Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

3rd Lecture / 3. Vorlesung

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Different Coordinate Systems / Verschiedene Koordinatensysteme

- Cartesian (Rectangular) Coordinate System / Kartesisches Koordinatensystem
- Cylindrical Coordinate System / Zylinderkoordinatensystem
- Spherical Coordinate System / Kugelkoordinatensystem

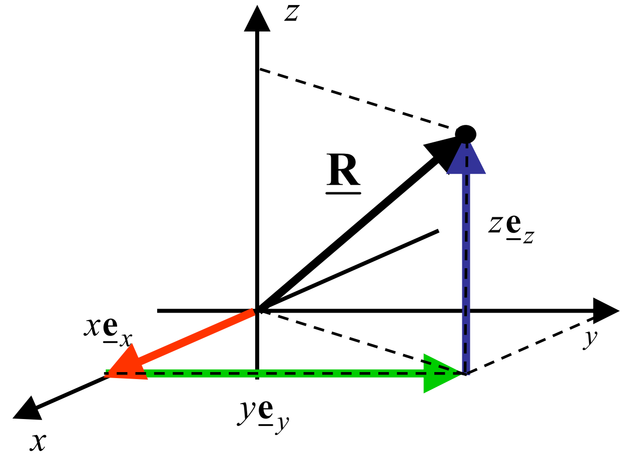
What is the benefit of the Use of a Problem Matched Coordinate Systems ? /
Was ist der Nutzen der Verwendung eines problemangepassten Koordinatensystemen ?

⇓

(Easier) Solution of the Problem under Concern! /
(Einfachere) Lösung des betrachteten Problems?

Position Vector / Ortsvektor (Positionsvektor)

Cartesian Coordinate System / Kartesisches Koordinatensystem



$$\underline{\mathbf{R}} = \underline{\mathbf{R}}_x(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_y(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_z(\underline{\mathbf{R}})$$
$$= R_x(\underline{\mathbf{R}})\underline{\mathbf{e}}_x + R_y(\underline{\mathbf{R}})\underline{\mathbf{e}}_y + R_z(\underline{\mathbf{R}})\underline{\mathbf{e}}_z$$
$$= x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$

Coordinates / Koordinaten $\quad x, y, z; \quad -\infty < x, y, z < \infty$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren $\quad \underline{\mathbf{e}}_x, \underline{\mathbf{e}}_y, \underline{\mathbf{e}}_z$

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z \qquad |\underline{\mathbf{e}}_x| = |\underline{\mathbf{e}}_y| = |\underline{\mathbf{e}}_z| = 1$$

Scalar Vector Components / Skalare Vektorkomponenten

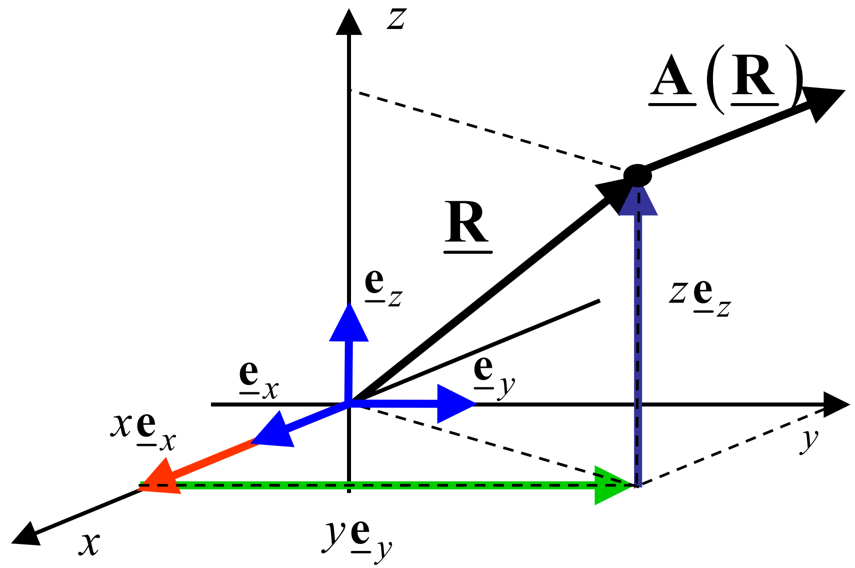
$$R_x(x, y, z) = x$$
$$R_y(x, y, z) = y$$
$$R_z(x, y, z) = z$$

Vectorial Vector Components / Vektorielle Vektorkomponenten

$$\underline{\mathbf{R}}_x(\underline{\mathbf{R}}) = R_x(x, y, z)\underline{\mathbf{e}}_x = x\underline{\mathbf{e}}_x$$
$$\underline{\mathbf{R}}_y(\underline{\mathbf{R}}) = R_y(x, y, z)\underline{\mathbf{e}}_y = y\underline{\mathbf{e}}_y$$
$$\underline{\mathbf{R}}_z(\underline{\mathbf{R}}) = R_z(x, y, z)\underline{\mathbf{e}}_z = z\underline{\mathbf{e}}_z$$

Field Vector / Feldvektor

Cartesian Coordinate System / Kartesisches Koordinatensystem



Coordinates / Koordinaten x, y, z

Limits / Grenzen

$$-\infty < x < \infty$$
$$-\infty < y < \infty$$
$$-\infty < z < \infty$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_x, \underline{\mathbf{e}}_y, \underline{\mathbf{e}}_z$$
$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z$$
$$|\underline{\mathbf{e}}_x| = |\underline{\mathbf{e}}_y| = |\underline{\mathbf{e}}_z| = 1$$

$\perp$: Perpendicular / Senkrecht

Arbitrary Vector Field / Beliebiges Vektorfeld

$$\underline{\mathbf{A}}(\underline{\mathbf{R}}) = \underline{\mathbf{A}}_x(\underline{\mathbf{R}}) + \underline{\mathbf{A}}_y(\underline{\mathbf{R}}) + \underline{\mathbf{A}}_z(\underline{\mathbf{R}})$$
$$= \mathrm{A}_x(x,y,z)\underline{\mathbf{e}}_x + \mathrm{A}_y(x,y,z)\underline{\mathbf{e}}_y + \mathrm{A}_z(x,y,z)\underline{\mathbf{e}}_z$$

