Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

5th Lecture / 5. Vorlesung

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Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Electrostatic Potential / Elektrostatisches Potential

Integral Form / Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{R}} = 0$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{S}} = \iiint \rho_e(\underline{\mathbf{R}}) \mathrm{d}V$$

Differential Form / Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_e(\underline{\mathbf{R}})$$

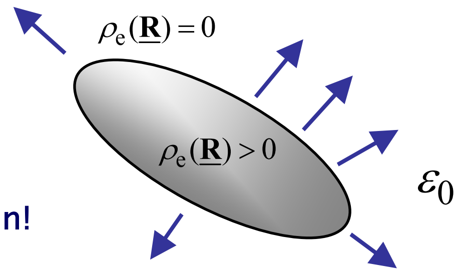
Unknown! / Unbekannt!

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}), \underline{\mathbf{D}}(\underline{\mathbf{R}}) = ?$$

Given, Prescribed! / Gegeben, vorgeschrieben!

Vacuum / Vakuum

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \, \underline{\mathbf{E}}(\underline{\mathbf{R}})$$



$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = 0, \quad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = 0, \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_e$$

Standard Way: Method of Potentials / Standard Weg: Methode der Potentiale

Electrostatics / Elektrostatik: $\Phi_e(\underline{\mathbf{R}})$ [V] Scalar Electrostatic Potential / Skalares elektrostatisches Potential

Del (Nabla), Grad, Div, and Curl Operator in Cartesian Coordinate System / Nabla-, Grad-, Div- und Rot-Operator im Kartesischen Koordinatensystem

Del (Nabla) Operator / Nabla-Operator

$$\nabla = \underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z}$$

Gradient / Gradient

$$\mathrm{grad} = \nabla = \underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z}$$

Divergence / Divergenz

$$\mathrm{div} = \nabla \bullet = \left(\underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z} \right) \bullet$$

Curl / Rotation

$$\mathrm{curl} / \mathrm{rot} = \nabla \times = \left(\underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z} \right) \times$$

Del (Nabla) Operator in Orthogonal Curvilinear Coordinate System / Nabla-Operator im orthogonal krummlinigen Koordinatensystem

Del (Nabla) Operator / Nabla-Operator

$$\nabla = \underline{\mathbf{e}}_{\xi_1} \frac{1}{h_{\xi_1}} \frac{\partial}{\partial \xi_1} + \underline{\mathbf{e}}_{\xi_2} \frac{1}{h_{\xi_2}} \frac{\partial}{\partial \xi_2} + \underline{\mathbf{e}}_{\xi_3} \frac{1}{h_{\xi_3}} \frac{\partial}{\partial \xi_3}$$

$$= \sum_{i=1}^{3} \underline{\mathbf{e}}_{\xi_i} \frac{1}{h_{\xi_i}} \frac{\partial}{\partial \xi_i}$$

$$= \underline{\mathbf{e}}_{\xi_i} \frac{1}{h_{\xi_i}} \frac{\partial}{\partial \xi_i}$$

Generalized Curvilinear Coordinates / Verallgemeinerte krummlinige Koordinaten

$$\xi_1, \xi_2, \xi_3 \quad \text{or} \quad \xi_i, i = 1, 2, 3$$

The del Operator / Der Nabla-Operator ∇ is a Vector / ist ein Vektor

Vector-analytical Expressions in the Different Coordinate Systems / Vektoranalytische Ausdrücke in den verschiedenen Koordinatensystemen

	Cartesian Coordinates / Kartesische Koordinaten	Cylindrical Coordinates / Zylinderkoordinaten	Spherical Coordinates / Kugelkoordinaten
$\underline{\mathbf{dR}}$	$\mathrm{d}x\underline{\mathbf{e}}_x + \mathrm{d}y\underline{\mathbf{e}}_y + \mathrm{d}z\underline{\mathbf{e}}_z$	$\mathrm{d}r\underline{\mathbf{e}}_r + r\mathrm{d}\varphi\underline{\mathbf{e}}_\varphi + \mathrm{d}z\underline{\mathbf{e}}_z$	$\mathrm{d}R\underline{\mathbf{e}}_R + R\mathrm{d}\vartheta\underline{\mathbf{e}}_\vartheta + R\sin\vartheta\,\mathrm{d}\varphi\underline{\mathbf{e}}_\varphi$
$\mathrm{grad}\,\Phi$ $= \nabla\Phi$	$\frac{\partial\Phi}{\partial x}\underline{\mathbf{e}}_x + \frac{\partial\Phi}{\partial y}\underline{\mathbf{e}}_y + \frac{\partial\Phi}{\partial z}\underline{\mathbf{e}}_z$	$\frac{\partial\Phi}{\partial r}\underline{\mathbf{e}}_r + \frac{1}{r}\frac{\partial\Phi}{\partial\varphi}\underline{\mathbf{e}}_\varphi + \frac{\partial\Phi}{\partial z}\underline{\mathbf{e}}_z$	$\frac{\partial\Phi}{\partial R}\underline{\mathbf{e}}_R + \frac{1}{R}\frac{\partial\Phi}{\partial\vartheta}\underline{\mathbf{e}}_\vartheta + \frac{1}{R\sin\vartheta}\frac{\partial\Phi}{\partial\varphi}\underline{\mathbf{e}}_\varphi$
$\mathrm{div}\,\underline{\mathbf{A}}$ $= \nabla\bullet\underline{\mathbf{A}}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r}\frac{\partial(r A_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{R^2}\frac{\partial(R^2 A_R)}{\partial R} + \frac{1}{R\sin\vartheta}\frac{\partial(\sin\vartheta A_\vartheta)}{\partial\vartheta} + \frac{1}{R\sin\vartheta}\frac{\partial A_\varphi}{\partial\varphi}$
$\mathrm{curl}\underline{\mathbf{A}}$ $= \mathrm{rot}\,\underline{\mathbf{A}}$ $= \nabla\times\underline{\mathbf{A}}$	$\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\underline{\mathbf{e}}_x$ $+\left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\underline{\mathbf{e}}_y$ $+\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\underline{\mathbf{e}}_z$	$\left[\frac{1}{r}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right]\underline{\mathbf{e}}_r$ $+\left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right]\underline{\mathbf{e}}_\varphi$ $+\frac{1}{r}\left[\frac{\partial(r A_\varphi)}{\partial r} - \frac{\partial A_r}{\partial\varphi}\right]\underline{\mathbf{e}}_z$	$\frac{1}{R\sin\vartheta}\left[\frac{\partial(\sin\vartheta A_\varphi)}{\partial\vartheta} - \frac{\partial A_\vartheta}{\partial\varphi}\right]\underline{\mathbf{e}}_R$ $+\frac{1}{R}\left[\frac{1}{\sin\vartheta}\frac{\partial A_R}{\partial\varphi} - \frac{\partial(R A_\varphi)}{\partial R}\right]\underline{\mathbf{e}}_\vartheta$ $+\frac{1}{R}\left[\frac{\partial(R A_\vartheta)}{\partial R} - \frac{\partial A_R}{\partial\vartheta}\right]\underline{\mathbf{e}}_\varphi$

Vector-Analytical Expressions in the Different Coordinate Systems / Vektoranalytische Ausdrücke in den verschiedenen Koordinatensystemen

