)



ERS-1, gestartet 1991, war der erste Erdbeobachtungssatellit der ESA; er trug eine umfangreiche Nutzlast, die einen Synthetic Aperture Radar (SAR), einen Radar-Altimeter und andere Instrumente zur Messung von Meeresoberflächen-Temperaturen und Seewinden umfasste. ERS-2, der sich mit ERS-1 überschnitt, wurde 1995 mit einem zusätzlichen Sensor für atmosphärische Ozonforschung gestartet.

Radar-Frequenzbänder – Frequenznamentabelle

Im zweiten Weltkrieg wurden Hochfrequenzen im GHz-Bereich, die für Radar-Ortung eingesetzt wurden, zur Geheimhaltung Buchstaben zugeordnet.

Das IEEE versucht, die Bezeichnungen zu vereinheitlichen, was nicht immer gelingt.

Band		Kurzbezeichnungen, oft bei Satellitenfunk
	Frequenzbereich	Frequenzbereich
P	220–300 MHz	
L	1–2,6 GHz	1–2 GHz
S	2,6–3,95 GHz	2–4 GHz
C	3,95–5,8 GHz	4–8 GHz
J	5,85–8,2 GHz	
X	8,2–12,4 GHz	8–12 GHz
Ku	12,4–18 GHz	12–18 GHz
K	18–26,5 GHz	18–27 GHz
Ka	26,5–40 GHz	27–40 GHz
Q	33–50 GHz	
U	40–60 GHz	
V	50–75 GHz	
E	60–90 GHz	
W	75–110 GHz	
F	90–140 GHz	
D	110–170 GHz	
G	140–220 GHz	
Y	170–260 GHz	
J	220–325 GHz	

Bezeichnungen bei Satellitenfunk mit Unterscheidung nach Diensten

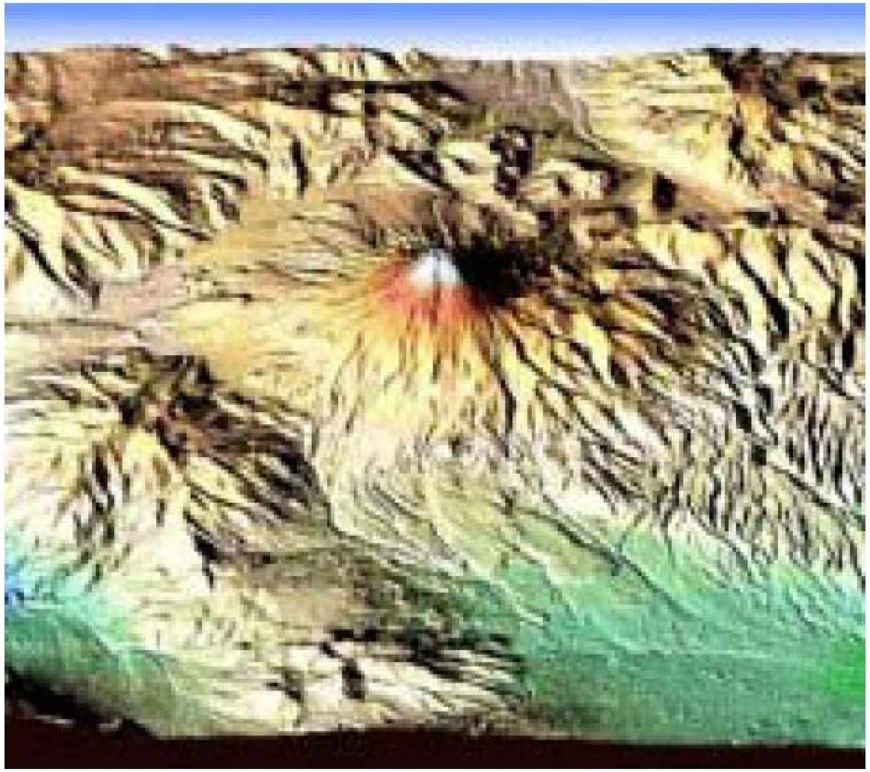
Slotted Waveguide Antenna / Hohlleiterschlitzantenne ERS-2 Misson

Slotted Waveguide Antenna / Hohlleiterschlitzantenne

(XSAR – X-Band SAR – X-Band bei einer Frequenz von f = 9.6 GHz)

XSAR / SRTM (Shuttle Radar Topography Mission)

SRTM in orbit, artist view



Cotopaxi - interferometric image generation resulting from SRTM data

Slotted Waveguide Antenna / Hohlleiterschlitzantenne

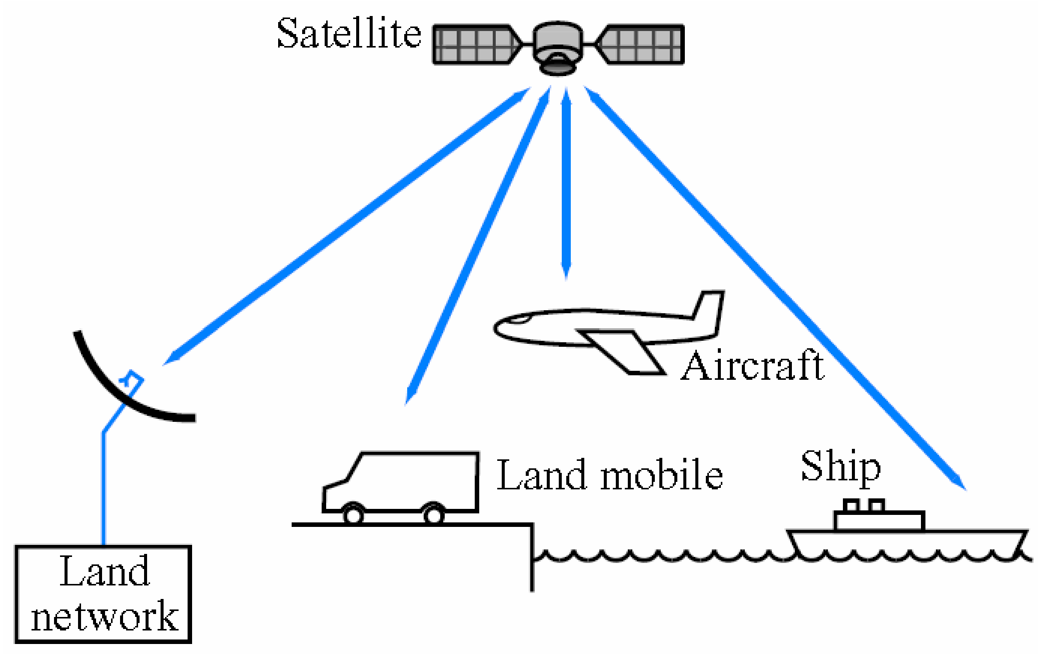
MESSENGER Mission

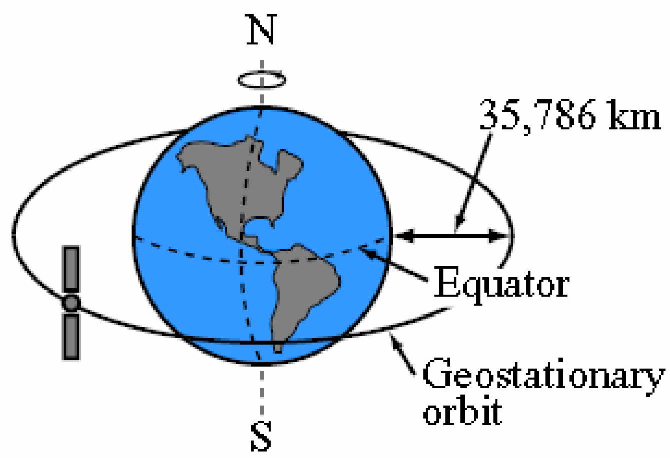
MESSENGER (MErcury Surface, Space ENvironment, GEochemistry and Ranging) ist eine NASA-Raumsonde im Rahmen des Discovery-Programms, startete am 3. August 2004 zum Merkur. Ursprünglich geplant war ein Start im Frühjahr 2004, dieser wurde jedoch aus technischen Gründen verschoben.

Figure 2 Flight Phased-Array and Fanbeam Antenna Assembly

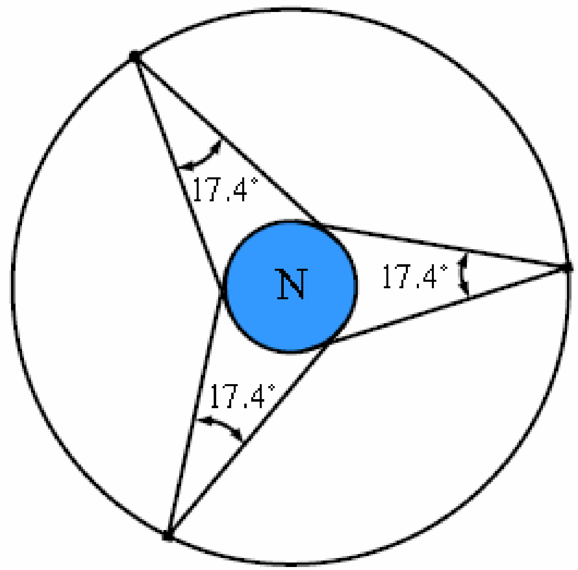
Zwei kohärente X-Band-Kommunikationssysteme aus zwei elektronisch phasengesteuerten Hohlleiterschlitzantennen mit hohem Gewinn

EM Fields – EM Waves – Satellite Communication / EM Felder – EM-Wellen- Satellitenkommunikation



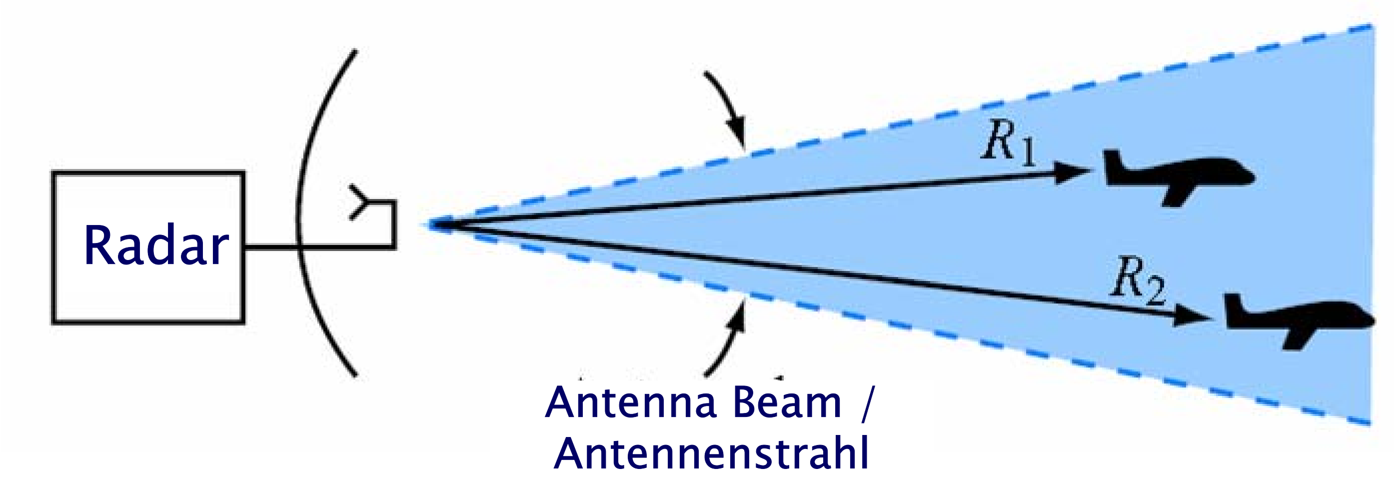


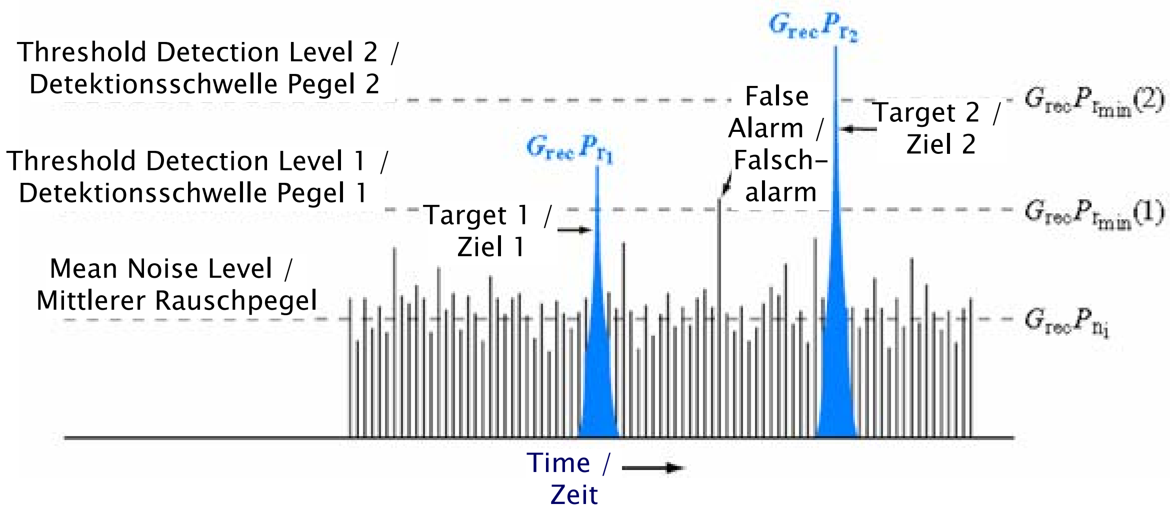
(a) Geostationary satellite orbit



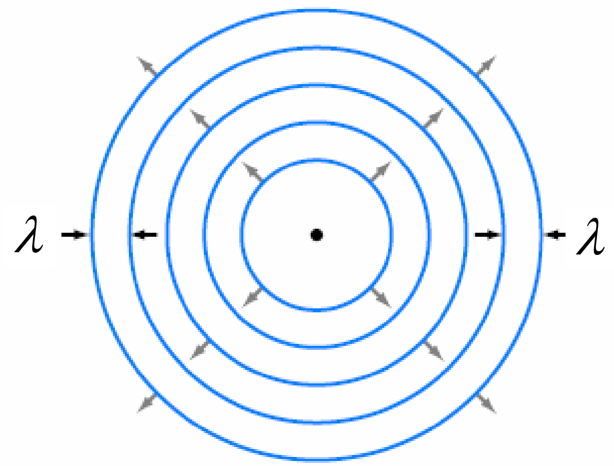
(b) Worldwide coverage by three satellites spaced 120° apart

