

Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

6th Lecture / 6. Vorlesung

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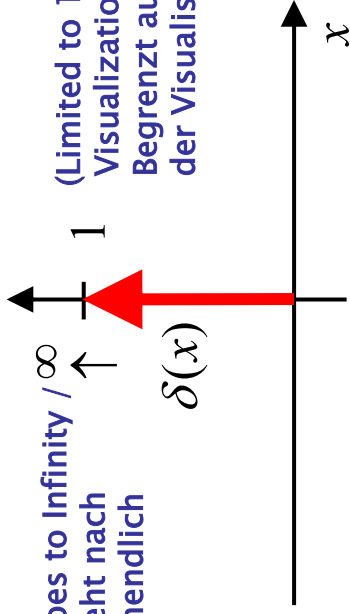
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Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution / 1D Delta-Distribution

Goes to Infinity / ∞
Geht nach Unendlich

(Limited to 1 only for Visualization /
Begrenzt auf 1 wegen der Visualisierung)



$$\delta(x) = \begin{cases} \text{"}\infty\text{"} & \text{for/} \\ \text{für} & x = 0 \\ 0 & \text{for/} \\ \text{für} & x \neq 0 \end{cases} \quad x \text{ [m]}, \delta(x) \left[\frac{1}{\text{m}} \right]$$

Delta-Function / Delta-Funktion
 δ -Distribution / δ -Distribution
 δ -Dirac-Pulse / δ -Dirac-Impuls

Distribution \rightarrow Generalized Function /
Verallgemeinerte Funktion

The Unit of the Delta-Distribution is the Inverse Unit of the Argument / Die Einheit der Delta-Distribution ist die inverse Einheit des Argumentes

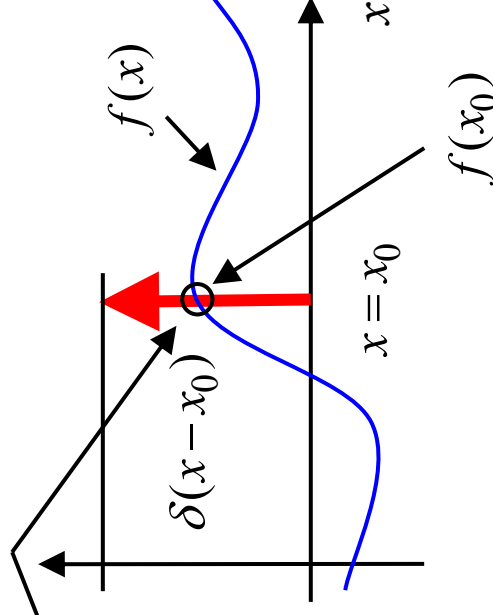
Definition of the δ -Distribution /
Definition der δ -Distribution

$$\int_{x=-\infty}^{\infty} \delta(x - x_0) dx = 1$$

$$\int_{x=-\infty}^{\infty} f(x) \delta(x - x_0) dx = f(x_0)$$

$$f(x) \delta(x - x_0) = f(x_0) \delta(x - x_0)$$

Sifting Property / Siebeigenschaft



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution / 1D Delta-Distribution

$$\int_{x=-\infty}^{\infty} f(x) \delta(x-x_0) dx = f(x_0)$$
$$\langle f(x), \delta(x-x_0) \rangle = f(x_0)$$

Properties: Algebraic and Calculus Properties /
Eigenschaften: Algebraische Eigenschaften und Rechenregeln

$$\alpha \delta(x-x_0) :$$

$$\int_{x=-\infty}^{\infty} \alpha \delta(x-x_0) f(x) dx = \alpha f(x_0)$$

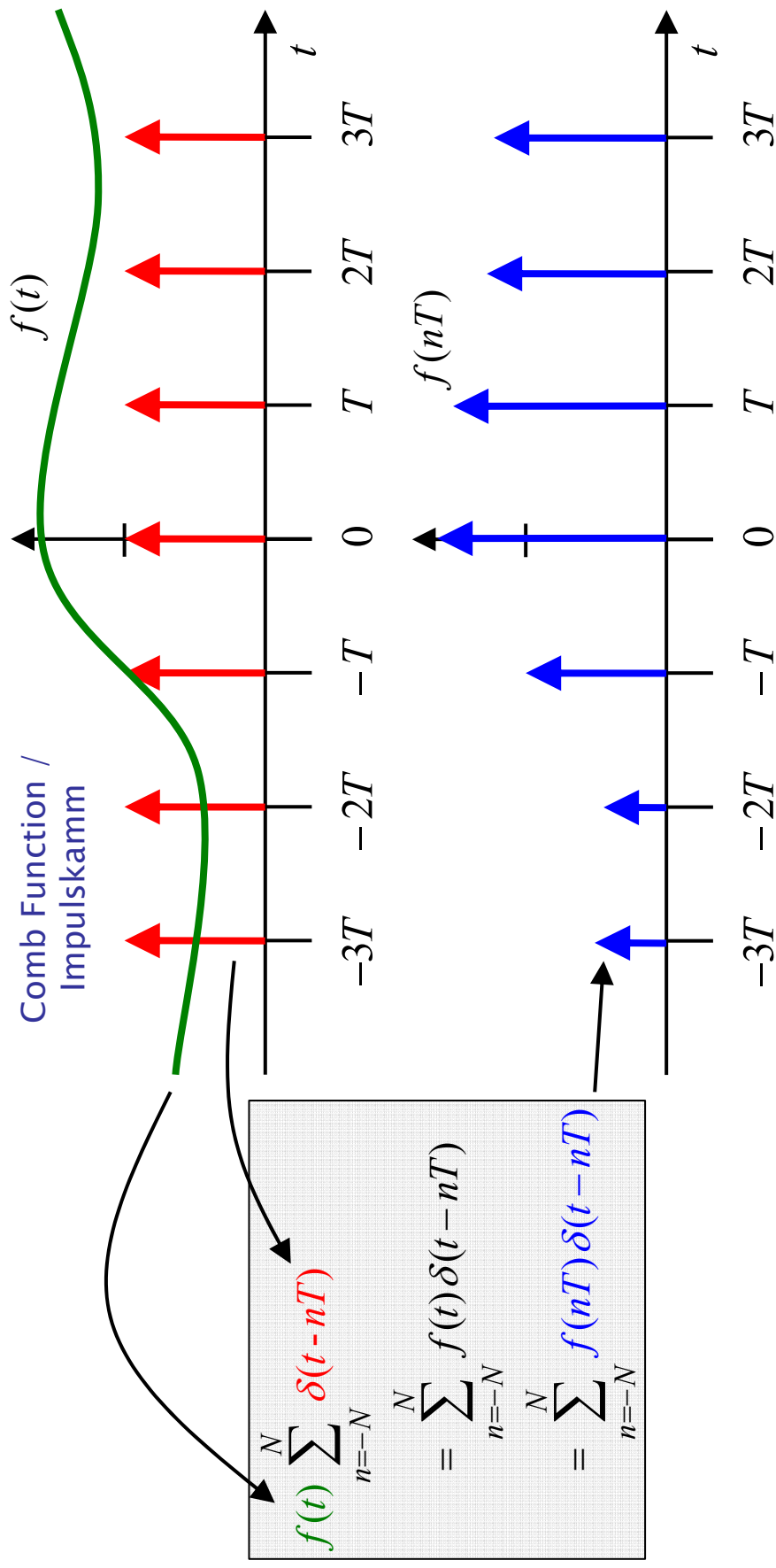
$$\alpha(x) \delta(x-x_0) :$$

$$\int_{x=-\infty}^{\infty} \alpha(x) \delta(x-x_0) f(x) dx = \alpha(x_0) f(x_0)$$

$$\alpha(x) \delta(x-x_0) = \alpha(x_0) \delta(x-x_0)$$

Electrostatic (ES) Fields – Point Charge Concept /
 Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution – Signal Processing – Sampling /
 1D Delta-Distribution – Signalverarbeitung – Abtastung



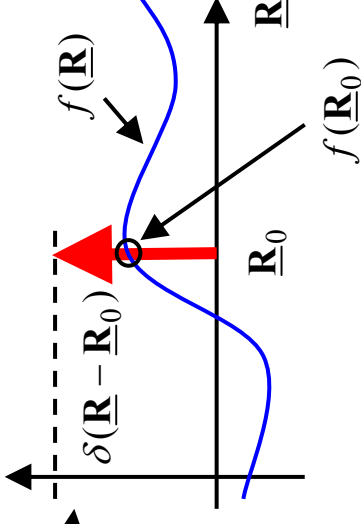
Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...) 3-D Delta-Distribution / 3D Delta-Distribution

Sifting Property / Siebeigenschaft

$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_0) d^3 \underline{\mathbf{R}} = 1$$

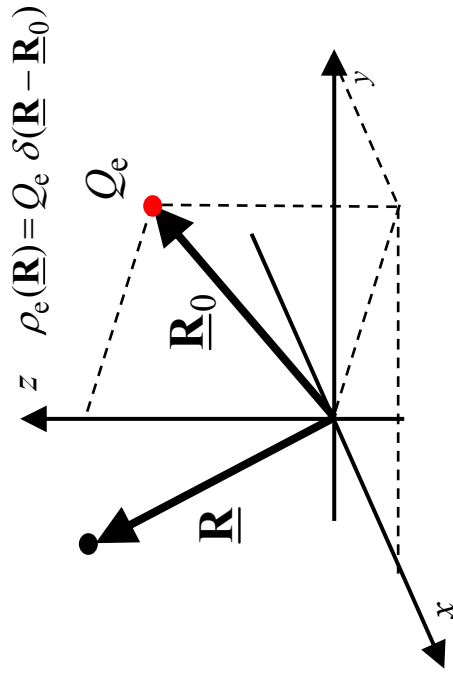
$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} f(\underline{\mathbf{R}}) \delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_0) d^3 \underline{\mathbf{R}} = f(\underline{\mathbf{R}}_0)$$

$$f(\underline{\mathbf{R}}) \delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_0) = f(\underline{\mathbf{R}}_0) \delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_0)$$



Distribution → Generalized Function / Verallgemeinerte Funktion

$$\begin{aligned} \iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \rho_e(\underline{\mathbf{R}}) d^3 \underline{\mathbf{R}} &= \iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} Q_e \delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_0) d^3 \underline{\mathbf{R}} \\ &= Q_e \underbrace{\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_0) d^3 \underline{\mathbf{R}}}_{=1} \\ &= Q_e \end{aligned}$$



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

3-D Delta-Distribution / 3D Delta-Distribution

$$\delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_0) = \delta(x - x_0)\delta(y - y_0)\delta(z - z_0)$$

Cartesian Coordinate System /
Kartesisches Koordinatensystem

$$= \delta(r - r_0) \frac{\delta(\varphi - \varphi_0)}{r} \delta(z - z_0) = \frac{\delta(r - r_0)\delta(\varphi - \varphi_0)\delta(z - z_0)}{r}$$

Cylindrical Coordinate System /
Zylinderkoordinatensystem

$$= \delta(R - R_0) \frac{\delta(\vartheta - \vartheta_0)}{R} \frac{\delta(\varphi - \varphi_0)}{R \sin \vartheta} = \frac{\delta(R - R_0)\delta(\vartheta - \vartheta_0)\delta(\varphi - \varphi_0)}{R^2 \sin \vartheta}$$

Spherical Coordinate System /
Kugelkoordinatensystem

General Case /
Allgemeiner Fall

$$= \frac{\delta(\xi_1 - \xi_{10})}{h_{\xi_1}} \frac{\delta(\xi_2 - \xi_{20})}{h_{\xi_2}} \frac{\delta(\xi_3 - \xi_{30})}{h_{\xi_3}}$$