	Cartesian Coordinates / Kartesische Koordinaten	Cylindrical Coordinates / Zylinderkoordinaten	Spherical Coordinates / Kugelkoordinaten
$\operatorname{div}\operatorname{grad}\Phi = \nabla\bullet\nabla\Phi = \nabla^2\Phi = \Delta\Phi$	$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}$	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\varphi^2}+\frac{\partial^2\Phi}{\partial z^2}$	$\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial\Phi}{\partial R}\right)+\frac{1}{R^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\Phi}{\partial\vartheta}\right)$ $+\frac{1}{R^2\sin^2\vartheta}\left(\frac{\partial^2\Phi}{\partial\varphi^2}\right)$
$\operatorname{div}\operatorname{grad}\underline{\mathbf{A}} = \nabla\bullet\nabla\underline{\mathbf{A}} = \nabla^2\underline{\mathbf{A}} = \Delta\underline{\mathbf{A}}$	$\Delta A_x\underline{\mathbf{e}}_x$ $+\Delta A_y\underline{\mathbf{e}}_y$ $+\Delta A_z\underline{\mathbf{e}}_z$	$\left[\Delta A_r-\frac{1}{r^2}A_r-\frac{2}{r^2}\frac{\partial A_\varphi}{\partial\varphi}\right]\underline{\mathbf{e}}_r$ $+\left[\Delta A_\varphi-\frac{1}{r^2}A_\varphi+\frac{2}{r^2}\frac{\partial A_r}{\partial\varphi}\right]\underline{\mathbf{e}}_\varphi$ $+\Delta A_z\underline{\mathbf{e}}_z$	$\left[\Delta A_R-\frac{2}{R^2}A_R-\frac{2\cot\vartheta}{R^2}A_\vartheta-\frac{2}{R^2}\frac{\partial A_\vartheta}{\partial\vartheta}-\frac{2}{R^2\sin\vartheta}\frac{\partial A_\varphi}{\partial\varphi}\right]\underline{\mathbf{e}}_R$ $+\left[\Delta A_\vartheta+\frac{2}{R^2}\frac{\partial A_R}{\partial\vartheta}-\frac{1}{R^2\sin^2\vartheta}A_\vartheta-\frac{2\cos\vartheta}{R^2\sin^2\vartheta}\frac{\partial A_\varphi}{\partial\varphi}\right]\underline{\mathbf{e}}_\vartheta$ $+\left[\Delta A_\varphi+\frac{2}{R^2\sin\vartheta}\frac{\partial A_R}{\partial\varphi}-\frac{1}{R^2\sin^2\vartheta}A_\varphi+\frac{2\cos\vartheta}{R^2\sin^2\vartheta}\frac{\partial A_\vartheta}{\partial\varphi}\right]\underline{\mathbf{e}}_\varphi$

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Electrostatic Potential / Elektrostatisches Potential

Integral Form /
Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dR}} = 0$$

Differential Form /
Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

Irrotational Field can be always Represented by a Gradient Field /
Rotationsfreies Feld kann immer als Gradientenfeld dargestellt werden

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}})$$

Electrostatic Potential /
Elektrostatisches Potential

$$\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) \; [\mathrm{V}]$$

because / weil

$$\begin{aligned} \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) &= \nabla \times \left[-\nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}}) \right] \\ &= -\nabla \times \nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}}) \\ &= \underline{\mathbf{0}} \end{aligned}$$

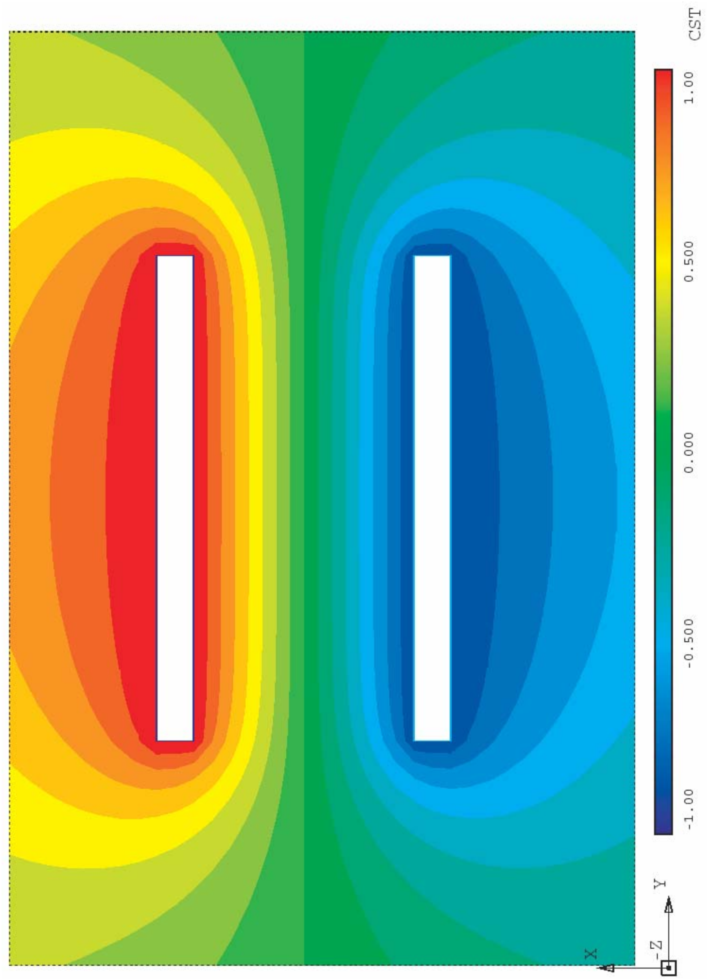
In General /
Im allgemeinen

$$\nabla \times \nabla \equiv \underline{\mathbf{0}}$$

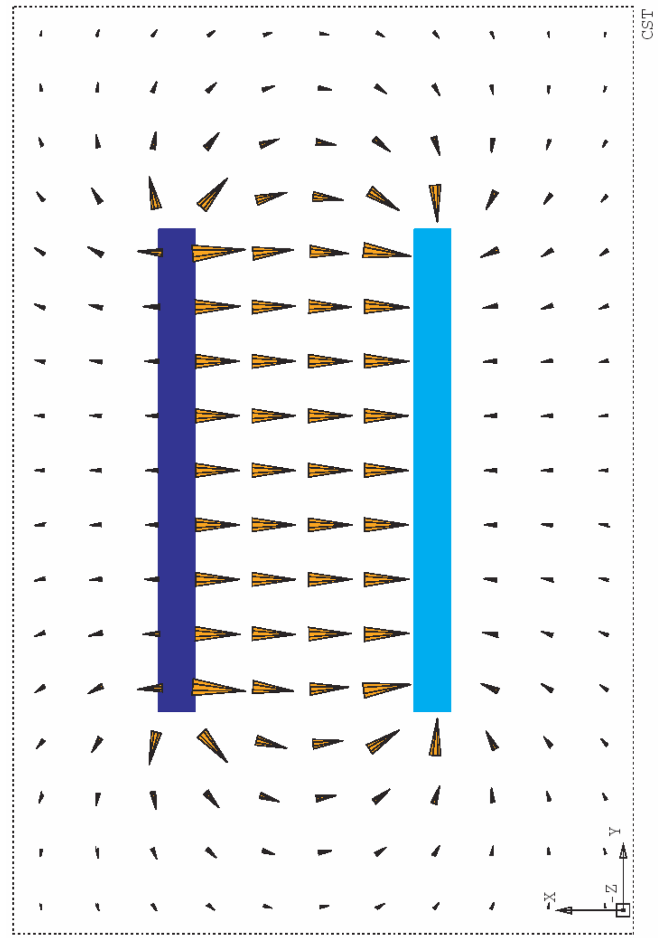
General Vector Analytic Property / Allgemeine Vektoridentität

Electrostatic Field Problem – Example: Parallel Plate Capacitor / Elektrostatisches Feldproblem – Beispiel: Paralleler Plattenkondensator

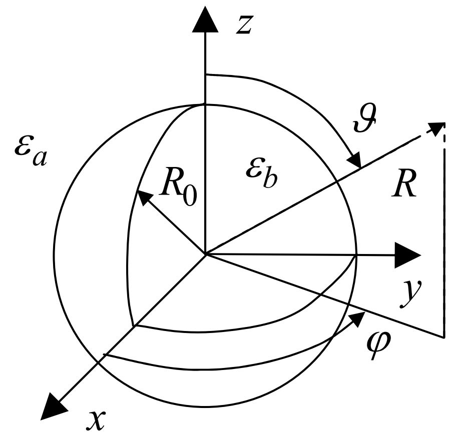
Scalar Field: Electrostatic Potential /
Skalarfeld: Elektrostatisches Potenzial