EM Fields – EM Wave – Radar Systems / EM Felder – EM-Wellen – Radar-Systeme

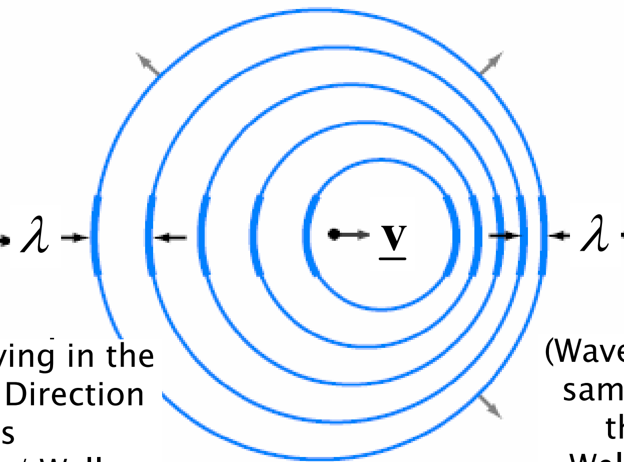




EM Fields – EM Waves – Doppler Radar Systems / EM Felder – EM–Wellen – Doppler Radar–Systeme



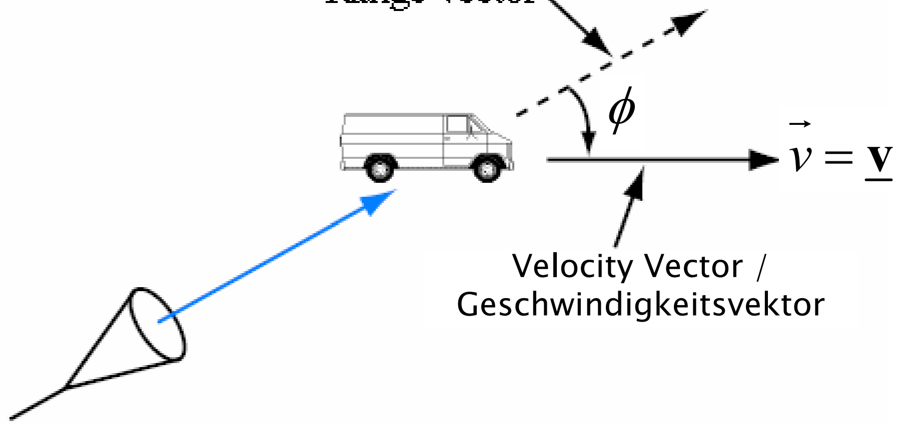
Stationary Source / Stationäre Quelle



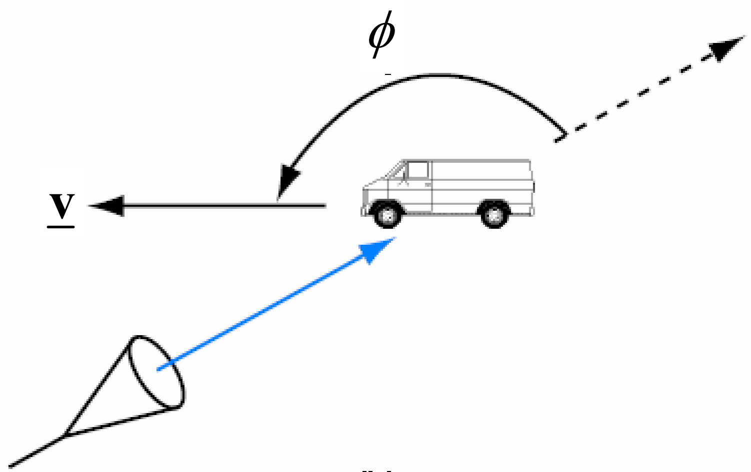
(Wave Moving in the Opposite Direction as the Source / Wellen–bewegung in die entgegengesetzte Richtung wie die Quelle

Moving Source / Bewegte Quelle

(Wave Moving in the same Direction as the Source / Wellenbewegung in die gleiche Richtung wie die Quelle

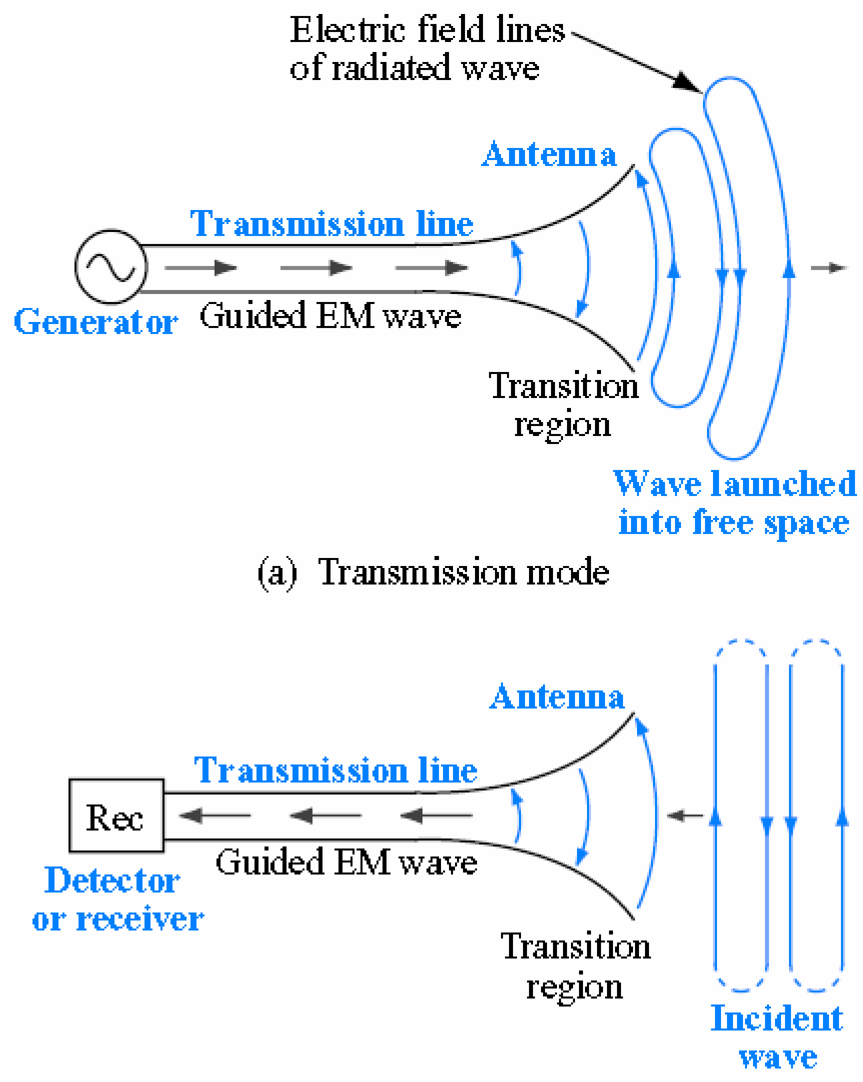


(a)



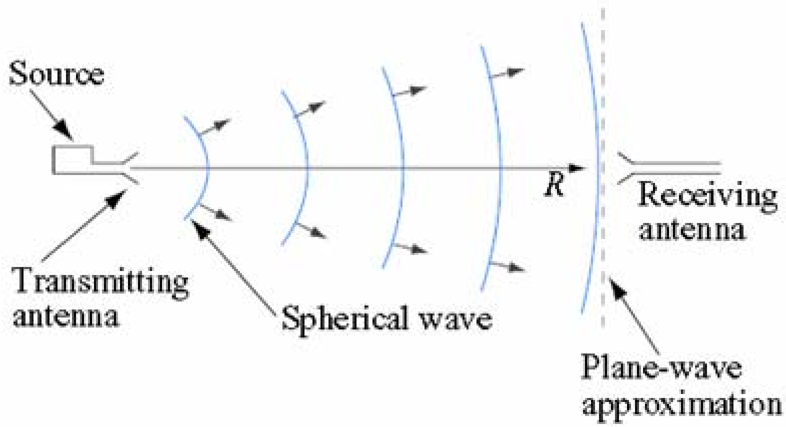
(b)

EM Fields – EM Waves – Antenna Systems / EM Felder – EM-Wellen – Antennensysteme

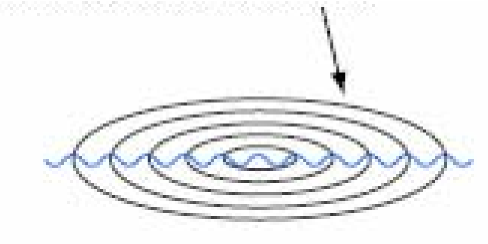


(a) Transmission mode

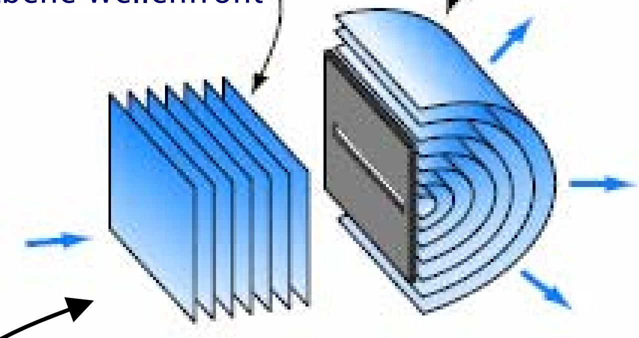
(b) Reception mode



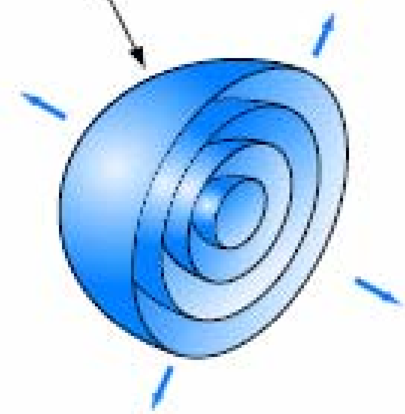
EM Fields – EM Waves – Elementary EM Waves / EM Felder – EM–Wellen – Elementare EM–Wellen



(a) Circular Wave / Zylinderwelle



(b) Plane and Cylindrical Waves / Ebene und zylindrische Wellen



(c) Spherical Wave / Kugelwelle

$$\overbrace{\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}_0(\underline{\mathbf{k}},\omega)\,\mathrm{e}^{\mathrm{j}\underline{\mathbf{k}}\cdot\underline{\mathbf{R}}}} \\ = \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}k\hat{\underline{\mathbf{k}}}\cdot\underline{\mathbf{R}}}$$

Plane Wave in the Frequency Domain Propagating in <u>k</u> Direction / Ebene Welle im Frequenzbereich, die sich in <u>k</u> Richtung ausbreitet

EM Fields – Maxwell’s Equations – Vector Wave Equation / EM Felder – Maxwellsche Gleichungen – Vektorielle Wellengleichung

Maxwell’s equations / Maxwellsche Gleichungen

$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Continuity equations / Kontinuitätsgleichungen

$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Constitutive Equations for Vacuum / Konstituierende Gleichungen (Materialgleichungen) für Vakuum