$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_0) d^3 \underline{\mathbf{R}} = \iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(x - x_0)\delta(y - y_0)\delta(z - z_0) d^3 \underline{\mathbf{R}} = \int_{z=-\infty}^{\infty} \int_{y=-\infty}^{\infty} \int_{x=-\infty}^{\infty} \delta(x - x_0)\delta(y - y_0)\delta(z - z_0) dx dy dz$$

$$= \left[\int_{z=-\infty}^{\infty} \underbrace{\left[\int_{y=-\infty}^{\infty} \underbrace{\left[\int_{x=-\infty}^{\infty} \delta(x - x_0) dx \right]}_{=1} \delta(y - y_0) dy \right]}_{=1} \delta(z - z_0) dz \right]_{=1} = 1$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

3-D Delta-Distribution / 3D Delta-Distribution

$$\begin{aligned} \iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0) d^3 \underline{\mathbf{R}} &= \iiint_{z=-\infty}^{\infty} \frac{\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(z-z_0)}{r} d^3 \underline{\mathbf{R}} = \int_{z=-\infty}^{\infty} \int_{\varphi=0}^{2\pi} \int_{r=0}^{\infty} \frac{\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(z-z_0)}{r} r dr d\varphi dz \\ &= \left[\int_{z=-\infty}^{\infty} \underbrace{\left[\int_{\varphi=0}^{2\pi} \underbrace{\left[\int_{r=0}^{\infty} \delta(r-r_0) dr \right]}_{=1} \frac{\delta(\varphi-\varphi_0)}{r} r d\varphi \right]}_{=1} \delta(z-z_0) dz \right]_{=1} = 1 \end{aligned}$$

$$\begin{aligned} \iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0) d^3 \underline{\mathbf{R}} &= \iiint_{\varphi=0}^{2\pi} \frac{\delta(R-R_0)\delta(\vartheta-\vartheta_0)\delta(\varphi-\varphi_0)}{R^2 \sin \vartheta} d^3 \underline{\mathbf{R}} = \int_{\varphi=0}^{2\pi} \int_{\vartheta=0}^{\pi} \int_{R=0}^{\infty} \frac{\delta(R-R_0)\delta(\vartheta-\vartheta_0)\delta(\varphi-\varphi_0)}{R^2 \sin \vartheta} R^2 \sin \vartheta dR d\vartheta d\varphi \\ &= \left[\int_{\varphi=0}^{2\pi} \underbrace{\left[\int_{\vartheta=0}^{\pi} \underbrace{\left[\int_{R=0}^{\infty} \delta(R-R_0) dR \right]}_{=1} \frac{\delta(\vartheta-\vartheta_0)}{R} R d\vartheta \right]}_{=1} \frac{\delta(\varphi-\varphi_0)}{R \sin \vartheta} R \sin \vartheta d\varphi \right]_{=1} = 1 \end{aligned}$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

Electrostatic Point Charge Density /
Elektrostatische Punktladung

$$Q_e = Q_e(x_0, y_0, z_0) \text{ [As]}$$

Electrostatic Volume Charge Density /
Elektrostatische Raumladungsdichte

$$\rho_e(x, y, z) = Q_e \delta(x - x_0) \delta(y - y_0) \delta(z - z_0)$$


Q_e ● Point / Punkt

Electrostatic Line Charge Density /
Elektrostatische Linienladungsdichte

$$\zeta_e(z) = \zeta_e(x_0, y_0, z) \text{ [As/m]}$$

Electrostatic Line Charge Density /
Elektrostatische Linienladungsdichte

$$\rho_e(x, y, z) = \zeta_e(z) \delta(x - x_0) \delta(y - y_0)$$

$\zeta_e(z)$  Line / Linie

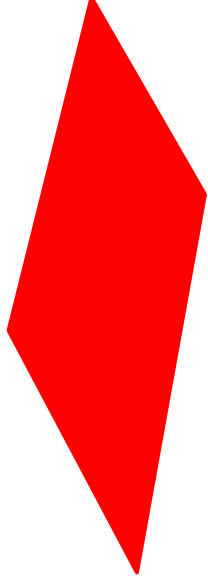
Electrostatic Surface Charge Density /
Elektrostatische Flächenladungsdichte

$$\eta_e(x, y) = \eta_e(x, y, z_0) \text{ [As/m}^2\text{]}$$

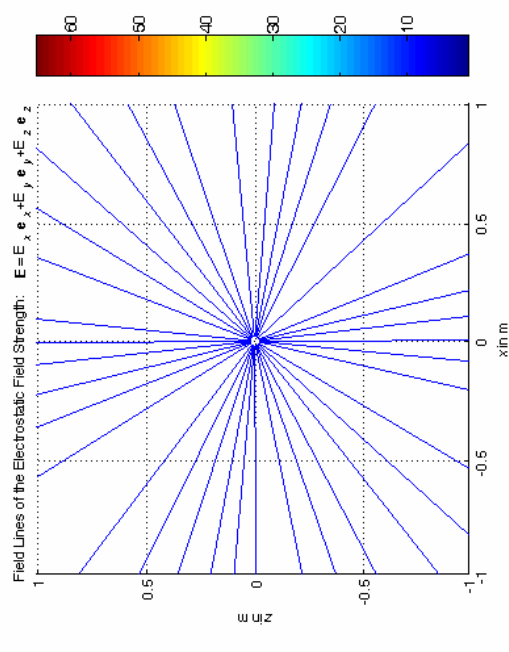
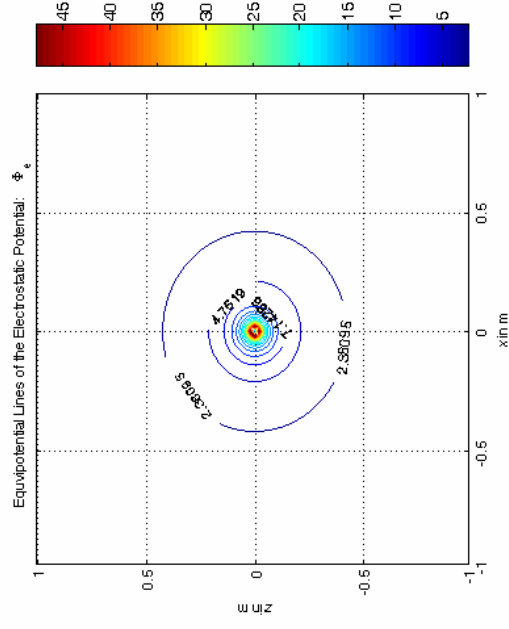
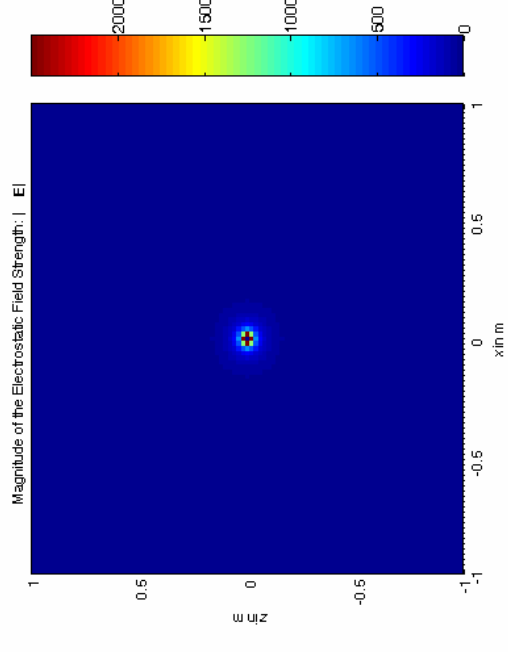
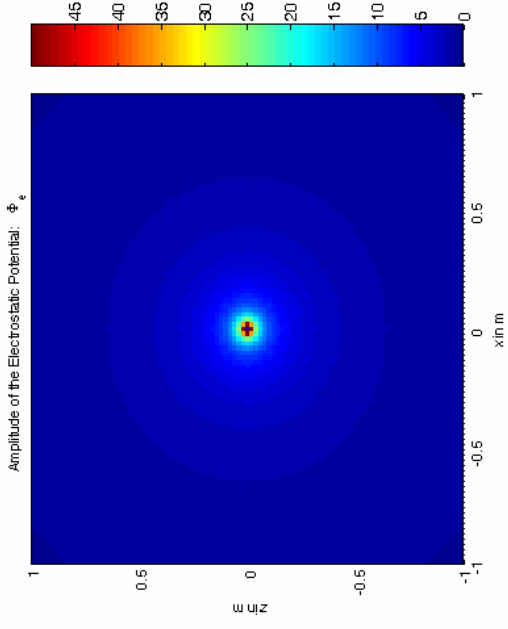
Electrostatic Charge Density /
Elektrostatische Ladungsdichte

$$\rho_e(x, y, z) = \eta_e(x, y) \delta(z - z_0)$$

Surface / Surface

$\eta_e(x, y, z_0)$ 

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)



ES Fields – Point Charge Concept / ES Felder – Konzept der Punktladung (....)

Electrostatic Charge Density /
Elektrostatische Ladungsdichte

$$\rho_e(\underline{\mathbf{R}}) = Q_e \delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_0)$$

Electrostatic Potential /
Elektrostatisches Potential

$$\Phi_e(\underline{\mathbf{R}}) = \frac{1}{4\pi\epsilon_0} \frac{Q_e}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0|}$$
$$\frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0|} = \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}$$

Electrostatic Field Strength /
Elektrostatische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_e(\underline{\mathbf{R}})$$
$$= \frac{Q_e}{4\pi\epsilon_0} \frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_0}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0|^3}$$
$$\frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0|^3} = \frac{1}{\left(\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}\right)^3}$$
$$= \frac{1}{\left[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\right]^{3/2}}$$

ES Fields – Coulomb Integral / ES Felder – Coulomb-Integral

Poisson and Laplace Equation / Poisson- und Laplace-Gleichung

$$\Delta\Phi_e(\underline{\mathbf{R}}) = \begin{cases} -\frac{\rho_e(\underline{\mathbf{R}})}{\varepsilon_0} & \text{for / für } \rho_e(\underline{\mathbf{R}}) \neq 0 \\ 0 & \text{for / für } \rho_e(\underline{\mathbf{R}}) = 0 \end{cases}$$

Poisson Equation / Poisson-Gleichung
Laplace Equation / Laplace-Gleichung

$$\Delta = \nabla^2 = \nabla \cdot \nabla : \text{Laplace Operator / Laplace-Operator}$$

Limited Source Volume /
Begrenztes Quellvolumen

$$\rho_e(\underline{\mathbf{R}}) \begin{cases} \neq 0 & \underline{\mathbf{R}} \in V_s \\ 0 & \underline{\mathbf{R}} \notin V_s \end{cases}$$

