Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

4th Lecture / 4. Vorlesung

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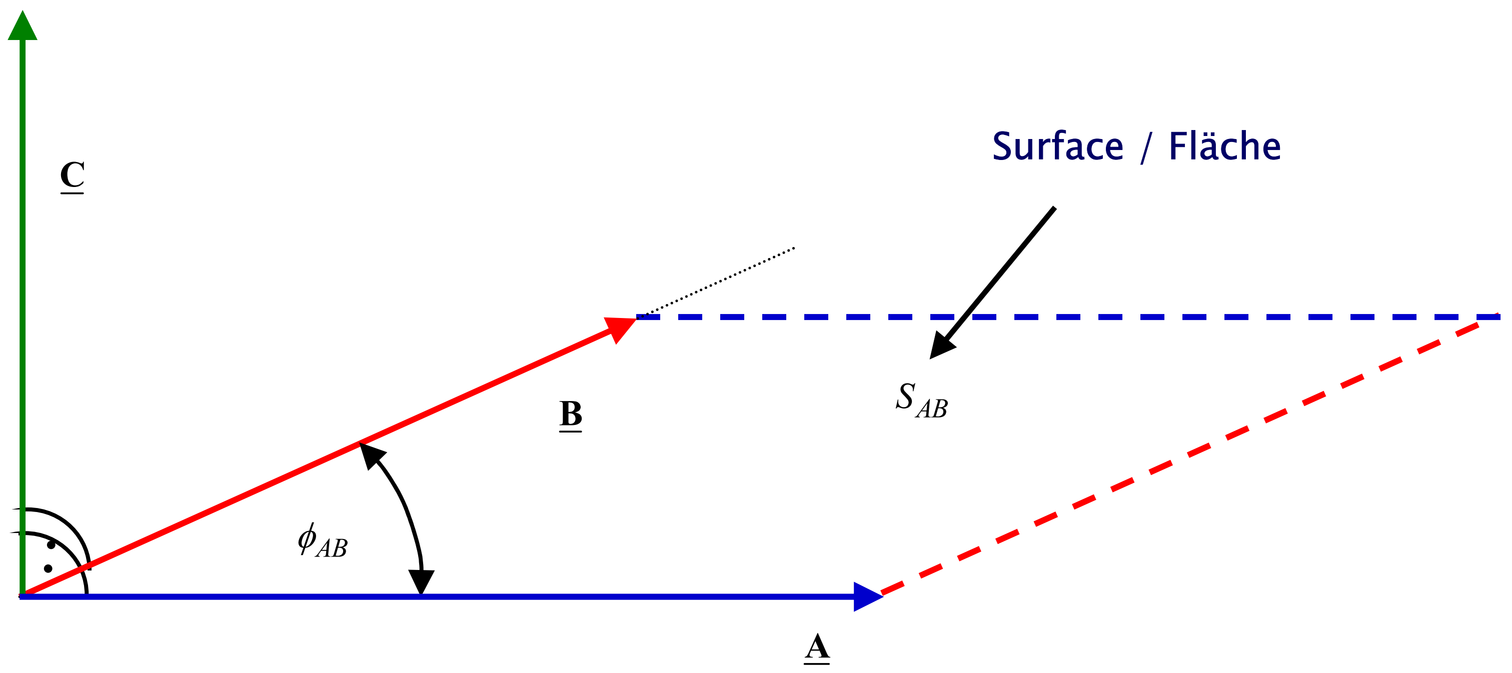
Vector Product (Cross or Outer Product) / Vektorprodukt (Kreuzprodukt oder äußeres Produkt) (1)



$$\underline{\mathbf{C}} = \underline{\mathbf{A}} \times \underline{\mathbf{B}}$$

$$C = |\underline{\mathbf{A}}||\underline{\mathbf{B}}| \sin \underbrace{\angle(\underline{\mathbf{A}}, \underline{\mathbf{B}})}_{\phi_{AB}}$$

$$= AB \sin \phi_{AB}$$

$$= S_{AB}$$

$\underline{\mathbf{C}} \perp \underline{\mathbf{A}}$ and / und $\underline{\mathbf{C}} \perp \underline{\mathbf{B}}$

Vector Product (Cross or Outer Product) / Vektorprodukt (Kreuzprodukt oder äußeres Produkt) (2)

$$
\begin{aligned}
\underline{\mathbf{A}} \times \underline{\mathbf{B}} &= (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \times (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z) \\
&= A_x B_x \underbrace{\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_x}_{=0} + A_x B_y \underbrace{\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y}_{=\underline{\mathbf{e}}_z} + A_x B_z \underbrace{\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_z}_{=-\underline{\mathbf{e}}_y} \\
&\quad + A_y B_x \underbrace{\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_x}_{=-\underline{\mathbf{e}}_z} + A_y B_y \underbrace{\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_y}_{=0} + A_y B_z \underbrace{\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z}_{=\underline{\mathbf{e}}_x} \\
&\quad + A_z B_x \underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x}_{=\underline{\mathbf{e}}_y} + A_z B_y \underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_y}_{=-\underline{\mathbf{e}}_x} + A_z B_z \underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_z}_{=0} \\
&= (A_y B_z \underline{\mathbf{e}}_x - A_z B_y) \underline{\mathbf{e}}_x + (A_z B_x - A_x B_z) \underline{\mathbf{e}}_y + (A_x B_y - A_y B_x) \underline{\mathbf{e}}_z
\end{aligned}
$$

$$\underline{\mathbf{A}} \times \underline{\mathbf{B}} = -\underline{\mathbf{B}} \times \underline{\mathbf{A}} \qquad \underline{\mathbf{A}} \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_x = \underline{\mathbf{0}}$$
$$\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y = \underline{\mathbf{e}}_z$$
$$\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_z = -\underline{\mathbf{e}}_y$$

$$\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_x = -\underline{\mathbf{e}}_z$$
$$\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_y = \underline{\mathbf{0}}$$
$$\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z = \underline{\mathbf{e}}_x$$

$$\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x = \underline{\mathbf{e}}_y$$
$$\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_y = -\underline{\mathbf{e}}_x$$
$$\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_z = \underline{\mathbf{0}}$$

Vector Product (Cross or Outer Product) / Vektorprodukt (Kreuzprodukt oder äußeres Produkt) (3)

$$\underline{\mathbf{A}} \times \underline{\mathbf{B}} = \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Add the first two Columns / Addiere die beiden ersten Spalten

$$= \left| \begin{matrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{matrix} \right| \begin{matrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y \\ A_x & A_y \\ B_x & B_y \end{matrix} \Bigg|$$

$$= (A_y B_z - A_z B_y)\underline{\mathbf{e}}_x + (A_z B_x - A_x B_z)\underline{\mathbf{e}}_y + (A_x B_y - A_y B_x)\underline{\mathbf{e}}_z$$

Sarrus Law /
Regel von Sarrus
[Pierre Frédéric Sarrus, 1831]
http://de.wikipedia.org/wiki/Regel_von_Sarrus

Dyadic Product / Dyadisches Produkt

$$\underline{\mathbf{A}}\,\underline{\mathbf{B}} = \sum_{i=1}^{3} A_{x_i}\,\underline{\mathbf{e}}_{x_i}\;\sum_{j=1}^{3} B_{x_j}\,\underline{\mathbf{e}}_{x_j}$$

$$= \sum_{i=1}^{3}\;\sum_{j=1}^{3} A_{x_i}\,\underline{\mathbf{e}}_{x_i}\;B_{x_j}\,\underline{\mathbf{e}}_{x_j}$$

$$= A_{x_i}\,\underline{\mathbf{e}}_{x_i}\;B_{x_j}\,\underline{\mathbf{e}}_{x_j}$$

$$= \underbrace{A_{x_i}\,B_{x_j}}_{=D_{x_i x_j}}\,\underline{\mathbf{e}}_{x_i}\,\underline{\mathbf{e}}_{x_j}$$

$$= D_{x_i x_j}\,\underline{\mathbf{e}}_{x_i}\,\underline{\mathbf{e}}_{x_j}$$

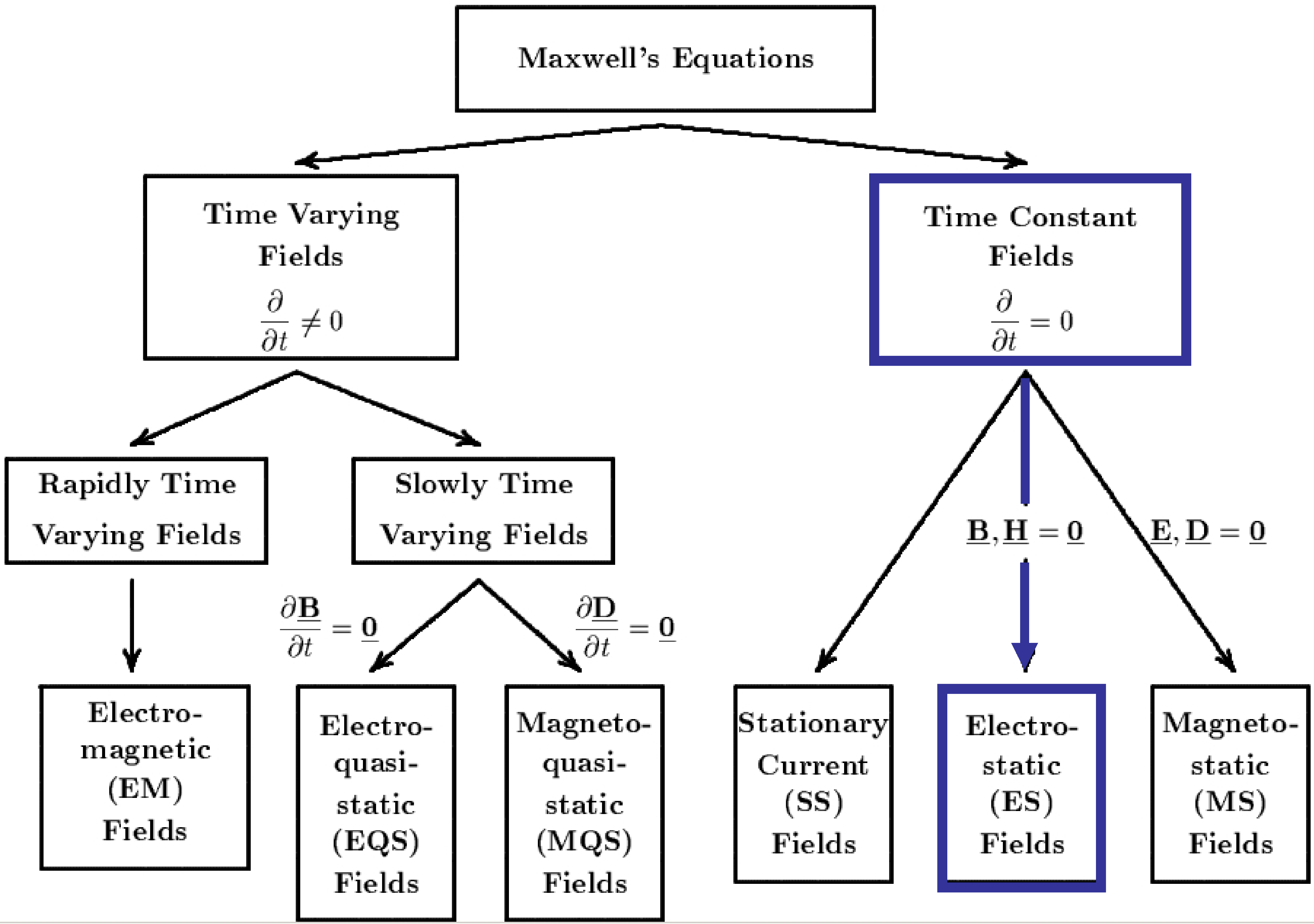
$$= \underline{\underline{\mathbf{D}}}$$

$$\underline{\mathbf{B}}\,\underline{\mathbf{A}} \neq \underline{\mathbf{A}}\,\underline{\mathbf{B}}$$

$$\underline{\mathbf{D}} = \underline{\underline{\boldsymbol{\varepsilon}}} \bullet \underline{\mathbf{E}}$$

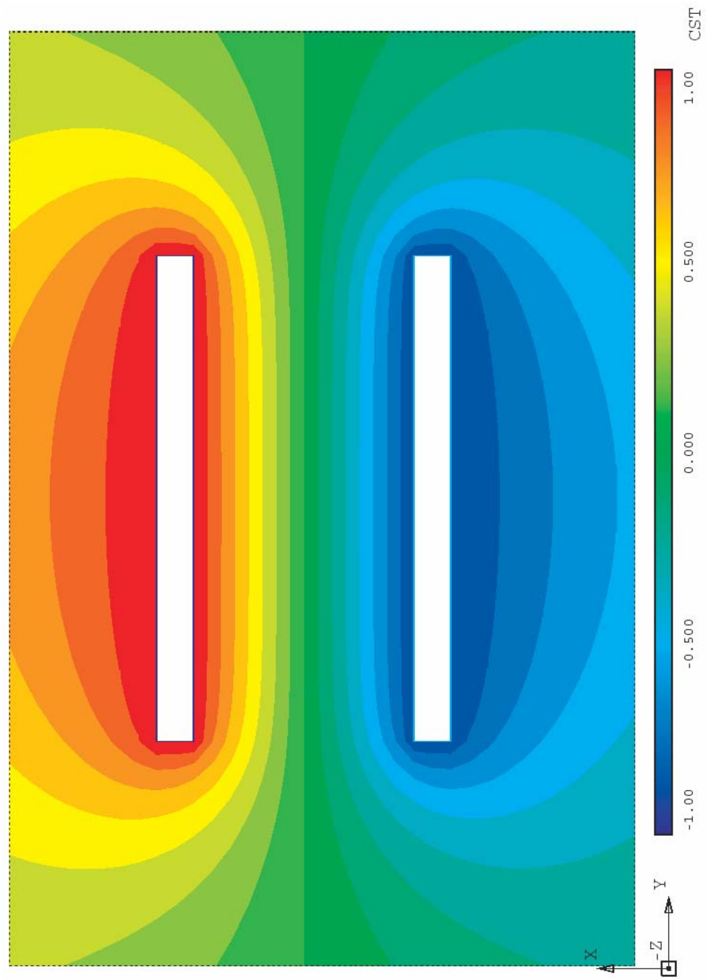
$$\underline{\mathbf{B}} = \underline{\underline{\boldsymbol{\mu}}} \bullet \underline{\mathbf{H}}$$