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \mu_0\,\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \varepsilon_0\,\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\mu_0}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{\varepsilon_0}\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM–Wellen – Vektorielle Wellengleichung

$$\frac{\partial}{\partial t}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\mu_0}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \quad (1)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{\varepsilon_0}\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \quad (2)$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\mu_0}\nabla\times\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \quad (3)$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{\varepsilon_0}\nabla\times\frac{\partial}{\partial t}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \quad (4)$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\mu_0}\nabla\times\left[\frac{1}{\varepsilon_0}\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\right] - \frac{1}{\mu_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \quad (5)$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{\varepsilon_0}\nabla\times\left[-\frac{1}{\mu_0}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\right] - \frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \quad (6)$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\varepsilon_0\mu_0}\nabla\times\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) + \frac{1}{\varepsilon_0\mu_0}\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \quad (7)$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{1}{\varepsilon_0\mu_0}\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0\mu_0}\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \quad (8)$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM–Wellen – Vektorielle Wellengleichung

$$\frac{1}{\varepsilon_0\mu_0}\nabla\times\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)+\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=+\frac{1}{\varepsilon_0\mu_0}\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)-\frac{1}{\mu_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \qquad (1)$$

$$\frac{1}{\varepsilon_0\mu_0}\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)+\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=-\frac{1}{\varepsilon_0\mu_0}\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)-\frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \qquad (2)$$

Velocity of Light in Vacuum / Lichtgeschwindigkeit in Vakuum

$$c_0=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$$-\nabla\times\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \qquad (3)$$

$$-\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \qquad (4)$$

Vector identity / Vektoridentität

$$\nabla\times\nabla\times=\nabla\nabla\bullet-\underbrace{\nabla\bullet\nabla}_{=\nabla^2=\Delta}=\nabla\nabla\bullet-\Delta$$

Short–hand notation / Abkürzende Schreibweise

$$\nabla\bullet\nabla=\nabla^2=\Delta$$

$$-\left[\nabla\nabla\bullet-\Delta\right]\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \qquad (5)$$

$$-\left[\nabla\nabla\bullet-\Delta\right]\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \qquad (6)$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM–Wellen – Vektorielle Wellengleichung

$$-\left[\nabla\nabla\bullet-\Delta\right]\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$-\left[\nabla\nabla\bullet-\Delta\right]\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

3rd and 4th Maxwell's equations / 3. und 4. Maxwellsche Gleichung

$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},t)=\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$
$$\nabla\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)=\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

+

Constitutive equations / Materialgleichungen

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},t)=\mu_0\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$
$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t)=\varepsilon_0\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

⇒

$$\nabla\bullet\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=\frac{1}{\mu_0}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$
$$\nabla\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\frac{1}{\varepsilon_0}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\nabla\underbrace{\nabla\bullet\underline{\mathbf{H}}(\underline{\mathbf{R}},t)}_{=\frac{1}{\mu_0}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)}-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\nabla\underbrace{\nabla\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)}_{=\frac{1}{\varepsilon_0}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)}-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM–Wellen – Vektorielle Wellengleichung

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\nabla\left[\frac{1}{\mu_0}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\right]-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\nabla\left[\frac{1}{\varepsilon_0}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\right]-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\frac{1}{\mu_0}\nabla\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\frac{1}{\varepsilon_0}\nabla\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Laplace operator in Cartesian coordinates /
Laplace–Operator in Kartesischen Koordinaten

$$\begin{aligned}\Delta&=\nabla\bullet\nabla\\&=\left(\underline{\mathrm{e}}_x\frac{\partial}{\partial x}+\underline{\mathrm{e}}_y\frac{\partial}{\partial y}+\underline{\mathrm{e}}_z\frac{\partial}{\partial z}\right)\bullet\left(\underline{\mathrm{e}}_x\frac{\partial}{\partial x}+\underline{\mathrm{e}}_y\frac{\partial}{\partial y}+\underline{\mathrm{e}}_z\frac{\partial}{\partial z}\right)\\&=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\end{aligned}$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM–Wellen – Vektorielle Wellengleichung

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \bullet \left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

$$= \left(\underline{\mathrm{e}}_x \frac{\partial}{\partial x} + \underline{\mathrm{e}}_y \frac{\partial}{\partial y} + \underline{\mathrm{e}}_z \frac{\partial}{\partial z}\right) \bullet \left[\left(\underline{\mathrm{e}}_x \frac{\partial}{\partial x} + \underline{\mathrm{e}}_y \frac{\partial}{\partial y} + \underline{\mathrm{e}}_z \frac{\partial}{\partial z}\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

Short–hand notation / Abkürzende Schreibweise $\quad \frac{\partial}{\partial x} = \partial_x \quad \frac{\partial}{\partial y} = \partial_y \quad \frac{\partial}{\partial z} = \partial_z \quad \frac{\partial}{\partial t} = \partial_t$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \bullet \left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

$$= \left(\underline{\mathrm{e}}_x\partial_x + \underline{\mathrm{e}}_y\partial_y + \underline{\mathrm{e}}_z\partial_z\right) \bullet \left[\left(\underline{\mathrm{e}}_x\partial_x + \underline{\mathrm{e}}_y\partial_y + \underline{\mathrm{e}}_z\partial_z\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

$$\left(\underline{\mathrm{e}}_x\partial_x + \underline{\mathrm{e}}_y\partial_y + \underline{\mathrm{e}}_z\partial_z\right)\left[E_x(\underline{\mathbf{R}},t)\underline{\mathrm{e}}_x + E_y(\underline{\mathbf{R}},t)\underline{\mathrm{e}}_y + E_z(\underline{\mathbf{R}},t)\underline{\mathrm{e}}_z\right]$$

$$\begin{aligned} = \; & \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_x E_z(\underline{\mathbf{R}},t) \\ & + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_y E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t) \\ & + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_x E_z(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_z E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t) \end{aligned}$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM–Wellen – Vektorielle Wellengleichung

$$
\begin{aligned}
\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \nabla\bullet\left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\\
&= \left(\underline{\mathrm{e}}_x\partial_x+\underline{\mathrm{e}}_y\partial_y+\underline{\mathrm{e}}_z\partial_z\right)\bullet\left[\left(\underline{\mathrm{e}}_x\partial_x+\underline{\mathrm{e}}_y\partial_y+\underline{\mathrm{e}}_z\partial_z\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\\
&= \left(\underline{\mathrm{e}}_x\partial_x+\underline{\mathrm{e}}_y\partial_y+\underline{\mathrm{e}}_z\partial_z\right)\bullet\Big[\ \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_x E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_x E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_x E_z(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_y E_z(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\Big]
\end{aligned}
$$

$$
\begin{aligned}
\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \nabla\bullet\left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\\
&= \underline{\mathrm{e}}_x\partial_x\bullet\Big[\ \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_x E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_y E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_z E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_z E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\Big]\\
&\quad +\underline{\mathrm{e}}_y\partial_y\bullet\Big[\ \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_y E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_z E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_z E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\Big]\\
&\quad +\underline{\mathrm{e}}_z\partial_z\bullet\Big[\ \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_x E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_y E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_z E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_z E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\Big]
\end{aligned}
$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM–Wellen – Vektorielle Wellengleichung

$$
\begin{aligned}
\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \nabla \bullet \left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right] \\
&= \Big[\ \underline{\mathrm{e}}_x \partial_x^2 E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y \partial_x^2 E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z \partial_x^2 E_z(\underline{\mathbf{R}},t) \\
&\quad + \Big[\ \underline{\mathrm{e}}_x \partial_y^2 E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y \partial_y^2 E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z \partial_y^2 E_z(\underline{\mathbf{R}},t) \\
&\quad + \Big[\ \underline{\mathrm{e}}_x \partial_z^2 E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y \partial_z^2 E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z \partial_z^2 E_z(\underline{\mathbf{R}},t) \\
&= \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right) \underbrace{\left[E_x(\underline{\mathbf{R}},t)\underline{\mathrm{e}}_x + E_y(\underline{\mathbf{R}},t)\underline{\mathrm{e}}_y + E_z(\underline{\mathbf{R}},t)\underline{\mathrm{e}}_z\right]}_{=\underline{\mathbf{E}}(\underline{\mathbf{R}},t)} \\
&= \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)
\end{aligned}
$$

$$
\begin{aligned}
\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t) \\
&= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)
\end{aligned}
$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM–Wellen – Vektorielle Wellengleichung

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\frac{1}{\mu_0}\nabla\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\frac{1}{\varepsilon_0}\nabla\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\left(\partial_x^2+\partial_y^2+\partial_z^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=\left(\partial_x^2+\partial_y^2+\partial_z^2\right)\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\frac{1}{\mu_0}\nabla\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\frac{1}{\varepsilon_0}\nabla\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)=-\frac{\partial}{\partial t}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)=-\frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

EM Fields – EM Waves – Homogeneous Vector Wave Equation / EM Felder – EM-Wellen – Homogene vektorielle Wellengleichung

Source-free Case / Quellenfreier Fall

$$\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

$$\rho_{\mathrm{e}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{m}}(\underline{\mathbf{R}},t) = 0$$

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \underbrace{-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) + \varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) + \frac{1}{\mu_0}\nabla\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)}_{=\underline{\mathbf{0}}}$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underbrace{\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) + \mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) + \frac{1}{\varepsilon_0}\nabla\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)}_{=\underline{\mathbf{0}}}$$

Homogeneous Vector Wave Equations / Homogene vektorielle Wellengleichungen

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

D'Alembert Operator / D'Alembert-Operator

$$\Box = \Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}$$

$$\Box\,\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

$$\Box\,\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

EM Fields – Homogeneous Vector Wave Equation – Fourier Transform / EM–Felder – Homogene vektorielle Wellengleichung – Fourier–Transformation

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

Fourier Transform with Regard to Time / Fourier–Transformation bzgl. der Zeit

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \int_{t=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\, \mathrm{e}^{+\mathrm{j}\omega t}\, \mathrm{d}t = \mathcal{FT}_{t\to\omega}\left\{\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right\} \qquad \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \overset{\omega\leftarrow t}{\bullet\!\!-\!\!\circ} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\, \mathrm{e}^{-\mathrm{j}\omega t}\, \mathrm{d}\omega = \mathcal{FT}^{-1}_{\omega\to t}\left\{\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\right\} \qquad \underline{\mathbf{E}}(\underline{\mathbf{R}},t) \overset{t\leftarrow \omega}{\circ\!\!-\!\!\bullet} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

Inverse Fourier Transform with Regard to Circular Frequency / Inverse Fourier–Transformation bzgl. der Kreisfrequenz

$$\frac{\partial}{\partial t} \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} -\mathrm{j}\omega$$

$$\frac{\partial^2}{\partial t^2} \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} -\omega^2$$

$$\left(\frac{\partial}{\partial t}\right)^n \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} (-\mathrm{j}\omega)^n$$

EM Fields – Fourier Transform / EM–Felder – Fourier–Transformation

Direct and Inverse Fourier Transform with Regard to Time and Circular Frequency / Direkte und Inverse Fourier–Transformation bzgl. der Zeit und Kreisfrequenz

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \int_{t=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\, e^{j\omega t}\, dt = \mathcal{FT}_{t\to\omega}\{\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\} \qquad \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \overset{\omega\leftarrow t}{\bullet\!\!-\!\!\circ} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\, e^{-j\omega t}\, d\omega = \mathcal{FT}^{-1}_{\omega\to t}\{\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\} \qquad \underline{\mathbf{E}}(\underline{\mathbf{R}},t) \overset{t\leftarrow\omega}{\circ\!\!-\!\!\bullet} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

$$\frac{\partial}{\partial t} \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} -j\omega$$

Derive the Following Identity / Leite die folgende Identität ab

$$\frac{\partial}{\partial t} \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} -j\omega$$

$$\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{\partial}{\partial t}\left[\frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\, e^{-j\omega t}\, d\omega\right]$$

$$= \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\left[\frac{\partial}{\partial t} e^{-j\omega t}\right] d\omega$$

$$\frac{\partial}{\partial t} e^{-j\omega t} = -j\omega\, e^{-j\omega t}$$

$$\left[\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right] = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \left[-j\omega \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\right] e^{-j\omega t}\, d\omega$$

$$\frac{\partial}{\partial t} \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} -j\omega$$

EM Fields – Homogeneous Vector Helmholtz Equation / EM–Felder – Homogene vektorielle Helmholtz–Gleichung

Vector Wave Equation / Vektorielle Wellengleichung

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=\underline{\mathbf{0}}$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\underline{\mathbf{0}}$$

$$-\omega^2 \bullet\!\!-\!\!\circ \frac{\partial^2}{\partial t^2}$$

Homogeneous Vector Helmholtz Equation / Homogene vektorielle Helmholtz–Gleichung

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)+\frac{\omega^2}{c_0^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)=\underline{\mathbf{0}}$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)+\frac{\omega^2}{c_0^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\underline{\mathbf{0}}$$

with the wave number / mit der Wellenzahl

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)+k^2\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)=\underline{\mathbf{0}}$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)+k^2\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\underline{\mathbf{0}}$$

in Operator Notation / in Operatorschreibweise

$$\left(\Delta+k^2\right)\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)=\underline{\mathbf{0}}$$

$$\left(\Delta+k^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\underline{\mathbf{0}}$$

Wave Number / Wellenzahl

$$k=\frac{\omega}{c_0}\quad\left[\frac{1}{\mathrm{m}}=\frac{1/\mathrm{s}}{\mathrm{m/s}}\right]$$

$$k=\frac{2\pi f}{c_0}=\frac{2\pi}{\lambda}$$

Wave Length / Wellenlänge

$$\lambda=\frac{c_0}{f}\quad\left[\mathrm{m}=\frac{\mathrm{m/s}}{1/\mathrm{s}}\right]$$