Notation and Field Quantities / Notation und Feldgrößen

Vector / Vektor: Electric Field Strength / Elektrische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underbrace{\underline{\mathbf{E}}_x(\underline{\mathbf{R}},t) + \underline{\mathbf{E}}_y(\underline{\mathbf{R}},t) + \underline{\mathbf{E}}_z(\underline{\mathbf{R}},t)}_{\text{3 Vector Components / 3 Vektorkomponenten}}$$

$$= \mathrm{E}_x(x,y,z,t)\underline{\mathbf{e}}_x + \mathrm{E}_y(x,y,z,t)\underline{\mathbf{e}}_y + \mathrm{E}_z(x,y,z,t)\underline{\mathbf{e}}_z$$

$$= \sum_{i=1}^{3} \mathrm{E}_{x_i}(x_1,x_2,x_3,t)\underline{\mathbf{e}}_{x_i}$$

$$= \mathrm{E}_{x_i}(x_1,x_2,x_3,t)\underline{\mathbf{e}}_{x_i}$$

Dyad / Dyade: Permittivity Dyad / Permittivitätsdyade

$$\underline{\underline{\varepsilon}}(\underline{\mathbf{R}},t) = \underbrace{\begin{aligned} &\underline{\underline{\varepsilon}}_{xx}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{xy}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{xz}(\underline{\mathbf{R}},t) \\ +&\underline{\underline{\varepsilon}}_{yx}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{yy}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{yz}(\underline{\mathbf{R}},t) \\ +&\underline{\underline{\varepsilon}}_{zx}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{zy}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{zz}(\underline{\mathbf{R}},t) \end{aligned}}_{\text{9 Dyadic Components / 9 dyadische Komponenten}}$$

$$\begin{aligned} =\ &\varepsilon_{xx}(x,y,z,t)\underline{\mathbf{e}}_x\underline{\mathbf{e}}_x + \varepsilon_{xy}(x,y,z,t)\underline{\mathbf{e}}_x\underline{\mathbf{e}}_y + \varepsilon_{xz}(x,y,z,t)\underline{\mathbf{e}}_x\underline{\mathbf{e}}_z \\ +&\varepsilon_{yx}(x,y,z,t)\underline{\mathbf{e}}_y\underline{\mathbf{e}}_x + \varepsilon_{yy}(x,y,z,t)\underline{\mathbf{e}}_y\underline{\mathbf{e}}_y + \varepsilon_{yz}(x,y,z,t)\underline{\mathbf{e}}_y\underline{\mathbf{e}}_z \\ +&\varepsilon_{zx}(x,y,z,t)\underline{\mathbf{e}}_z\underline{\mathbf{e}}_x + \varepsilon_{zy}(x,y,z,t)\underline{\mathbf{e}}_z\underline{\mathbf{e}}_y + \varepsilon_{zz}(x,y,z,t)\underline{\mathbf{e}}_z\underline{\mathbf{e}}_z \end{aligned}$$

$$= \sum_{i=1}^{3}\sum_{j=1}^{3} \varepsilon_{x_i x_j}(x_1,x_2,x_3,t)\underline{\mathbf{e}}_{x_i}\underline{\mathbf{e}}_{x_j}$$

$$= \varepsilon_{x_i x_j}(x_1,x_2,x_3,t)\underline{\mathbf{e}}_{x_i}\underline{\mathbf{e}}_{x_j}$$

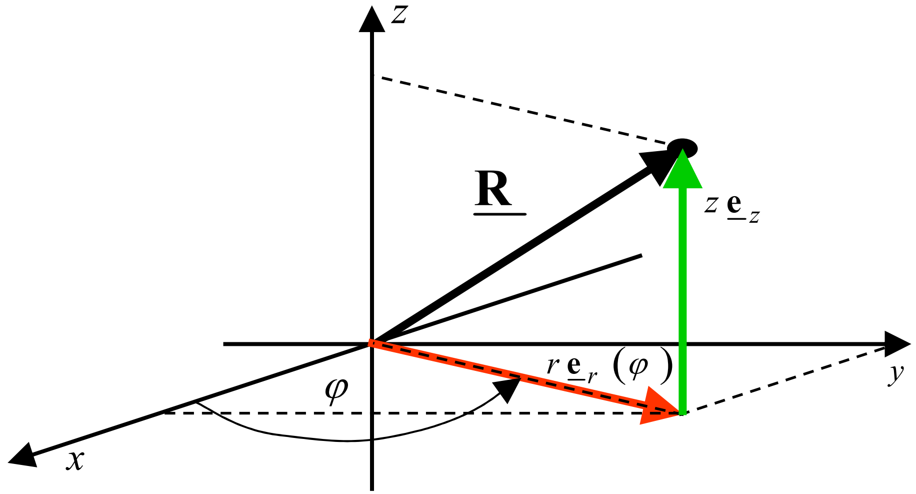
with Einstein's Summation Convention / mit Einsteinscher Summationskonvention

Einstein's Summation Convention: If a index appears two times at one side of an equation (and not at the other side), the index is automatically summed over 1 to 3. / *Einsteinsche Summenkonvention*: Wenn ein Index auf einer Seite einer Gleichung zweimal vorkommt (und auf der anderen nicht), wird darüber von 1 bis 3 summiert.

Position Vector / Ortsvektor (Positionsvektor)

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$\underline{\mathbf{R}} = \underline{\mathbf{R}}_r(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_\varphi(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_z(\underline{\mathbf{R}})$$
$$= R_r(\underline{\mathbf{R}})\underline{\mathbf{e}}_r(\varphi) + R_\varphi(\underline{\mathbf{R}})\underline{\mathbf{e}}_\varphi(\varphi) + R_z(\underline{\mathbf{R}})\underline{\mathbf{e}}_z$$
$$= r\,\underline{\mathbf{e}}_r(\varphi) + z\,\underline{\mathbf{e}}_z$$



Coordinates / Koordinaten: $r, \varphi, z; \quad 0 \le r < \infty,\ 0 \le \varphi < 2\pi,\ -\infty < z < \infty$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren:

$$\underline{\mathbf{e}}_r(\varphi), \underline{\mathbf{e}}_\varphi(\varphi), \underline{\mathbf{e}}_z$$
$$\underline{\mathbf{e}}_r(\varphi) \perp \underline{\mathbf{e}}_\varphi(\varphi) \perp \underline{\mathbf{e}}_z \qquad |\underline{\mathbf{e}}_r(\varphi)| = |\underline{\mathbf{e}}_\varphi(\varphi)| = |\underline{\mathbf{e}}_z| = 1$$