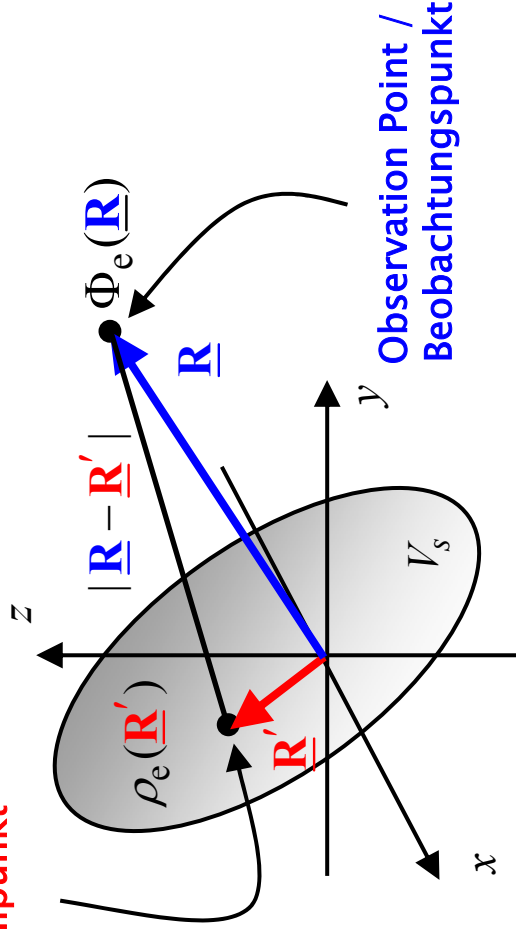
Source Point /
Quellpunkt

Coulomb Integral / Coulomb-Integral:

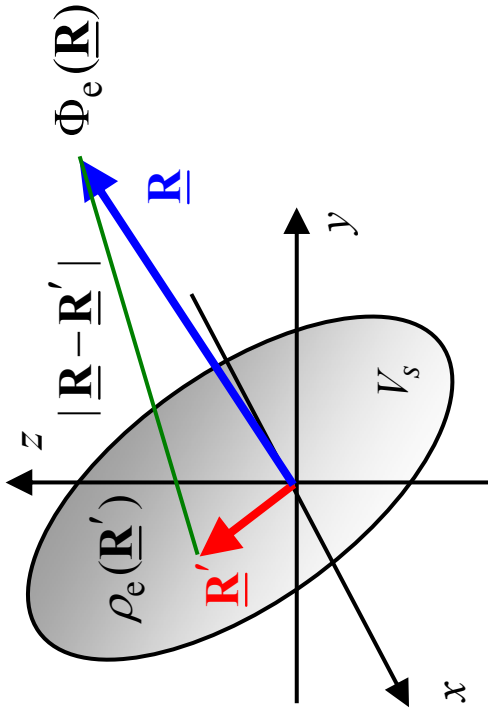
$$\Phi_e(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0} \iiint_{V_s} \frac{\rho_e(\underline{\mathbf{R}'})}{|\underline{\mathbf{R}} - \underline{\mathbf{R}'}|} d^3\underline{\mathbf{R}'}$$

$\rho_e(\underline{\mathbf{R}'})$: known / bekannt

$\Phi_e(\underline{\mathbf{R}})$: unknown / unbekannt



ES Fields – Coulomb Integral / ES Felder – Coulomb–Integral (...)



Coulomb Integral / Coulomb–Integral:

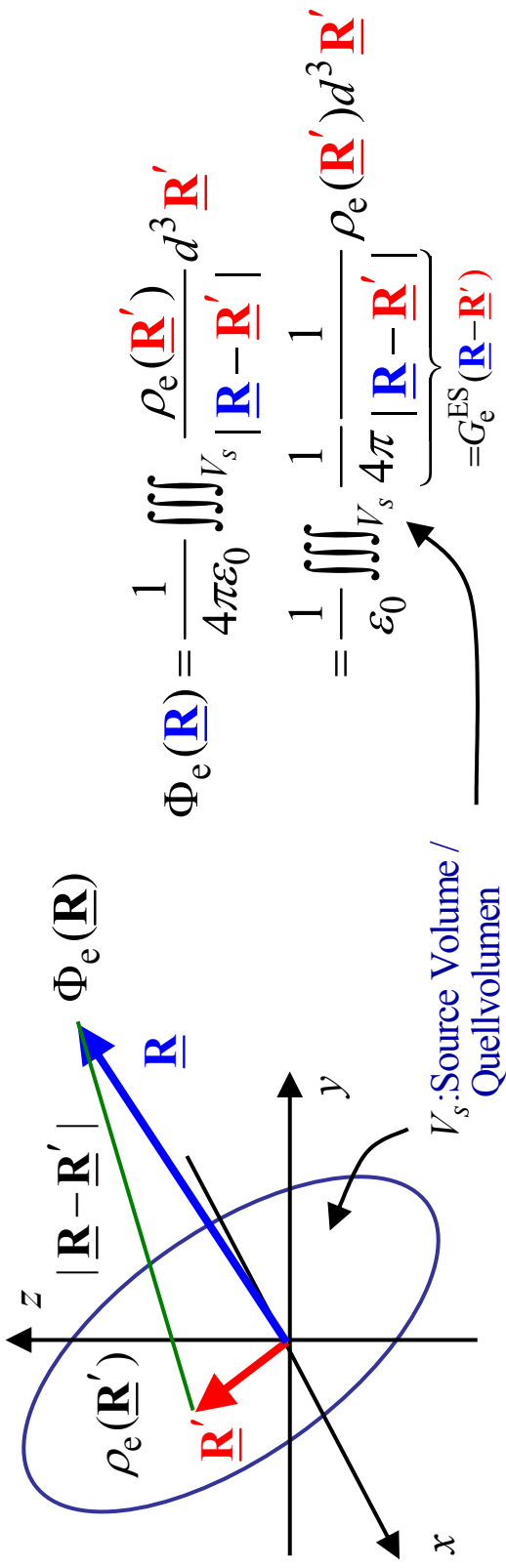
$$\Phi_e(\underline{\mathbf{R}}) = \frac{1}{4\pi\epsilon_0} \iiint_{V_s} \frac{\rho_e(\underline{\mathbf{R}'})}{|\underline{\mathbf{R}} - \underline{\mathbf{R}'}|} d^3\underline{\mathbf{R}'}$$

$\rho_e(\underline{\mathbf{R}'})$: known / bekannt

$\Phi_e(\underline{\mathbf{R}})$: unknown / unbekannt

$$\begin{aligned} \Delta\Phi_e(\underline{\mathbf{R}}) &= \frac{1}{4\pi\epsilon_0} \Delta \iiint_{V_s} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}'}|} \rho_e(\underline{\mathbf{R}'}) d^3\underline{\mathbf{R}'} \\ &= \frac{1}{4\pi\epsilon_0} \iiint_{V_s} \underbrace{\left[\Delta \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}'}|} \right]}_{=-4\pi\delta(\underline{\mathbf{R}} - \underline{\mathbf{R}'})} \rho_e(\underline{\mathbf{R}'}) d^3\underline{\mathbf{R}'} \quad \text{with } \Delta \frac{1}{4\pi|\underline{\mathbf{R}} - \underline{\mathbf{R}'}|} = -\delta(\underline{\mathbf{R}} - \underline{\mathbf{R}'}) \\ &= \underbrace{-\frac{1}{4\pi\epsilon_0} \iiint_{V_s} 4\pi\delta(\underline{\mathbf{R}} - \underline{\mathbf{R}'}) \rho_e(\underline{\mathbf{R}'}) d^3\underline{\mathbf{R}'}}_{=\rho_e(\underline{\mathbf{R}})} = -\frac{1}{\epsilon_0} \rho_e(\underline{\mathbf{R}}) \end{aligned}$$

ES Fields – Green’s Function / ES Felder – Greensche Funktion



$$\begin{aligned} \Phi_e(\underline{\mathbf{R}}) &= \frac{1}{4\pi\epsilon_0} \iiint_{V_s} \frac{\rho_e(\underline{\mathbf{R}'})}{|\underline{\mathbf{R}} - \underline{\mathbf{R}'})|} d^3\underline{\mathbf{R}'} \\ &= \frac{1}{\epsilon_0} \iiint_{V_s} \underbrace{\frac{1}{4\pi} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}'})|}}_{=G_e^{\text{ES}}(\underline{\mathbf{R}} - \underline{\mathbf{R}'})} \rho_e(\underline{\mathbf{R}'}) d^3\underline{\mathbf{R}'} \end{aligned}$$

Electrostatic Green’s Function / Elektrostatische Greensche Funktion

$$G_e^{\text{ES}}(\underline{\mathbf{R}} - \underline{\mathbf{R}'}) = \frac{1}{4\pi} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}'})|}$$

for / für $\underline{\mathbf{R}} \neq \underline{\mathbf{R}'}$

with $\Delta G_e^{\text{ES}}(\underline{\mathbf{R}} - \underline{\mathbf{R}'}) = -\delta(\underline{\mathbf{R}} - \underline{\mathbf{R}'})$

Normalized Potential of
a Point Charge /
Normiertes Potential
einer Punktladung

Electrostatic Potential of an Electrostatic Point Charge /
Elektrostatisches Potential einer elektrostatischen Punktladung

$$\Phi_e(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}'})|}$$

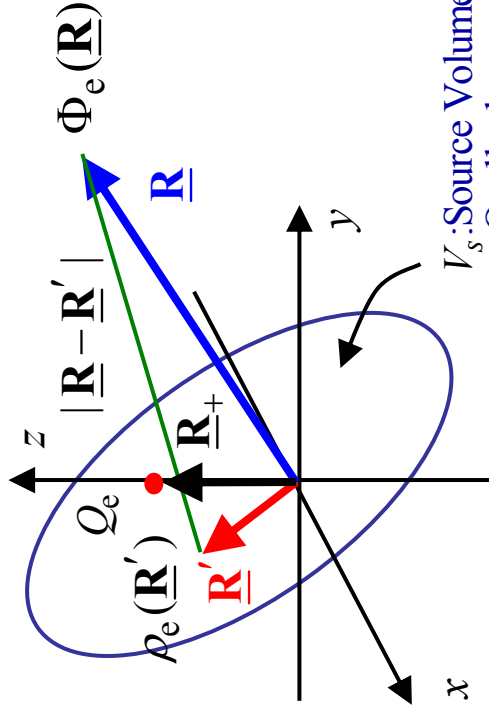
for / für $\rho_e(\underline{\mathbf{R}}) = Q_e \delta(\underline{\mathbf{R}} - \underline{\mathbf{R}'})$

ES Fields – Potential of a Point Charge / ES Felder – Potential einer Punktladung

Electrostatic Volume Charge Density /
Elektrostatistisches Raumladungsdichte

$$\rho_e(\underline{\mathbf{R}}) = Q_e \delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_+)$$

with / mit $\underline{\mathbf{R}} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z$ $\underline{\mathbf{R}}_+ = x_+\mathbf{e}_x + y_+\mathbf{e}_y + z_+\mathbf{e}_z$



$$\begin{aligned} \Phi_e(\underline{\mathbf{R}}) &= \frac{1}{4\pi\epsilon_0} \iiint_{V_s} \frac{Q_e \delta(\underline{\mathbf{R}}' - \underline{\mathbf{R}}_+)}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|} d^3 \underline{\mathbf{R}}' \\ &= \frac{Q_e}{4\pi\epsilon_0} \iiint_{V_s} \frac{\delta(\underline{\mathbf{R}}' - \underline{\mathbf{R}}_+)}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|} d^3 \underline{\mathbf{R}}' \\ &= \frac{Q_e}{4\pi\epsilon_0} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|} \end{aligned}$$

$$\Phi_e(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\epsilon_0} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|}$$

Electrostatic (ES) Fields / Elektrostatistische (ES) Felder

Field of an Electrostatic Point Charge / Feld einer elektrostatischen Punktladung