EM Fields – Homogeneous Vector Helmholtz Equation – Plane Wave / EM–Felder – Homogene vektorielle Helmholtz–Gleichung – Ebene Welle

Homogeneous Vector Helmholtz Equation /
Homogene vektorielle Helmholtz–Gleichung

$$\left(\Delta + k^2\right)\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

$$\left(\Delta + k^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

Elementary Solution of the Homogeneous Vector Helmholtz Equation: Plane Wave /
Elementare Lösung der homogenen vektoriellen Helmholtz–Gleichung: Ebene Welle

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{H}}_0(\hat{\underline{\mathbf{k}}},\omega) \quad e^{j\underline{\mathbf{k}}\cdot\underline{\mathbf{R}}}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underbrace{\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)}_{\text{Amplitude / Amplitude}} \quad \underbrace{e^{j\underline{\mathbf{k}}\cdot\underline{\mathbf{R}}}}_{\text{Phase Function / Phasenterm}}$$

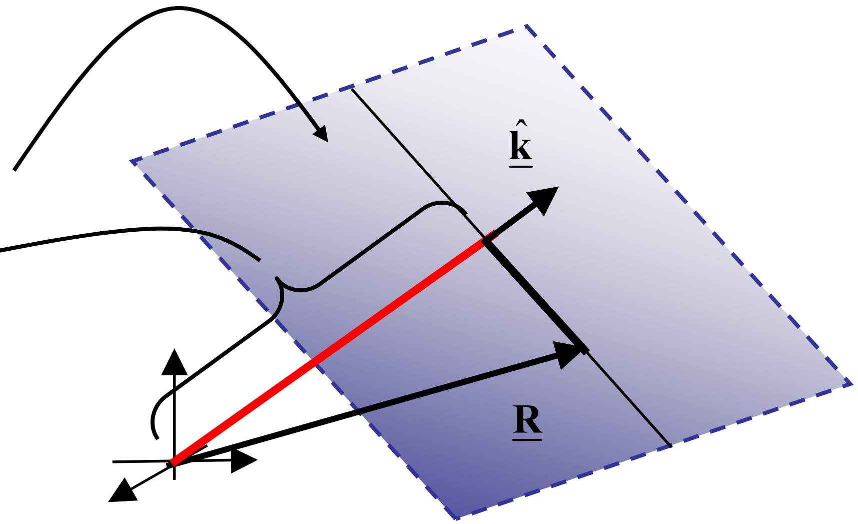
Plane Wave: Because the Phase is Constant on a Plane! /
Ebene Welle: Weil die Phase auf einer Ebene konstant ist.

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{H}}_0(\hat{\underline{\mathbf{k}}},\omega)\, e^{jk\hat{\underline{\mathbf{k}}}\cdot\underline{\mathbf{R}}}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\, e^{jk\hat{\underline{\mathbf{k}}}\cdot\underline{\mathbf{R}}}$$

Plane of constant phase / Ebene konstanter Phase

$$\hat{\underline{\mathbf{k}}}\cdot\underline{\mathbf{R}} = \text{const.}$$



EM Fields – Plane Wave / EM–Felder – Ebene Welle

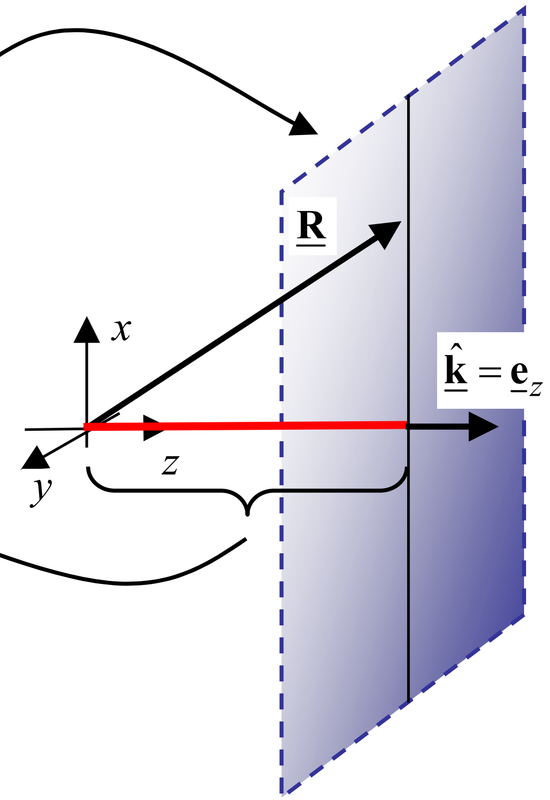
Example: Plane Wave Propagating in Positive z Direction /
Beispiel: ebene Welle, die sich in positive z Richtung ausbreitet $\underline{\mathbf{k}} = k\hat{\underline{\mathbf{k}}} = k\underline{\mathbf{e}}_z$

$$
\begin{aligned}
\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}\underline{\mathbf{k}}\bullet\underline{\mathbf{R}}} \\
&= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}k\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}} \\
&= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}k\underline{\mathbf{e}}_z\bullet\underline{\mathbf{R}}} \\
&= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}kz}
\end{aligned}
$$

Plane Wave: Because the Phase is Constant on a Plane! /
Ebene Welle: Weil die Phase auf einer Ebene konstant ist.

Plane of constant phase / Ebene konstanter Phase

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}kz} \longrightarrow \hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}} = z = \text{const.}$$

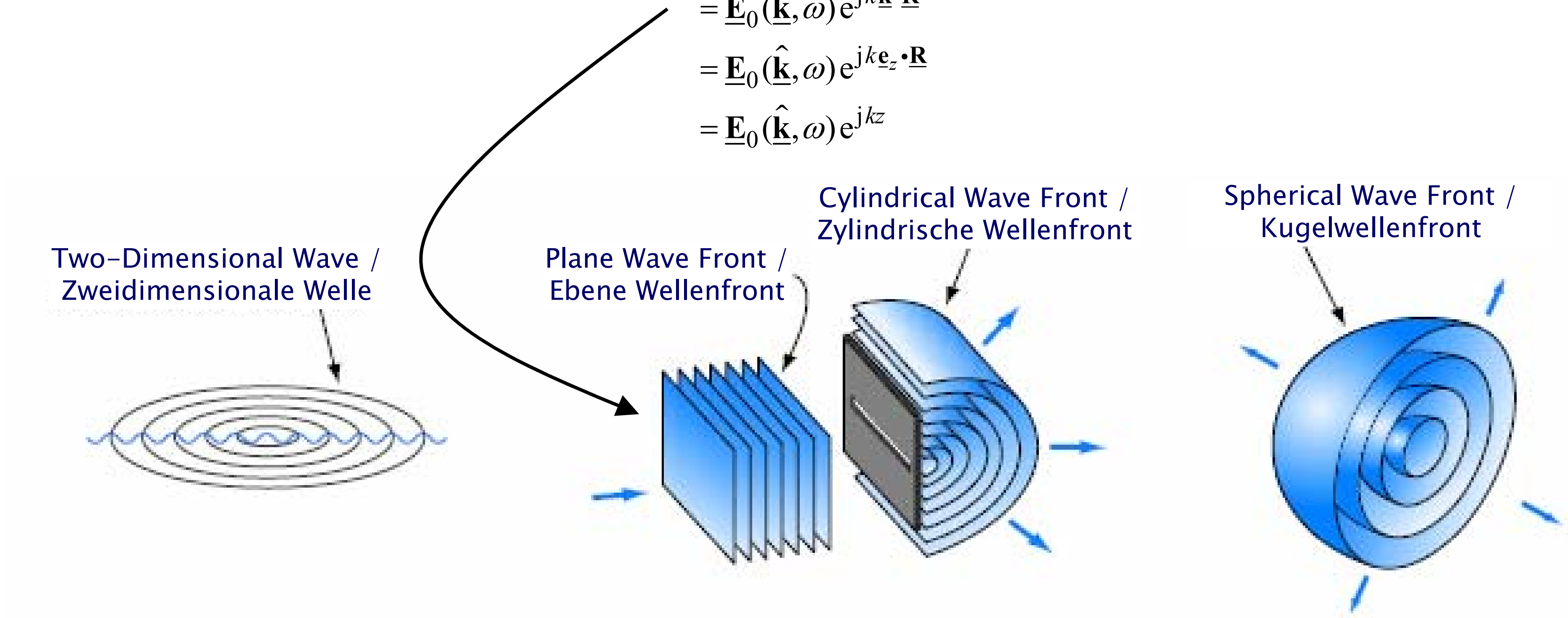


EM Fields – Plane Wave / EM–Felder – Ebene Welle

Wave Vector of a Plane Wave in the Frequency Domain Propagating in Positive z Direction /
Wellenvektor für eine ebene Welle im Frequenzbereich, die sich in positive z–Richtung ausbreitet

$$\underline{\mathbf{k}} = k\hat{\underline{\mathbf{k}}} = k\underline{\mathbf{e}}_z$$

$$\begin{aligned}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}\underline{\mathbf{k}}\bullet\underline{\mathbf{R}}} \\ &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}k\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}} \\ &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}k\underline{\mathbf{e}}_z\bullet\underline{\mathbf{R}}} \\ &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}kz}\end{aligned}$$



(a) Circular Wave / Zylinderwelle

(b) Plane and Cylindrical Waves / Ebene und zylindrische Wellen

(c) Spherical Wave / Kugelwelle

EM Fields – 3-D and 1-D Plane EM Wave – Frequency and Time Domain / EM-Felder – 3D und 1D ebene EM-Welle – Frequenz- und Zeitbereich

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{\mathrm{j}k\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}}\,\underline{\mathbf{e}}_x$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\,\mathrm{e}^{-\mathrm{j}\omega \mathrm{t}}\,\mathrm{d}\omega$$

$$= \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty} E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{\mathrm{j}k\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}}\,\underline{\mathbf{e}}_x\,\mathrm{e}^{-\mathrm{j}\omega \mathrm{t}}\,\mathrm{d}\omega$$

$$= \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty} E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{\mathrm{j}\omega\frac{\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}}{c_0}}\,\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}\omega\,\underline{\mathbf{e}}_x$$

$$= E_0\left(t - \frac{\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}}{c_0}\right)\underline{\mathbf{e}}_x$$

$$E_0\left(\hat{\underline{\mathbf{k}}},t\right) = \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty} E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{-\mathrm{j}\omega \mathrm{t}}\,\mathrm{d}\omega$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{\mathrm{j}kz}\,\underline{\mathbf{e}}_x$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\,\mathrm{e}^{-\mathrm{j}\omega \mathrm{t}}\,\mathrm{d}\omega$$

$$= \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty} E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{\mathrm{j}kz}\,\underline{\mathbf{e}}_x\,\mathrm{e}^{-\mathrm{j}\omega \mathrm{t}}\,\mathrm{d}\omega$$

$$= \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty} E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{\mathrm{j}\omega\frac{z}{c_0}}\,\underline{\mathbf{e}}_x\,\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}\omega$$

$$= E_0\left(t - \frac{z}{c_0}\right)\underline{\mathbf{e}}_x$$

$$E_0\left(\hat{\underline{\mathbf{k}}},t\right) = \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty} E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{-\mathrm{j}\omega \mathrm{t}}\,\mathrm{d}\omega$$

EM Fields – 1-D Plane EM Wave / EM-Felder – 1D ebene EM-Welle

Plane Wave Propagating in Positive z Direction /
Ebene Welle, die sich in positive z Richtung ausbreitet

$$\underline{\mathbf{k}} = k\hat{\underline{\mathbf{k}}} = k\underline{\mathbf{e}}_z$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}_0(\underline{\mathbf{k}},\omega)\mathrm{e}^{\mathrm{j}kz}$$

Maxwell's Equations in the Time Domain for the Source-Free Case /
Maxwellsche Gleichungen im Zeitbereich für den quellenfreien Fall

$$\partial/\partial t\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$
$$\partial/\partial t\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$
$$\nabla\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = 0$$
$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = 0$$

$$\frac{\partial}{\partial t} \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} -\mathrm{j}\omega$$

Maxwell's Equations in the Frequency Domain for the Source-Free Case /
Maxwellsche Gleichungen im Frequenzbereich für den quellenfreien Fall

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \mathrm{j}\omega\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega)$$
$$\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = -\mathrm{j}\omega\underline{\mathbf{D}}(\underline{\mathbf{R}},\omega)$$
$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},\omega) = 0$$
$$\nabla\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) = 0$$

$$\begin{aligned}
\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) &= \frac{1}{\mathrm{j}\omega}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \\
&= \frac{1}{\mathrm{j}\omega}\nabla\times\left[\underline{\mathbf{E}}_0(\underline{\mathbf{k}},\omega)\mathrm{e}^{\mathrm{j}kz}\right] \\
&= \frac{1}{\mathrm{j}\omega}\left[\mathrm{e}^{\mathrm{j}kz}\underbrace{\nabla\times\underline{\mathbf{E}}_0(\underline{\mathbf{k}},\omega)}_{=\underline{\mathbf{0}}} + \left(\nabla\mathrm{e}^{\mathrm{j}kz}\right)\underline{\mathbf{E}}_0(\underline{\mathbf{k}})\right]
\end{aligned}$$