Scalar Vector Components / Skalare Vektorkomponenten:

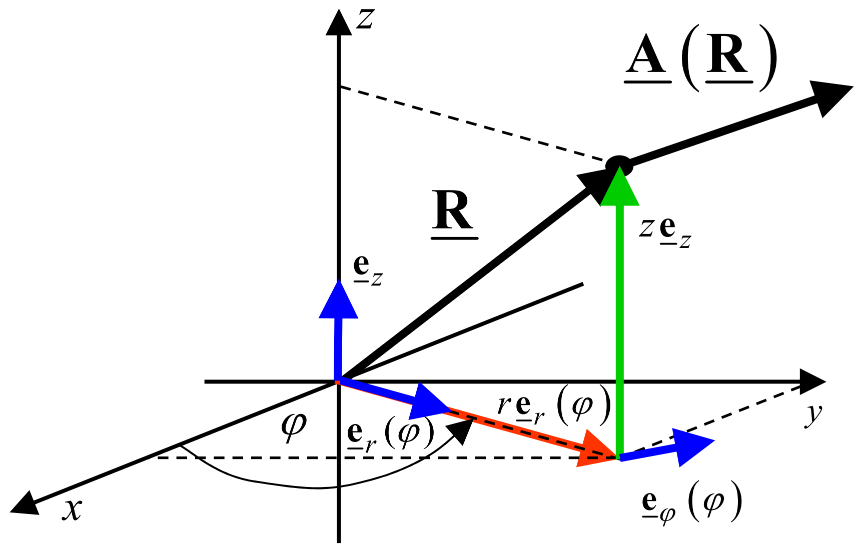
$$R_r(r,\varphi,z) = r\,\underline{\mathbf{e}}_r(\varphi)$$
$$R_\varphi(r,\varphi,z) = 0$$
$$R_z(r,\varphi,z) = z\,\underline{\mathbf{e}}_z$$

Vectorial Vector Components / Vektorielle Vektorkomponenten:

$$\underline{\mathbf{R}}_r(\underline{\mathbf{R}}) = R_r(r)\underline{\mathbf{e}}_r(\varphi) = r\,\underline{\mathbf{e}}_r(\varphi)$$
$$\underline{\mathbf{R}}_\varphi(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$
$$\underline{\mathbf{R}}_z(\underline{\mathbf{R}}) = R_z(z)\underline{\mathbf{e}}_z = z\,\underline{\mathbf{e}}_z$$

Field Vector / Feldvektor

Cylindrical Coordinate System / Zylinderkoordinatensystem



Coordinates / Koordinaten r, φ, z

Limits / Grenzen

$$0 \le r < \infty$$
$$0 \le \varphi < 2\pi$$
$$-\infty < z < \infty$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_r(\varphi), \underline{\mathbf{e}}_\varphi(\varphi), \underline{\mathbf{e}}_z$$
$$\underline{\mathbf{e}}_r(\varphi) \perp \underline{\mathbf{e}}_\varphi(\varphi) \perp \underline{\mathbf{e}}_z$$
$$|\underline{\mathbf{e}}_r(\varphi)| = |\underline{\mathbf{e}}_\varphi(\varphi)| = |\underline{\mathbf{e}}_z| = 1$$

$\perp$: Perpendicular / Senkrecht

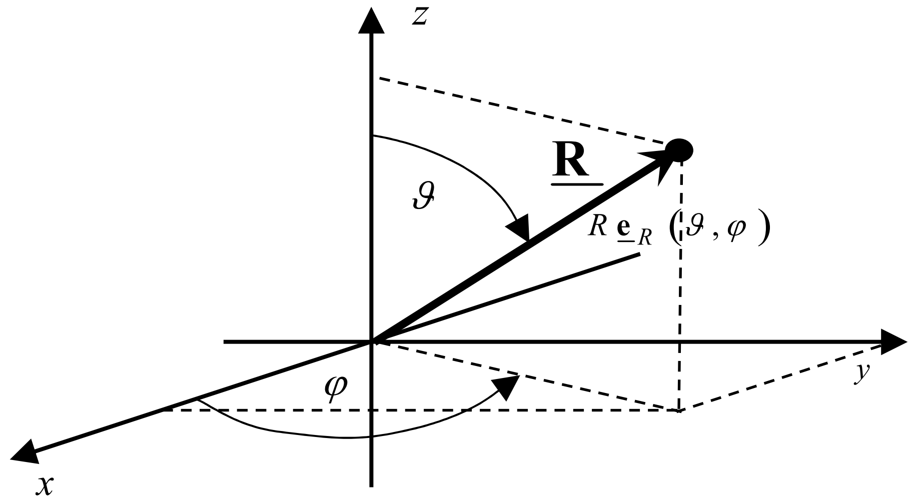
Arbitrary Vector Field / Beliebiges Vektorfeld

$$\underline{\mathbf{A}}(\underline{\mathbf{R}}) = \underline{\mathbf{A}}_r(\underline{\mathbf{R}}) + \underline{\mathbf{A}}_\varphi(\underline{\mathbf{R}}) + \underline{\mathbf{A}}_z(\underline{\mathbf{R}})$$
$$= \mathrm{A}_r(r,\varphi,z)\underline{\mathbf{e}}_r(\varphi) + A_\varphi(r,\varphi,z)\underline{\mathbf{e}}_\varphi(\varphi) + A_z(r,\varphi,z)\underline{\mathbf{e}}_z$$

Position Vector / Ortsvektor (Positionsvektor)

Spherical Coordinate System / Kugelkoordinatensystem

$$\underline{\mathbf{R}} = \underline{\mathbf{R}}_R(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_\vartheta(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_\varphi(\underline{\mathbf{R}})$$
$$= R_R(\underline{\mathbf{R}})\underline{\mathbf{e}}_R(\vartheta,\varphi) + R_\vartheta(\underline{\mathbf{R}})\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi)$$
$$+ R_\varphi(\underline{\mathbf{R}})\underline{\mathbf{e}}_\varphi(\varphi)$$
$$= R\underline{\mathbf{e}}_R(\vartheta,\varphi)$$