Electrostatic Potential of a Point Charge /
Elektrostatistisches Potential einer Punktladung

$$\Phi_e(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\epsilon_0} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|}$$

with $\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$

$$\underline{\mathbf{R}}_+ = x_+\underline{\mathbf{e}}_x + y_+\underline{\mathbf{e}}_y + z_+\underline{\mathbf{e}}_z$$

$$\begin{aligned} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|} &= \frac{1}{\sqrt{(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2}} \\ &= \frac{1}{\left[(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2 \right]^{1/2}} \end{aligned}$$

Electrostatic Field Strength of a Point Charge /
Elektrostatistische Feldstärke einer Punktladung

$$\begin{aligned} \underline{\mathbf{E}}(\underline{\mathbf{R}}) &= -\nabla\Phi_e(\underline{\mathbf{R}}) \\ &= \frac{Q_e}{4\pi\epsilon_0} \frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_+}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|^3} \end{aligned}$$

with $\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$

$$\underline{\mathbf{R}}_+ = x_+\underline{\mathbf{e}}_x + y_+\underline{\mathbf{e}}_y + z_+\underline{\mathbf{e}}_z$$

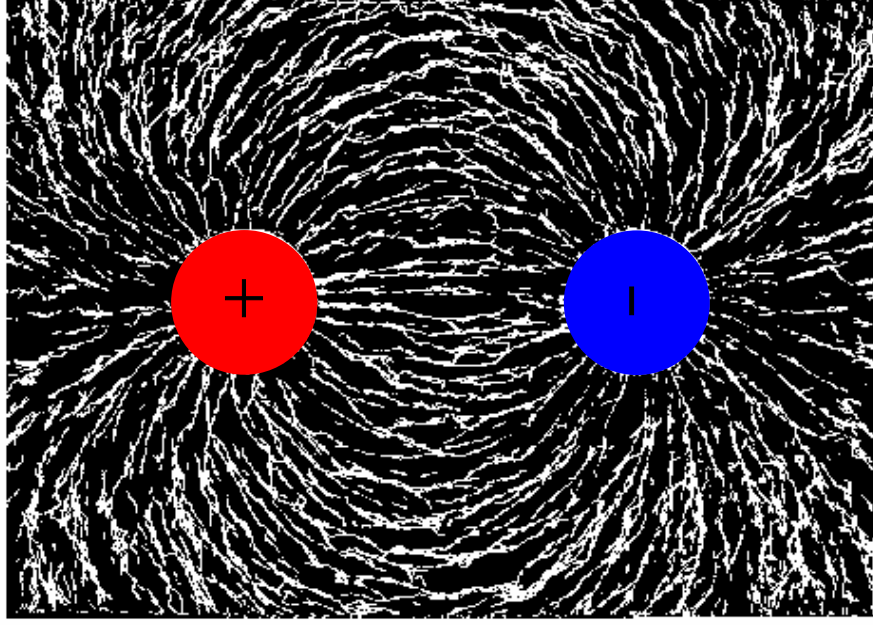
$$\begin{aligned} \frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_+}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|^3} &= \frac{(x-x_+)\underline{\mathbf{e}}_x + (y-y_+)\underline{\mathbf{e}}_y + (z-z_+)\underline{\mathbf{e}}_z}{\left[\sqrt{(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2} \right]^3} \\ &= \frac{(x-x_+)\underline{\mathbf{e}}_x + (y-y_+)\underline{\mathbf{e}}_y + (z-z_+)\underline{\mathbf{e}}_z}{\left[(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2 \right]^{3/2}} \end{aligned}$$

Electrostatic (ES) Fields / Elektrostatistische (ES) Felder

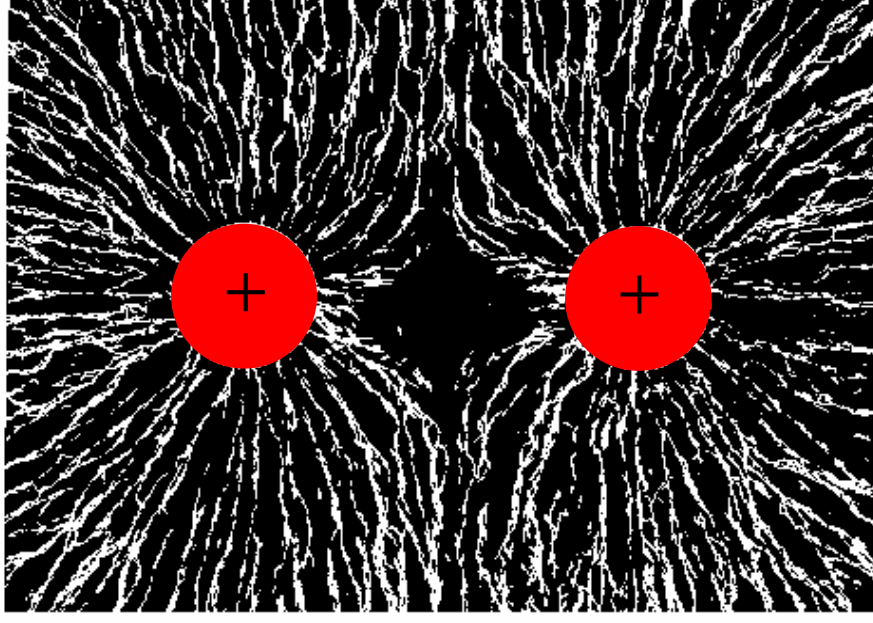
Field of Two Electrostatic Point Charges – Electrostatic Dipole /

Feld von zwei elektrostatischen Punktladungen – Elektrostatistischen Dipol

Field Lines of the Electric Field Strength of Two Spheres
Carrying Charges of Opposite Sign / Feldlinien der
elektrischen Feldstärke zweier ungleich geladener Kugeln



Electric Field Lines of Two Spheres Carrying Charges of the
Same Sign / Feldlinien der elektrischen Feldstärke zweier
gleich geladener Kugeln



Electrostatic (ES) Fields / Elektrostatistische (ES) Felder

Field of Two Electrostatic Point Charges – Electrostatic Dipole / Feld von zwei elektrostatischen Punktladungen – Elektrostatistischen Dipol

Electrostatic Potential /
Elektrostatistisches Potential

$$\Phi_e(\underline{\mathbf{R}}) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_{e+}}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|} + \frac{Q_{e-}}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_-|} \right)$$

with/mit

$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$

$$\underline{\mathbf{R}}_{\pm} = x_{\pm}\underline{\mathbf{e}}_x + y_{\pm}\underline{\mathbf{e}}_y + z_{\pm}\underline{\mathbf{e}}_z$$

$$\frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_{\pm}|} = \frac{1}{\sqrt{(x-x_{\pm})^2 + (y-y_{\pm})^2 + (z-z_{\pm})^2}}$$

$$= \frac{1}{[(x-x_{\pm})^2 + (y-y_{\pm})^2 + (z-z_{\pm})^2]^{1/2}}$$

Electrostatic Field Strength /
Elektrostatistische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_e(\underline{\mathbf{R}})$$

$$= \frac{1}{4\pi\epsilon_0} \left(Q_{e+} \frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_+}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|^3} + Q_{e-} \frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_-}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_-|^3} \right)$$

with/mit

$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$

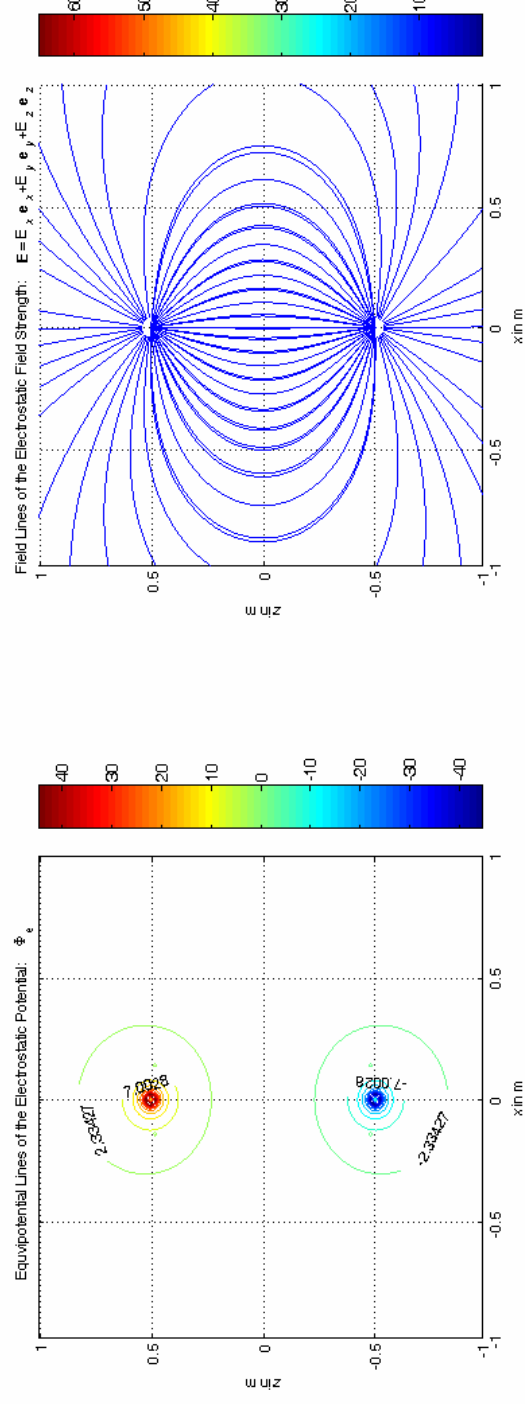
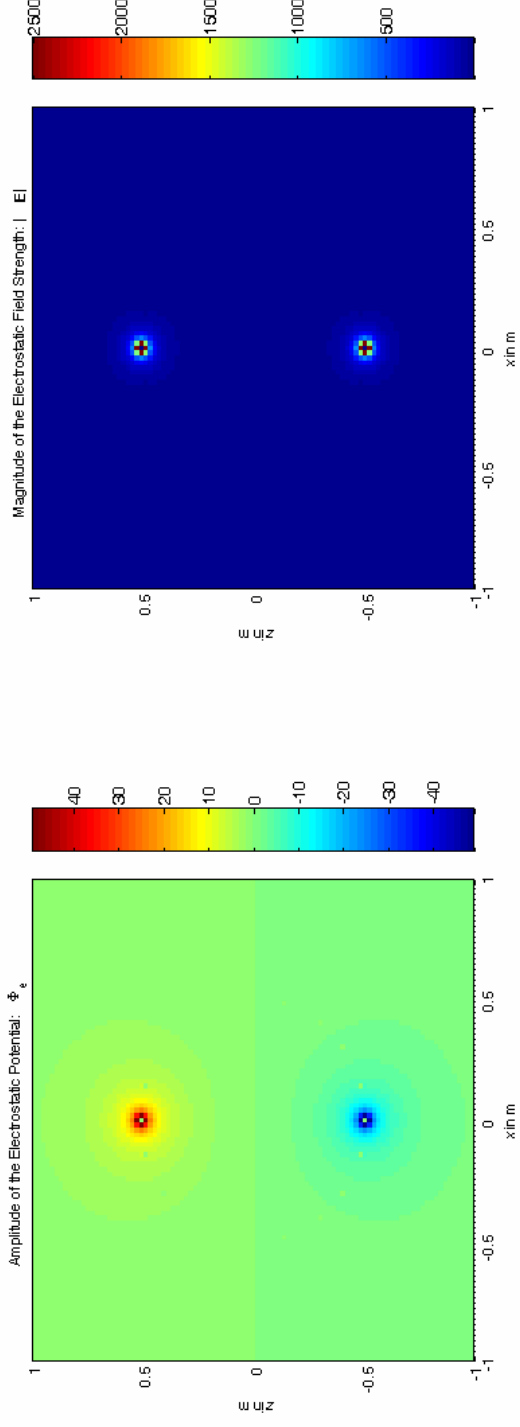
$$\underline{\mathbf{R}}_{\pm} = x_{\pm}\underline{\mathbf{e}}_x + y_{\pm}\underline{\mathbf{e}}_y + z_{\pm}\underline{\mathbf{e}}_z$$

$$\frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_{\pm}}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_{\pm}|^3} = \frac{(x-x_{\pm})\underline{\mathbf{e}}_x + (y-y_{\pm})\underline{\mathbf{e}}_y + (z-z_{\pm})\underline{\mathbf{e}}_z}{\left[\sqrt{(x-x_{\pm})^2 + (y-y_{\pm})^2 + (z-z_{\pm})^2} \right]^3}$$

$$= \frac{(x-x_{\pm})\underline{\mathbf{e}}_x + (y-y_{\pm})\underline{\mathbf{e}}_y + (z-z_{\pm})\underline{\mathbf{e}}_z}{\left[(x-x_{\pm})^2 + (y-y_{\pm})^2 + (z-z_{\pm})^2 \right]^{3/2}}$$