EM Fields – 1-D Plane EM Wave / EM-Felder – 1D ebene EM-Welle

Plane Wave Propagating in Positive z Direction / Ebene Welle, die sich in positive z Richtung ausbreitet

$$\underline{\mathbf{k}} = k\hat{\underline{\mathbf{k}}} = k\underline{\mathbf{e}}_z$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\mathrm{e}^{\mathrm{j}kz}$$

$$\begin{aligned}
\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) &= \frac{1}{\mathrm{j}\omega}\left[\mathrm{e}^{\mathrm{j}kz}\underbrace{\nabla\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)}_{=\underline{\mathbf{0}}} + \left(\nabla\mathrm{e}^{\mathrm{j}kz}\right)\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\right] \\
&= \frac{1}{\mathrm{j}\omega}\underbrace{\left(\nabla\mathrm{e}^{\mathrm{j}kz}\right)}\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \\
&= \frac{1}{\mathrm{j}\omega}\mathrm{j}k\underbrace{\underline{\mathbf{e}}_z}_{=\hat{\underline{\mathbf{k}}}}\mathrm{e}^{\mathrm{j}kz}\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \\
&= \frac{k}{\omega}\hat{\underline{\mathbf{k}}}\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\mathrm{e}^{\mathrm{j}kz} \\
&= \frac{k}{\omega}\hat{\underline{\mathbf{k}}}\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)
\end{aligned}$$

$$\begin{aligned}
\nabla\mathrm{e}^{\mathrm{j}kz} &= \left(\underline{\mathbf{e}}_x\frac{\partial}{\partial x} + \underline{\mathbf{e}}_y\frac{\partial}{\partial y} + \underline{\mathbf{e}}_z\frac{\partial}{\partial z}\right)\mathrm{e}^{\mathrm{j}kz} \\
&= \left(\underline{\mathbf{e}}_x\underbrace{\frac{\partial}{\partial x}\mathrm{e}^{\mathrm{j}kz}}_{=0} + \underline{\mathbf{e}}_y\underbrace{\frac{\partial}{\partial y}\mathrm{e}^{\mathrm{j}kz}}_{=0} + \underline{\mathbf{e}}_z\underbrace{\frac{\partial}{\partial z}\mathrm{e}^{\mathrm{j}kz}}_{\mathrm{j}k\mathrm{e}^{\mathrm{j}kz}}\right) \\
&= \mathrm{j}k\underbrace{\underline{\mathbf{e}}_z}_{=\hat{\underline{\mathbf{k}}}}\mathrm{e}^{\mathrm{j}kz} \\
&= \mathrm{j}k\hat{\underline{\mathbf{k}}}\mathrm{e}^{\mathrm{j}kz}
\end{aligned}$$

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) = \frac{k}{\omega}\hat{\underline{\mathbf{k}}}\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

EM Fields – 1-D Plane EM Wave / EM-Felder – 1D ebene EM-Welle

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \frac{1}{\mu_0}\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) = \frac{k}{\omega}\hat{\underline{\mathbf{k}}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

$$\begin{aligned}
\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) &= \frac{1}{\mu_0}\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) \\
&= \frac{1}{\mu_0}\frac{k}{\omega} e^{jkz}\, \underline{\mathbf{e}}_z \times \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \\
&= \frac{1}{\mu_0}\frac{\omega / c_0}{\omega} e^{jkz}\, \underline{\mathbf{e}}_z \times \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \\
&= \frac{1}{\mu_0}\frac{1}{c_0} e^{jkz}\, \underline{\mathbf{e}}_z \times \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \\
&= \frac{1}{\mu_0}\sqrt{\varepsilon_0 \mu_0}\, e^{jkz}\, \underline{\mathbf{e}}_z \times \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \\
&= \sqrt{\frac{\varepsilon_0}{\mu_0}} e^{jkz} \underbrace{\underline{\mathbf{e}}_z}_{=\hat{\underline{\mathbf{k}}}} \times \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \\
&= \underbrace{Y_0 \hat{\underline{\mathbf{k}}} \times \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)}_{=\underline{\mathbf{H}}_0(\hat{\underline{\mathbf{k}}},\omega)} e^{jkz} \\
&= \underline{\mathbf{H}}_0(\hat{\underline{\mathbf{k}}},\omega) e^{jkz}
\end{aligned}$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = Y_0 \hat{\underline{\mathbf{k}}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

Wave Impedance of Free Space (Vacuum) / Wellenimpedanz des Freiraumes (Vakuum)

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = Z_0 = 376.7\ \Omega \approx 377\ \Omega \approx 120\pi\ \Omega$$

Wave Admittance of Free Space (Vacuum) / Wellenadmittanz des Freiraumes (Vakuum)

$$\frac{1}{Y_0} = Z_0$$

EM Fields – 1-D Plane EM Wave / EM-Felder – 1D ebene EM-Welle

Homogeneous Vector Wave Equation →
Proof: The Divergence of a Plane Wave must be Zero /

Homogene vektorielle Wellengleichung →
Beweis: Die Divergenz einer ebenen Wellen muss null sein

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}},\omega) = 0$$

$$\begin{aligned}
\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}},\omega) &= \nabla \bullet \left[\varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\right] \\
&= \varepsilon_0 \nabla \bullet \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \\
&= \varepsilon_0 \nabla \bullet \left[\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \mathrm{e}^{\mathrm{j}kz}\right] \\
&= \varepsilon_0 \left[\underbrace{\nabla \bullet \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)}_{=0} \mathrm{e}^{\mathrm{j}kz} + \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \bullet \underbrace{\nabla \mathrm{e}^{\mathrm{j}kz}}_{=\mathrm{j}k\underline{\mathbf{e}}_z \mathrm{e}^{\mathrm{j}kz}}\right] \\
&= \mathrm{j}k\varepsilon_0 \mathrm{e}^{\mathrm{j}kz} \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \bullet \underbrace{\underline{\mathbf{e}}_z}_{=\hat{\underline{\mathbf{k}}}} \\
&= 0
\end{aligned}$$

$$\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \bullet \hat{\underline{\mathbf{k}}} = 0 \quad \text{for / für} \quad \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \perp \hat{\underline{\mathbf{k}}}$$

The Divergence of a Plane Wave is Zero, if the Field is Perpendicular to the Propagation Direction! /
Die Divergenz einer ebenen Wellen ist null, wenn das Feld senkrecht auf der Ausbreitungsrichtung steht!

EM Fields – 1-D Plane EM Wave – TEM Wave / EM-Felder – 1D ebene EM-Welle – TEM-Welle

Example: Linear Polarized in x Direction /
Beispiel: Linear polarisiert in x Richtung

$$\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) = \underline{\mathbf{E}}_0(\omega) = E_0(\omega)\underline{\mathbf{e}}_x$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = E_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_x$$

$$\begin{aligned}\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) &= \underline{\mathbf{H}}_0(\underline{\mathbf{k}})\mathrm{e}^{\mathrm{j}kz} \\ &= Y_0\underline{\mathbf{e}}_z \times \underline{\mathbf{E}}_0(\underline{\mathbf{k}})\mathrm{e}^{\mathrm{j}kz} \\ &= Y_0\underline{\mathbf{e}}_z \times E_0(\omega)\underline{\mathbf{e}}_x\mathrm{e}^{\mathrm{j}kz} \\ &= Y_0E_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x}_{=\underline{\mathbf{e}}_y} \\ &= \underbrace{Y_0E_0(\omega)}_{=H_0(\omega)}\underline{\mathbf{e}}_y\mathrm{e}^{\mathrm{j}kz} \\ &= H_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_y\end{aligned}$$

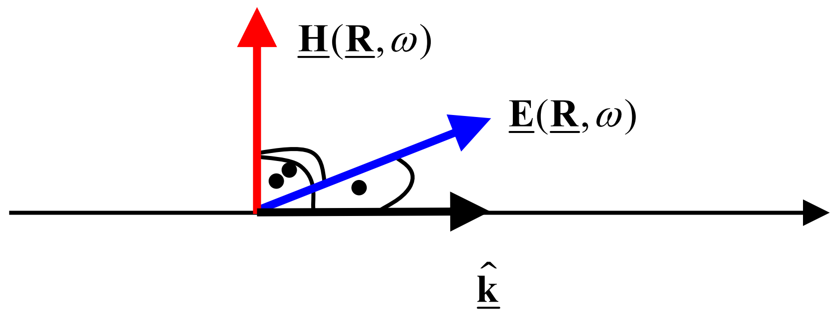
$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = H_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_y$$

TEM Wave (TEM: Transversal Electromagnetic Wave)
TEM-Welle (TEM: transversale elektromagnetische Welle

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = E_0(\omega)\,\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_x$$
$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = H_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_y$$

TEM Wave (TEM: Transversal Electromagnetic Wave)
TEM-Welle (TEM: transversale elektromagnetische Welle

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = -Z_0\hat{\underline{\mathbf{k}}} \times \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \perp \hat{\underline{\mathbf{k}}}$$
$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \frac{1}{Z_0}\hat{\underline{\mathbf{k}}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) \perp \hat{\underline{\mathbf{k}}}$$



EM Fields – Plane Wave – Energy Flow – Poynting Vector / EM–Felder – Ebene Welle – Energiefluss – Poynting–Vektor

Plane Wave Traveling in z Direction /
Ebene Welle, die sich in z Richtung ausbreitet

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = E_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_x$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = H_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_y$$

Complex Poynting Vector /
Komplexer Poynting–Vektor

$$\begin{aligned}
\underline{\mathbf{S}}_\mathrm{C}(\underline{\mathbf{R}},\omega) &= \frac{1}{2}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\times\underline{\mathbf{H}}^*(\underline{\mathbf{R}},\omega) \\
&= \frac{1}{2}E_{x0}(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_x \times H_{y0}^*(\omega)\mathrm{e}^{-\mathrm{j}kz}\underline{\mathbf{e}}_y \\
&= \frac{1}{2}E_{x0}(\omega)H_{y0}^*(\omega)\underbrace{\underline{\mathbf{e}}_x\times\underline{\mathbf{e}}_y}_{=\underline{\mathbf{e}}_z}\underbrace{\mathrm{e}^{\mathrm{j}kz}\mathrm{e}^{-\mathrm{j}kz}}_{=1} \\
&= \frac{1}{2}E_{x0}(\omega)\left[Y_0E_{x0}(\omega)\right]^*\underline{\mathbf{e}}_z \\
&= \frac{1}{2}Y_0E_{x0}(\omega)E_{x0}^*(\omega)\underline{\mathbf{e}}_z \\
&= \underbrace{\frac{1}{2}Y_0\left|E_{x0}(\omega)\right|^2}_{=S_z(\omega)}\underline{\mathbf{e}}_z \\
&= S_z(\omega)\underline{\mathbf{e}}_z
\end{aligned}$$

$$\frac{\mathrm{V}}{\mathrm{m}}\frac{\mathrm{A}}{\mathrm{m}} = \frac{\mathrm{W(att)}}{\mathrm{m}^2}$$

The Energy Propagates in Positive z Direction! /
Die Energie breitet sich in positive z Richtung aus!