Coordinates / Koordinaten $R,\vartheta,\varphi; \quad 0 \le R < \infty, 0 \le \vartheta \le \pi; 0 \le \varphi < 2\pi$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren $\underline{\mathbf{e}}_R, \underline{\mathbf{e}}_\vartheta, \underline{\mathbf{e}}_\varphi$

$$\underline{\mathbf{e}}_R \perp \underline{\mathbf{e}}_\vartheta \perp \underline{\mathbf{e}}_\varphi \qquad |\underline{\mathbf{e}}_R| = |\underline{\mathbf{e}}_\vartheta| = |\underline{\mathbf{e}}_\varphi| = 1$$

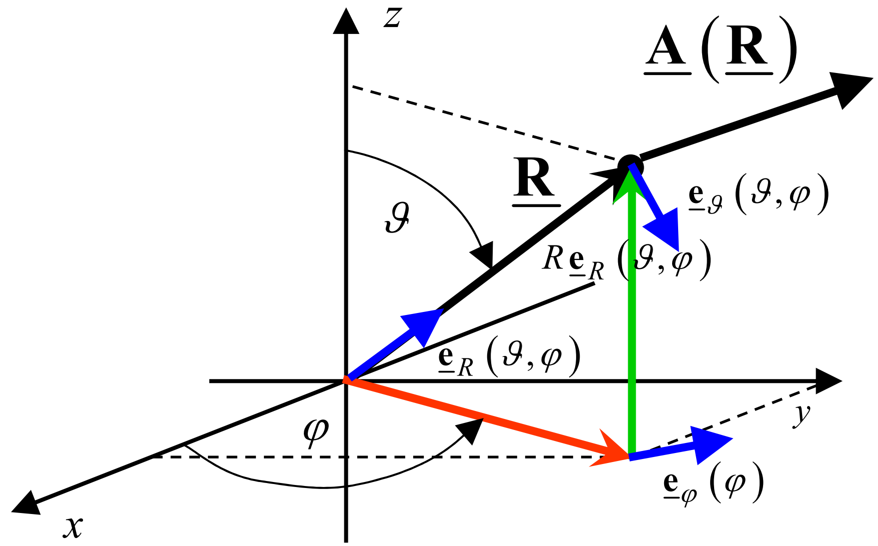
Scalar Vector Components / Skalare Vektorkomponenten $R_R(R,\vartheta,\varphi), R_\vartheta(R,\vartheta,\varphi), R_\varphi(R,\vartheta,\varphi)$

Vectorial Vector Components / Vektorielle Vektorkomponenten

$$\underline{\mathbf{R}}_R(\underline{\mathbf{R}}) = R_R(R,\vartheta,\varphi)\underline{\mathbf{e}}_R(\vartheta,\varphi) = R\underline{\mathbf{e}}_R(\vartheta,\varphi)$$
$$\underline{\mathbf{R}}_\vartheta(\underline{\mathbf{R}}) = R_\vartheta(R,\vartheta,\varphi)\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi) = \underline{\mathbf{0}}$$
$$\underline{\mathbf{R}}_\varphi(\underline{\mathbf{R}}) = R_\varphi(R,\vartheta,\varphi)\underline{\mathbf{e}}_\varphi(\varphi) = \underline{\mathbf{0}}$$

Field Vector / Feldvektor

Spherical Coordinate System / Kugelkoordinatensystem



Coordinates / Koordinaten $\quad R,\vartheta,\varphi$

Limits / Grenzen

$$0 \le R < \infty$$
$$0 \le \vartheta \le \pi$$
$$0 \le \varphi < 2\pi$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_R(\vartheta,\varphi),\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi),\underline{\mathbf{e}}_\varphi(\varphi)$$

$$\underline{\mathbf{e}}_R(\vartheta,\varphi)\perp\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi)\perp\underline{\mathbf{e}}_\varphi(\varphi)$$

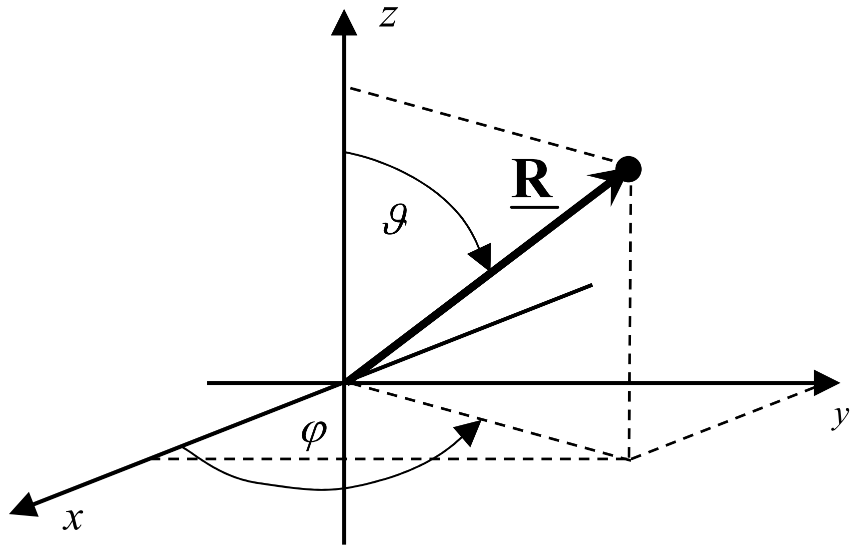
$\perp$: Perpendicular / Senkrecht

$$|\underline{\mathbf{e}}_R(\vartheta,\varphi)|=|\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi)|=|\underline{\mathbf{e}}_\varphi(\varphi)|=1$$

Arbitrary Vector Field / Beliebiges Vektorfeld

$$\underline{\mathbf{A}}(\underline{\mathbf{R}},t)=\underline{\mathbf{A}}_R(\underline{\mathbf{R}})+\underline{\mathbf{A}}_\vartheta(\underline{\mathbf{R}})+\underline{\mathbf{A}}_\varphi(\underline{\mathbf{R}})$$
$$=\mathrm{A}_R(R,\vartheta,\varphi)\underline{\mathbf{e}}_R(\vartheta,\varphi)+A_\vartheta(R,\vartheta,\varphi)\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi)+A_\varphi(R,\vartheta,\varphi)\underline{\mathbf{e}}_\varphi(\varphi)$$

Coordinates of Different Coordinate Systems / Koordinaten verschiedenen Koordinatensystemen