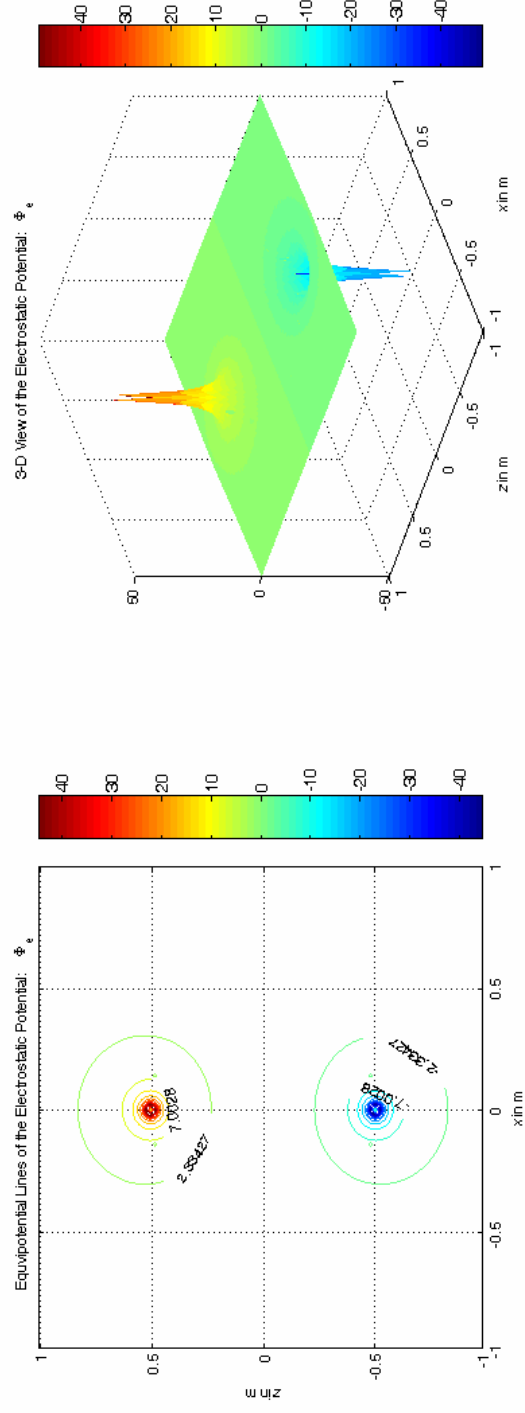
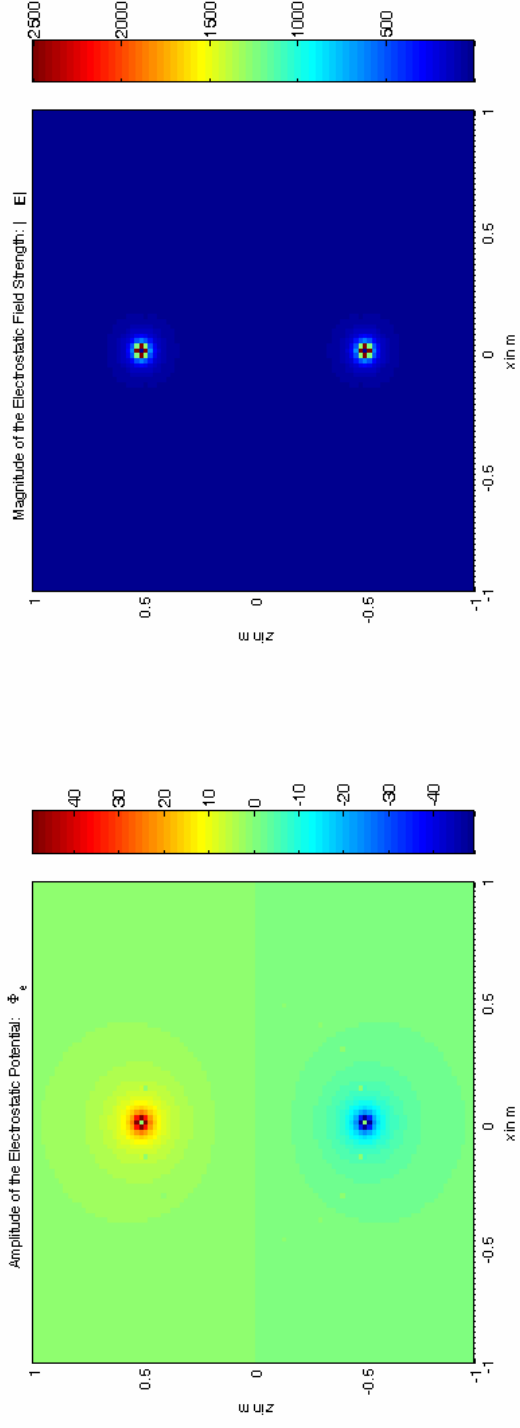
Electrostatic (ES) Fields / Elektrostatistische (ES) Felder

Field of Two Electrostatic Point Charges – Electrostatic Dipole / Feld von zwei elektrostatischen Punktladungen – Elektrostatistischer Dipol

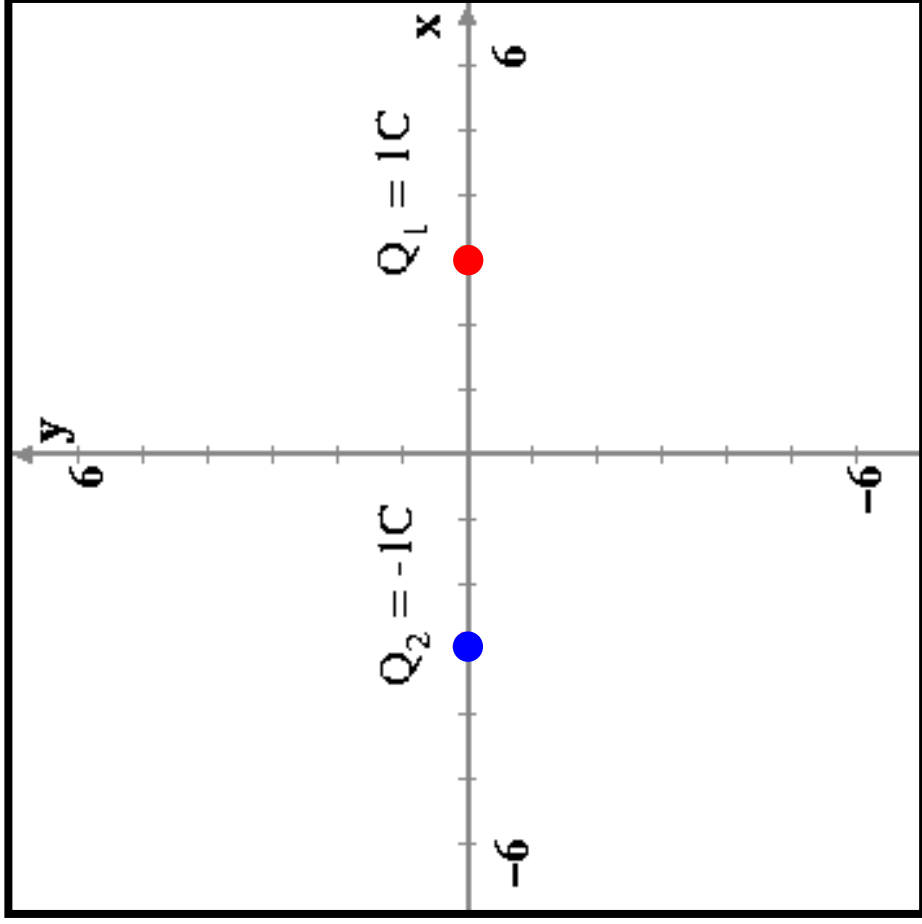


Electrostatic (ES) Fields / Elektrostatistische (ES) Felder

Field of Two Electrostatic Point Charges – Electrostatic Dipole / Feld von zwei elektrostatischen Punktladungen – Elektrostatistischer Dipol

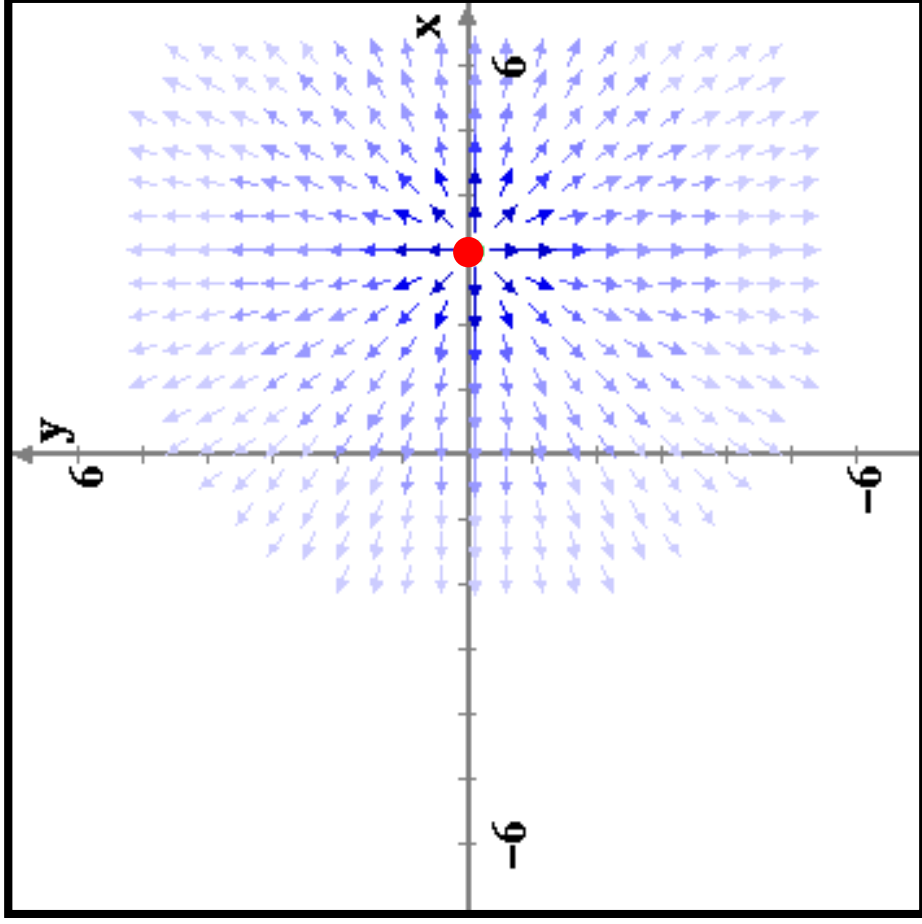


Electrostatic Field Due To Two Point Charges / Elektrostatistische Feld von zwei Punktladungen