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

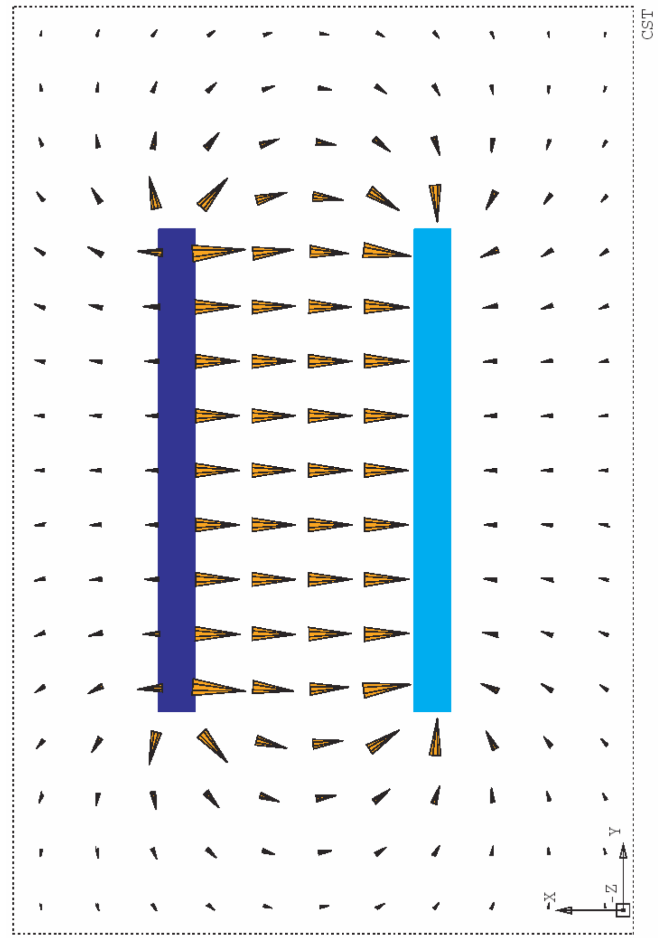


Electrostatic Field Problem – Example: Parallel Plate Capacitor / Elektrostatisches Feldproblem – Beispiel: Paralleler Plattenkondensator

Scalar Field: Electrostatic Potential /
Skalarfeld: Elektrostatisches Potenzial



Vector Field: Electrostatic Field Strength /
Vektorfeld: Elektrostatische Feldstärke



Electrostatic (ES) Fields – Governing Equations / Elektrostatische (ES) Felder – Grundgleichungen

Electrostatic / Elektrostatik $\quad \frac{\partial}{\partial t} \equiv 0 \quad$ No Time Dependence and No Magnetic Field Quantities / Keine Zeitabhängigkeit und keine magnetischen Feldgrößen

$\underline{\mathbf{E}}(\underline{\mathbf{R}})$: Electric Field Strength / Elektrische Feldstärke
$\underline{\mathbf{D}}(\underline{\mathbf{R}})$: Electric Flux Density / Elektrische Flussdichte
$\rho_\mathrm{e}(\underline{\mathbf{R}})$: Electric Charge Density / Elektrische Raumladungsdichte

Integral Form / Integralform	Differential Form / Differentialform
$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dR}} = 0$	$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$
$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V$	$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_\mathrm{e}(\underline{\mathbf{R}})$

Curl-Free E-Field / Rotationsfreies E-Feld

Divergence of D Represents Electric Charge Density / Quellstärke von D entspricht der elektrischen Raumladungsdichte

Electrostatic (ES) Fields – Governing Equations / Elektrostatische (ES) Felder – Grundgleichungen

Integral Form / Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{R}} = 0$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{S}} = \iiint_V \rho_e(\underline{\mathbf{R}})\, \mathrm{d}V = Q_e$$

$\underline{\mathbf{E}}(\underline{\mathbf{R}})$ [V/m = Newton/Coulomb = N/C]

$\underline{\mathbf{D}}(\underline{\mathbf{R}})$ [As/m^2]

$\rho_e(\underline{\mathbf{R}})$ [As/m^3]

Vacuum / Vakuum

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}})$$

Electric Field Constant / Elektrische Feldkonstante (IEEE, VDE)
Permittivity of Free Space / Permittivität des Freiraumes

Differential Form / Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_e(\underline{\mathbf{R}})$$

Side Remark: In some Cases / Nebenbemerkung: In einigen Fällen

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \varepsilon_r \underline{\mathbf{E}}(\underline{\mathbf{R}})$$

Permittivity / Permittivität

Material	ε_r
Air / Luft	1.006
Paper / Papier	2...4
Wet Earth / Nasse Erde	5...15
Gallium Arsenide / Gallium Arsenid	13
Seawater / Seewasser	70

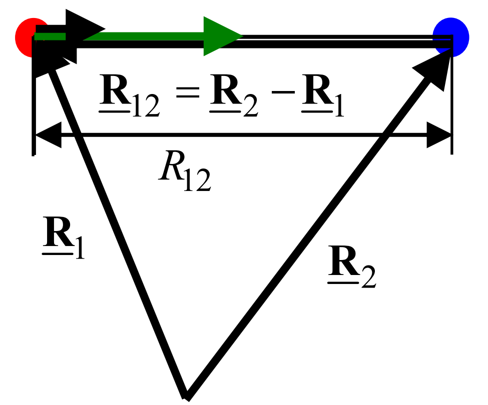
ES Fields – Electric Points Charge and Electric Field Strength – Coulomb's Law / ES Felder – Elektrische Punktladung und elektrische Feldstärke – Coulombsches Gesetz

Coulomb's Law / Coulombsches Gesetz

Charles Augustin de Coulomb (1736 – 1806)

$$\underline{\mathbf{F}}(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon}\frac{Q_e^{(1)}Q_e^{(2)}}{R_{12}^2}\hat{\underline{\mathbf{R}}}_{12} \qquad [\mathrm{N}]$$

Force / Kraft	$\underline{\mathbf{F}}(\underline{\mathbf{R}})$	[N]
Electric Point Charge / Elektrische Punktladung	$Q_e^{(1)}$	[As]
Electric Point Charge / Elektrische PunktLadung	$Q_e^{(2)}$	[As]
Distance / Abstand	R	[m]
Distance Unit Vector / Abstandseinheitsvektor	$\hat{\underline{\mathbf{R}}}$	[1]
Permittivity of Free-Space / Permittivität des Freiraumes	ε	[As/Vm]



$$\hat{\underline{\mathbf{R}}} = \frac{\underline{\mathbf{R}}}{|\underline{\mathbf{R}}|} = \frac{\underline{\mathbf{R}}}{R} \qquad [1] \qquad R = |\underline{\mathbf{R}}| = \sqrt{\underline{\mathbf{R}}\bullet\underline{\mathbf{R}}} \quad [\mathrm{m}]$$

$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$

$$R = \sqrt{\underline{\mathbf{R}}\bullet\underline{\mathbf{R}}} = \sqrt{x^2+y^2+z^2}$$

$$\hat{\underline{\mathbf{R}}} = \frac{x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z}{\sqrt{x^2+y^2+z^2}} = \underline{\mathbf{e}}_R(\vartheta,\varphi)$$

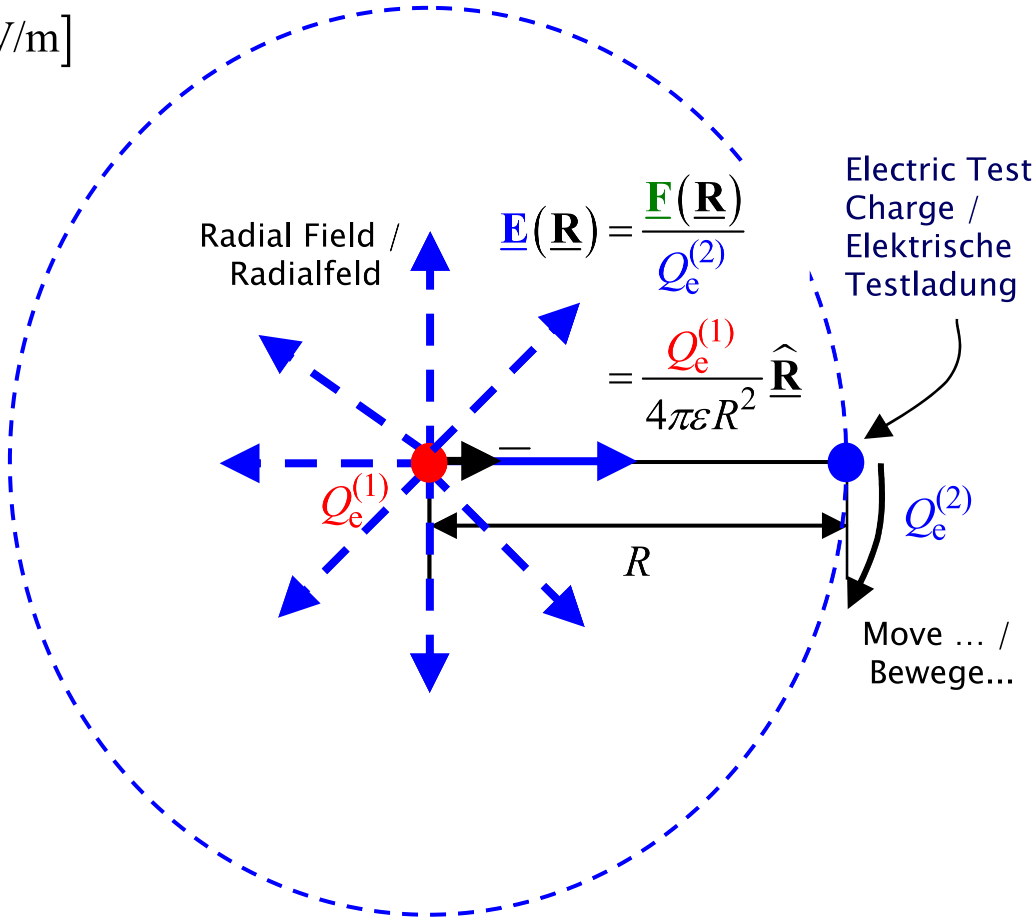
ES Fields – Electric Charge and Electric Field Strength – Coulomb's Law / ES Felder – Elektrische Ladung und elektrische Feldstärke – Coulombsches Gesetz

Electric Field Strength: Force Per Unit Charge / Elektrische Feldstärke: Kraft pro Einheitsladung

$Q_e^{(2)}$ Electric Test Charge / Elektrische Testladung

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{\underline{\mathbf{F}}(\underline{\mathbf{R}})}{Q_e^{(2)}} = \frac{Q_e^{(1)}}{4\pi\varepsilon R^2}\hat{\underline{\mathbf{R}}} \qquad [\mathrm{N/C \ or \ V/m}]$$

Electric Field Strength / Elektische Feldstärke	$\underline{\mathbf{E}}(\underline{\mathbf{R}})$	[V/m]
Force / Kraft	$\underline{\mathbf{F}}(\underline{\mathbf{R}})$	[N]
Electric Charge / Elektrische Ladung	$Q_e^{(1)}$	[As]
Electric Test Charge / Elektrische Testladung	$Q_e^{(2)}$	[As]
Distance / Abstand	R	[m]
Distance Unit Vector / Abstandseinheitsvektor	$\hat{\underline{\mathbf{R}}}$	[1]
Permittivity of Free-Space / Permittivität des Freiraumes	ε	[As/Vm]



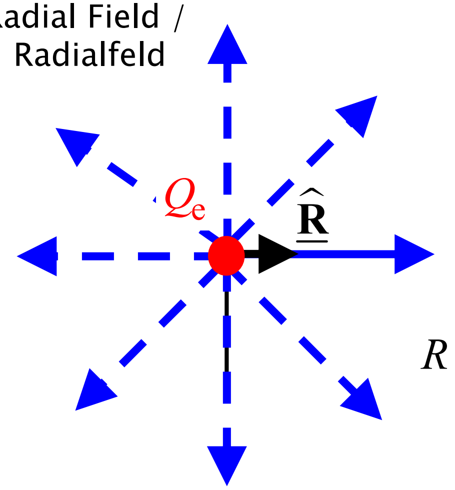
ES Fields – Electric Charge and Electric Field Strength – Coulomb's Law / ES Felder – Elektrische Ladung und elektrische Feldstärke – Coulombsches Gesetz

Electric Field Strength: Force Per Unit Charge / Elektrische Feldstärke: Kraft pro Einheitsladung

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon R^2}\hat{\underline{\mathbf{R}}} = \frac{Q_e}{4\pi\varepsilon R}\underline{\mathbf{R}} \qquad [\mathrm{V/m}]$$

$$\underline{\mathbf{R}} = R\hat{\underline{\mathbf{R}}}$$

Electric Field Strength / Elektische Feldstärke	$\underline{\mathbf{E}}(\underline{\mathbf{R}})$	[V/m]
Electric Charge / Elektrische Ladung	Q_e	[As]
Distance / Abstand	R	[m]
Distance Unit Vector / Abstandseinheitsvektor	$\hat{\underline{\mathbf{R}}}$	[1]
Permittivity of Free-Space / Permittivität des Freiraumes	ε	[As/Vm]



$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon R^2}\hat{\underline{\mathbf{R}}}$$

$$= \frac{Q_e}{4\pi\varepsilon R}\underline{\mathbf{R}}$$

Electrostatic (ES) Fields – Governing Equations / Elektrostatische (ES) Felder – Grundgleichungen

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$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{S}} = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V$	$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_\mathrm{e}(\underline{\mathbf{R}})$

Curl-Free E-Field / Rotationsfreies E-Feld

Divergence of D Represents Electric Charge Density / Quellstärke von D entspricht der elektrischen Raumladungsdichte

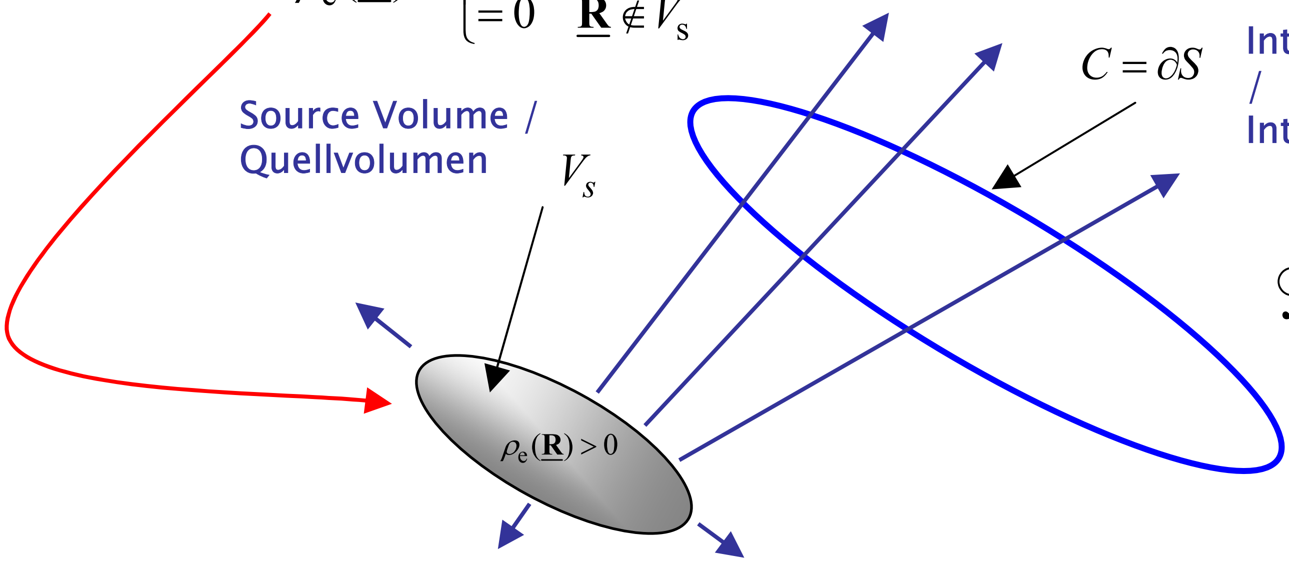
⬇

Method of Gauss' Electric Law / Methode des Gaußschen elektrischen Gesetzes

ES Fields – Method of Electric Gauss' Law / ES–Felder – Methode des elektrischen Gaußschen Gesetzes