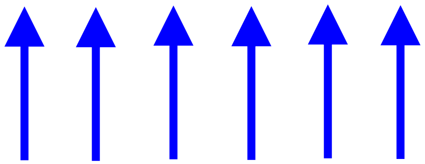
Vector Field: Electrostatic Field Strength /
Vektorfeld: Elektrostatische Feldstärke



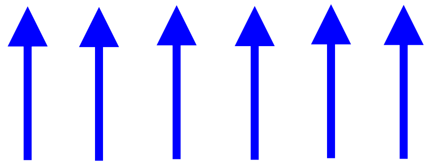
Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (1)



$$\underline{\mathbf{E}}_0 = E_0 \underline{\mathbf{e}}_z$$



$$\underline{\mathbf{E}}_0 = E_0 \underline{\mathbf{e}}_z$$



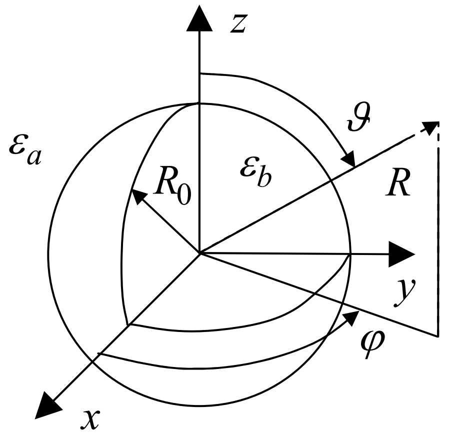
$$\varepsilon(\underline{\mathbf{R}}) = \varepsilon_a$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = E_0 \hat{\mathbf{E}}_0$$

$$\varepsilon(\underline{\mathbf{R}}) = \begin{cases} \varepsilon_b & 0 < R \le R_0 \\ \varepsilon_a & R > R_0 \end{cases}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = E_0 \underline{\mathbf{e}}_z + \underline{\mathbf{E}}_b(\underline{\mathbf{R}})$$

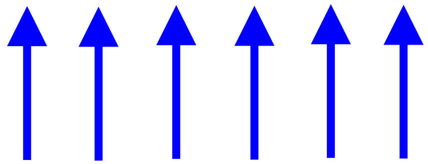
Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (2)



$$\varepsilon(\underline{\mathbf{R}}) = \begin{cases} \varepsilon_b & 0 < R \le R_0 \\ \varepsilon_a & R > R_0 \end{cases}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = E_0 \underline{\mathbf{e}}_z + \underline{\mathbf{E}}_b(\underline{\mathbf{R}})$$

$$\underline{\mathbf{E}}_0 = E_0 \underline{\mathbf{e}}_z$$



$$\Phi_\mathrm{e}(\underline{\mathbf{R}}) = \begin{cases} -E_0 \beta R \cos\vartheta & 0 < R \le R_0 \\ -E_0 \left[1 - \dfrac{\alpha}{R^3}\right] R \cos\vartheta & R > R_0 \end{cases}$$

$$\alpha = \frac{\varepsilon_b - \varepsilon_a}{\varepsilon_b + 2\varepsilon_a} R_0^3$$

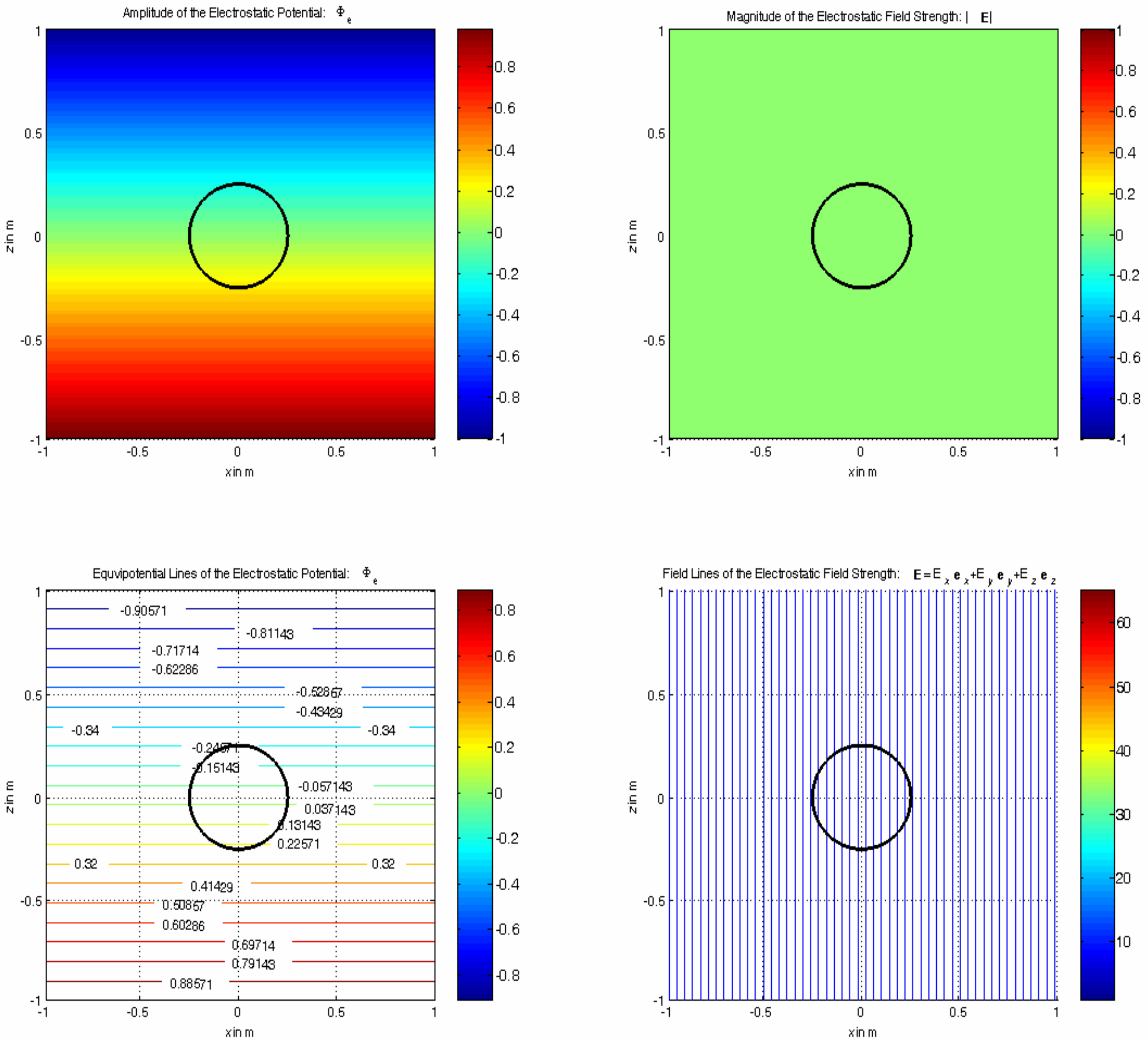
$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \begin{cases} E_0 \beta \left[\cos\vartheta \underline{\mathbf{e}}_R - \cos\vartheta \underline{\mathbf{e}}_\vartheta\right] & 0 < R \le R_0 \\ E_0 \left[\left(1 - \dfrac{2\alpha}{R^3}\right) \cos\vartheta \underline{\mathbf{e}}_R - \left(1 - \dfrac{\alpha}{R^3}\right) \sin\vartheta \underline{\mathbf{e}}_\vartheta\right] & R > R_0 \end{cases}$$

$$\beta = \frac{3\varepsilon_a}{\varepsilon_b + 2\varepsilon_a}$$

Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (3)