Transformation Table / Umrechnungstabelle

Cartesian Coordinates / Kartesische Koordinaten	Cylindrical Coordinates / Zylinderkoordinaten	Spherical Coordinates / Kugelkoordinaten
x y z	$r\cos\varphi$ $r\sin\varphi$ z	$R\sin\vartheta\cos\varphi$ $R\sin\vartheta\sin\varphi$ $R\cos\vartheta$
$\sqrt{x^2+y^2}$ $\arctan\frac{y}{x}$ z	r φ z	$R\sin\vartheta$ φ $R\cos\vartheta$
$\sqrt{x^2+y^2+z^2}$ $\arctan\frac{\sqrt{x^2+y^2}}{z}$ $\arctan\frac{y}{x}$	$\sqrt{r^2+z^2}$ $\arctan\frac{r}{z}$ φ	R ϑ φ

Examples / Beispiele

1. Formulate x as a function of the cylinder and spherical coordinates. / Formuliere x als Funktion der Zylinder- und Kugelkoordinaten.

$$x = r\cos\varphi = R\sin\vartheta\cos\varphi$$

2. Formulate r as a function of the Cartesian and spherical coordinates. / Formuliere r als Funktion der Kartesischen und Kugelkoordinaten.

$$r = \sqrt{x^2 + y^2} = R\sin\vartheta$$

3. Formulate $\sqrt{x^2 + y^2}$ as a function of the cylinder coordinates. / Formuliere $\sqrt{x^2 + y^2}$ als Funktion der Zylinderkoordinaten.

$$\sqrt{x^2 + y^2} = \sqrt{(r\cos\varphi)^2 + (r\sin\varphi)^2} = r\sqrt{\underbrace{\cos^2\varphi + \sin^2\varphi}_{=1}} = r$$

Scalar Vector Components in Different Coordinate Systems / Skalare Vektorkomponenten in verschiedenen Koordinatensystemen

Transformation Table / Umrechnungstabelle

Cartesian Coordinates / Kartesische Koordinaten	Cylindrical Coordinates / Zylinderkoordinaten	Spherical Coordinates / Kugelkoordinaten
$\underline{\mathbf{A}} = A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z$	$\underline{\mathbf{A}} = A_r \underline{\mathbf{e}}_r + A_\varphi \underline{\mathbf{e}}_\varphi + A_z \underline{\mathbf{e}}_z$	$\underline{\mathbf{A}} = A_R \underline{\mathbf{e}}_R + A_\vartheta \underline{\mathbf{e}}_\vartheta + A_\varphi \underline{\mathbf{e}}_\varphi$
A_x A_y A_z	$A_r \cos\varphi - A_\varphi \sin\varphi$ $A_r \sin\varphi + A_\varphi \cos\varphi$ A_z	$A_R \sin\vartheta\cos\varphi + A_\vartheta \cos\vartheta\cos\varphi - A_\varphi \sin\varphi$ $A_R \sin\vartheta\sin\varphi + A_\vartheta \cos\vartheta\sin\varphi + A_\varphi \cos\varphi$ $A_R \cos\vartheta - A_\vartheta \sin\vartheta$
$A_x \cos\varphi + A_y \sin\varphi$ $-A_x \sin\varphi + A_y \cos\varphi$ A_z	A_r A_φ A_z	$A_R \sin\vartheta + A_\vartheta \cos\vartheta$ A_φ $A_R \cos\vartheta - A_\vartheta \sin\vartheta$
$A_x \sin\vartheta\cos\varphi + A_y \sin\vartheta\sin\varphi + A_z \cos\vartheta$ $A_x \cos\vartheta\cos\varphi + A_y \cos\vartheta\sin\varphi - A_z \sin\vartheta$ $-A_x \sin\varphi + A_y \cos\varphi$	$A_r \sin\vartheta + A_z \cos\vartheta$ $A_r \cos\vartheta - A_z \sin\vartheta$ A_φ	A_R A_ϑ A_φ

Example: Coordinate Transformation of the Position Vector / Beispiel: Koordinatentransformation des Ortsvektor

Position Vector in the Cartesian Coordinate System / Ortsvektor im Kartesischen Koordinatensystem

$$\underline{\mathbf{R}} = \underbrace{x}_{R_x(x,y,z)} \underline{\mathbf{e}}_x + \underbrace{y}_{R_y(x,y,z)} \underline{\mathbf{e}}_y + \underbrace{z}_{R_z(x,y,z)} \underline{\mathbf{e}}_z$$

Transformation of the Coordinates / Transformation der Koordinaten

$$R_x(r,\varphi,z) = x(r,\varphi,z) = r\cos\varphi$$
$$R_y(r,\varphi,z) = y(r,\varphi,z) = r\sin\varphi$$
$$R_z(r,\varphi,z) = z(r,\varphi,z) = z$$

Position Vector in the Cylinder Coordinate System / Ortsvektor im Zylinderkoordinatensystem

?

$$\underline{\mathbf{R}}(r,\varphi,y,R_r,R_\varphi,R_z)$$
$$= R_r(r,\varphi,y)\underline{\mathbf{e}}_r(\varphi) + R_\varphi(r,\varphi,y)\underline{\mathbf{e}}_\varphi(\varphi) + R_z(r,\varphi,y)\underline{\mathbf{e}}_z$$

Position Vector in the Cartesian Coordinate System as a Function of Cylinder Coordinates / Ortsvektor im Kartesischen Koordinatensystem als Funktion der Zylinderkoordinaten

$$\underline{\mathbf{R}} = \underbrace{r\cos\varphi}_{R_x(r,\varphi,z)} \underline{\mathbf{e}}_x + \underbrace{r\sin\varphi}_{R_y(r,\varphi,z)} \underline{\mathbf{e}}_y + \underbrace{z}_{R_z(r,\varphi,z)} \underline{\mathbf{e}}_z$$

Transformation of the Scalar Vector Components / Transformation der skalaren Vektorkomponenten

$$R_r(r,\varphi,z,R_x,R_y,R_z) = R_x\cos\varphi + R_y\sin\varphi$$
$$R_\varphi(r,\varphi,z,R_x,R_y,R_z) = -R_x\sin\varphi + R_y\cos\varphi$$
$$R_z(r,\varphi,z,R_x,R_y,R_z) = R_z$$

$$R_r = r\cos\varphi\cos\varphi + r\sin\varphi\sin\varphi$$
$$= r\underbrace{(\cos^2\varphi + \sin^2\varphi)}_{=1} = r$$
$$R_\varphi = -r\cos\varphi\sin\varphi + r\sin\varphi\cos\varphi$$
$$= 0$$
$$R_z = R_z$$