Source Distribution / Quellverteilung

$$\rho_e(\underline{\mathbf{R}}) = \begin{cases} \neq 0 & \underline{\mathbf{R}} \in V_s \\ = 0 & \underline{\mathbf{R}} \notin V_s \end{cases}$$



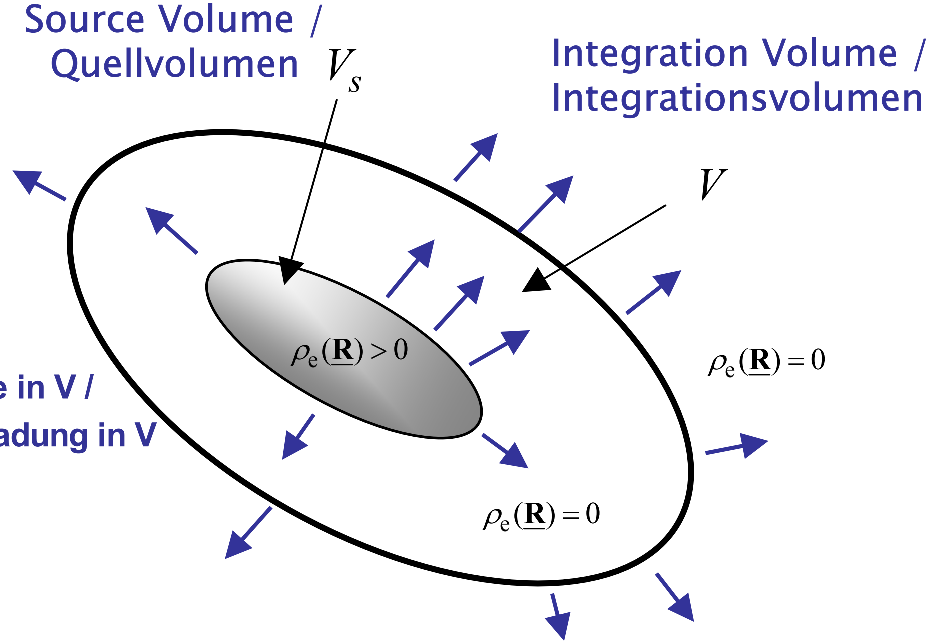
Integration Contour / Integrationskontur

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dR}} = 0$$

ES Fields – Method of Electric Gauss' Law / ES-Felder – Methode des elektrischen Gaußschen Gesetzes

Source Distribution / Quellverteilung

$$\rho_e(\underline{\mathbf{R}}) = \begin{cases} \neq 0 & \underline{\mathbf{R}} \in V_s \\ = 0 & \underline{\mathbf{R}} \notin V_s \end{cases}$$



$$\psi_e = \oint\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}} = \iiint_V \rho_e(\underline{\mathbf{R}})\, \mathrm{d}V$$

$$= Q_e$$

Total Electric Charge in V / Elektrische Gesamtladung in V

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}} = \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{n}}}_{D_n(\underline{\mathbf{R}})}\, \mathrm{d}S$$

$$\underbrace{\underbrace{\oint\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}}}\ \oint\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{n}}}_{D_n(\underline{\mathbf{R}})} \mathrm{d}S} = \underbrace{\underbrace{\iiint_V \rho_e(\underline{\mathbf{R}})\, \mathrm{d}V}_{Q_e}}$$

Summation of all $D_n = \underline{\mathbf{n}} \cdot \underline{\mathbf{D}}$ Contributions / Summation aller $D_n = \underline{\mathbf{n}} \cdot \underline{\mathbf{D}}$-Beiträge

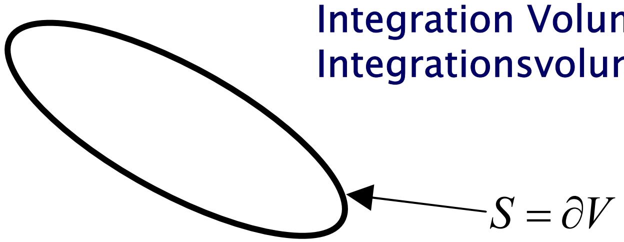
Total electric charge inside the volume V with the closed surface $S=\partial V$ / Gesamte elektrische Ladung im Volumen V mit der geschlossenen Oberfläche $S=\partial V$

Flux of $\underline{\mathbf{D}}$ through $S = Q_e$ in V /
Fluss von $\underline{\mathbf{D}}$ durch $S = Q_e$ in V

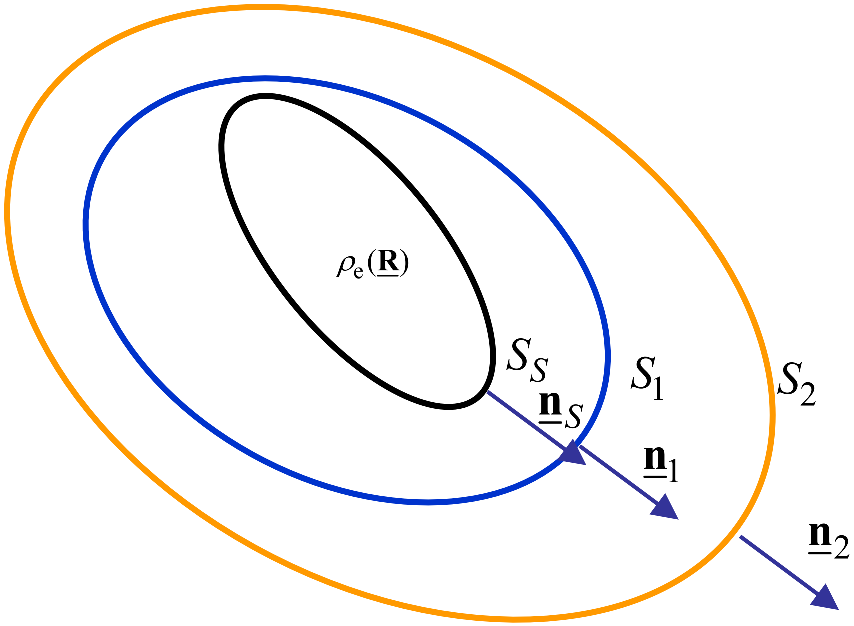
Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Method of Electric Gauss' Law / Methode des elektrischen Gaußschen Gesetzes

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \iiint_V \rho_e(\underline{\mathbf{R}})\,\mathrm{d}V$$
$$= Q_e$$

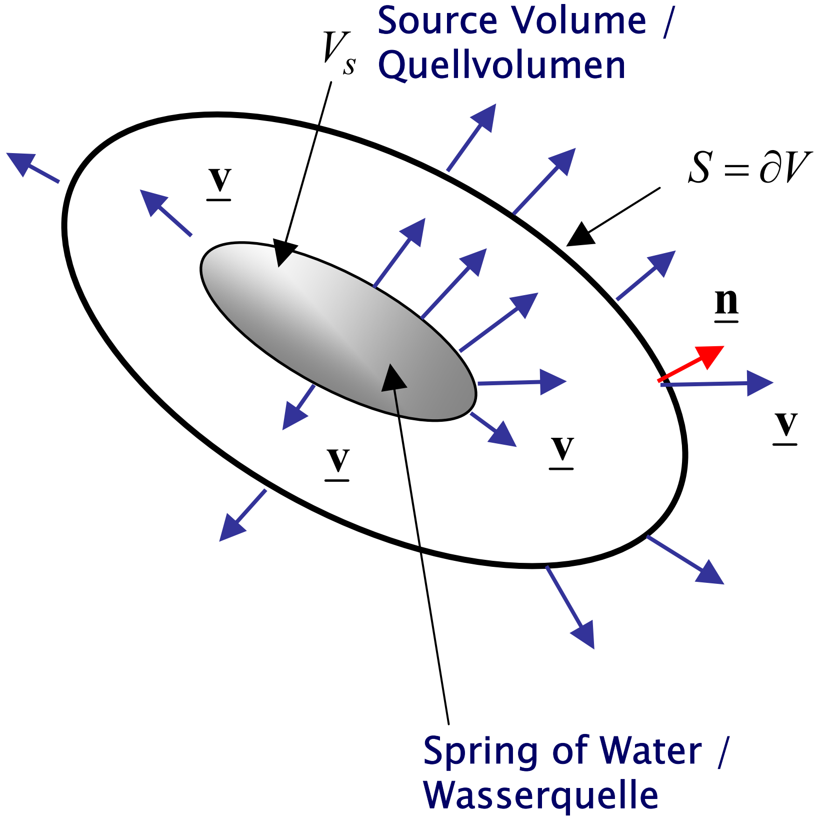


$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} \begin{cases} = 0 & \text{source-free / quellenfrei} \\ > 0 & \text{Source / Quelle} \\ < 0 & \text{Sink / Senke} \end{cases}$$



$$\oiint_{S_S=\partial V_S} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}_S \mathrm{d}S$$
$$= \oiint_{S_1=\partial V_1} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}_1 \mathrm{d}S$$
$$= \oiint_{S_2=\partial V_2} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}_2 \mathrm{d}S$$
$$= Q_e$$

Example: Fluid Mechanics – Spring of Water / Beispiel: Strömungsmechanik – Wasserquelle



Integration Surface (Closed Surface) /
Integrationsfläche (geschlossene Oberfläche)

Total Flux through the Closed Surface /
Gesamtfluss durch die geschlossene Oberfläche

$$\oiint_{S=\partial V} \underline{\mathbf{v}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \oiint_{S=\partial V} \underbrace{\underline{\mathbf{v}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=\mathrm{v}_n(\underline{\mathbf{R}})} \mathrm{d}S$$

$$= \oiint_{S=\partial V} \mathrm{v}_n(\underline{\mathbf{R}}) \, \mathrm{d}S$$

$$= \Phi_{\mathrm{v}}$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

Consider the Electrostatic (ES) Case / Betrachte den elektrostatischen (ES) Fall

$$\oint\!\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}} = \oint\!\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} \mathrm{d}S = \underbrace{\iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V}_{=Q_\mathrm{e}}$$

Prescribed: Electric Charge Density / Vorgegeben: Elektrische Raumladungsdichte

$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = \rho_\mathrm{e}(R) = \begin{cases} \rho_{\mathrm{e}0} \dfrac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases}$$

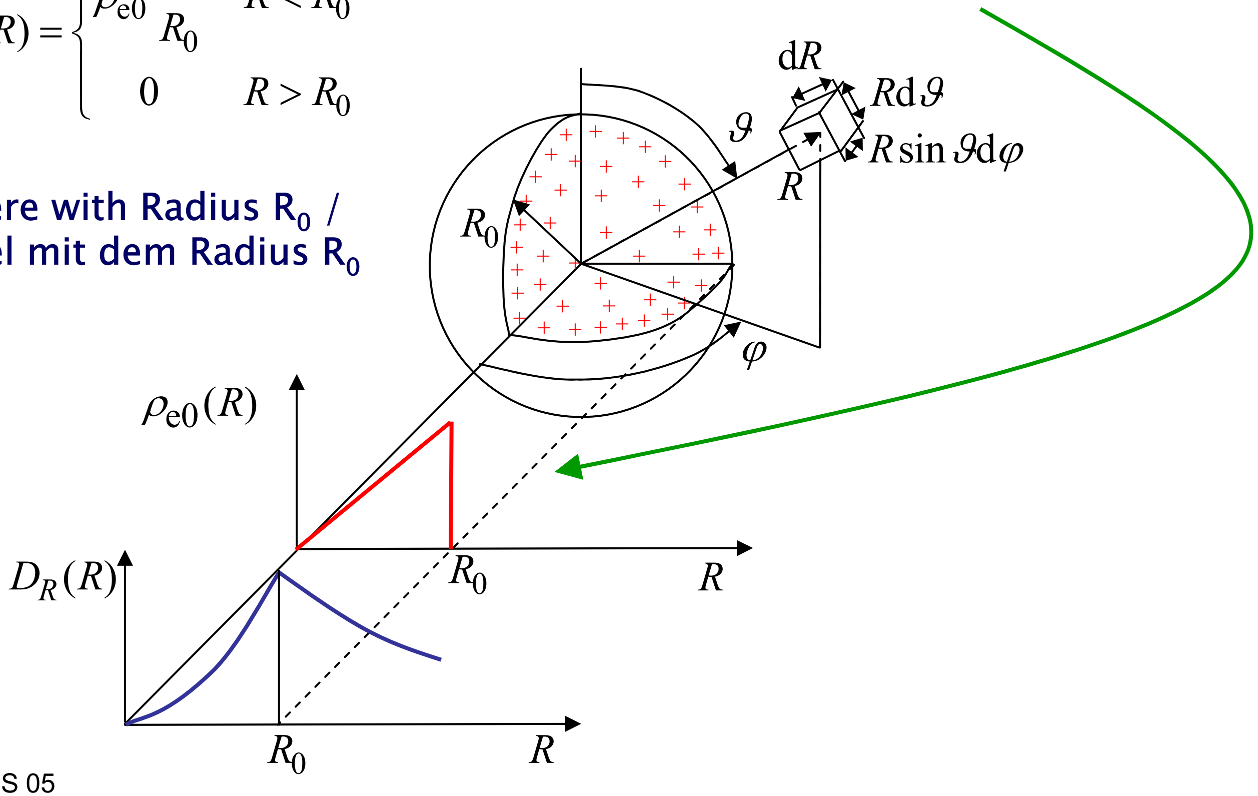
Radial Symmetry / Radialsymmetrie **!**

$$\underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} = \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{e}}_R}_{=D_R(\underline{\mathbf{R}})}$$

$$D_n(\underline{\mathbf{R}}) = D_R(\underline{\mathbf{R}})$$

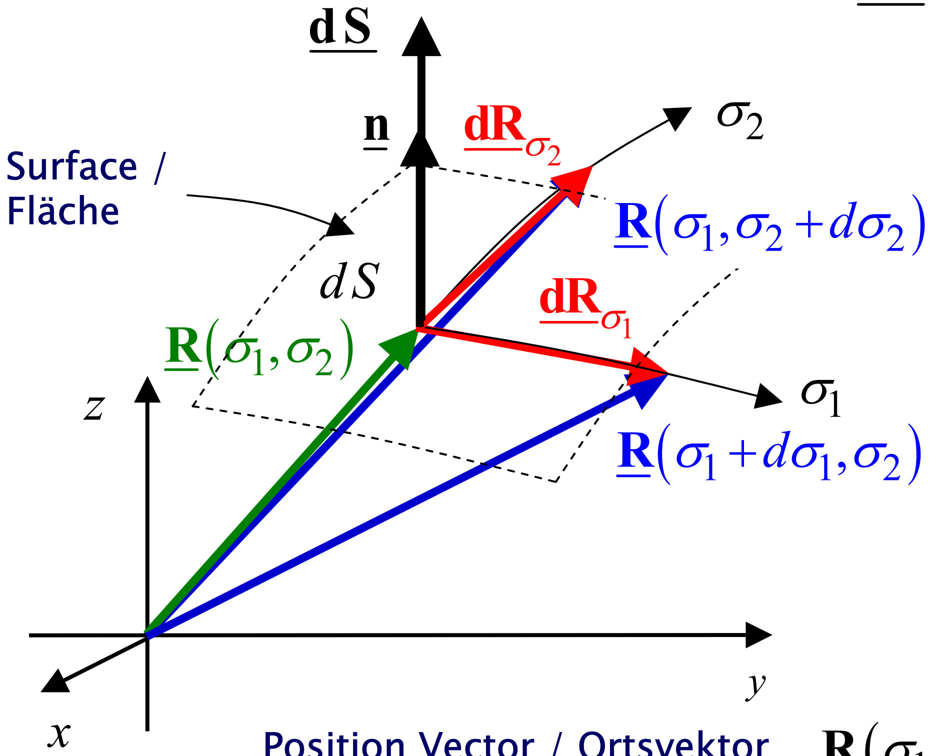
Charged Sphere with Radius R_0 / Geladene Kugel mit dem Radius R_0