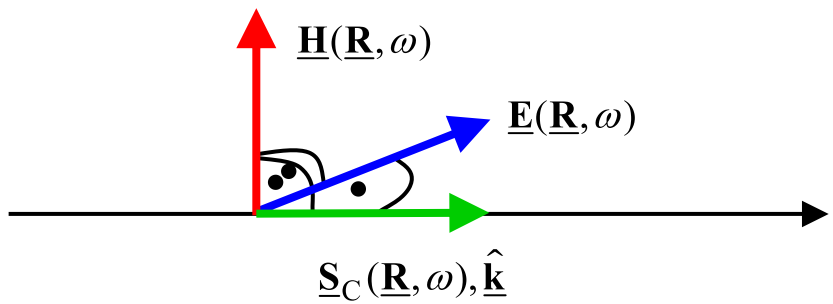
EM Fields – 1-D Plane EM Wave – TEM Wave / EM-Felder – 1D ebene EM-Welle – TEM-Welle

TEM Wave (TEM: Transversal Electromagnetic Wave)
TEM-Welle (TEM: transversal elektromagnetische Welle

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = -Z_0 \hat{\underline{\mathbf{k}}} \times \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \perp \hat{\underline{\mathbf{k}}}$$

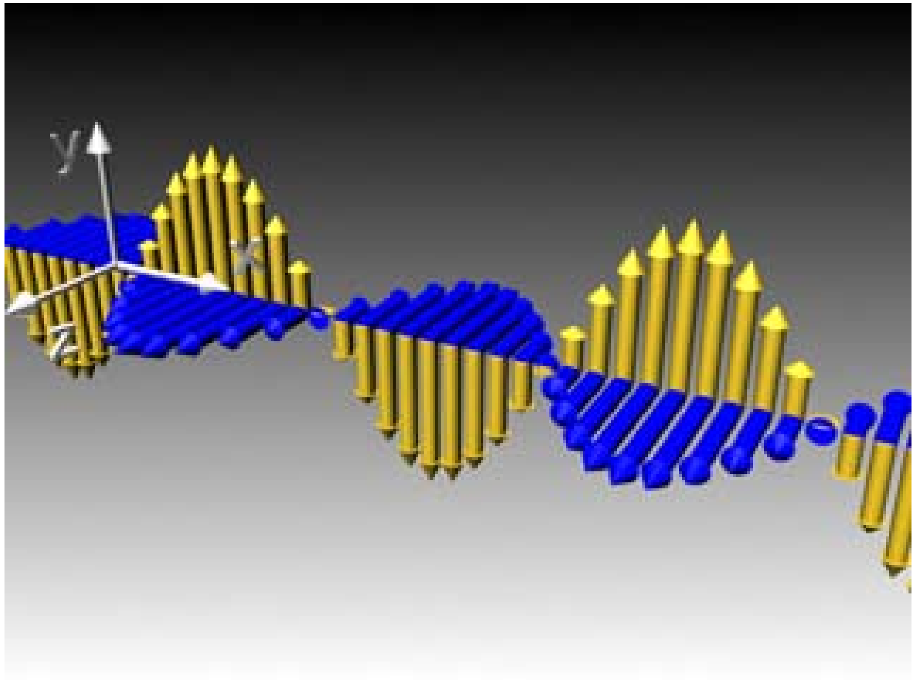
$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \frac{1}{Z_0} \hat{\underline{\mathbf{k}}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) \perp \hat{\underline{\mathbf{k}}}$$

$$\begin{aligned}\underline{\mathbf{S}}_C(\underline{\mathbf{R}},\omega) &= \frac{1}{2}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \times \underline{\mathbf{H}}^*(\underline{\mathbf{R}},\omega)\\ &= \underbrace{\frac{1}{2} Y_0 \left|E_{x0}(\omega)\right|^2}_{=S_z(\omega)} \underbrace{\underline{\mathbf{e}}_z}_{=\hat{\underline{\mathbf{k}}}}\\ &= S_z(\omega)\underline{\mathbf{e}}_z\\ &= S_z(\omega)\hat{\underline{\mathbf{k}}}\end{aligned}$$



EM Fields – 1-D Plane EM Wave – TEM Wave / EM-Felder – 1D ebene EM-Welle – TEM-Welle

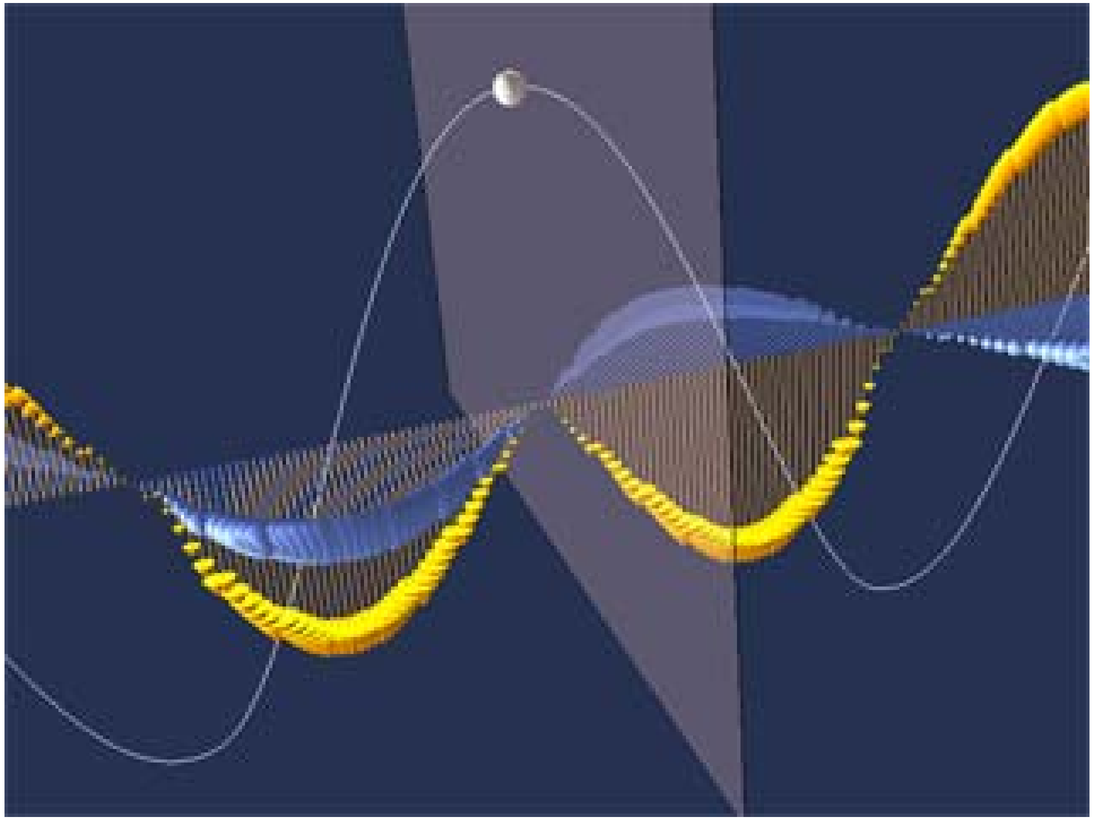
TEM Wave (TEM: Transversal Electromagnetic Wave)
TEM-Welle (TEM: transversal elektromagnetische Welle



Description: The electric and magnetic fields of an infinite plane wave. The magnetic field and the electric field are mutually perpendicular, and are both perpendicular to the direction of propagation /
Beschreibung: Das elektrische und magnetische Feld einer ebenen Welle. Das magnetische Feld und das elektrische Feld stehen senkrecht aufeinander und beide Vektorfelder stehen senkrecht auf der Ausbreitungsrichtung

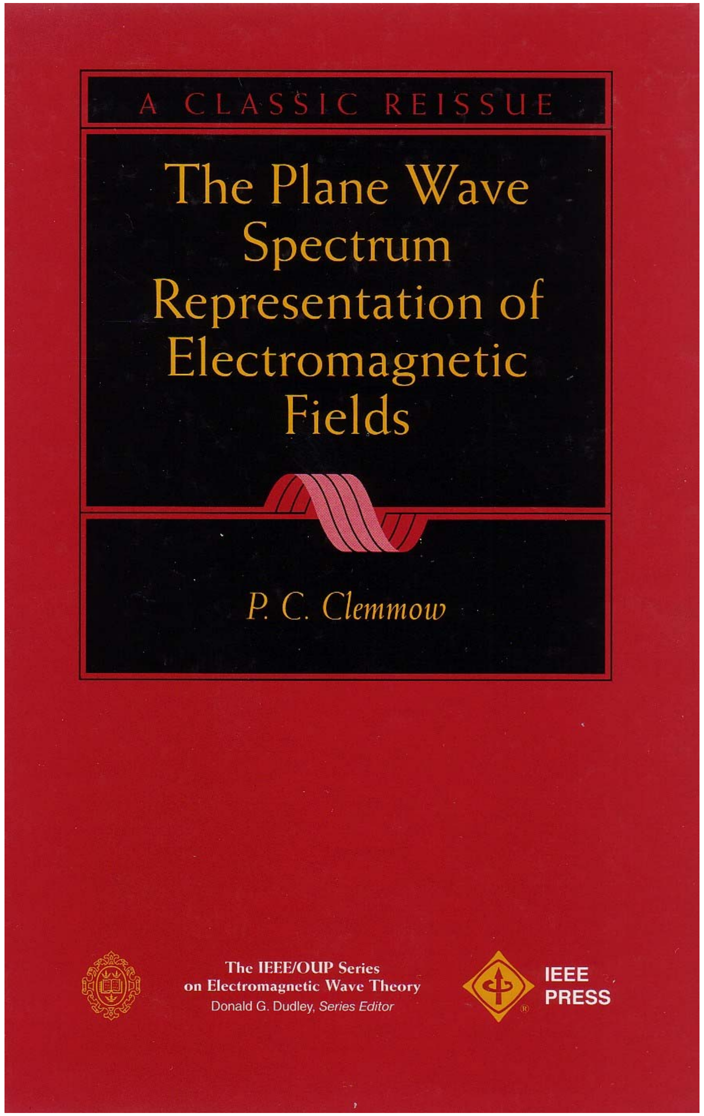
EM Fields – 1-D Plane EM Wave – TEM Wave / EM-Felder – 1D ebene EM-Welle – TEM-Welle

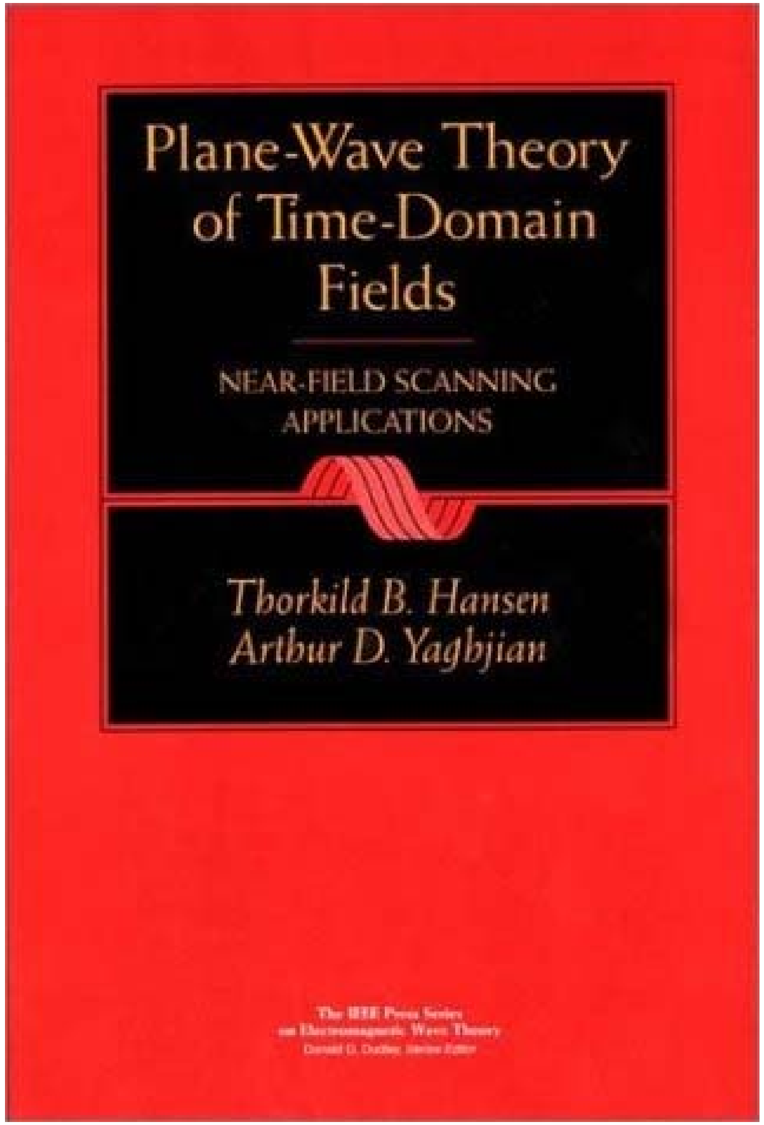
TEM Wave (TEM: Transversal Electromagnetic Wave)
TEM-Welle (TEM: transversal elektromagnetische Welle



This applet simulates the electromagnetic radiation generated by an oscillating sheet of charge. /
Dieses Apple simuliert die elektromagnetische Abstrahlung von einer oszillierenden Flächenladung

EM Fields – Plane Wave – Theory – Frequency and Time Domain / EM–Felder – Ebene Welle – Theorie – Frequenz– und Zeitbereich



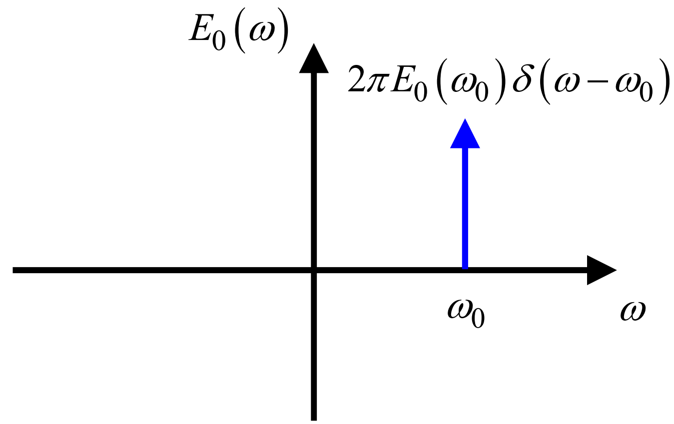


EM Fields – 1–D Plane EM Wave – Frequency and Time Domain / EM–Felder – 1D ebene EM–Welle – Frequenz– und Zeitbereich

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = E_0(\omega)\,\mathrm{e}^{\mathrm{j}kz}\,\underline{\mathbf{e}}_x$$