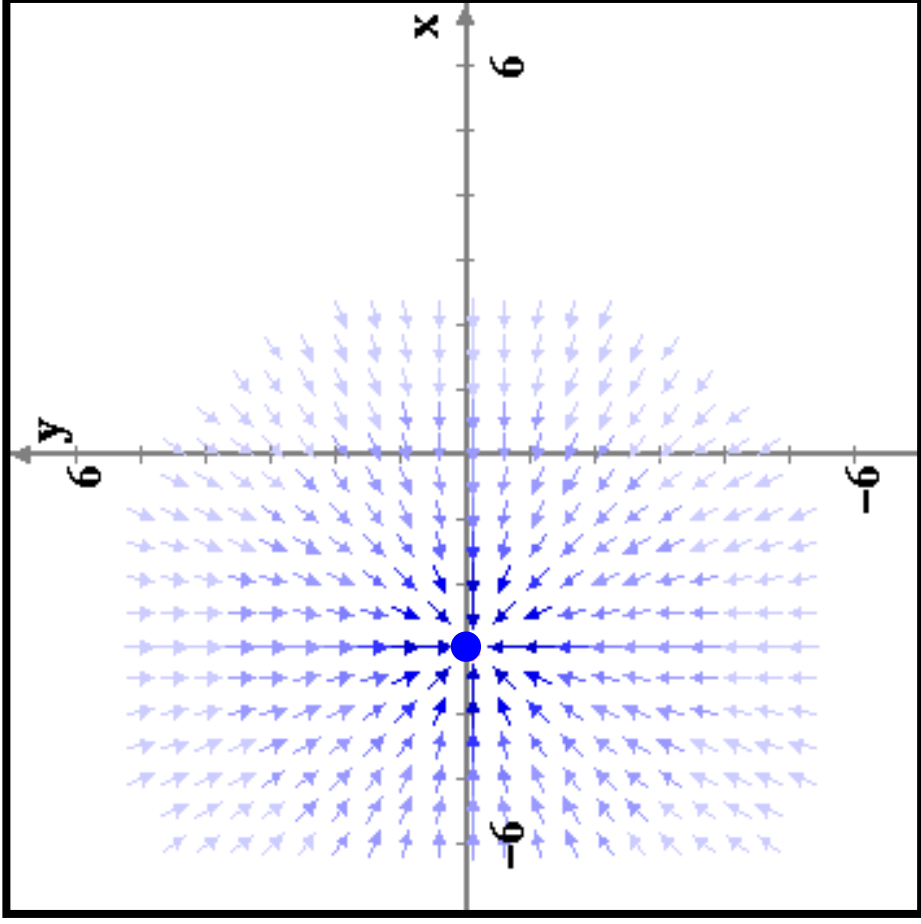
$Q_1 = 1$ As located at $R(x,y,z) = (3,0,0)$ and $Q_2 = -1$ As located at $R(x,y,z) = (-3,0,0)$	
In this demo, arrows are used to sketch the electric field pattern in the x - y plane due to:	
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone
Press	Q_1 and Q_2 / Q_1 and Q_2
Note: Color Intensity is Proportional to the Magnitude of the Electric Field Strength. / Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.	

Electrostatic Field... / Elektrostatische Feld... Q_1 alone / Q_1 alleine



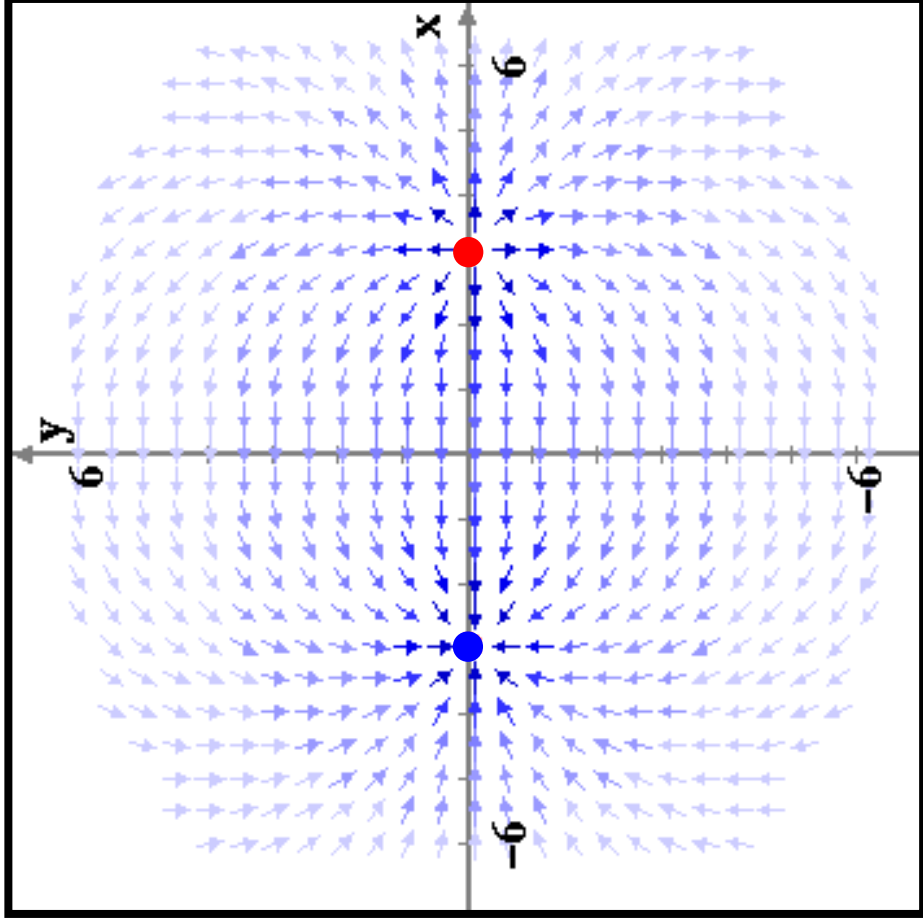
$Q_1 = 1$ As located at $R(x,y,z) = (3,0,0)$ and $Q_2 = -1$ As located at $R(x,y,z) = (-3,0,0)$	
In this demo, arrows are used to sketch the electric field pattern in the xy plane due to:	
Press	Geometry / Geometrie
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone / Q_2 alleine
Press	Q_1 and Q_2 / Q_1 und Q_2
Note: Color Intensity is Proportional to the Magnitude of the Electric Field Strength. / Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.	

Electrostatic Field... / Elektrostatische Feld... Q_2 alone / Q_2 alleine



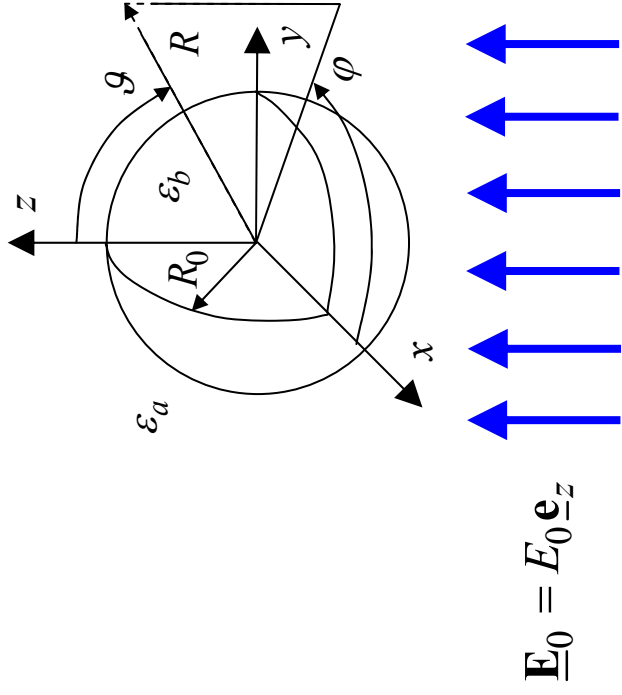
$Q_1 = 1$ As located at $R(x,y,z) = (3,0,0)$ and $Q_2 = -1$ As located at $R(x,y,z) = (-3,0,0)$	
In this demo, arrows are used to sketch the electric field pattern in the x - y plane due to:	
Press	Geometry / Geometrie
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone / Q_2 alleine
Press	Q_1 and Q_2 / Q_1 und Q_2
Note: Color Intensity is Proportional to the Magnitude of the Electric Field Strength. / Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.	

Electrostatic Field... / Elektrostatische Feld... Q_1 and Q_2 / Q_1 und Q_2



$Q_1 = 1$ As located at $R(x,y,z) = (3,0,0)$ and $Q_2 = -1$ As located at $R(x,y,z) = (-3,0,0)$	
In this demo, arrows are used to sketch the electric field pattern in the x - y plane due to:	
Press	Geometry / Geometrie
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone / Q_2 alleine
Press	Q_1 and Q_2 / Q_1 und Q_2
Note: Color Intensity is Proportional to the Magnitude of the Electric Field Strength. / Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.	

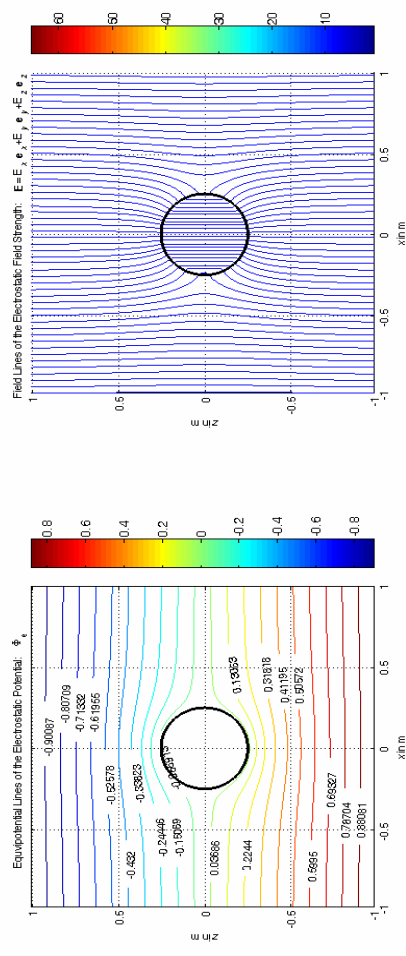
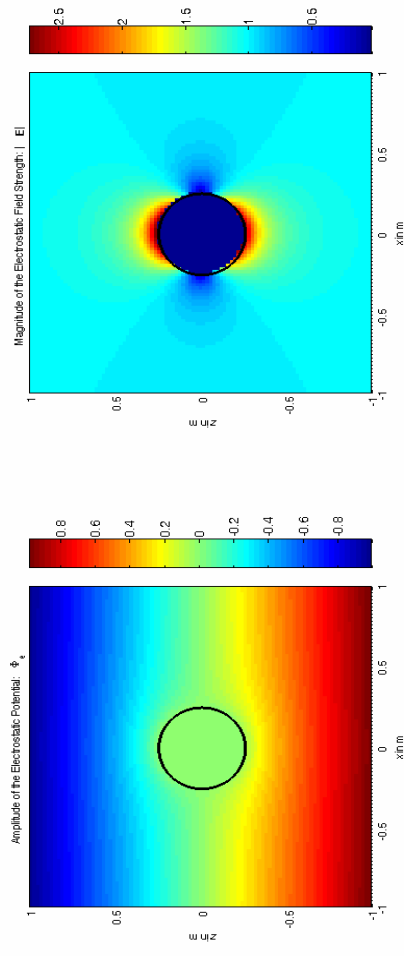
Transition Conditions = ? / Übergangsbedingungen = ?