Solution for D(R) / Lösung für D(R)



Vector Differential Surface Element / Vektorielles differentielles Flächenelement (1)

Definition: $\underline{\mathbf{d\,S}} = \underline{\mathbf{n}}\,\mathrm{d}S$



σ_1,σ_2	Surface Parameters / Flächenparameter
$\underline{\mathbf{R}}(\sigma_1,\sigma_2)$	Position Vector / Ortsvektor
$\underline{\mathbf{R}}(\sigma_1+\mathrm{d}\sigma_1,\sigma_2)$	Position Vector / Ortsvektor
$\underline{\mathbf{R}}(\sigma_1,\sigma_2+\mathrm{d}\sigma_2)$	Position Vector / Ortsvektor
$\underline{\mathbf{dR}}_{\sigma_1}$, $\underline{\mathbf{dR}}_{\sigma_2}$	Vector Differential Line Elements / Vektorielle differentielle Linienelemente

Position Vector / Ortsvektor $\underline{\mathbf{R}}(\sigma_1,\sigma_2)$

Tangential Vectors / Tangentialvektoren

$$\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2) = \frac{\partial}{\partial\sigma_1}\underline{\mathbf{R}}(\sigma_1,\sigma_2)$$

$$\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2) = \frac{\partial}{\partial\sigma_2}\underline{\mathbf{R}}(\sigma_1,\sigma_2)$$

Vector Differential Surface Element / Vektorielles differentielles Flächenelement (2)

Vector Differential Line Elements / Vektorielles differentielles Linienelement

$$\underline{\mathbf{dR}}_{\sigma_1} = \underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_1$$

$$\underline{\mathbf{dR}}_{\sigma_2} = \underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_2$$

Scalar Differential Surface Elements / Skalares differentielles Flächenelement

$$\begin{aligned}\mathrm{d}S &= \left|\underline{\mathbf{dR}}_{\sigma_1} \times \underline{\mathbf{dR}}_{\sigma_2}\right| \\ &= \left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|\mathrm{d}\sigma_1\mathrm{d}\sigma_2\end{aligned}$$

Normal Unit-Vector / Normaleneinheitsvektor

$$\underline{\mathbf{n}} = \frac{\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)}{\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|}$$

Vector Differential Surface Element / Vektorielles differentielles Flächenelement

$$\begin{aligned}\underline{\mathbf{dS}} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= \frac{\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)}{\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|}\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|\mathrm{d}\sigma_1\mathrm{d}\sigma_2 \\ &= \underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_1\mathrm{d}\sigma_2\end{aligned}$$

Gauss' Electric Law / Gaußsches elektrisches Gesetz

$$\underbrace{\oint\!\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}})\bullet \mathrm{d}\underline{\mathbf{S}}}_{\text{Closed Surface Integral / Geschlossenes Flächenintegral}} = \underbrace{\oint\!\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})}\,\mathrm{d}S}_{=\psi_e} = \underbrace{\iiint_V \rho_e(\underline{\mathbf{R}})\,\mathrm{d}V}_{=Q_e}$$

Closed Surface Integral /
Geschlossenes Flächenintegral

Summation of all Normal Componentes of **D** at the Closed Surface $S=\partial V$ of the Volume V /
Summation aller Normalkomponenten von **D** auf der geschlossenen Oberfläche $S=\partial V$ des Volumens V

$=\psi_e$
Flux Through the Colsed Surface /
Fluss durch die geschlossene Oberfläche

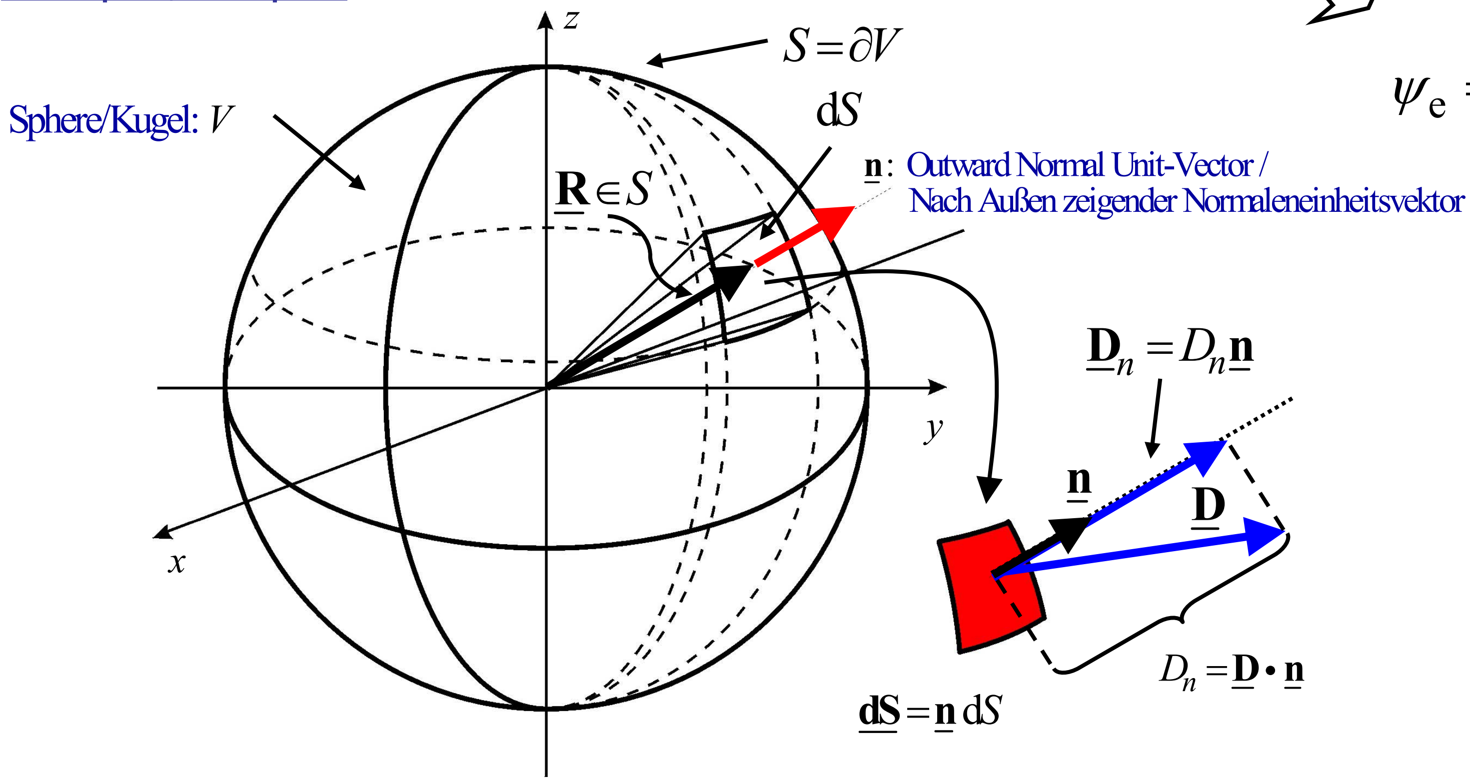
Volume Integral /
Volumenintegral

Summation of all charges inside the Volume V /
Summation aller Ladungen in dem Volumen V

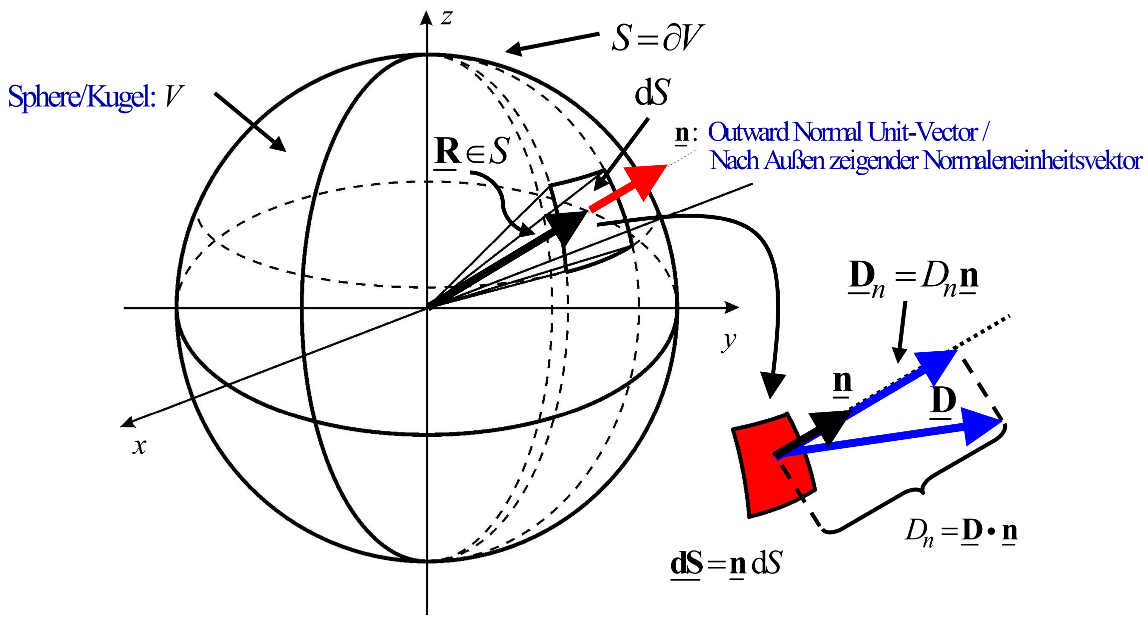
$=Q_e$

$$\psi_e = Q_e$$

Example / Beispiel:



Example: Sphere with Radius *a* / Beispiel: Kugel mit Radius *a* (1)



$$\oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} \mathrm{d}S = \iiint_V \rho_e(\underline{\mathbf{R}})\,\mathrm{d}V$$

$$\underline{\mathbf{dS}} = \underline{\mathbf{n}}dS \quad (= \underline{\mathbf{n}}_{\vartheta\varphi}\, h_\vartheta\, h_\varphi\, d\vartheta\, d\varphi)$$

$$= \underbrace{\underline{\mathbf{e}}_R(\vartheta,\varphi)}_{\underline{\mathbf{n}}} \underbrace{R^2 \sin\vartheta\, d\vartheta\, d\varphi}_{dS}\Big|_{R=a} = \underbrace{\underline{\mathbf{e}}_R(\vartheta,\varphi)}_{\underline{\mathbf{n}}} \underbrace{a^2 \sin\vartheta\, d\vartheta\, d\varphi}_{dS}$$

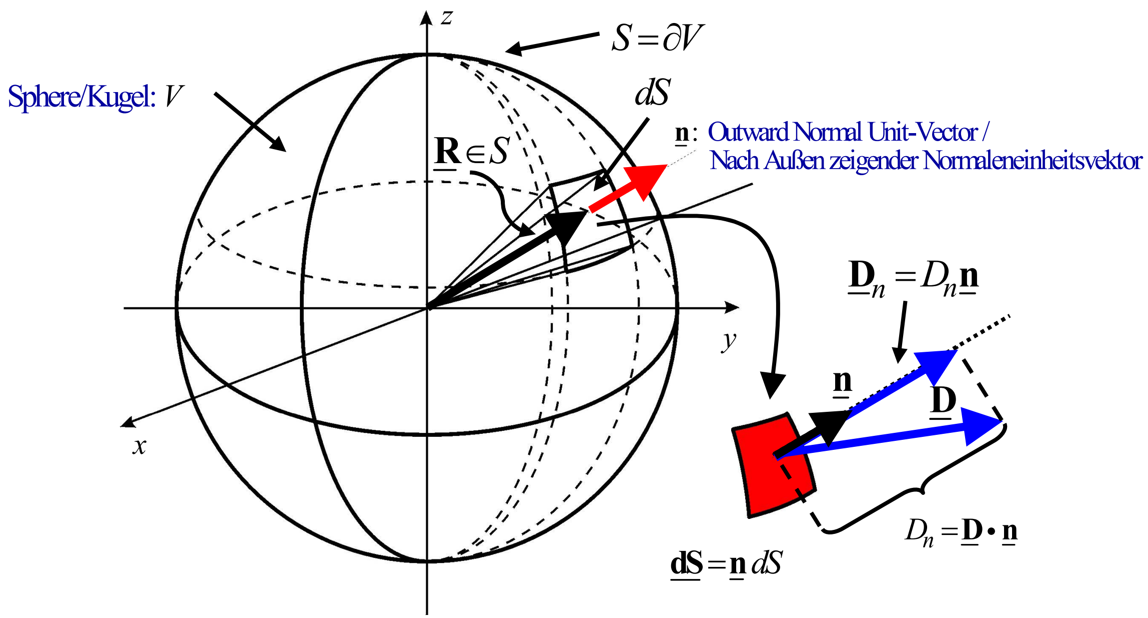
$$0 \le \vartheta \le \pi$$

$$0 \le \varphi < 2\pi$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{dS}} = \oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} \mathrm{d}S = \int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi} \underbrace{\underline{\mathbf{D}}[\underline{\mathbf{R}}(R=a,\vartheta,\varphi)]\bullet\underline{\mathbf{e}}_R(\vartheta,\varphi)}_{\substack{=D_R[\underline{\mathbf{R}}(R=a,\vartheta,\varphi)] \\ =D_n[\underline{\mathbf{R}}(R=a,\vartheta,\varphi)]}} a^2 \sin\vartheta\; \mathrm{d}\vartheta\; \mathrm{d}\varphi$$

$$= \psi_e$$

Example: Sphere with Radius *a* / Beispiel: Kugel mit Radius *a* (2)



$$\oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{n}}}_{D_n(\underline{\mathbf{R}})}\,\mathrm{d}S = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V$$

$$\mathrm{d}V = R^2 \sin\vartheta \,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \quad \left(= h_R h_\vartheta h_\varphi \;\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi\right)$$

$$\iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V = \int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi}\int_{R=0}^{a} \rho_\mathrm{e}[\underline{\mathbf{R}}(R,\vartheta,\varphi)]\,R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$$

$$= Q_\mathrm{e}$$

$$0 \le R \le a$$
$$0 \le \vartheta \le \pi$$
$$0 \le \varphi < 2\pi$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