Position Vector in the Cylinder Coordinate System / Ortsvektor in dem Zylinderkoordinatensystem

$$\underline{\mathbf{R}} = \underbrace{r}_{R_r} \underline{\mathbf{e}}_r(\varphi) + \underbrace{z}_{R_z} \underline{\mathbf{e}}_z$$

Faraday‘s Induction Law in Integral Form / Faradaysches Induktionsgesetz in Integralform (1)

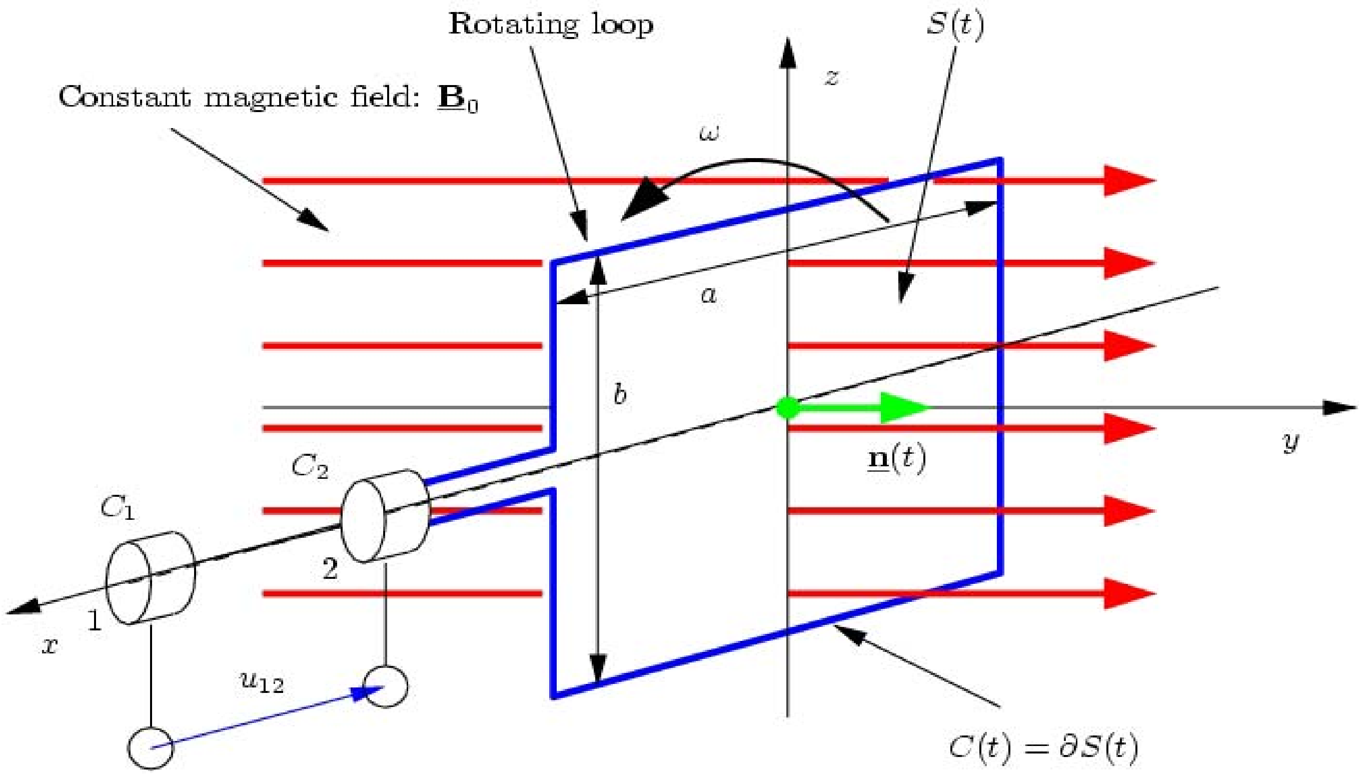
Faraday‘s Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C(t)=\partial S(t)} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{S(t)} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} - \iint_{S(t)} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}$$

Time Dependent Surface / Zeitabhängige Fläche

$$S(t) \quad C(t) = \partial S(t)$$

Time Dependent Contour / Zeitabhängige Kontur



Faraday‘s Induction Law in Integral Form / Faradaysches Induktionsgesetz in Integralform (2)

Faraday‘s Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C(t)=\partial S(t)} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{S(t)} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} - \iint_{S(t)} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}$$

$\oint_{C(t)=\partial S(t)} [\circ]\bullet\underline{\mathbf{dR}}$	[m]	Closed Contour Integral / Geschlossenes Kurvenintegral
$\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$	[V/m]	Electric Field Strength / Elektrische Feldstärke
$\underline{\mathbf{dR}}$	[m]	Vectorial Differential Line Element / Vektorielles differentielles Linienelement
$\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}}$	[V]	Scalar Product of E and dR = tangential projection of E onto dR / Skalarprodukt von E auf dR = Tangentialprojektion von E auf dR

Vectorial Differential Line Element / Vektorielles differentielles Linienelement

$$\underline{\mathbf{dR}} = \underline{\mathbf{s}}\ \mathrm{d}R$$

Tangential Unit Vector / Tangentialer Einheitsvektor

Scalar Differential Line Element / Skalares differentielles Linienelement

Different Products / Verschiedene Produkte

Scalar Product / Skalarprodukt $C = \underline{\mathbf{A}} \bullet \underline{\mathbf{B}}$

Vector Product / Vektorprodukt $\underline{\mathbf{C}} = \underline{\mathbf{A}} \times \underline{\mathbf{B}}$