$$\varepsilon(\underline{\mathbf{R}}) = \begin{cases} \varepsilon_b & 0 < R \leq R_0 \\ \varepsilon_a & R > R_0 \end{cases}$$

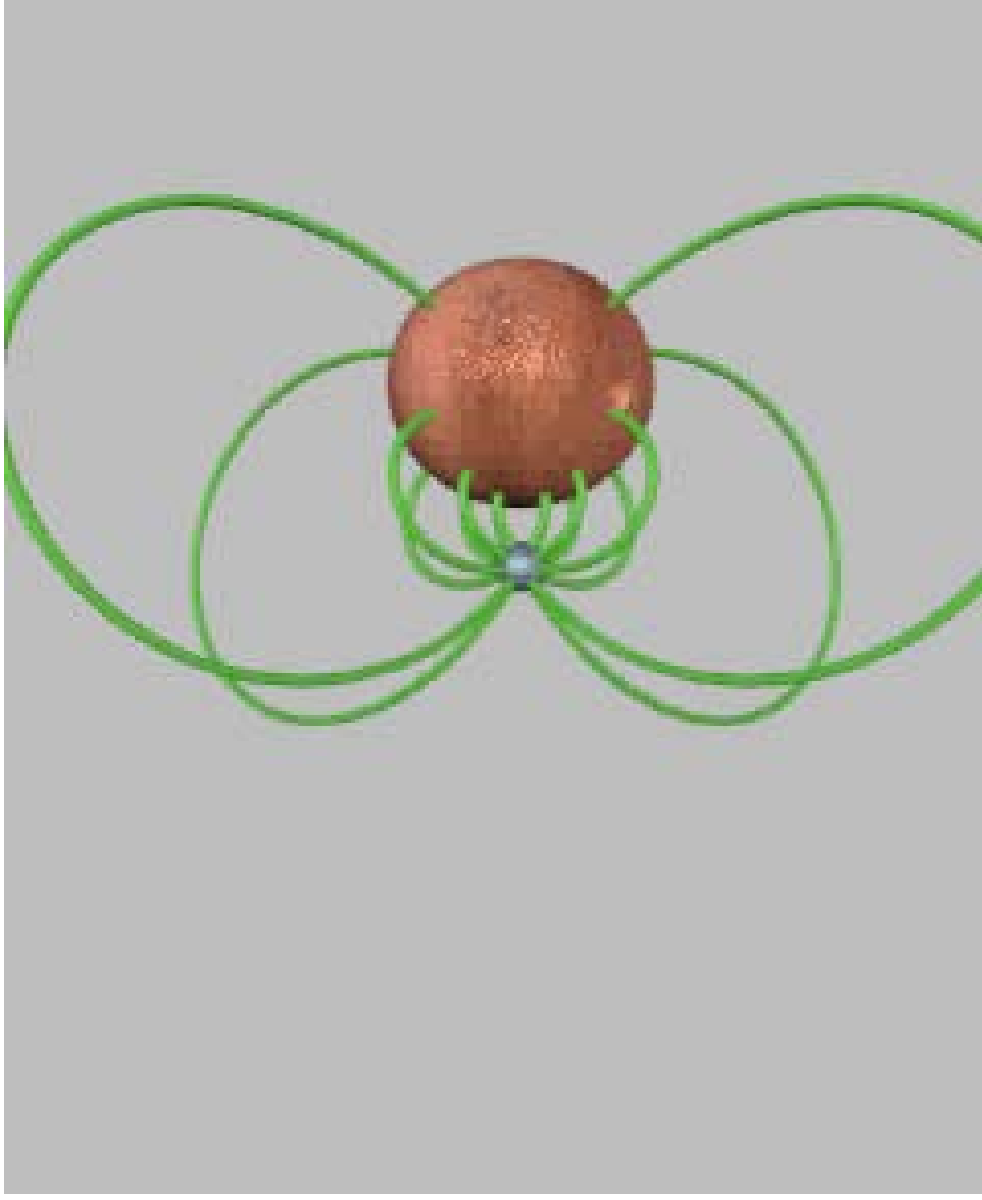
$$\varepsilon_a = \varepsilon_0$$

$$\varepsilon_b = 100\varepsilon_0$$



Boundary Conditions = ? / Randbedingungen = ?

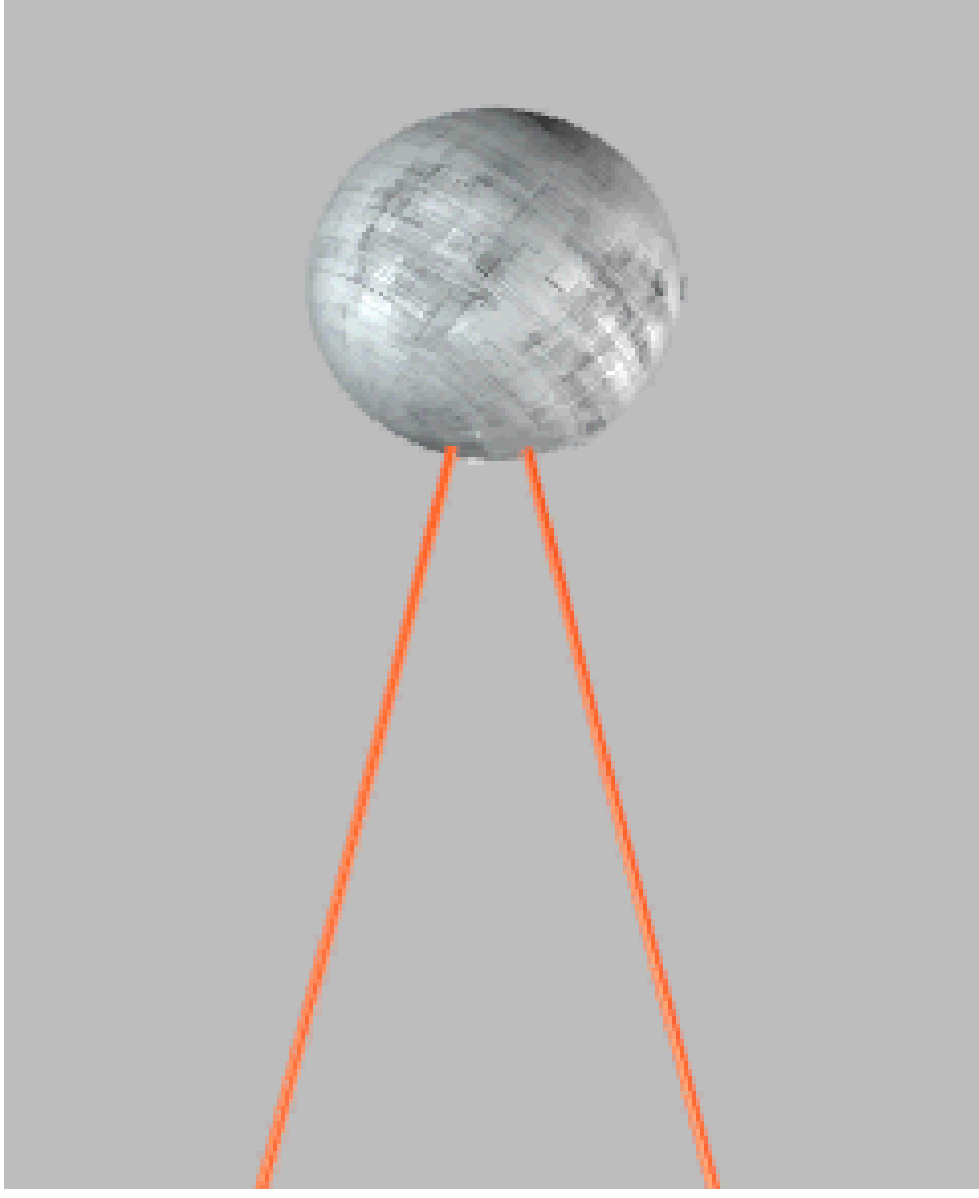
Point Charge Attracted to a Electrically Charged Sphere /
Punktladung angezogen von einer elektrisch geladenen Kugel



<http://web.mit.edu/jbelcher/www/att.html>

Boundary Conditions = ? / Randbedingungen = ?

Point Charge Repulsed By A Charged Sphere /
Punktladung abgestoßen von einer elektrisch geladenen Kugel



ES Fields: Transition and Boundary Conditions / ES-Felder: Übergangs- und Randbedingungen

Governing Equations in Integral Form /
Grundgleichungen in Integralform

$$\oint_{C=\partial S} \underline{E}(\underline{R}) \cdot d\underline{R} = 0$$

$$\oiint_{S=\partial V} \underline{D}(\underline{R}, t) \cdot d\underline{S} = \iiint_V \rho_e(\underline{R}, t) dV$$

Transition Conditions / Übergangsbedingungen



$$\underline{n} \times [\underline{E}^{(2)}(\underline{R}, t) - \underline{E}^{(1)}(\underline{R}, t)] = \underline{0}$$

$$\underline{n} \cdot [\underline{D}^{(2)}(\underline{R}, t) - \underline{D}^{(1)}(\underline{R}, t)] = \begin{cases} \eta_e(\underline{R}, t) & \text{ws / mq} \\ 0 & \text{sf / qf} \end{cases}$$

ws: with sources; sf = source-free /
mq = mit Quellen; qf = quellenfrei

Boundary Conditions / Randbedingungen



$$\underline{n} \times \underline{E}(\underline{R}, t) = \underline{0} \quad \text{pec / iel}$$

$$\underline{n} \cdot \underline{D}(\underline{R}, t) = \eta_e(\underline{R}, t) \quad \text{pec / iel}$$

pec = perfectly electric conducting; pmc = perfectly magnetic
conducting / iel = ideal elektrisch leitend; iml = ideal
magnetisch leitend

ES Fields: Transition and Boundary Conditions / ES-Felder: Übergangs- und Randbedingungen

Transition Conditions / Übergangsbedingungen



$$\underline{n} \times [\underline{E}^{(2)}(\underline{R}) - \underline{E}^{(1)}(\underline{R})] = \underline{0}$$

$$\underline{n} \cdot [\underline{D}^{(2)}(\underline{R}) - \underline{D}^{(1)}(\underline{R})] = \eta_e(\underline{R})$$

ws: with sources; sf = source-free /
mq = mit Quellen; qf = quellenfrei

$$\underline{n} \times \underline{E}(\underline{R}) = \underline{E}_{tan}(\underline{R}) \\ = E_{tan}(\underline{R}) \underline{e}_{tan}$$

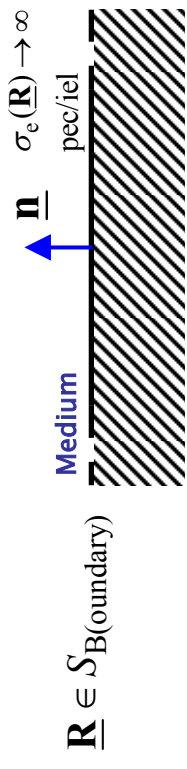
$\underline{E}_{tan}(\underline{R})$: Vector Tangential Component of $\underline{E}(\underline{R})$
Vektorielle Tangentialkomponente von $\underline{E}(\underline{R})$