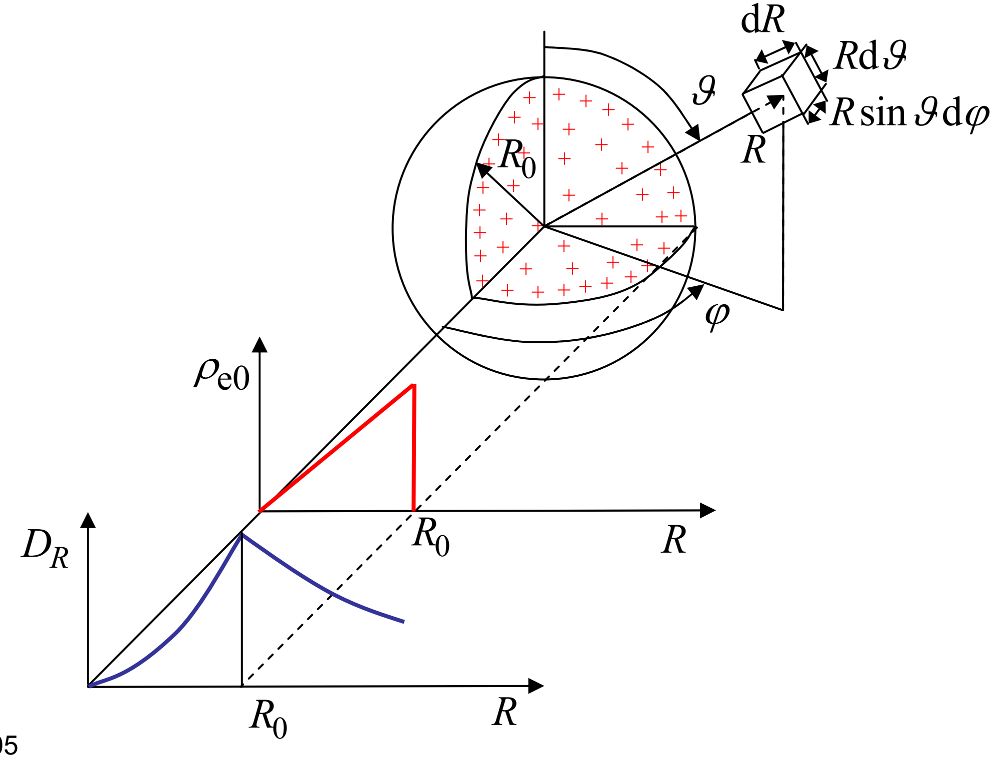
Consider the Electrostatic (ES) Case / Betrachte den elektrostatischen Fall

$$\oint\!\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\cdot\underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})}\,\mathrm{d}S = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V$$

Electric Charge Density / Elektrische Raumladungsdichte

$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = \begin{cases} \rho_{\mathrm{e}0}\dfrac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases}$$

Radial Symmetry / Radialsymmetrisch !



Metric Coefficients / Metrische Koeffizienten

Cartesian Coordinates / Kartesische Koordinaten

$$x, y, z = x_1, x_2, x_3$$

$$\underline{\mathbf{e}}_x, \underline{\mathbf{e}}_y, \underline{\mathbf{e}}_z = \underline{\mathbf{e}}_{x_1}, \underline{\mathbf{e}}_{x_2}, \underline{\mathbf{e}}_{x_3}$$

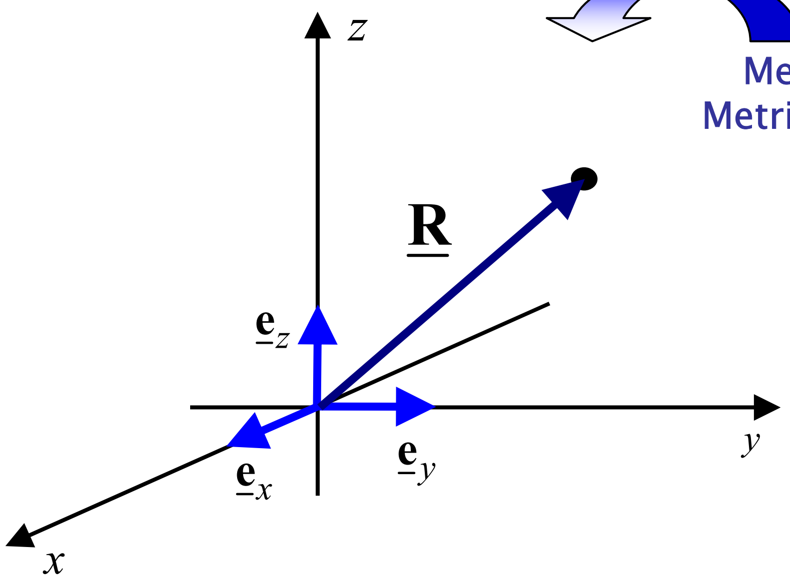
$$\underline{\mathbf{e}}_x, \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z \; ; \; \underline{\mathbf{e}}_{x_1} \perp \underline{\mathbf{e}}_{x_2} \perp \underline{\mathbf{e}}_{x_3}$$

Orthogonal Curvilinear Coordinates / Orthogonale Krummlinige Koordinaten

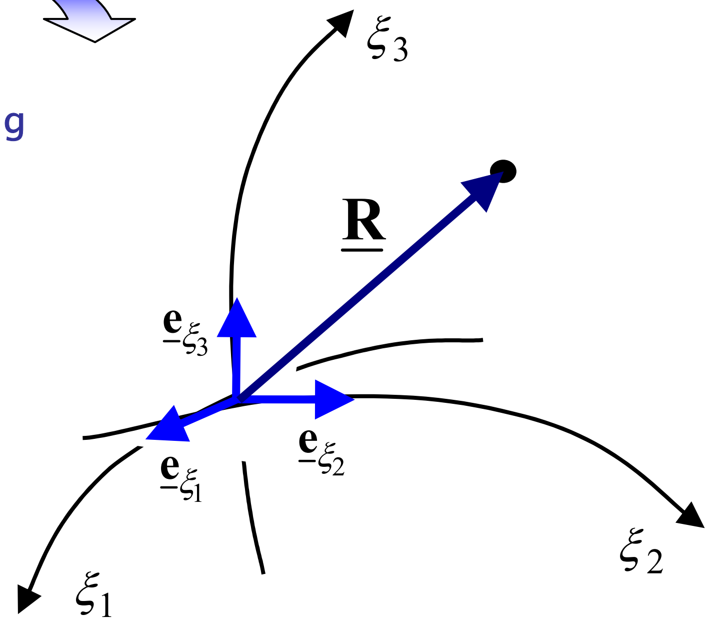
$$\xi_1, \xi_2, \xi_3$$

$$\underline{\mathbf{e}}_{\xi_1}, \underline{\mathbf{e}}_{\xi_2}, \underline{\mathbf{e}}_{\xi_3}$$

$$\underline{\mathbf{e}}_{\xi_1} \perp \underline{\mathbf{e}}_{\xi_2} \perp \underline{\mathbf{e}}_{\xi_3}$$



Metric Scaling / Metrische Skalierung

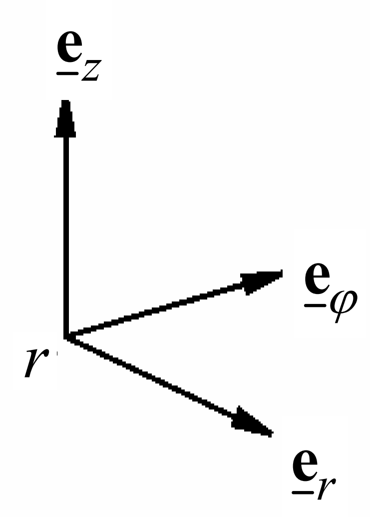


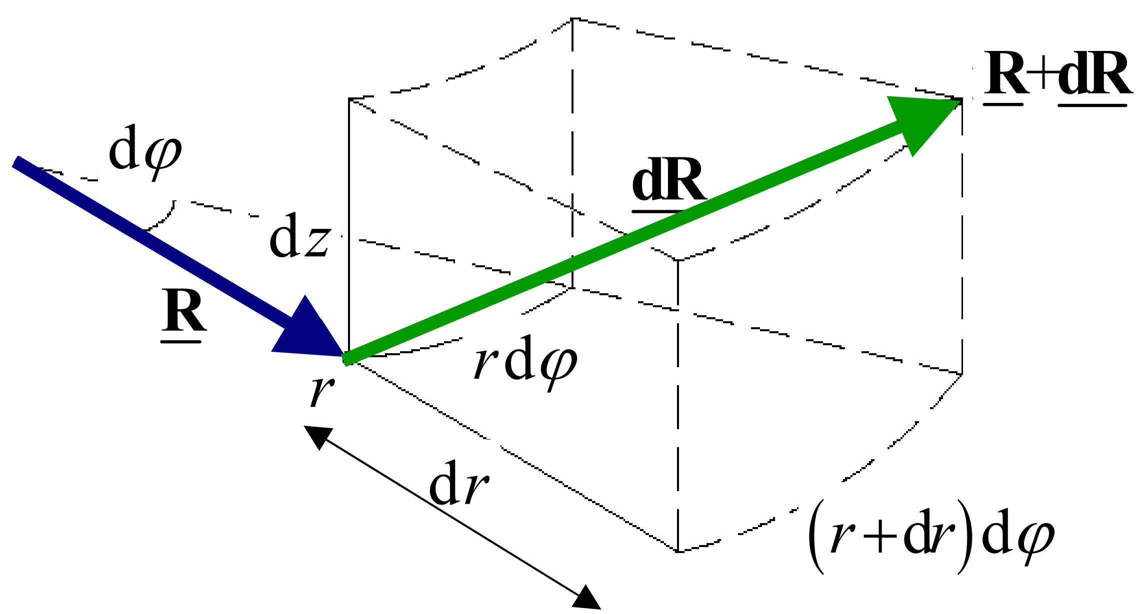
Cartesian Coordinates / Kartesische Koordinaten

$$x = x(\xi_1, \xi_2, \xi_3)$$
$$y = y(\xi_1, \xi_2, \xi_3)$$
$$z = z(\xi_1, \xi_2, \xi_3)$$

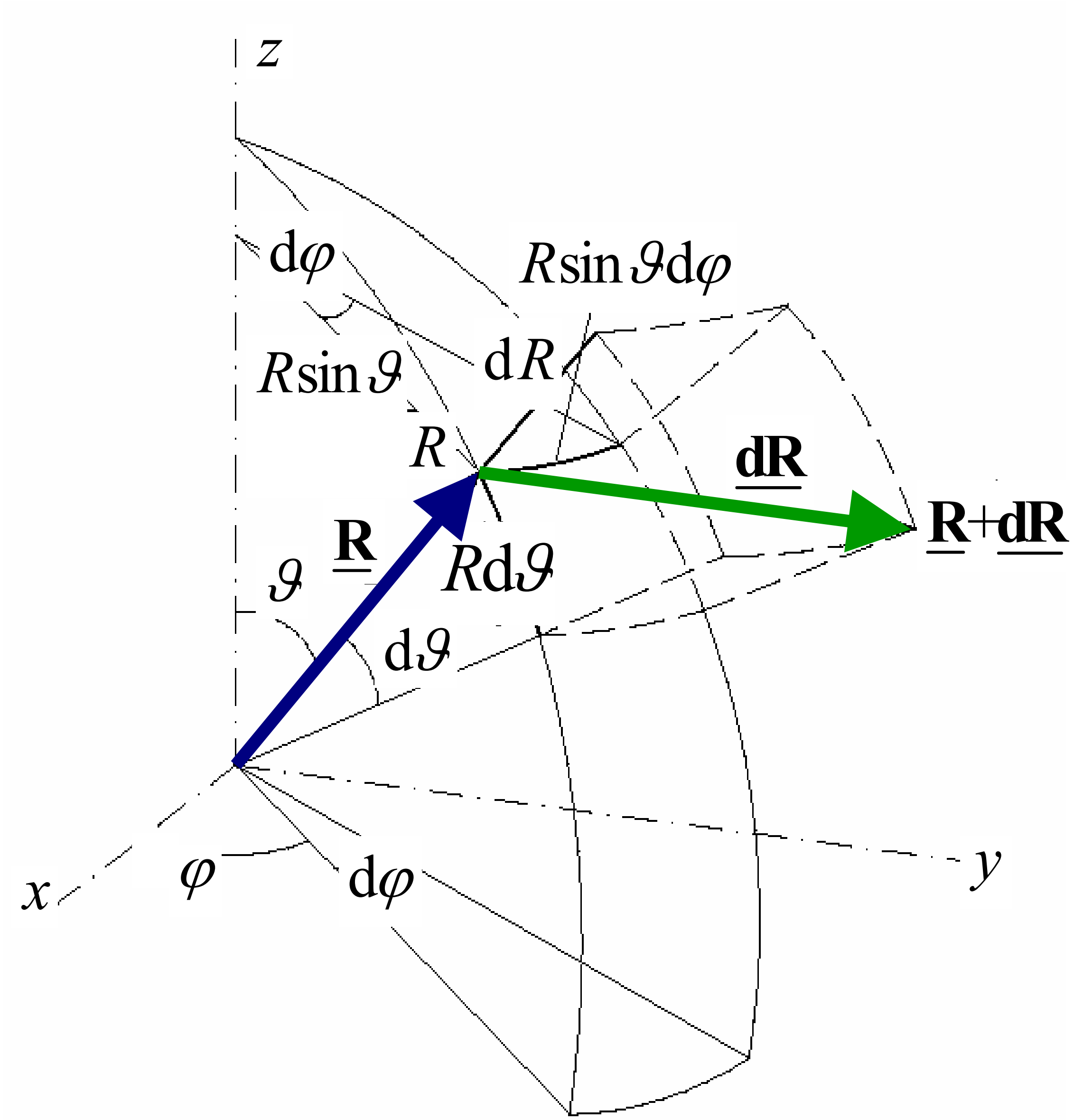
$$\xi_1 = \xi_1(x, y, z)$$
$$\xi_2 = \xi_2(x, y, z)$$
$$\xi_3 = \xi_3(x, y, z)$$