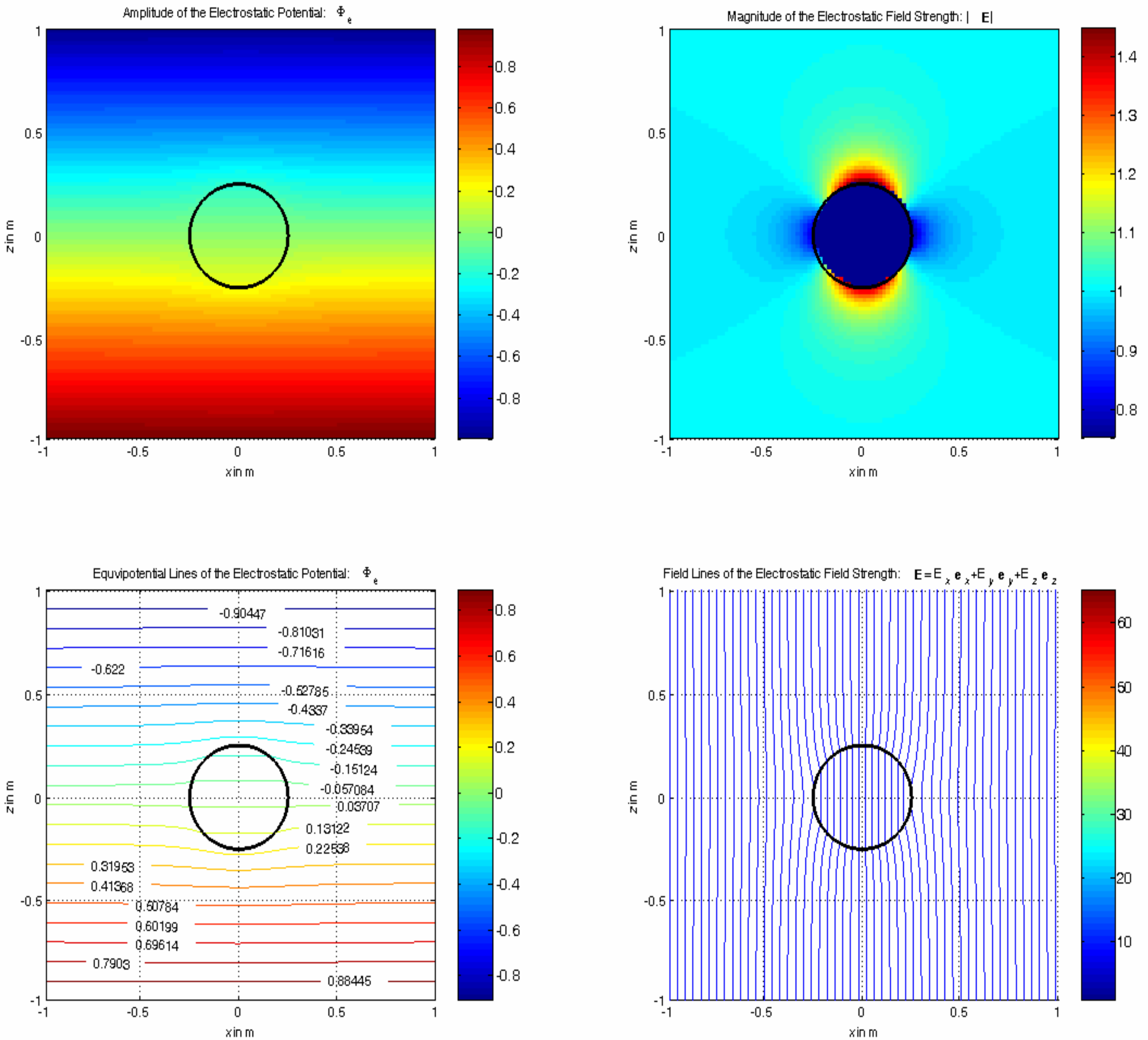
$$\varepsilon_a = \varepsilon_0$$

$$\varepsilon_b = \varepsilon_0$$



Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (4)

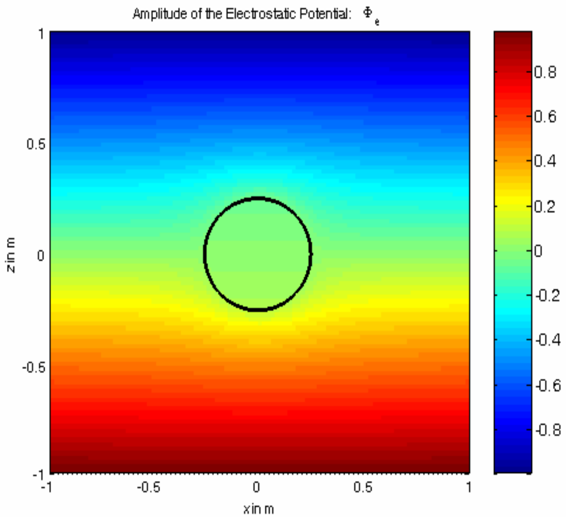
$$\varepsilon_a = \varepsilon_0$$
$$\varepsilon_b = 2\varepsilon_0$$

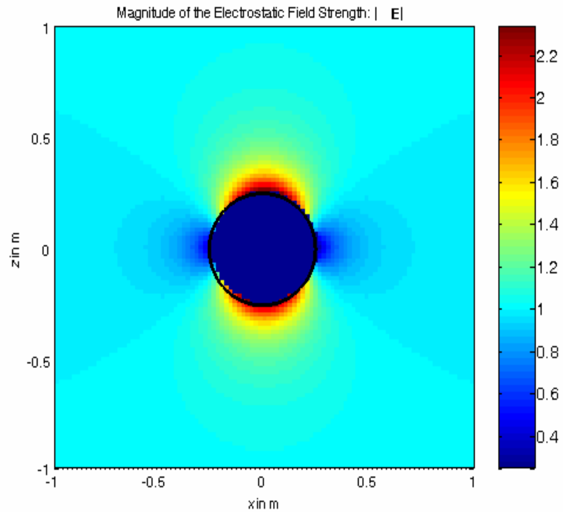


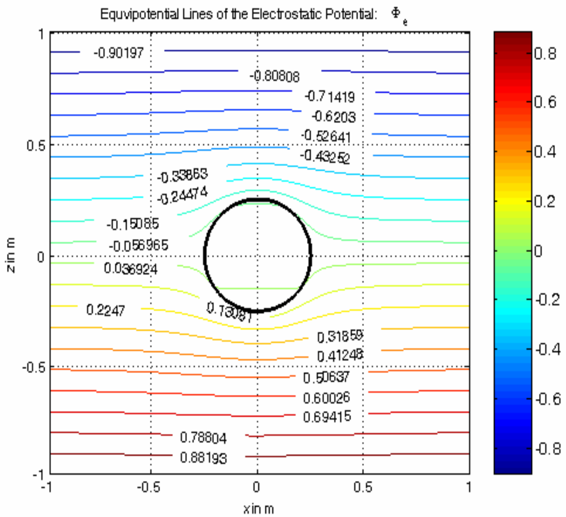
Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (5)