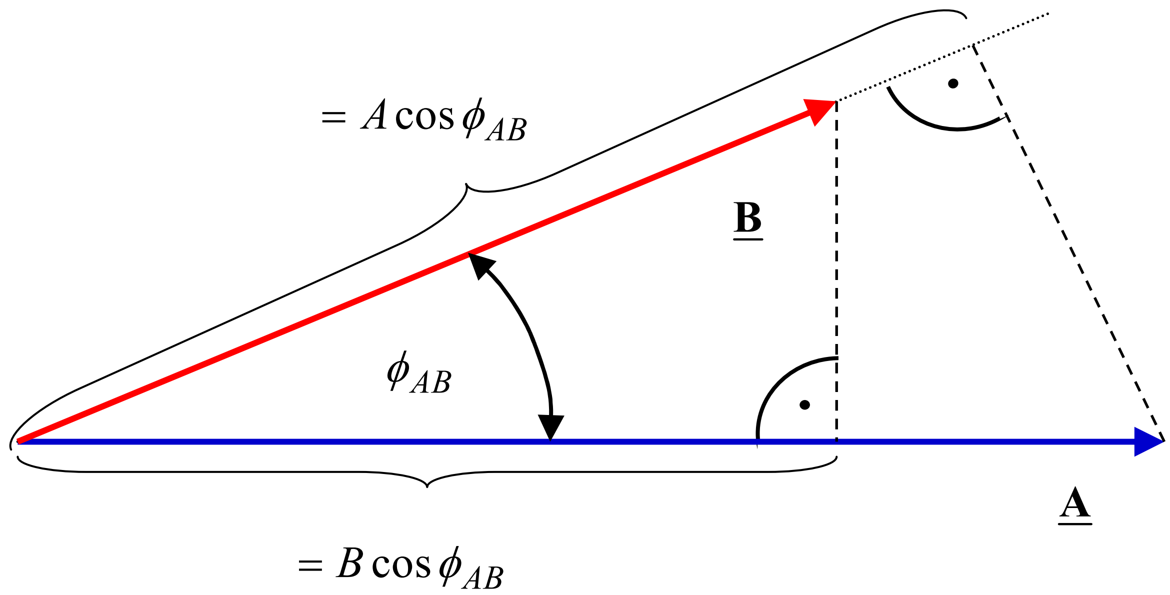
Dyadic Product / Dyadisches Produkt $\underline{\underline{\mathbf{C}}} = \underline{\mathbf{A}}\,\underline{\mathbf{B}}$

Scalar Product (Dot or Inner Product) / Skalarprodukt (Punktprodukt oder inneres Produkt) (1)

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = |\underline{\mathbf{A}}||\underline{\mathbf{B}}| \cos \underbrace{\angle(\underline{\mathbf{A}}, \underline{\mathbf{B}})}_{\phi_{AB}}$$

$$= AB \cos\phi_{AB}$$

Enclosed Angle / Eingeschlossener Winkel ϕ_{AB}



$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = \underline{\mathbf{B}} \bullet \underline{\mathbf{A}}$$

$$= BA\cos\phi_{BA}$$

$$= AB\cos\phi_{AB}$$

$$\cos(\phi_{AB}) = \cos(-\phi_{AB})$$

$$\cos\phi_{AB} = \frac{\underline{\mathbf{A}} \bullet \underline{\mathbf{B}}}{|\underline{\mathbf{A}}||\underline{\mathbf{B}}|}$$

$$\phi_{AB} = \arccos\left(\frac{\underline{\mathbf{A}} \bullet \underline{\mathbf{B}}}{|\underline{\mathbf{A}}||\underline{\mathbf{B}}|}\right)$$

Scalar Product (Dot or Inner Product) / Skalarprodukt (Punktprodukt oder inneres Produkt) (2)

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \bullet (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z)$$

$$= A_x B_x \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_x}_{=1} + A_x B_y \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_y}_{=0} + A_x B_z \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_z}_{=0}$$

$$+ A_y B_x \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_x}_{=0} + A_y B_y \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_y}_{=1} + A_y B_z \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_z}_{=0}$$

$$+ A_z B_x \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_x}_{=0} + A_z B_y \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_y}_{=0} + A_z B_z \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_z}_{=1}$$

$$= A_x B_x + A_y B_y + A_z B_z$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z$$

$\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_x = 1$	$\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_x = 0$	$\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_x = 0$
$\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_y = 0$	$\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_y = 1$	$\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_y = 0$
$\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_z = 0$	$\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_z = 0$	$\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_z = 1$

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \bullet (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z)$$

$$= A_x B_x + A_y B_y + A_z B_z$$

$$= (A_{x_1} \underline{\mathbf{e}}_{x_1} + A_{x_2} \underline{\mathbf{e}}_{x_2} + A_{x_3} \underline{\mathbf{e}}_{x_3}) \bullet (B_{x_1} \underline{\mathbf{e}}_{x_1} + B_{x_2} \underline{\mathbf{e}}_{x_2} + B_{x_3} \underline{\mathbf{e}}_{x_3})$$

$$= A_{x_1} B_{x_1} + A_{x_2} B_{x_2} + A_{x_3} B_{x_3}$$

$$= \sum_{i=1}^{3} A_{x_i} B_{x_i}$$

Cartesian Coordinates / Kartesische Koordinaten

$$x = x_1$$
$$y = x_2$$
$$z = x_3$$

Scalar Product (Dot or Inner Product) / Skalarprodukt (Punktprodukt oder inneres Produkt) (3)

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \bullet (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z)$$

$$= \sum_{i=1}^{3} A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet \sum_{j=1}^{3} B_{x_j} \underline{\mathbf{e}}_{x_j}$$

$$= \sum_{i=1}^{3} \sum_{j=1}^{3} A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet B_{x_j} \underline{\mathbf{e}}_{x_j}$$

$$= \sum_{i=1}^{3} \sum_{j=1}^{3} A_{x_i} B_{x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{=\delta_{ij}}$$

Kronecker Delta / Kronecker-Delta

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$= A_{x_i} B_{x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{=\delta_{ij}}$$

with Einstein's Summation Convention / mit Einsteinscher Summationskonvention

$$= A_{x_i} \underbrace{B_{x_j} \delta_{ij}}_{=B_{x_i}} \quad \left(\text{or/oder} \quad \underbrace{\underbrace{A_{x_i} \delta_{ij}}_{=A_{x_j}} B_{x_j}}_{=A_{x_j} B_{x_j}} \right)$$

$$= A_{x_i} B_{x_i}$$

Einstein's Summation Convention: If a index appears two times at one side of an equation (and not at the other side), the index is automatically summed over 1 to 3. /
Einsteinsche Summenkonvention: Wenn ein Index auf einer Seite einer Gleichung zweimal vorkommt (und auf der anderen nicht), wird darüber von 1 bis 3 summiert.