Metric Coefficients – Cylindrical Coordinate System / Metrische Koeffizienten – Zylinderkoordinatensystem





Metric Coefficients – Spherical Coordinate System / Metrische Koeffizienten – Kugelkoordinatensystem



Metric Coefficients / Metrische Koeffizienten

Cartesian Coordinates / Kartesische Koordinaten

$$x,y,z = x_1,x_2,x_3 = \xi_1,\xi_2,\xi_3$$
$$x = x(\xi_1,\xi_2,\xi_3)$$
$$y = y(\xi_1,\xi_2,\xi_3)$$
$$z = z(\xi_1,\xi_2,\xi_3)$$

Orthogonal Curvilinear Coordinates / Orthogonale Krummlinige Koordinaten

$$\xi_1,\xi_2,\xi_3$$
$$\xi_1 = \xi_1(x,y,z)$$
$$\xi_2 = \xi_2(x,y,z)$$
$$\xi_3 = \xi_3(x,y,z)$$

$$\underline{\mathbf{R}} = x(\xi_1,\xi_2,\xi_3)\underline{\mathbf{e}}_x + y(\xi_1,\xi_2,\xi_3)\underline{\mathbf{e}}_y + z(\xi_1,\xi_2,\xi_3)\underline{\mathbf{e}}_z$$

$$\frac{\partial \underline{\mathbf{R}}}{\partial \xi_i} = \frac{\partial x(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\underline{\mathbf{e}}_x + \frac{\partial y(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\underline{\mathbf{e}}_y + \frac{\partial z(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\underline{\mathbf{e}}_z$$

$i = 1,2,3$

$$\frac{\partial \underline{\mathbf{R}}}{\partial \xi_i} = \underbrace{\underbrace{\left|\frac{\partial \underline{\mathbf{R}}(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\right|}_{\text{Magnitude / Betrag}}}_{=h_{\xi_i}} \underbrace{\underbrace{\frac{\dfrac{\partial \underline{\mathbf{R}}(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}}{\left|\dfrac{\partial \underline{\mathbf{R}}(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\right|}}_{\text{Direction / Richtung}}}_{=\underline{\mathbf{e}}_{\xi_i}}, \quad i = 1,2,3$$

Metric Coefficients / Metrische Koeffizienten

$$h_{\xi_i} = \left|\frac{\partial \underline{\mathbf{R}}(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\right|$$
$$= \sqrt{\frac{\partial \underline{\mathbf{R}}(\xi_1,\xi_2,\xi_3)}{\partial \xi_i} \bullet \frac{\partial \underline{\mathbf{R}}(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}}$$
$$= \sqrt{\left(\frac{\partial x(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\right)^2 + \left(\frac{\partial y(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\right)^2 + \left(\frac{\partial z(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\right)^2}$$

$i = 1,2,3$

Metric Coefficients / Metrische Koeffizienten

$$\xi_1 = x$$
$$\xi_2 = y$$
$$\xi_3 = z$$

$$
\begin{aligned}
h_x &= \left| \frac{\partial \underline{\mathbf{R}}(x,y,z)}{\partial x} \right| \\
&= \sqrt{\frac{\partial \underline{\mathbf{R}}(x,y,z)}{\partial x} \bullet \frac{\partial \underline{\mathbf{R}}(x,y,z)}{\partial x}} \\
&= \sqrt{\left(\frac{\partial x(x,y,z)}{\partial x}\right)^2 + \left(\frac{\partial y(x,y,z)}{\partial x}\right)^2 + \left(\frac{\partial z(x,y,z)}{\partial x}\right)^2} \\
&= \sqrt{\underbrace{\left(\frac{\partial x}{\partial x}\right)^2}_{1} + \underbrace{\left(\frac{\partial y}{\partial x}\right)^2}_{=0} + \underbrace{\left(\frac{\partial z}{\partial x}\right)^2}_{=0}} \\
&= 1 \\
h_y &= 1 \\
h_z &= 1
\end{aligned}
$$

Metric Coefficients / Metrische Koeffizienten

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$h_x = 1$$
$$h_y = 1$$
$$h_z = 1$$

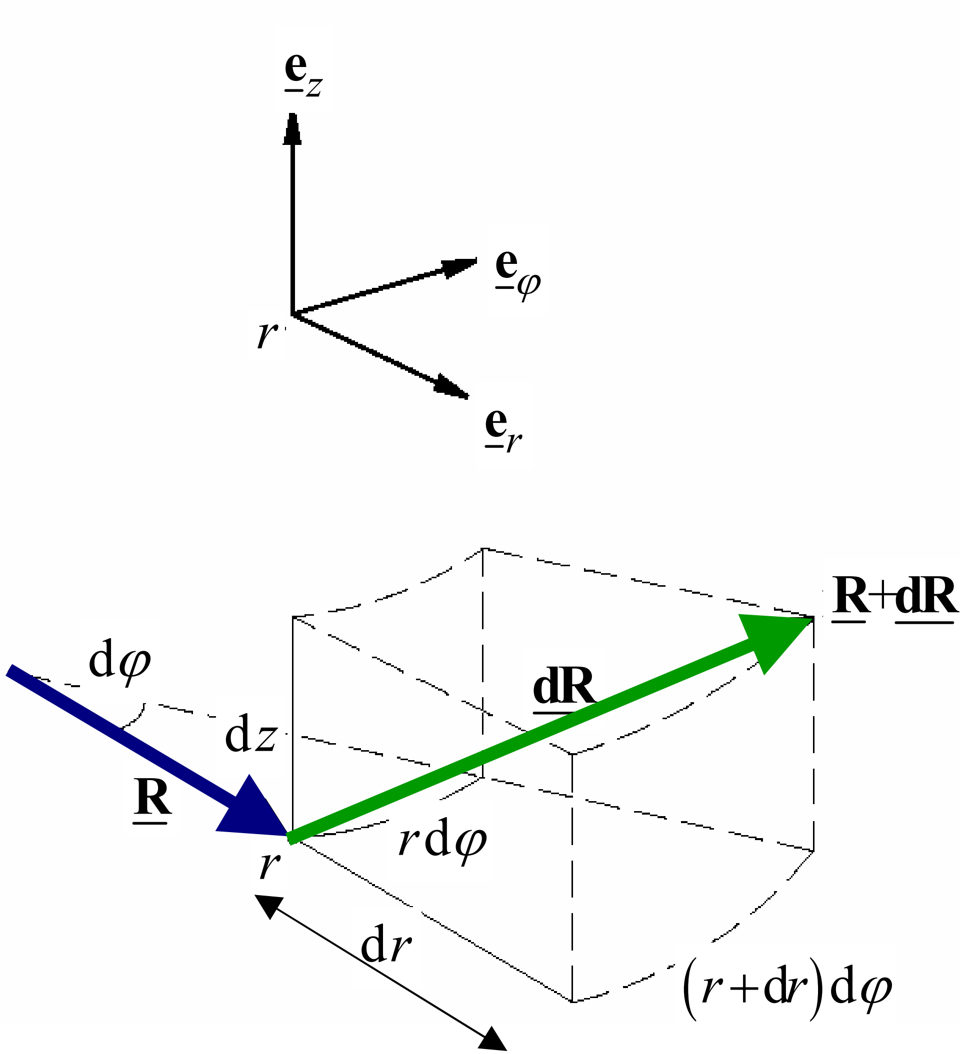
Cylindrical Coordinate System / Zylinderkoordinatensystem

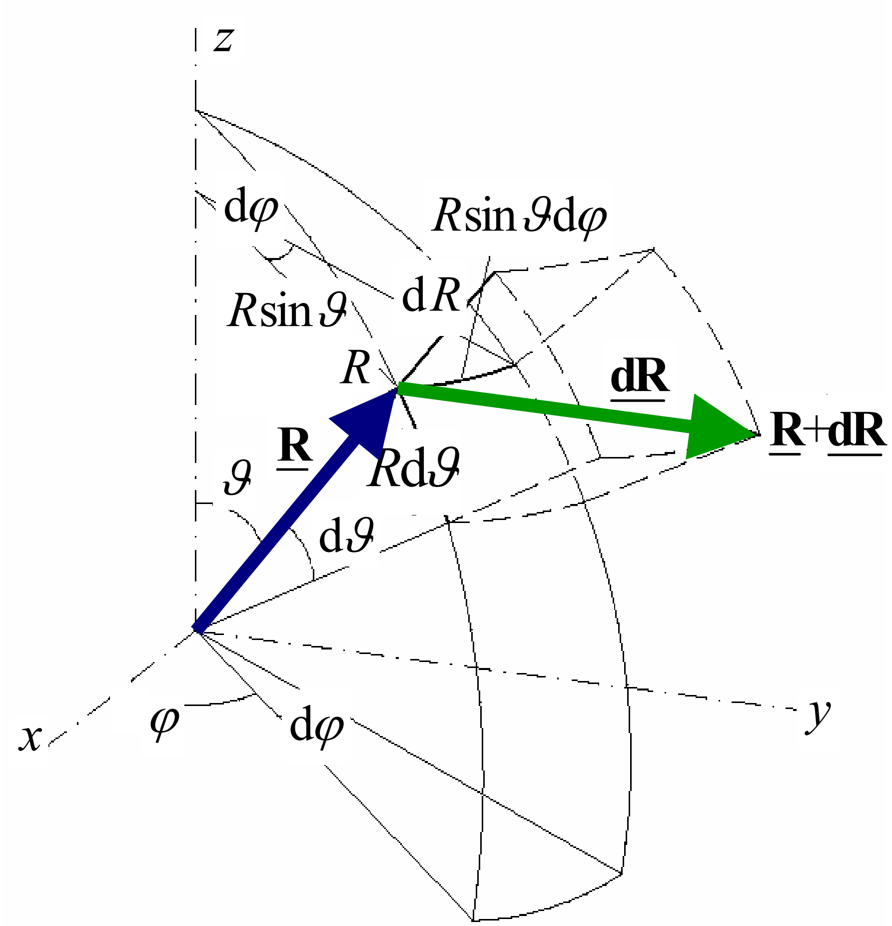
$$h_r = 1$$
$$h_\varphi = r$$
$$h_z = 1$$

Spherical Coordinate System / Kugelkoordinatensystem

$$h_R = 1$$
$$h_\vartheta = R$$
$$h_\varphi = R \sin \vartheta$$

Metric Coefficients – Cylindrical and Spherical Coordinate System / Metrische Koeffizienten – Zylinder- und Kugelkoordinatensystem





Metric Coefficients and Vector Differential Line Elements / Metrische Koeffizienten und vektorielle differentielle Linienelemente

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$h_x = 1, \quad h_y = 1, \quad h_z = 1$$

$$\underline{\mathbf{dR}}_x = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_x\, h_x\, \mathrm{d}x = \underline{\mathbf{e}}_x\, \mathrm{d}x$$

$$\underline{\mathbf{dR}}_y = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_y\, h_y\, \mathrm{d}y = \underline{\mathbf{e}}_y\, \mathrm{d}y$$

$$\underline{\mathbf{dR}}_z = \underline{\mathbf{n}}\,\mathrm{d}R = \underline{\mathbf{e}}_z\, h_z\, \mathrm{d}z = \underline{\mathbf{e}}_z\, \mathrm{d}z$$

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$h_r = 1, \quad h_\varphi = r, \quad h_z = 1$$

$$\underline{\mathbf{dR}}_r = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_r\, h_r\, \mathrm{d}r = \underline{\mathbf{e}}_r\, \mathrm{d}r$$

$$\underline{\mathbf{dR}}_\varphi = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_\varphi\, h_\varphi\, \mathrm{d}\varphi = \underline{\mathbf{e}}_\varphi\, r\,\mathrm{d}\varphi$$

$$\underline{\mathbf{dR}}_z = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_z\, h_z\, \mathrm{d}z = \underline{\mathbf{e}}_z\, \mathrm{d}z$$

Spherical Coordinate System / Kugelkoordinatensystem

$$h_R = 1, \quad h_\vartheta = R, \quad h_\varphi = R\sin\vartheta$$

$$\underline{\mathbf{dR}}_R = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_R\, h_R\, \mathrm{d}R = \underline{\mathbf{e}}_R\, \mathrm{d}R$$

$$\underline{\mathbf{dR}}_\vartheta = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_\vartheta\, h_\vartheta\, \mathrm{d}\vartheta = \underline{\mathbf{e}}_\vartheta\, R\,\mathrm{d}\vartheta$$

$$\underline{\mathbf{dR}}_\varphi = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_\varphi\, h_\varphi\, \mathrm{d}\varphi = \underline{\mathbf{e}}_\varphi\, R\sin\vartheta\,\mathrm{d}\varphi$$

Metric Coefficients and Differential Volume and Surface Elements / Metrische Koeffizienten und differentielle Volumen- und Flächenelemente

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$h_x = 1, \quad h_y = 1, \quad h_z = 1$$

$$\mathrm{d}V = h_x\,\mathrm{d}x\,h_y\,\mathrm{d}y\,h_z\,\mathrm{d}z = h_x h_y h_z\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{yz} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z) h_y h_z\,\mathrm{d}y\,\mathrm{d}z = \underline{\mathbf{e}}_x\,\mathrm{d}y\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{xz} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x) h_x h_z\,\mathrm{d}x\,\mathrm{d}z = \underline{\mathbf{e}}_y\,\mathrm{d}x\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{xy} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y) h_x h_y\,\mathrm{d}x\,\mathrm{d}y = \underline{\mathbf{e}}_z\,\mathrm{d}x\,\mathrm{d}y$$

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$h_r = 1, \quad h_\varphi = r, \quad h_z = 1$$

$$\mathrm{d}V = h_r\,\mathrm{d}r\,h_\varphi\,\mathrm{d}\varphi\,h_z\,\mathrm{d}z = h_r h_\varphi h_z\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}y = r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{\varphi z} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_\varphi \times \underline{\mathbf{e}}_z) h_\varphi h_z\,\mathrm{d}\varphi\,\mathrm{d}z = \underline{\mathbf{e}}_r\,r\,\mathrm{d}y\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{rz} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_r) h_r h_z\,\mathrm{d}r\,\mathrm{d}z = \underline{\mathbf{e}}_\varphi\,\mathrm{d}r\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{r\varphi} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_r \times \underline{\mathbf{e}}_\varphi) h_r h_\varphi\,\mathrm{d}r\,\mathrm{d}\varphi = \underline{\mathbf{e}}_z\,r\,\mathrm{d}r\,\mathrm{d}\varphi$$