Assume the following Frequency Spectrum / Nehme das folgende Frequenzspektrum an

$$E_0(\omega) = 2\pi E_0(\omega_0)\,\delta(\omega-\omega_0)$$



Complex Monochromatic Plane Wave / Komplex monochromatische ebene Welle

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = E_0(\omega_0)\,\mathrm{e}^{-\mathrm{j}\omega_0\left(t-\frac{z}{c_0}\right)}\,\underline{\mathbf{e}}_x$$

$$\begin{aligned}
\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\,\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}\omega \\
&= \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} E_0(\omega)\,\mathrm{e}^{\mathrm{j}kz}\,\underline{\mathbf{e}}_x\,\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}\omega \\
&= \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} E_0(\omega)\,\mathrm{e}^{\mathrm{j}\omega\frac{z}{c_0}}\,\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}\omega\,\underline{\mathbf{e}}_x \\
&= \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} E_0(\omega)\,\mathrm{e}^{-\mathrm{j}\omega\left(t-\frac{z}{c_0}\right)}\,\mathrm{d}\omega\,\underline{\mathbf{e}}_x \\
&= \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \left\{2\pi E_0(\omega_0)\,\delta(\omega-\omega_0)\right\}\mathrm{e}^{-\mathrm{j}\omega\left(t-\frac{z}{c_0}\right)}\,\mathrm{d}\omega\,\underline{\mathbf{e}}_x \\
&= E_0(\omega_0)\,\mathrm{e}^{-\mathrm{j}\omega_0\left(t-\frac{z}{c_0}\right)}\,\underline{\mathbf{e}}_x
\end{aligned}$$

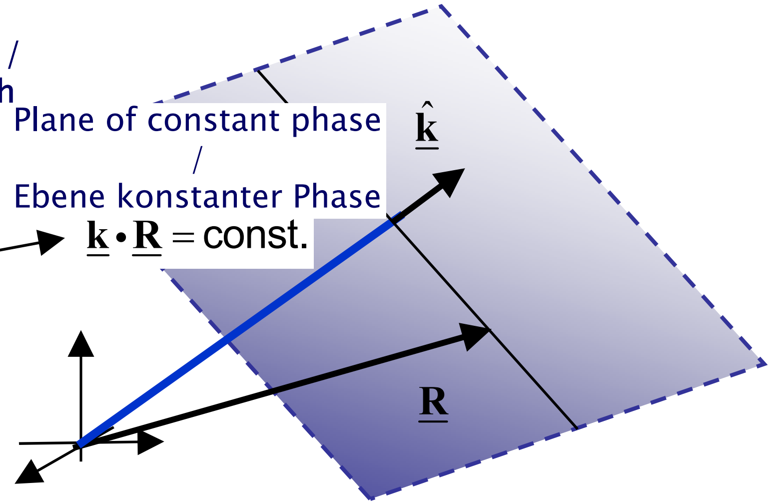
EM Fields – Complex Monochromatic Plane Wave / EM–Felder – Komplexe monochromatische Ebene Welle

Monofrequent (monochromatic) plane wave in the time domain /
Monofrequente (monochromatische) ebene Welle im Zeitbereich

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega_0)\,\mathrm{e}^{-\mathrm{j}\left(\omega_0 t - k\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}\right)}$$

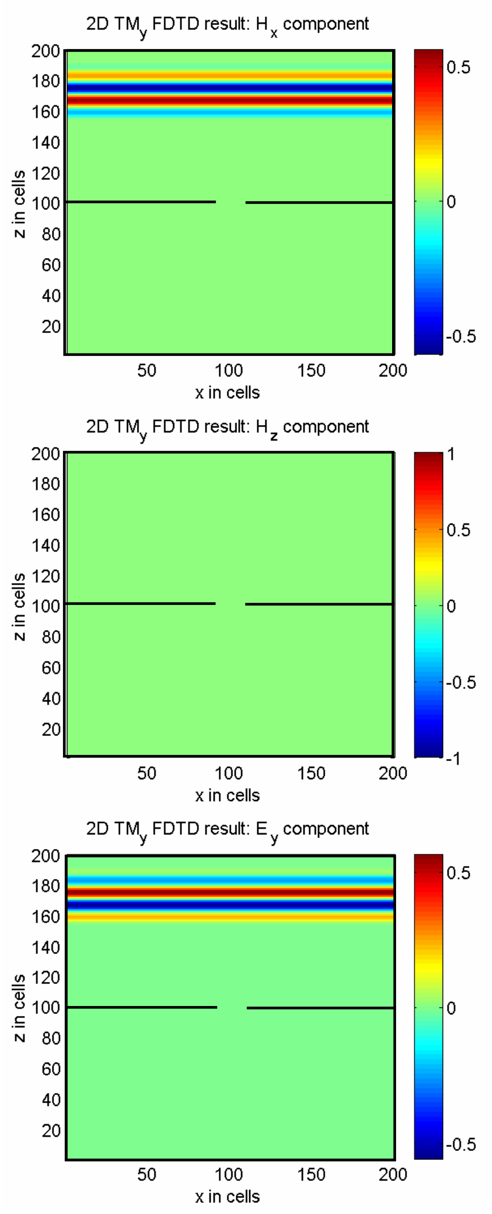
$$= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega_0)\,\mathrm{e}^{-\mathrm{j}\omega_0 t}\;\mathrm{e}^{\mathrm{j}k\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}}$$

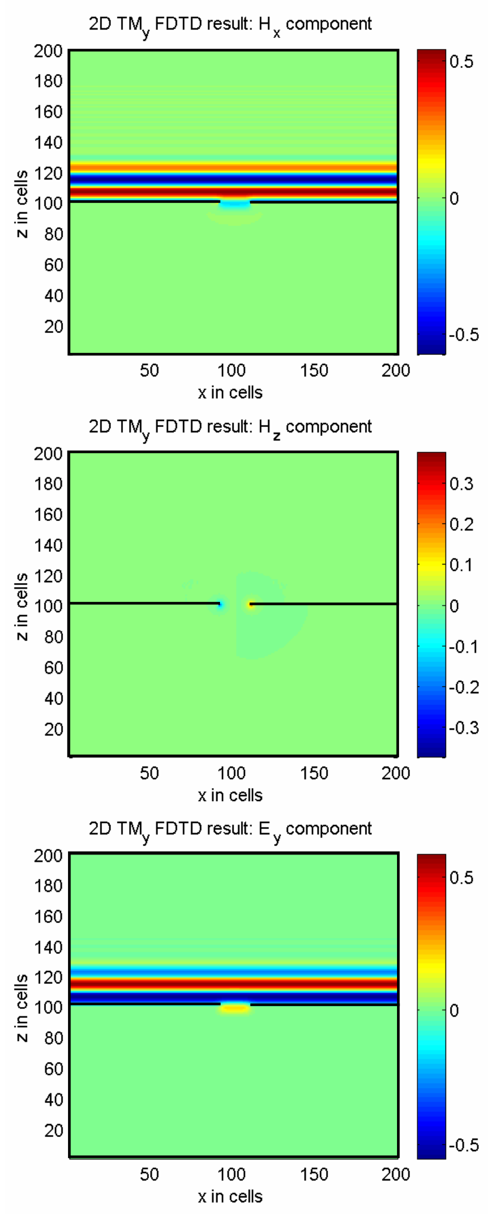
$$= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega_0)\,\mathrm{e}^{-\mathrm{j}\omega_0 t}\;\mathrm{e}^{\mathrm{j}\underline{\mathbf{k}}\bullet\underline{\mathbf{R}}}$$

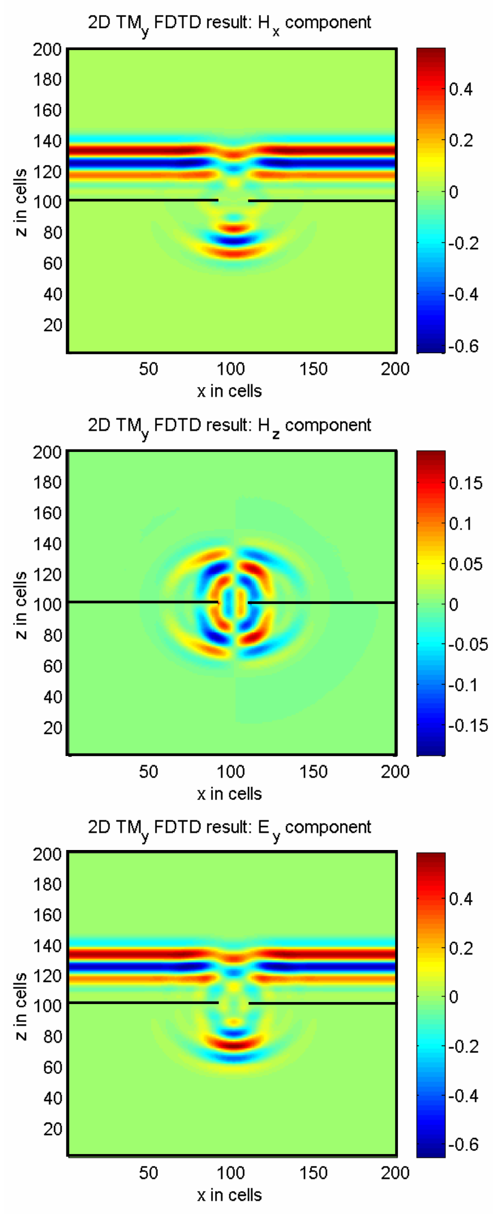


Wave vector / Wellenvektor	$\underline{\mathbf{k}} = \mathrm{k}_x\underline{\mathbf{e}}_x + \mathrm{k}_z\underline{\mathbf{e}}_z + \mathrm{k}_z\underline{\mathbf{e}}_z = \mathrm{k}_z\underline{\mathbf{e}}_z$
Magnitude of the wave vector / Betrag des Wellenvektors	$\lvert\underline{\mathbf{k}}\rvert = \sqrt{\underline{\mathbf{k}}\bullet\underline{\mathbf{k}}} = \sqrt{\mathrm{k}_x^2 + \mathrm{k}_y^2 + \mathrm{k}_z^2} = k$
Wavenumber / Wellenzahl	$k = \dfrac{\omega_0}{c}$
Circular frequency / Kreisfrequenz	$\omega_0 = 2\pi f_0$
Propagation direction / Ausbreitungsrichtung	$\hat{\underline{\mathbf{k}}} = \dfrac{\underline{\mathbf{k}}}{\lvert\underline{\mathbf{k}}\rvert}$
Phase of the plane wave / Phase der ebenen Welle	$\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}} = \mathrm{const.}$

2-D TM FDTD – Diffraction on a Single Slit /
2D-TM-FDTD – Beugung an einem Spalt







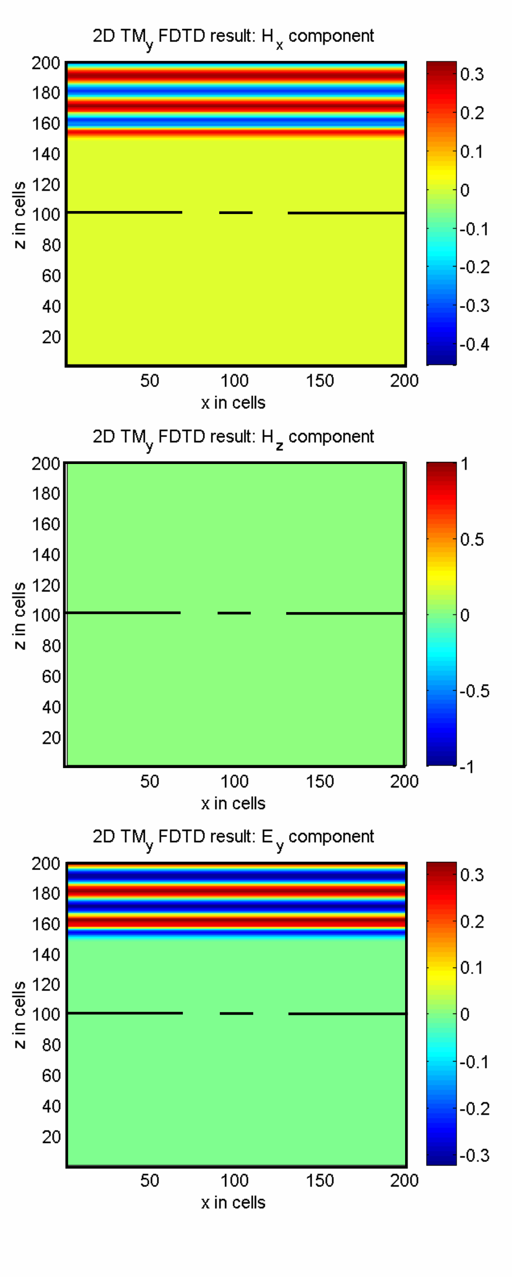
2-D TM FDTD – Diffraction on a Single Slit / 2D-TM-FDTD – Beugung am Spalt