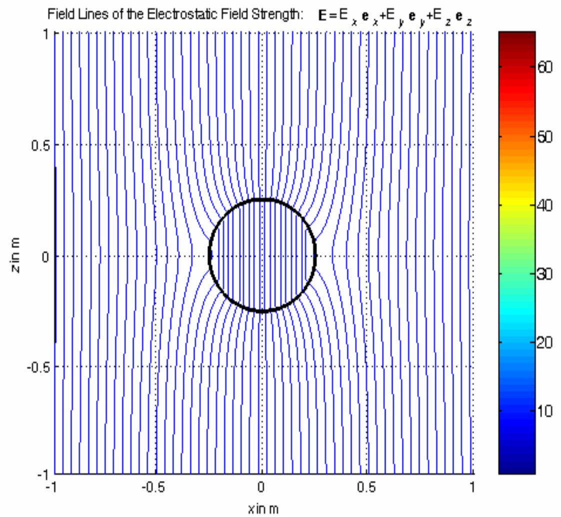
$\varepsilon_a = \varepsilon_0$

$\varepsilon_b = 10\varepsilon_0$



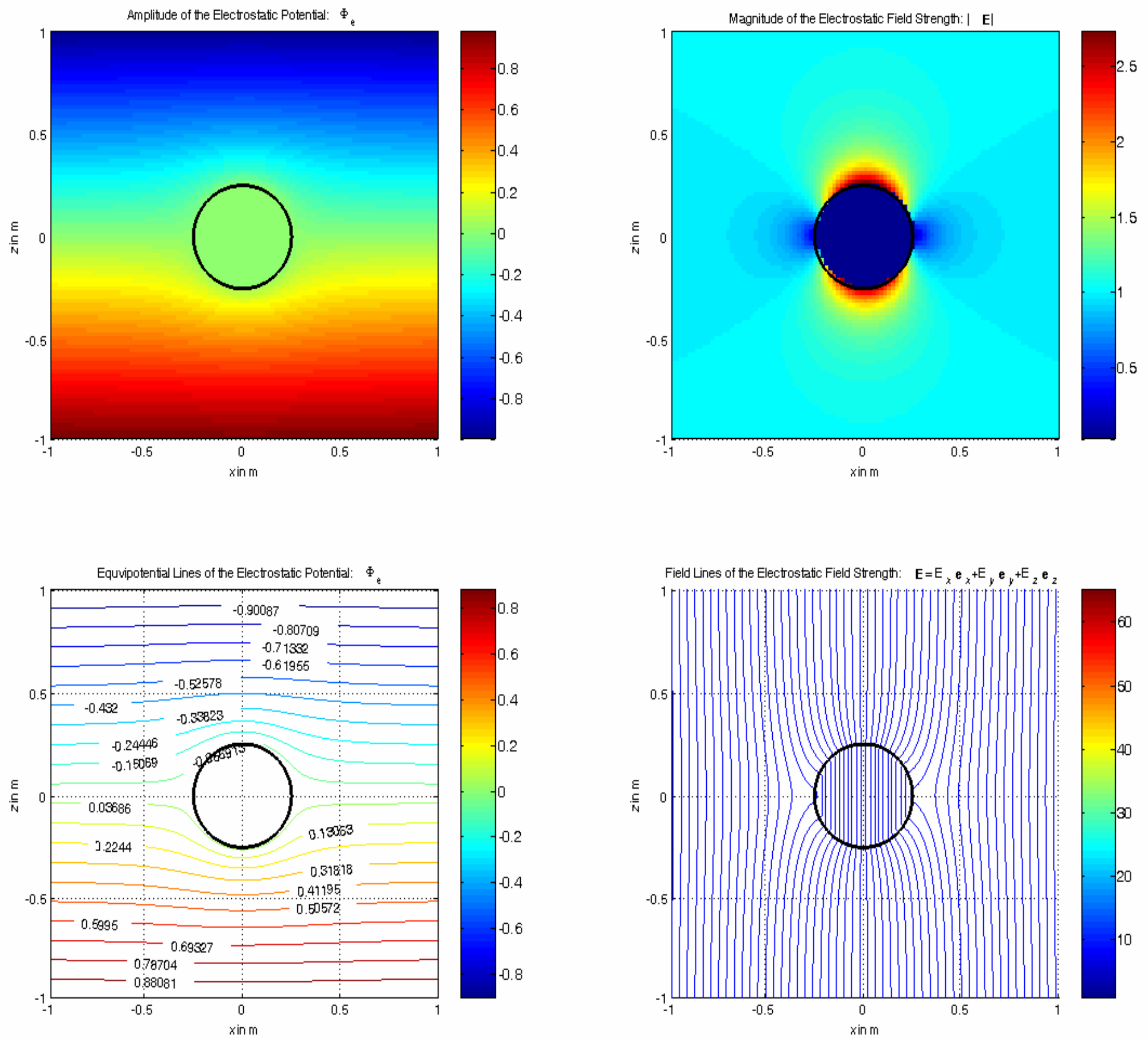






Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (6/1)

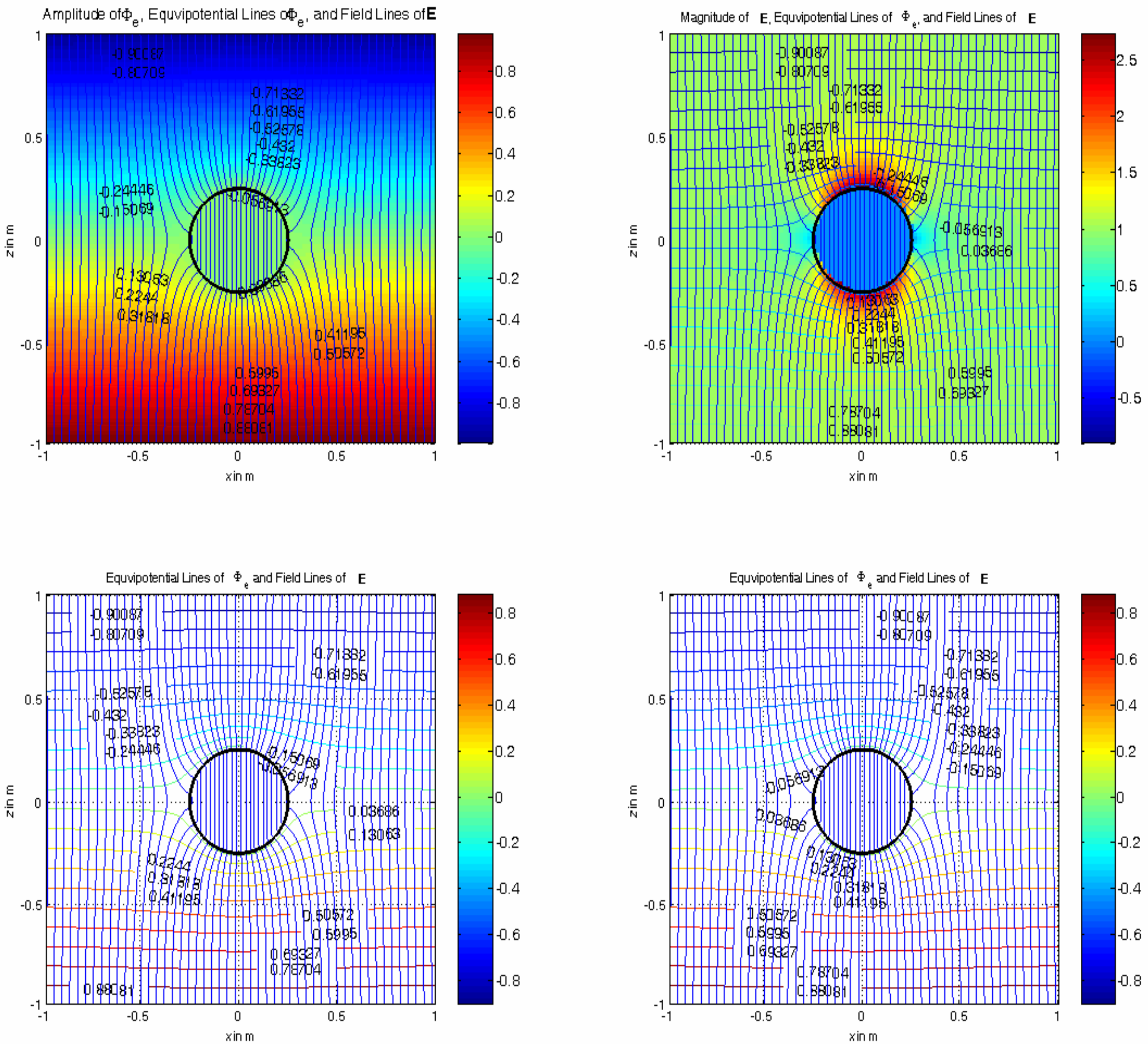
$$\varepsilon_a = \varepsilon_0$$
$$\varepsilon_b = 100\varepsilon_0$$



Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (6/2)