Spherical Coordinate System / Kugelkoordinatensystem

$$h_R = 1, \quad h_\vartheta = R, \quad h_\varphi = R\sin\vartheta$$

$$\mathrm{d}V = h_R\,\mathrm{d}R\,h_\vartheta\,\mathrm{d}\vartheta\,h_\varphi\,\mathrm{d}\varphi = h_R h_\vartheta h_\varphi\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi = R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$$

$$\underline{\mathbf{dS}}_{\vartheta\varphi} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_\vartheta \times \underline{\mathbf{e}}_\varphi) h_\vartheta h_\varphi\,\mathrm{d}\vartheta\,\mathrm{d}\varphi = \underline{\mathbf{e}}_R\,R^2\sin\vartheta\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$$

$$\underline{\mathbf{dS}}_{r\varphi} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_\varphi \times \underline{\mathbf{e}}_R) h_R h_\varphi\,\mathrm{d}R\,\mathrm{d}\varphi = \underline{\mathbf{e}}_\vartheta\,R\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\varphi$$

$$\underline{\mathbf{dS}}_{R\vartheta} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_R \times \underline{\mathbf{e}}_\vartheta) h_R h_\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta = \underline{\mathbf{e}}_\varphi\,R\,\mathrm{d}R\,\mathrm{d}\vartheta$$

Del (Nabla), Grad, Div, and Curl Operator in Cartesian Coordinate System / Nabla-, Grad-, Div- und Rot-Operator im Kartesischen Koordinatensystem

Del (Nabla) Operator / Nabla-Operator

$$\nabla = \underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z}$$

Gradient / Gradient

$$\mathrm{grad} = \nabla = \underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z}$$

Divergence / Divergenz

$$\mathrm{div} = \nabla \bullet = \left(\underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z} \right) \bullet$$

Curl / Rotation

$$\mathrm{curl} / \mathrm{rot} = \nabla \times = \left(\underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z} \right) \times$$

Del (Nabla) Operator in Orthogonal Curvilinear Coordinate System / Nabla-Operator im orthogonal krummlinigen Koordinatensystem

Del (Nabla) Operator / Nabla-Operator

$$\nabla = \underline{\mathbf{e}}_{\xi_1} \frac{1}{h_{\xi_1}} \frac{\partial}{\partial \xi_1} + \underline{\mathbf{e}}_{\xi_2} \frac{1}{h_{\xi_2}} \frac{\partial}{\partial \xi_2} + \underline{\mathbf{e}}_{\xi_3} \frac{1}{h_{\xi_3}} \frac{\partial}{\partial \xi_3}$$

$$= \sum_{i=1}^{3} \underline{\mathbf{e}}_{\xi_i} \frac{1}{h_{\xi_i}} \frac{\partial}{\partial \xi_i}$$

$$= \underline{\mathbf{e}}_{\xi_i} \frac{1}{h_{\xi_i}} \frac{\partial}{\partial \xi_i}$$

Generalized Curvilinear Coordinates / Verallgemeinerte krummlinige Koordinaten

$$\xi_1, \xi_2, \xi_3 \quad \text{or} \quad \xi_i, i = 1, 2, 3$$

The del Operator / Der Nabla-Operator ∇ is a Vector / ist ein Vektor

Vector-analytical Expressions in the Different Coordinate Systems / Vektoranalytische Ausdrücke in den verschiedenen Koordinatensystemen

	Cartesian Coordinates / Kartesische Koordinaten	Cylindrical Coordinates / Zylinderkoordinaten	Spherical Coordinates / Kugelkoordinaten
$\mathrm{d}\underline{\mathbf{R}}$	$\mathrm{d}x\,\underline{\mathbf{e}}_x + \mathrm{d}y\,\underline{\mathbf{e}}_y + \mathrm{d}z\,\underline{\mathbf{e}}_z$	$\mathrm{d}r\,\underline{\mathbf{e}}_r + r\mathrm{d}\varphi\,\underline{\mathbf{e}}_\varphi + \mathrm{d}z\,\underline{\mathbf{e}}_z$	$\mathrm{d}R\,\underline{\mathbf{e}}_R + R\mathrm{d}\vartheta\,\underline{\mathbf{e}}_\vartheta + R\sin\vartheta\,\mathrm{d}\varphi\,\underline{\mathbf{e}}_\varphi$
$\mathrm{grad}\,\Phi$ $= \nabla\Phi$	$\frac{\partial\Phi}{\partial x}\underline{\mathbf{e}}_x + \frac{\partial\Phi}{\partial y}\underline{\mathbf{e}}_y + \frac{\partial\Phi}{\partial z}\underline{\mathbf{e}}_z$	$\frac{\partial\Phi}{\partial r}\underline{\mathbf{e}}_r + \frac{1}{r}\frac{\partial\Phi}{\partial\varphi}\underline{\mathbf{e}}_\varphi + \frac{\partial\Phi}{\partial z}\underline{\mathbf{e}}_z$	$\frac{\partial\Phi}{\partial R}\underline{\mathbf{e}}_R + \frac{1}{R}\frac{\partial\Phi}{\partial\vartheta}\underline{\mathbf{e}}_\vartheta + \frac{1}{R\sin\vartheta}\frac{\partial\Phi}{\partial\varphi}\underline{\mathbf{e}}_\varphi$
$\mathrm{div}\,\underline{\mathbf{A}}$ $= \nabla\bullet\underline{\mathbf{A}}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{R^2}\frac{\partial(R^2A_R)}{\partial R} + \frac{1}{R\sin\vartheta}\frac{\partial(\sin\vartheta A_\vartheta)}{\partial\vartheta} + \frac{1}{R\sin\vartheta}\frac{\partial A_\varphi}{\partial\varphi}$
$\mathrm{curl}\,\underline{\mathbf{A}}$ $= \mathrm{rot}\,\underline{\mathbf{A}}$ $= \nabla\times\underline{\mathbf{A}}$	$\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\underline{\mathbf{e}}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\underline{\mathbf{e}}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\underline{\mathbf{e}}_z$	$\left[\frac{1}{r}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right]\underline{\mathbf{e}}_r + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right]\underline{\mathbf{e}}_\varphi + \frac{1}{r}\left[\frac{\partial(rA_\varphi)}{\partial r} - \frac{\partial A_r}{\partial\varphi}\right]\underline{\mathbf{e}}_z$	$\frac{1}{R\sin\vartheta}\left[\frac{\partial(\sin\vartheta A_\varphi)}{\partial\vartheta} - \frac{\partial A_\vartheta}{\partial\varphi}\right]\underline{\mathbf{e}}_R + \frac{1}{R}\left[\frac{1}{\sin\vartheta}\frac{\partial A_R}{\partial\varphi} - \frac{\partial(RA_\varphi)}{\partial R}\right]\underline{\mathbf{e}}_\vartheta + \frac{1}{R}\left[\frac{\partial(RA_\vartheta)}{\partial R} - \frac{\partial A_R}{\partial\vartheta}\right]\underline{\mathbf{e}}_\varphi$

Vector-Analytical Expressions in the Different Coordinate Systems / Vektoranalytische Ausdrücke in den verschiedenen Koordinatensystemen

	Cartesian Coordinates / Kartesische Koordinaten	Cylindrical Coordinates / Zylinderkoordinaten	Spherical Coordinates / Kugelkoordinaten
$\operatorname{div}\operatorname{grad}\Phi$ $= \nabla \bullet \nabla\Phi$ $= \nabla^2\Phi$ $= \Delta\Phi$	$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}$	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\varphi^2}+\frac{\partial^2\Phi}{\partial z^2}$	$\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial\Phi}{\partial R}\right)+\frac{1}{R^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\Phi}{\partial\vartheta}\right)$ $+\frac{1}{R^2\sin^2\vartheta}\left(\frac{\partial^2\Phi}{\partial\varphi^2}\right)$
$\operatorname{div}\operatorname{grad}\underline{\mathbf{A}}$ $=\nabla\bullet\nabla\underline{\mathbf{A}}$ $=\nabla^2\underline{\mathbf{A}}$ $=\Delta\underline{\mathbf{A}}$	$\Delta A_x\underline{\mathbf{e}}_x$ $+\Delta A_y\underline{\mathbf{e}}_y$ $+\Delta A_z\underline{\mathbf{e}}_z$	$\left[\Delta A_r-\frac{1}{r^2}A_r-\frac{2}{r^2}\frac{\partial A_\varphi}{\partial\varphi}\right]\underline{\mathbf{e}}_r$ $+\left[\Delta A_\varphi-\frac{1}{r^2}A_\varphi+\frac{2}{r^2}\frac{\partial A_r}{\partial\varphi}\right]\underline{\mathbf{e}}_\varphi$ $+\Delta A_z\underline{\mathbf{e}}_z$	$\left[\Delta A_R-\frac{2}{R^2}A_R-\frac{2\cot\vartheta}{R^2}A_\vartheta-\frac{2}{R^2}\frac{\partial A_\vartheta}{\partial\vartheta}-\frac{2}{R^2\sin\vartheta}\frac{\partial A_\varphi}{\partial\varphi}\right]\underline{\mathbf{e}}_R$ $+\left[\Delta A_\vartheta+\frac{2}{R^2}\frac{\partial A_R}{\partial\vartheta}-\frac{1}{R^2\sin^2\vartheta}A_\vartheta-\frac{2\cos\vartheta}{R^2\sin^2\vartheta}\frac{\partial A_\varphi}{\partial\varphi}\right]\underline{\mathbf{e}}_\vartheta$ $+\left[\Delta A_\varphi+\frac{2}{R^2\sin\vartheta}\frac{\partial A_R}{\partial\varphi}-\frac{1}{R^2\sin^2\vartheta}A_\varphi+\frac{2\cos\vartheta}{R^2\sin^2\vartheta}\frac{\partial A_\varphi}{\partial\varphi}\right]\underline{\mathbf{e}}_\varphi$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

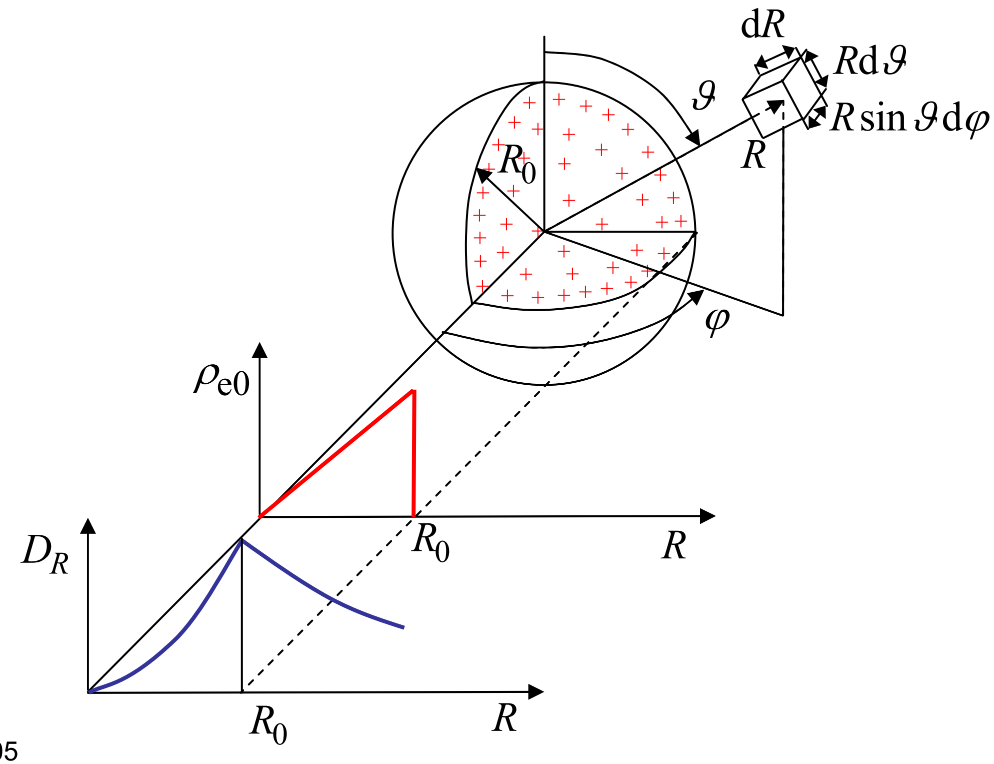
Consider the Electrostatic (ES) Case /
Betrachte den elektrostatischen Fall

$$\oiint_{S=\partial V} \underbrace{\underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})}}_{=D_R(\underline{\mathbf{R}})} \, \mathrm{d}S = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}}) \, \mathrm{d}V$$

Electric Charge Density /
Elektrische Raumladungsdichte

$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = \begin{cases} \rho_{\mathrm{e}0} \dfrac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases}$$

Radial Symmetry /
Radialsymmetrisch **!**



Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

$$\oiint_{S=\partial V} \underbrace{\underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\cdot\underline{\mathbf{n}}}_{=D_n(R)}}_{=D_R(R)} \mathrm{d}S = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\mathrm{d}V = Q_\mathrm{e}$$

$$\rho_\mathrm{e}(R) = \begin{cases} \rho_{\mathrm{e}0}\dfrac{R}{R_0} & R \le R_0 \\ 0 & R > R_0 \end{cases}$$

2 Cases / 2 Fälle

$0 \le R < R_0$

$$\oiint_{S=\partial V} D_R(R)\mathrm{d}S = D_R(R)\oiint_{S=\partial V}\mathrm{d}S = D_R(R)\int_{\vartheta'=0}^{\pi}\int_{\varphi'=0}^{2\pi} R^2\sin\vartheta\,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

$$\iiint_V \rho_\mathrm{e}(R)\mathrm{d}V = \int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\int_{R=0}^{R} \rho_\mathrm{e}(R)R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

$R > R_0$

$$\oiint_{S=\partial V} D_R(R)\mathrm{d}S = D_R(R)\oiint_{S=\partial V}\mathrm{d}S = D_R(R)\int_{\vartheta'=0}^{\pi}\int_{\varphi'=0}^{2\pi} R^2\sin\vartheta\,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

$$\iiint_V \rho_\mathrm{e}(R)\mathrm{d}V = \int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\int_{R=0}^{R_0} \rho_\mathrm{e}(R)R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

2 Cases / 2 Fälle

$R < R_0$

$$\oiint_{S=\partial V} D_R(R)\,\mathrm{d}S = D_R(R)\oiint_{S=\partial V}\mathrm{d}S$$
$$= D_R(R)\int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi} R^2\sin\vartheta\,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$
$$= D_R(R)R^2\underbrace{\int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\sin\vartheta\,\mathrm{d}\varphi\,\mathrm{d}\vartheta}_{=4\pi}$$
$$= D_R(R)4\pi R^2$$

$R > R_0$

$$\oiint_{S=\partial V} D_R(R)\,\mathrm{d}S = D_R(R)\oiint_{S=\partial V}\mathrm{d}S$$
$$= D_R(R)\int_{\vartheta'=0}^{\pi}\int_{\varphi'=0}^{2\pi} R^2\sin\vartheta\,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$
$$= D_R(R)R^2\underbrace{\int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\sin\vartheta\,\mathrm{d}\varphi\,\mathrm{d}\vartheta}_{=4\pi}$$
$$= D_R(R)4\pi R^2$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