Magnitude of a Vector / Betrag eines Vektors

$$\left|\underline{\mathbf{A}}\right| = \sqrt{\underline{\mathbf{A}} \bullet \underline{\mathbf{A}}}$$
$$= \sqrt{(\mathrm{A}_x \underline{\mathbf{e}}_x + \mathrm{A}_y \underline{\mathbf{e}}_y + \mathrm{A}_z \underline{\mathbf{e}}_z) \bullet (\mathrm{A}_x \underline{\mathbf{e}}_x + \mathrm{A}_y \underline{\mathbf{e}}_y + \mathrm{A}_z \underline{\mathbf{e}}_z)}$$
$$= \Big(\mathrm{A}_x \mathrm{A}_x \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_x}_{=1} + \mathrm{A}_x \mathrm{A}_y \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_y}_{=0} + \mathrm{A}_x \mathrm{A}_z \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_z}_{=0}$$
$$+ \mathrm{A}_y \mathrm{A}_x \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_x}_{=0} + \mathrm{A}_y \mathrm{A}_y \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_y}_{=1} + \mathrm{A}_y \mathrm{A}_z \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_z}_{=0}$$
$$+ \mathrm{A}_z \mathrm{A}_x \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_x}_{=0} + \mathrm{A}_z \mathrm{A}_y \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_y}_{=0} + \mathrm{A}_z \mathrm{A}_z \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_z}_{=1} \Big)^{\frac{1}{2}}$$
$$= \sqrt{\mathrm{A}_x \mathrm{A}_x + \mathrm{A}_y \mathrm{A}_y + \mathrm{A}_z \mathrm{A}_z}$$
$$= \sqrt{\mathrm{A}_x^2 + \mathrm{A}_y^2 + \mathrm{A}_z^2}$$
$$= \mathrm{A}$$

$$\left|\underline{\mathbf{A}}\right| = \sqrt{\underline{\mathbf{A}} \bullet \underline{\mathbf{A}}}$$
$$= \sqrt{\sum_{i=1}^{3} A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet \sum_{j=1}^{3} B_{x_j} \underline{\mathbf{e}}_{x_j}}$$
$$= \sqrt{A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet A_{x_j} \underline{\mathbf{e}}_{x_j}}$$
$$= \sqrt{A_{x_i} A_{x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{=\delta_{ij}}}$$
$$= \sqrt{A_{x_i}^2}$$

Example: Position Vector and Electric Field Strength Vector / Beispiel: Ortsvektor und elektrischer Feldstärkevektor

Cartesian Coordinate System / Kartesisches Koordinatensystem

Position Vector / Ortsvektor

$$\underline{\mathbf{R}}(x,y,z) = \mathrm{R}_x(x,y,z)\underline{\mathbf{e}}_x + \mathrm{R}_y(x,y,z)\underline{\mathbf{e}}_y + \mathrm{R}_z(x,y,z)\underline{\mathbf{e}}_z$$
$$= x\,\underline{\mathbf{e}}_x + y\,\underline{\mathbf{e}}_y + z\,\underline{\mathbf{e}}_z$$

Electric Field Strength Vector / Elektrische Feldstärkevektor

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{E}}(x,y,z,t)$$
$$= \mathrm{E}_x(x,y,z,t)\,\underline{\mathbf{e}}_x + \mathrm{E}_y(x,y,z,t)\,\underline{\mathbf{e}}_y + \mathrm{E}_z(x,y,z,t)\,\underline{\mathbf{e}}_z$$

Magnitude of the Position Vector (Distance) / Betrag des Ortsvektor (Abstand)

$$\left|\underline{\mathbf{R}}(x,y,z)\right| = \sqrt{\underline{\mathbf{R}}(x,y,z)\bullet\underline{\mathbf{R}}(x,y,z)}$$
$$= \sqrt{\left(x\,\underline{\mathbf{e}}_x + y\,\underline{\mathbf{e}}_y + z\,\underline{\mathbf{e}}_z\right)\bullet\left(x\,\underline{\mathbf{e}}_x + y\,\underline{\mathbf{e}}_y + z\,\underline{\mathbf{e}}_z\right)}$$
$$= \sqrt{x^2 + y^2 + z^2}$$

Magnitude of the Electric Field Strength Vector (Strength) / Betrag des elektrische Feldstärkevektors (Stärke)

$$\left|\underline{\mathbf{E}}(x,y,z)\right| = \sqrt{\underline{\mathbf{E}}(x,y,z)\bullet\underline{\mathbf{E}}(x,y,z)}$$
$$= \sqrt{\left(\mathrm{E}_x\,\underline{\mathbf{e}}_x + \mathrm{E}_y\,\underline{\mathbf{e}}_y + \mathrm{E}_z\,\underline{\mathbf{e}}_z\right)\bullet\left(\mathrm{E}_x\,\underline{\mathbf{e}}_x + \mathrm{E}_y\,\underline{\mathbf{e}}_y + \mathrm{E}_z\,\underline{\mathbf{e}}_z\right)}$$
$$= \sqrt{\mathrm{E}_x^2 + \mathrm{E}_y^2 + \mathrm{E}_z^2}$$

Position Unit Vector (Direction) / Ortseinheitsvektor (Richtung)

$$\hat{\underline{\mathbf{R}}}(x,y,z) = \frac{\underline{\mathbf{R}}(x,y,z)}{\left|\underline{\mathbf{R}}(x,y,z)\right|} = \frac{x\,\underline{\mathbf{e}}_x + y\,\underline{\mathbf{e}}_y + z\,\underline{\mathbf{e}}_z}{\sqrt{x^2 + y^2 + z^2}}$$

Electric Field Strength Unit Vector (Direction) / Elektrische Feldstärkeeinheitsvektor (Richtung)

$$\hat{\underline{\mathbf{E}}}(x,y,z) = \frac{\underline{\mathbf{E}}(x,y,z)}{\left|\underline{\mathbf{E}}(x,y,z)\right|} = \frac{\mathrm{E}_x\,\underline{\mathbf{e}}_x + \mathrm{E}_y\,\underline{\mathbf{e}}_y + \mathrm{E}_z\,\underline{\mathbf{e}}_z}{\sqrt{\mathrm{E}_x^2 + \mathrm{E}_y^2 + \mathrm{E}_z^2}}$$

End of Lecture 3 /
Ende der 3. Vorlesung