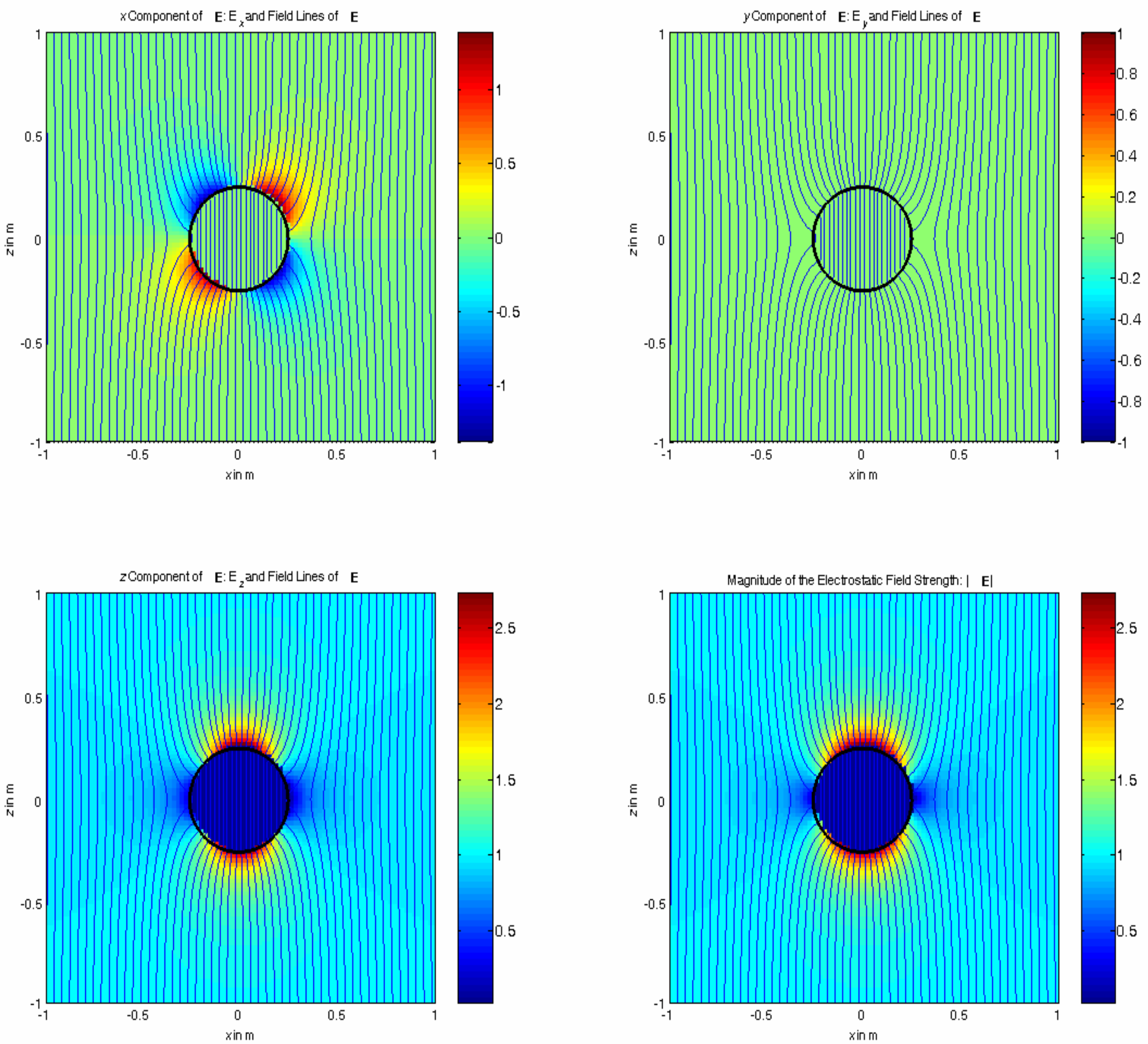
$\varepsilon_a = \varepsilon_0$

$\varepsilon_b = 100\varepsilon_0$



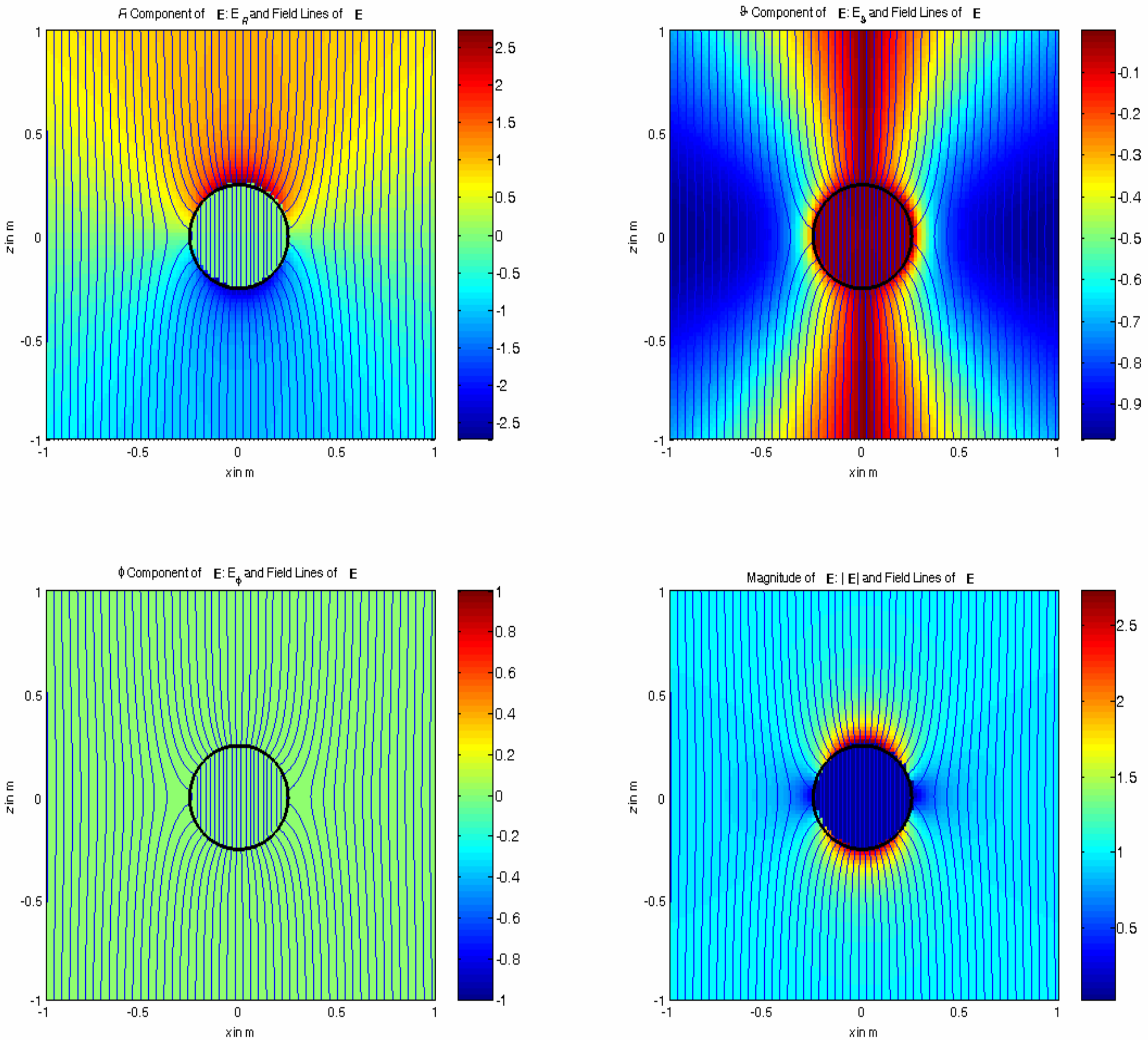
Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (6/3)

$$\varepsilon_a = \varepsilon_0$$
$$\varepsilon_b = 100\varepsilon_0$$



Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (6/4)

$$\varepsilon_a = \varepsilon_0$$
$$\varepsilon_b = 100\varepsilon_0$$



Electrostatic (ES) Fields – Poisson and Laplace Equation / Elektrostatische (ES) Felder – Poisson– und Laplace–Gleichung (1)

Differential Form / Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$
$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}})$$
$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_{\mathrm{e}}(\underline{\mathbf{R}})$$