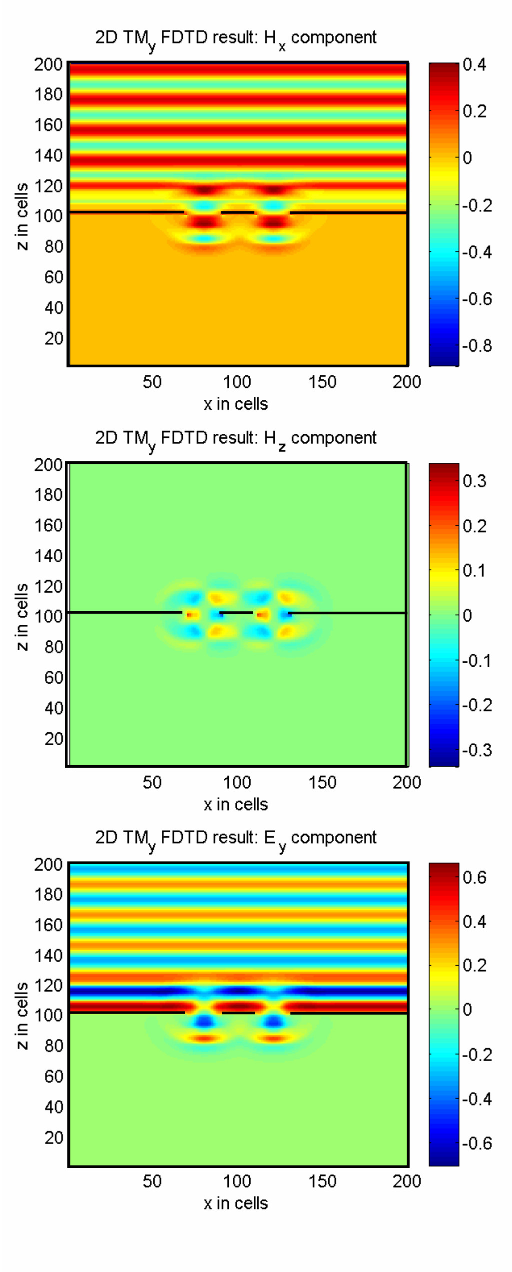
Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente

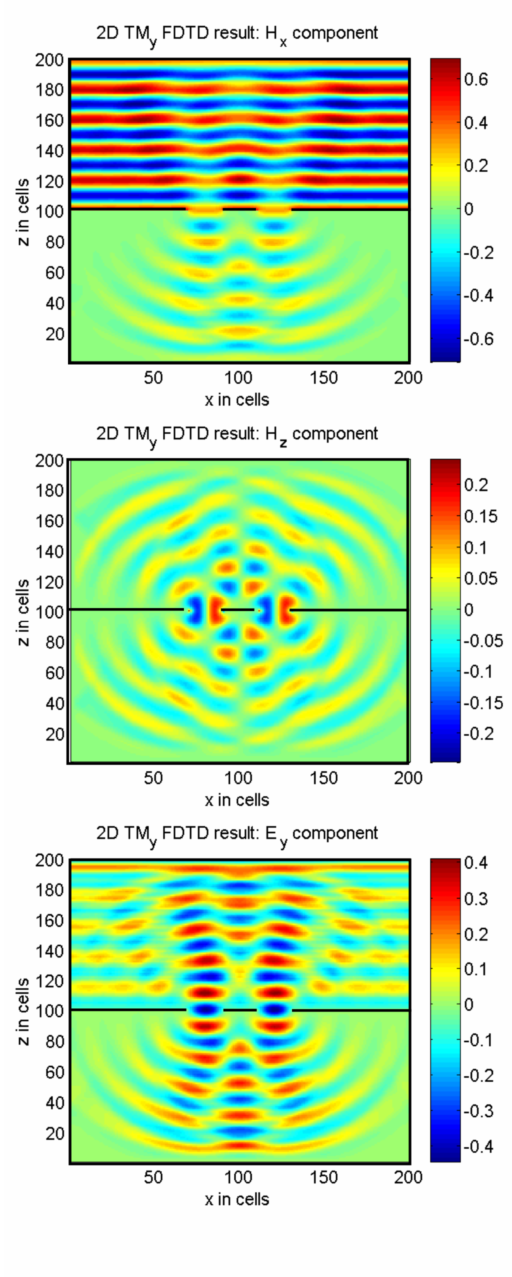
Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente

Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente

2-D TM FDTD – Diffraction on a Double Slit / 2D-TM-FDTD – Beugung am Doppelspalt







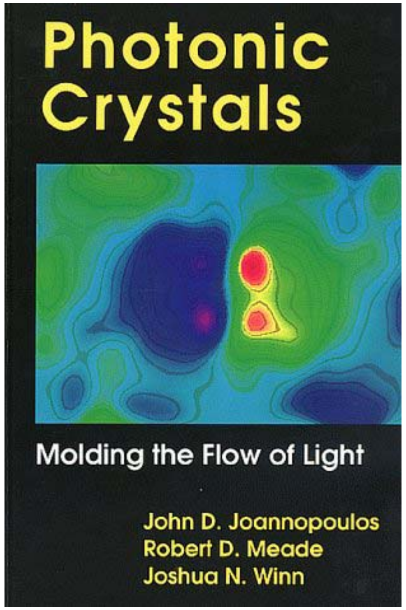
2-D TM FDTD – Diffraction on a Double Slit / 2D-TM-FDTD – Beugung am Doppelspalt

Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente

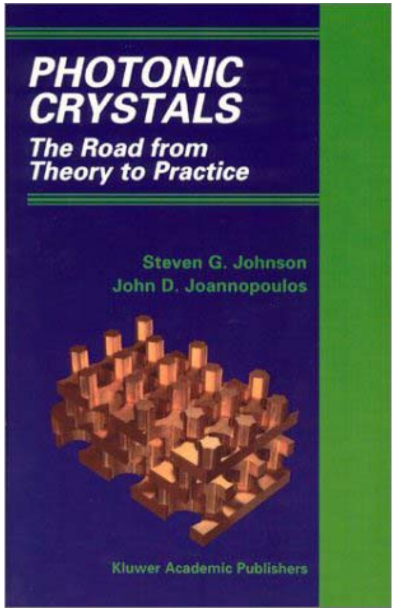
Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente

Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente

Photonic Crystals / Photonische Kristalle



Joannopoulos, J. D., R. D. Meade, J. N. Winn: *Photonic Crystals – Molding the Flow of Light.* *Princeton University Press, Princeton, 1995.*



Johnson, S. G.: *Photonic Crystals: The Road from Theory to Practice*. Kluwer Academic Press, 2001.

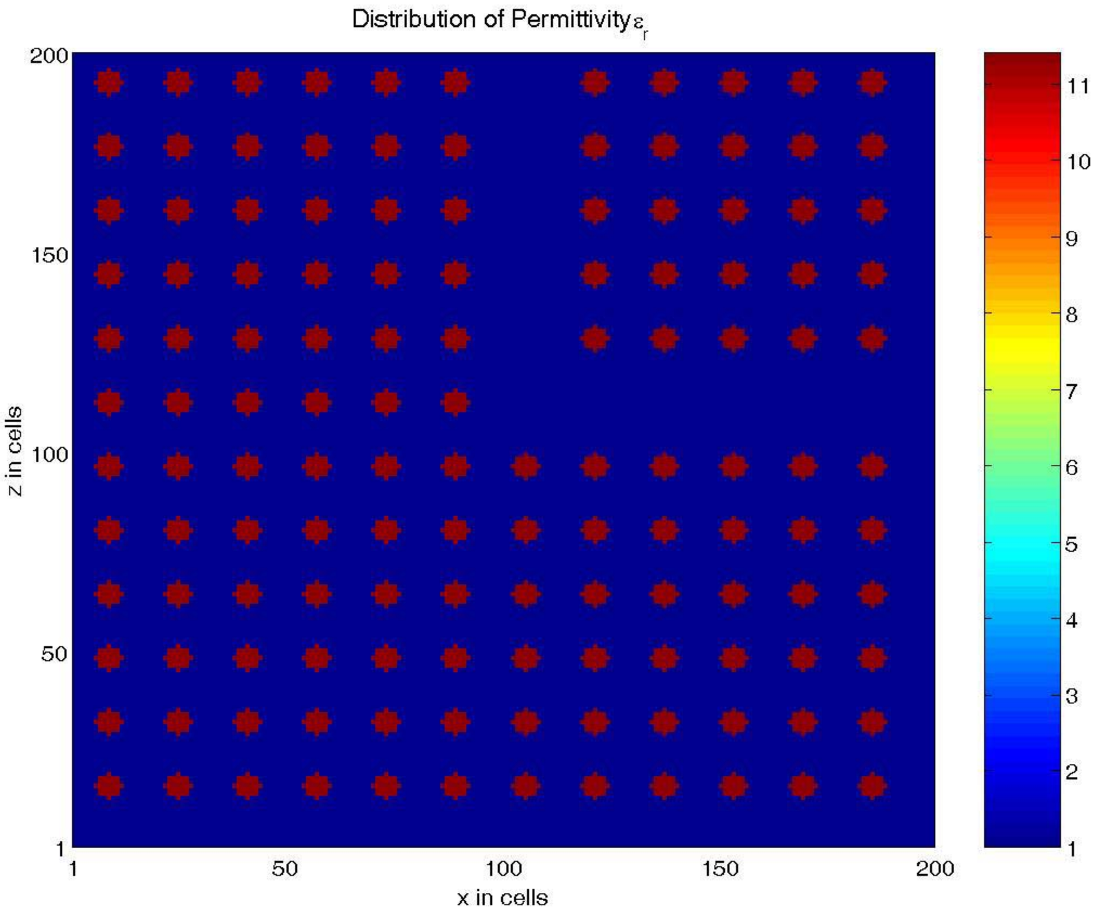
Links:

Photonic Crystals Research at MIT
Homepage of Prof. Sajeev John, University of Toronto, Canada

2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

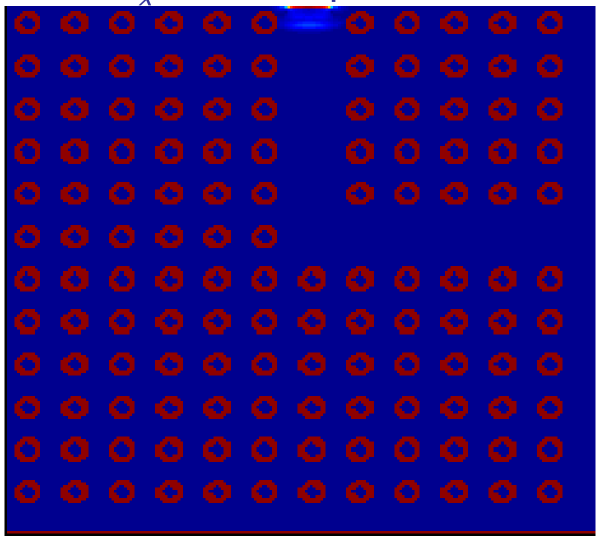
Relative permittivity of the background
Relative Permittivität des Hintergrundes $\varepsilon_{\mathrm{r}}^{(b)} = 1$

Relative permittivity of the rods
Relative Permittivität der Stäbe $\varepsilon_{\mathrm{r}}^{(r)} = 11.4$

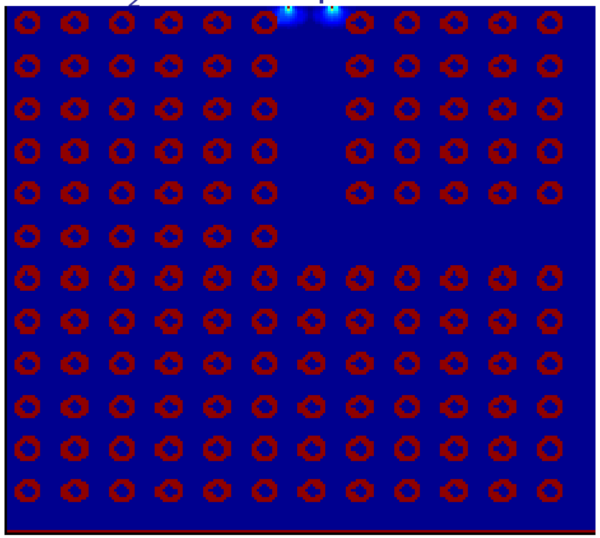


2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

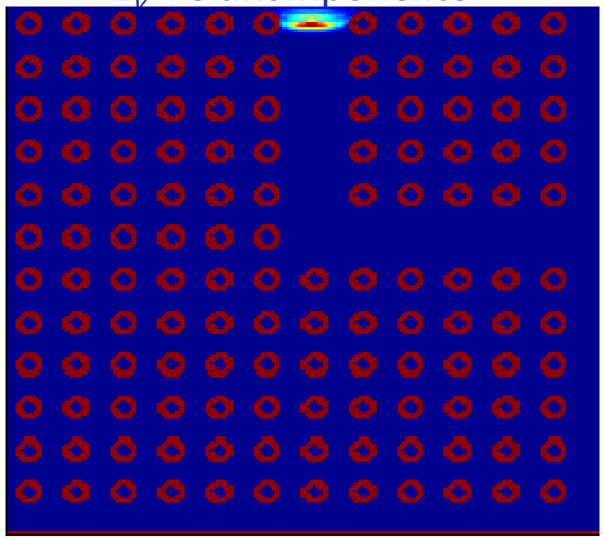
Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente



Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente



Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente



2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente

Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente

Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente

End of the 12th Lecture / Ende der 12. Vorlesung