2 Cases / 2 Fälle

$R < R_0$

$$\iiint_V \rho_\mathrm{e}(R)\mathrm{d}V = \int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\int_{R=0}^{R} \rho_\mathrm{e}(R)\,R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

$$= \rho_{\mathrm{e}0}\int_{R=0}^{R}\frac{R}{R_0}R^2\underbrace{\left[\int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\sin\vartheta\,\mathrm{d}\varphi\,\mathrm{d}\vartheta\right]}_{=4\pi}\mathrm{d}R$$

$$= \frac{4\pi\rho_{\mathrm{e}0}}{R_0}\int_{R=0}^{R}R^3\,\mathrm{d}R$$

$$= \frac{4\pi\rho_{\mathrm{e}0}}{R_0}\left.\frac{R^4}{4}\right|_{R=0}^{R}$$

$$= \frac{4\pi\rho_{\mathrm{e}0}}{R_0}\frac{R^4}{4}$$

$$= \pi\rho_{\mathrm{e}0}\frac{R^4}{R_0}$$

$R > R_0$

$$\iiint_V \rho_\mathrm{e}(R)\mathrm{d}V = \int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\int_{R=0}^{R_0} \rho_\mathrm{e}(R)\,R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

$$= \rho_{\mathrm{e}0}\int_{R=0}^{R_0}\frac{R}{R_0}R^2\underbrace{\left[\int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\sin\vartheta\,\mathrm{d}\varphi\,\mathrm{d}\vartheta\right]}_{=4\pi}\mathrm{d}R$$

$$= \frac{4\pi\rho_{\mathrm{e}0}}{R_0}\int_{R=0}^{R_0}R^3\,\mathrm{d}R'$$

$$= \frac{4\pi\rho_{\mathrm{e}0}}{R_0}\left.\frac{R^4}{4}\right|_{R'=0}^{R_0}$$

$$= \frac{4\pi\rho_{\mathrm{e}0}}{R_0}\frac{R_0^4}{4}$$

$$= \pi\rho_{\mathrm{e}0}\,R_0^3$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

2 Cases / 2 Fälle

$R < R_0$

$$\underbrace{\oiint_{S=\partial V} D_R(R)\,\mathrm{d}S}_{=D_R(R)4\pi R^2} = \underbrace{\iiint_V \rho_\mathrm{e}(R)\mathrm{d}V}_{=\pi\rho_{\mathrm{e}0}\frac{R^4}{R_0}}$$

$$D_R(R)4\pi R^2 = \pi\rho_{\mathrm{e}0}\frac{R^4}{R_0}$$

$$D_R(R) = \frac{\pi\rho_{\mathrm{e}0}\frac{R^4}{R_0}}{4\pi R^2}$$

$$= \frac{\rho_{\mathrm{e}0}}{4}\frac{R^2}{R_0}$$

$R > R_0$

$$\underbrace{\oiint_{S=\partial V} D_R(R)\,\mathrm{d}S}_{=D_R(R)4\pi R^2} = \underbrace{\iiint_V \rho_\mathrm{e}(R)\mathrm{d}V}_{=\pi\rho_{\mathrm{e}0} R_0^3}$$

$$D_R(R)4\pi R^2 = \pi\rho_{\mathrm{e}0} R_0^3$$

$$D_R(R) = \frac{\pi\rho_{\mathrm{e}0} R_0^3}{4\pi R^2}$$

$$= \frac{\rho_{\mathrm{e}0}}{4}\frac{R_0^3}{R^2}$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

Electric Charge Density /
Elektrische Raumladungsdichte

$$\rho_{\mathrm{e}}(\underline{\mathbf{R}}) = \begin{cases} \rho_{\mathrm{e}0} \dfrac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases}$$

Radial Symmetry /
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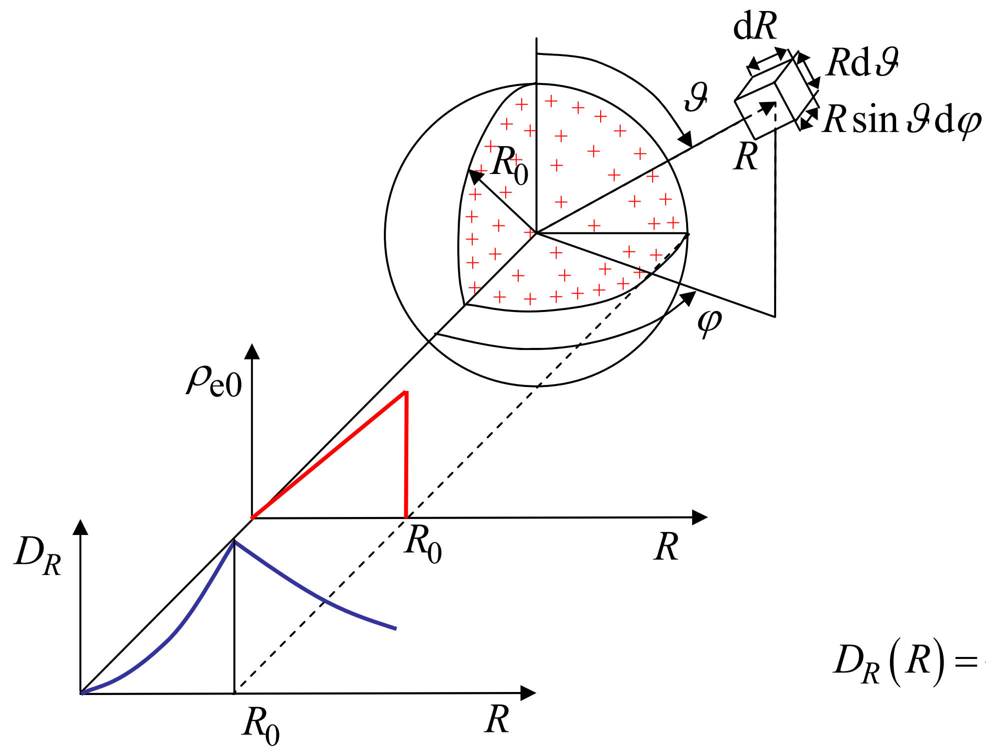


$$D_R(R) = \frac{\rho_{\mathrm{e}0}}{4} \begin{cases} \dfrac{R^2}{R_0} & R < R_0 \\ \dfrac{R_0^3}{R^2} & R > R_0 \end{cases}$$

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Electrostatic Potential / Elektrostatisches Potential

Integral Form / Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{R}} = 0$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{S}} = \iiint \rho_e(\underline{\mathbf{R}}) \mathrm{d}V$$

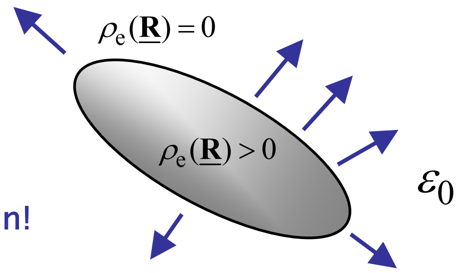
Differential Form / Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_e(\underline{\mathbf{R}})$$

Vacuum / Vakuum

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \, \underline{\mathbf{E}}(\underline{\mathbf{R}})$$



Unknown! / Unbekannt!

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}), \underline{\mathbf{D}}(\underline{\mathbf{R}}) = ?$$

Given, Prescribed! / Gegeben, vorgeschrieben!

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = 0, \quad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = 0, \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_e$$

Standard Way: Method of Potentials / Standard Weg: Methode der Potentiale

Electrostatics / Elektrostatik: $\Phi_e(\underline{\mathbf{R}})$ [V] Scalar Electrostatic Potential / Skalares elektrostatisches Potential

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Electrostatic Potential / Elektrostatisches Potential

Integral Form / Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dR}} = 0$$

Differential Form / Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

Irrotational Field can be always Represented by a Gradient Field / Rotationsfreies Feld kann immer als Gradientenfeld dargestellt werden

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}})$$

Electrostatic Potential / Elektrostatisches Potential

$$\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) \; [\mathrm{V}]$$

because / weil

$$\begin{aligned} \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) &= \nabla \times \left[-\nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}})\right] \\ &= -\nabla \times \nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}}) \\ &= \underline{\mathbf{0}} \end{aligned}$$

In General / Im allgemeinen

$$\nabla \times \nabla \equiv \underline{\mathbf{0}}$$

General Vector Analytic Property / Allgemeine Vektoridentität

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Electrostatic Potential / Elektrostatisches Potential

Integral Form / Integralform Differentialform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \cdot \mathbf{d}\underline{\mathbf{R}} = 0$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \mathbf{d}\underline{\mathbf{S}} = \iiint \rho_e(\underline{\mathbf{R}}) \mathrm{d}V$$

Differential Form /

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

$$\nabla \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_e(\underline{\mathbf{R}})$$

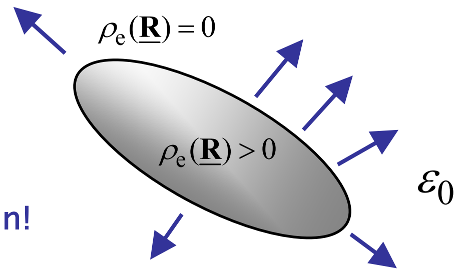
Unknown! / Unbekannt!

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}), \underline{\mathbf{D}}(\underline{\mathbf{R}}) = ?$$

Given, Prescribed! / Gegeben, vorgeschrieben!

Vacuum / Vakuum

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \, \underline{\mathbf{E}}(\underline{\mathbf{R}})$$



$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = 0, \quad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = 0, \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_e$$

Standard Way: Method of Potentials / Standard Weg: Methode der Potentiale

Electrostatics / Elektrostatik: $\Phi_e(\underline{\mathbf{R}})$ [V] Scalar Electrostatic Potential / Skalares elektrostatisches Potential

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Electrostatic Potential / Elektrostatisches Potential

Integral Form / Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{R}} = 0$$

Differential Form / Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

Irrotational Field can be always Represented by a Gradient Field /
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$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}})$$

Electrostatic Potential / Elektrostatisches Potential

$$\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) \; [\mathrm{V}]$$

because / weil

$$\begin{aligned} \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) &= \nabla \times \left[-\nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}}) \right] \\ &= -\nabla \times \nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}}) \\ &= \underline{\mathbf{0}} \end{aligned}$$

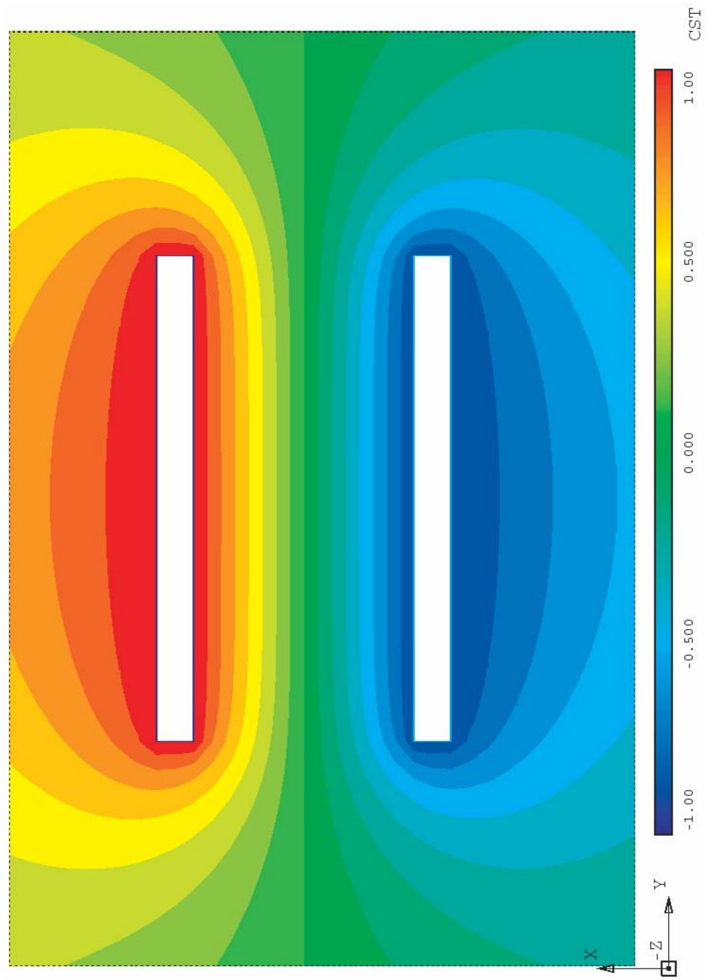
In General / Im allgemeinen

$$\nabla \times \nabla \equiv \underline{\mathbf{0}}$$

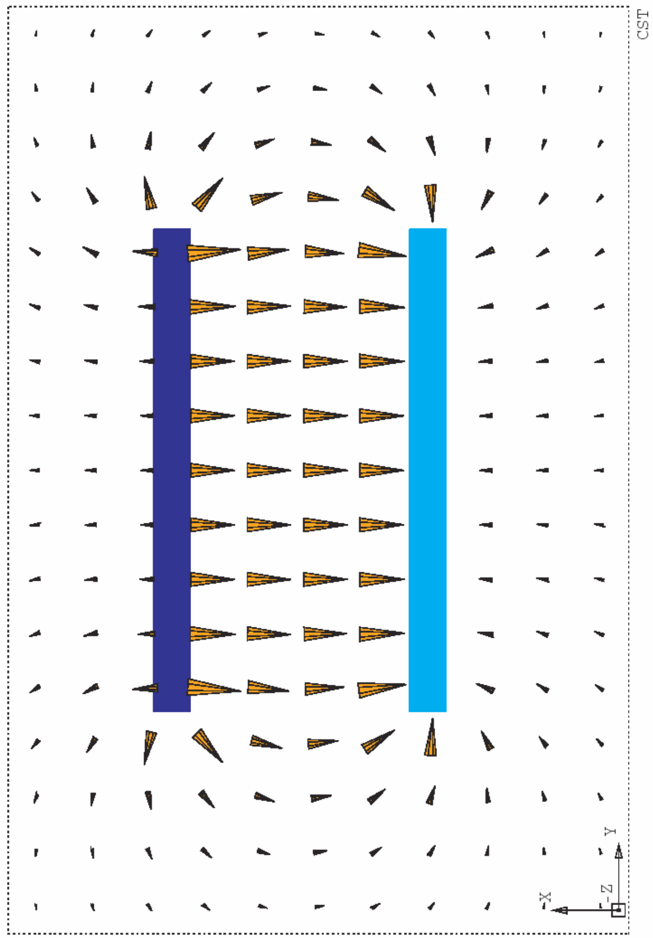
General Vector Analytic Property / Allgemeine Vektoridentität

Electrostatic Field Problem – Example: Parallel Plate Capacitor / Elektrostatisches Feldproblem – Beispiel: Paralleler Plattenkondensator

Scalar Field: Electrostatic Potential /
Skalarfeld: Elektrostatisches Potenzial



Vector Field: Electrostatic Field Strength /
Vektorfeld: Elektrostatische Feldstärke



End of Lecture 4 /
Ende der 4. Vorlesung