Vacuum / Vakuum

$$\begin{aligned}\underline{\mathbf{D}}(\underline{\mathbf{R}}) &= \varepsilon_0\, \underline{\mathbf{E}}(\underline{\mathbf{R}}) \\ &= -\varepsilon_0 \nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}})\end{aligned}$$

because / weil

$$\begin{aligned}\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) &= \varepsilon_0 \nabla \bullet \underline{\mathbf{E}}(\underline{\mathbf{R}}) \\ &= -\varepsilon_0 \nabla \bullet \nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}}) \\ &= \rho_{\mathrm{e}}(\underline{\mathbf{R}})\end{aligned}$$

or / oder

$$\underbrace{\nabla \bullet \nabla}_{\nabla^2 = \Delta} \Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = \begin{cases} -\dfrac{\rho_{\mathrm{e}}(\underline{\mathbf{R}})}{\varepsilon_0} & \text{for / für} \quad \rho_{\mathrm{e}}(\underline{\mathbf{R}}) \neq 0 \\ 0 & \text{for / für} \quad \rho_{\mathrm{e}}(\underline{\mathbf{R}}) = 0 \end{cases}$$

Poisson Equation / Poisson-Gleichung (for $\rho_{\mathrm{e}}(\underline{\mathbf{R}}) \neq 0$)

Laplace Equation / Laplace-Gleichung (for $\rho_{\mathrm{e}}(\underline{\mathbf{R}}) = 0$)

Laplace Operator / Laplace–Operator

$$\nabla \bullet \nabla = \nabla^2 = \Delta$$

Electrostatic (ES) Fields – Poisson and Laplace Equation / Elektrostatische (ES) Felder – Poisson- und Laplace-Gleichung (2)

$$\underbrace{\nabla \bullet \nabla}_{\nabla^2 = \Delta} \Phi_\mathrm{e}(\underline{\mathbf{R}}) = \begin{cases} -\dfrac{\rho_\mathrm{e}(\underline{\mathbf{R}})}{\varepsilon_0} & \text{for / für} \quad \rho_\mathrm{e}(\underline{\mathbf{R}}) \neq 0 \quad \text{Poisson Equation / Poisson-Gleichung} \\ 0 & \text{for / für} \quad \rho_\mathrm{e}(\underline{\mathbf{R}}) = 0 \quad \text{Laplace Equation / Laplace-Gleichung} \end{cases}$$

Laplace Operator / Laplace-Operator $\quad \nabla \bullet \nabla = \nabla^2 = \Delta$

Laplace Operator in Cartesian Coordinates / Laplace-Operator in Kartesischen Koordinaten

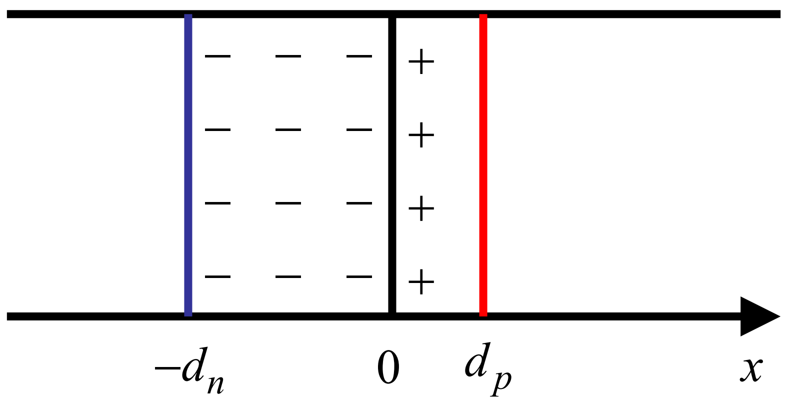
$$\begin{aligned}
\nabla \bullet \nabla &= \left(\underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z} \right) \bullet \left(\underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z} \right) \\
&= \underline{\mathbf{e}}_{x_i} \frac{\partial}{\partial x_i} \bullet \underline{\mathbf{e}}_{x_j} \frac{\partial}{\partial x_j} \\
&= \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{\delta_{ij}} \\
&= \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_i} \\
&= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \nabla^2 = \Delta
\end{aligned}$$

Electrostatic (ES) Fields – Poisson and Laplace Equation / Elektrostatische (ES) Felder – Poisson- und Laplace-Gleichung (3)

Laplace Operator in Cartesian Coordinates / Laplace-Operator in Kartesischen Koordinaten

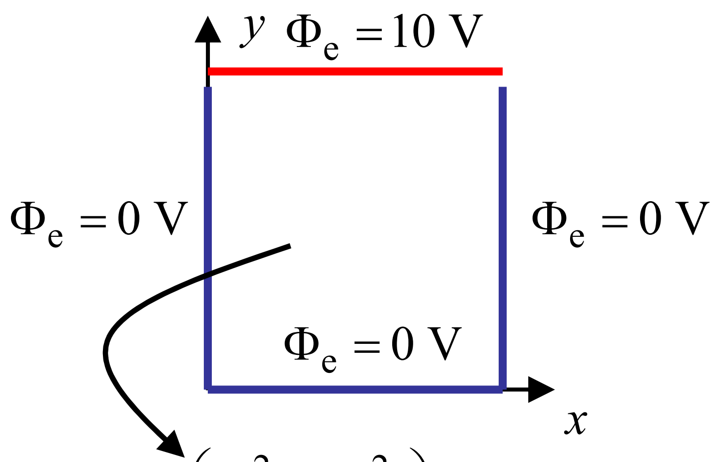
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi_\mathrm{e}(x,y,z)=\begin{cases}-\dfrac{\rho_\mathrm{e}(x,y,z)}{\varepsilon_0} & \text{for / für } \rho_\mathrm{e}(x,y,z)\neq 0 \quad \text{Poisson Equation / Poisson-Gleichung}\\ 0 & \text{for / für } \rho_\mathrm{e}(x,y,z)=0 \quad \text{Laplace Equation / Laplace-Gleichung}\end{cases}$$

Example: pn Junction – pn Diode / Beispiel: pn-Übergang – pn Diode



$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\Phi_\mathrm{e}(x)=\frac{e}{\varepsilon}\begin{cases}-n_e & \text{for / für } -d_n\leq x\leq 0\\ n_e & \text{for / für } 0\leq x\leq d_p\end{cases}$$

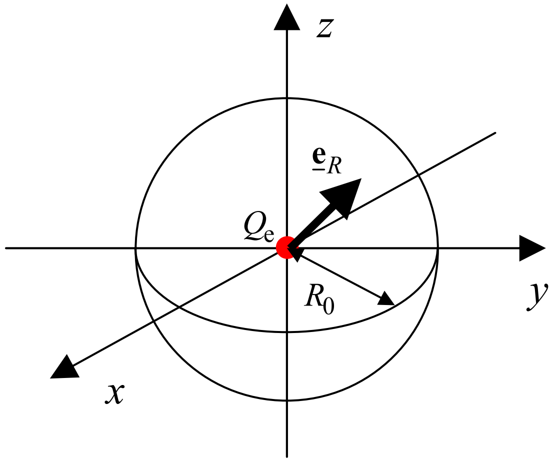
Example: / Beispiel:

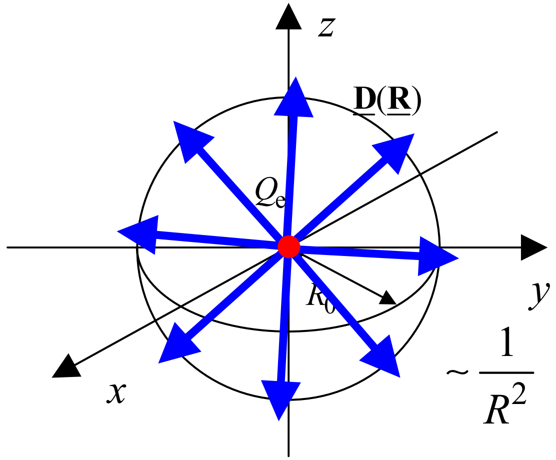


$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi_\mathrm{e}(x,y)=0$$

➡ **Separation of Variables / Separation der Variablen !**

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung





$$\oiint_{\underbrace{S=\partial V}_{=\text{Sphere } R_0}} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}\, \mathrm{d}S = \iiint_V \underbrace{\rho_e(\underline{\mathbf{R}})}_{=Q_e \delta(\underline{\mathbf{R}})} \,\mathrm{d}V = Q_e$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla \Phi_e(\underline{\mathbf{R}}) = -\nabla \Phi_e(R)$$

$$= -\frac{\partial}{\partial R} \Phi_e(R)\, \underline{\mathbf{e}}_R(\varphi, \vartheta) = \alpha \frac{1}{R^2} \underline{\mathbf{e}}_R(\varphi, \vartheta)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \underline{\mathbf{D}}(R) = \varepsilon_0 \alpha \frac{1}{R^2} \underline{\mathbf{e}}_R(\varphi, \vartheta)$$

$$\iint_{\underbrace{S=\partial V}_{\text{Sphere } R_0}} \varepsilon_0 \alpha \frac{1}{R_0^2} \underbrace{\underline{\mathbf{e}}_R(\varphi, \vartheta) \bullet \underline{\mathbf{e}}_R(\varphi, \vartheta)}_{=1} \mathrm{d}S = \varepsilon_0 \alpha \frac{1}{R_0^2} \underbrace{\iint_{\underbrace{S=\partial V}_{\text{Sphere } R_0}} \mathrm{d}S}_{4\pi R_0^2}$$

$$= 4\pi \varepsilon_0 \alpha$$

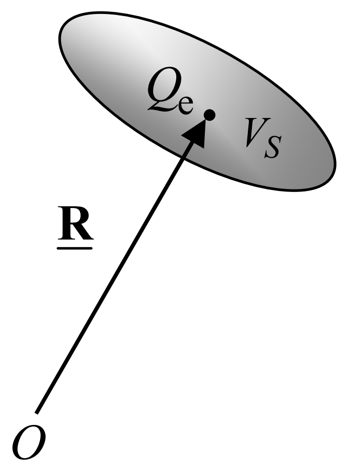
$$= Q_e$$

$$\Phi_e(R) = \frac{Q_e}{4\pi \varepsilon_0 R} \qquad \alpha = \frac{Q_e}{4\pi \varepsilon_0}$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

Point Source / Punktquelle

Q_e [As/m^3 = Coulomb]



$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = ?$$

$$= \text{infinite} \ / \ \text{unendlich}$$

$$\iiint_{V_S} \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V = Q_\mathrm{e}$$

Mathematically Nonsense / Mathematischer Unsinn

$$V_S \to 0$$

Integration Theory of Riemann / Riemannsche Integralrechnung:

$$\iiint_{V_S} \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V = 0$$

To Define Something New / Definiere etwas Neues

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

Electrostatic Charge /
Elektrostatisches Ladung

$$Q_\mathrm{e} = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\, dV$$

Electrostatic Volume Charge Density /
Elektrostatisches Raumladungsdichte

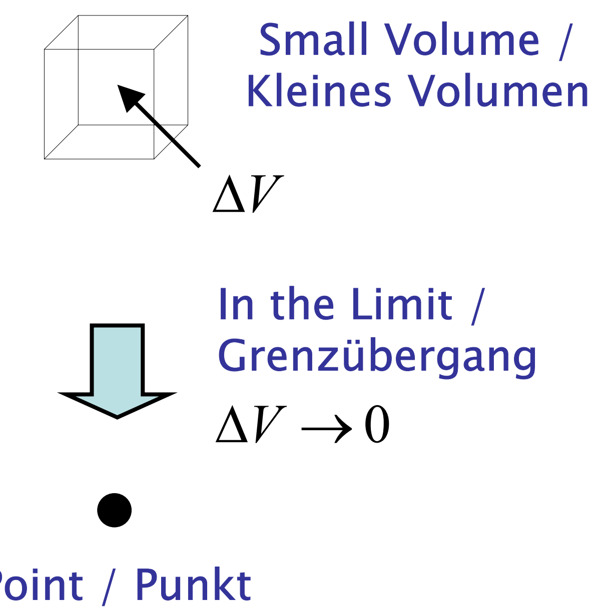
$$\rho_\mathrm{e} = \frac{\Delta Q_\mathrm{e}}{\Delta V}$$

Electrostatic Charge /
Elektrostatisches Ladung

$$\Delta Q_\mathrm{e} = \rho_\mathrm{e} \Delta V$$

Constant / Konstant

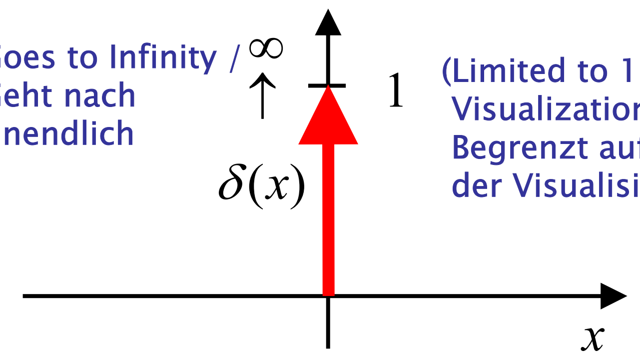
$$\Delta Q_\mathrm{e} = \lim_{\substack{\Delta V \to 0 \\ \rho_\mathrm{e} \to \infty}} \rho_\mathrm{e} \Delta V$$



ΔQ_e =Constant if ΔV Goes to Zero, than the Volume Charge Density must go to Infinity. /
ΔQ_e=konstant bleiben soll wenn ΔV nach null geht, dann muss die Raumladungsdichte nach unendlich gehen.

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution / 1D Delta-Distribution



(Limited to 1 only for Visualization / Begrenzt auf 1 wegen der Visualisierung)

Delta-Function / Delta-Funktion
δ-Distribution / δ-Distribution
δ-Dirac-Pulse / δ-Dirac-Impuls

Distribution ➡ Generalized Function / Verallgemeinerte Funktion

$$\delta(x) = \begin{cases} "\infty" & \text{for/für} \quad x = 0 \\ 0 & \text{for/für} \quad x \neq 0 \end{cases}$$

$$x\,[\mathrm{m}],\ \delta(x)\left[\frac{1}{\mathrm{m}}\right]$$

The Unit of the Delta-Distribution is the Inverse Unit o the Argument / Die Einheit der Delta-Distribution ist (inverse Einheit des Argumentes

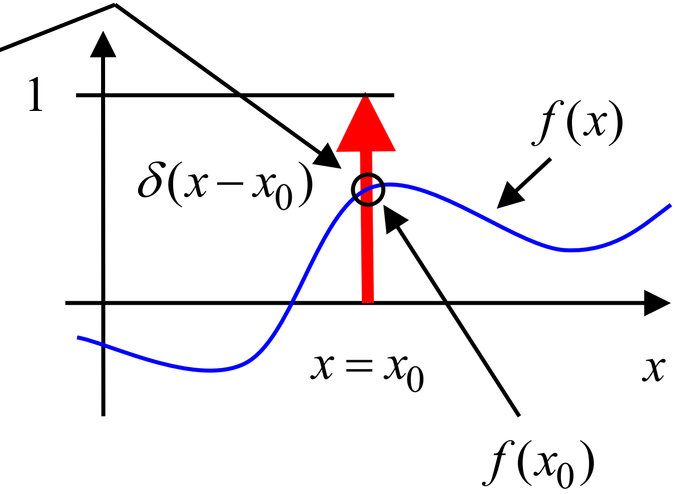
Definition of the δ -Distribution / Definition der δ -Distribution

Sifting Property / Siebeigenschaft

$$\int_{x=-\infty}^{\infty} \delta(x-x_0)\,\mathrm{d}x = 1$$

$$\int_{x=-\infty}^{\infty} f(x)\,\delta(x-x_0)\,\mathrm{d}x = f(x_0)$$

$$f(x)\,\delta(x-x_0) = f(x_0)\,\delta(x-x_0)$$



End of Lecture 5 /
Ende der 5. Vorlesung