$E_{tan}(\underline{R})$: Scalar Tangential Component of $\underline{E}(\underline{R})$
Skalare Tangentialkomponente von $\underline{E}(\underline{R})$

$$E_{tan}^{(2)}(\underline{R}) - E_{tan}^{(1)}(\underline{R}) = 0$$

$$D_n^{(2)}(\underline{R}) - D_n^{(1)}(\underline{R}) = \eta_e(\underline{R})$$

Boundary Conditions / Randbedingungen



$$\underline{n} \times \underline{E}(\underline{R}) = \underline{0} \quad \text{pec / iel}$$

$$\underline{n} \cdot \underline{D}(\underline{R}) = \eta_e(\underline{R}) \quad \text{pec / iel}$$

pec = perfectly electric conducting /
iel = ideal elektrisch leitend

$$\underline{n} \cdot \underline{D}(\underline{R}) = D_n(\underline{R})$$

$D_n(\underline{R})$: Scalar Normal Component of $\underline{D}(\underline{R})$
Skalare Normalkomponente von $\underline{D}(\underline{R})$

$$E_{tan}(\underline{R}) = 0 \quad \text{pec / iel}$$

$$D_n(\underline{R}) = \eta_e(\underline{R}) \quad \text{pec / iel}$$

ES Fields: Transition and Boundary Conditions / ES-Felder: Übergangs- und Randbedingungen

Transition Conditions / Übergangsbedingungen

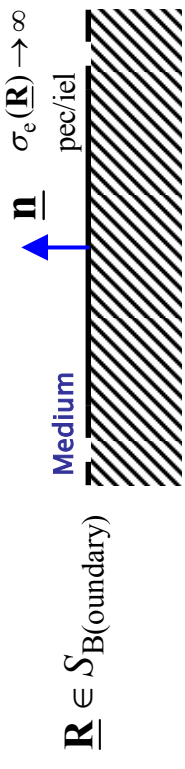


$$E_{tan}^{(2)}(\underline{\mathbf{R}}) - E_{tan}^{(1)}(\underline{\mathbf{R}}) = 0$$

$$D_n^{(2)}(\underline{\mathbf{R}}) - D_n^{(1)}(\underline{\mathbf{R}}) = \eta_e(\underline{\mathbf{R}})$$

ws: with sources; sf = source-free /
mq = mit Quellen; qf = quellenfrei

Boundary Conditions / Randbedingungen



$$E_{tan}(\underline{\mathbf{R}}) = 0 \quad \text{pec / iel}$$

$$D_n(\underline{\mathbf{R}}) = \eta_e(\underline{\mathbf{R}}) \quad \text{pec / iel}$$

pec = perfectly electric conducting /
iel = ideal elektrisch leitend

$$\underline{\mathbf{D}}^{(i)}(\underline{\mathbf{R}}) = \epsilon_0 \epsilon_r^{(i)} \underline{\mathbf{E}}^{(i)}(\underline{\mathbf{R}}) \quad i = 1, 2$$

$$= -\epsilon_0 \epsilon_r^{(i)} \nabla \Phi_e^{(i)}(\underline{\mathbf{R}})$$

$$\underline{\mathbf{n}} \times \nabla \Phi_e(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

$$\Phi_e(\underline{\mathbf{R}}) = 0$$

$$\epsilon_0 \epsilon_r \underbrace{\underline{\mathbf{n}} \cdot \nabla \Phi_e(\underline{\mathbf{R}})}_{= \frac{\partial}{\partial n}} = -\eta_e(\underline{\mathbf{R}})$$

$$\frac{\partial}{\partial n} \Phi_e(\underline{\mathbf{R}}) = -\frac{\eta_e(\underline{\mathbf{R}})}{\epsilon_0 \epsilon_r}$$

$$\underline{\mathbf{E}}^{(i)}(\underline{\mathbf{R}}) = -\nabla \Phi^{(i)}(\underline{\mathbf{R}})$$

$$\underline{\mathbf{n}} \times \nabla [\Phi_e^{(2)}(\underline{\mathbf{R}}) - \Phi_e^{(1)}(\underline{\mathbf{R}})] = \underline{\mathbf{0}}$$

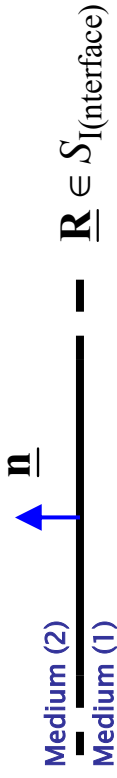
$$\Phi_e^{(2)}(\underline{\mathbf{R}}) - \Phi_e^{(1)}(\underline{\mathbf{R}}) = \Phi_e$$

$$\underbrace{\underline{\mathbf{n}} \cdot \nabla}_{= \frac{\partial}{\partial n}} [\epsilon_r^{(2)} \Phi_e^{(2)}(\underline{\mathbf{R}}) - \epsilon_r^{(1)} \Phi_e^{(1)}(\underline{\mathbf{R}})] = -\frac{\eta_e(\underline{\mathbf{R}})}{\epsilon_0}$$

$$\frac{\partial}{\partial n} \Phi_e^{(2)}(\underline{\mathbf{R}}) - \frac{\epsilon_r^{(1)}}{\epsilon_r^{(2)}} \frac{\partial}{\partial n} \Phi_e^{(1)}(\underline{\mathbf{R}}) = -\frac{\eta_e(\underline{\mathbf{R}})}{\epsilon_0 \epsilon_r^{(2)}}$$

ES Fields: Transition and Boundary Conditions / ES-Felder: Übergangs- und Randbedingungen

Transition Conditions / Übergangsbedingungen



$$E_{tan}^{(2)}(\underline{\mathbf{R}}) - E_{tan}^{(1)}(\underline{\mathbf{R}}) = 0$$

$$D_n^{(2)}(\underline{\mathbf{R}}) - D_n^{(1)}(\underline{\mathbf{R}}) = \eta_e(\underline{\mathbf{R}})$$

$$E_{tan}^{(2)}(\underline{\mathbf{R}}) - E_{tan}^{(1)}(\underline{\mathbf{R}}) = 0$$

⇓

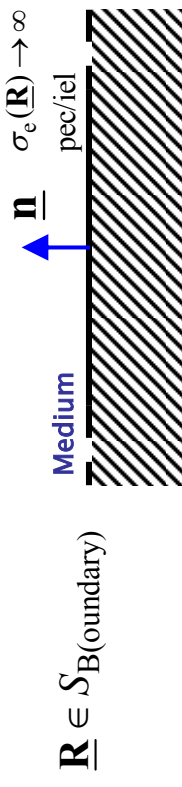
$$\Phi_e^{(2)}(\underline{\mathbf{R}}) - \Phi_e^{(1)}(\underline{\mathbf{R}}) = \Phi_{e0} = \text{const.}$$

$$D_n^{(2)}(\underline{\mathbf{R}}) - D_n^{(1)}(\underline{\mathbf{R}}) = \eta_e(\underline{\mathbf{R}})$$

⇓

$$\frac{\partial}{\partial n} \Phi_e^{(2)}(\underline{\mathbf{R}}) - \frac{\varepsilon_r^{(1)}}{\varepsilon_r^{(2)}} \frac{\partial}{\partial n} \Phi_e^{(1)}(\underline{\mathbf{R}}) = -\frac{1}{\varepsilon_0 \varepsilon_r^{(2)}} \eta_e(\underline{\mathbf{R}})$$

Boundary Conditions / Randbedingungen



$$E_{tan}(\underline{\mathbf{R}}) = 0 \quad \text{pec / iel}$$

$$D_n(\underline{\mathbf{R}}) = \eta_e(\underline{\mathbf{R}}) \quad \text{pec / iel}$$

$$E_{tan}(\underline{\mathbf{R}}) = 0$$

⇓

$$\Phi_e(\underline{\mathbf{R}}) = \Phi_{e0} = \text{const.}$$

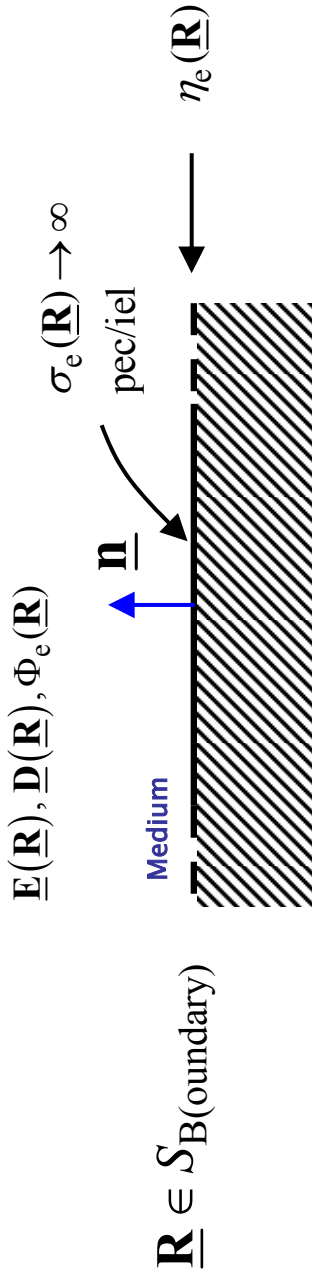
$$D_n(\underline{\mathbf{R}}) = \eta_e(\underline{\mathbf{R}})$$

⇓

$$\frac{\partial}{\partial n} \Phi_e(\underline{\mathbf{R}}) = -\frac{1}{\varepsilon_0 \varepsilon_r} \eta_e(\underline{\mathbf{R}})$$

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Boundary Conditions / Randbedingungen



$$\Phi_e(\underline{\mathbf{R}}) = \Phi_{e0} = \text{const.} \quad (\Phi_{e0} = 0 \text{ V})$$

$$\frac{\partial}{\partial n} \Phi_e(\underline{\mathbf{R}}) = -\frac{1}{\epsilon_0 \epsilon_r} \eta_e(\underline{\mathbf{R}})$$

Neumann Boundary Conditions for Φ_e /
Neumann-Randbedingung für Φ_e

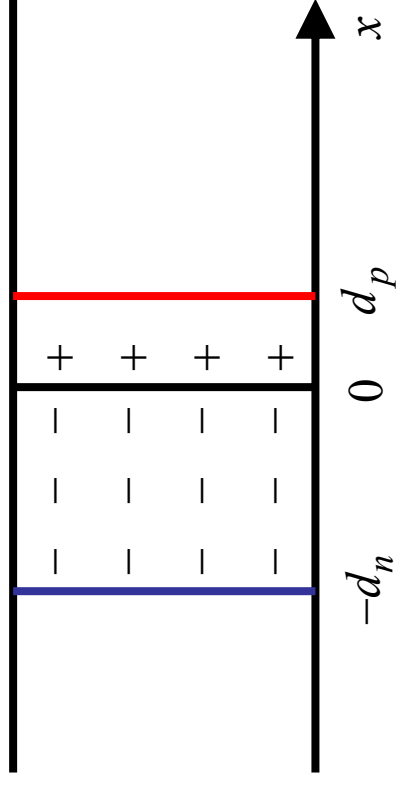
Dirichlet Boundary Conditions for Φ_e /
Dirichlet-Randbedingung für Φ_e

Electrostatic (ES) Fields – Poisson and Laplace Equation / Elektrostatistische (ES) Felder – Poisson- und Laplace-Gleichung (3)

Laplace Operator in Cartesian Coordinates / Laplace-Operator in Kartesischen Koordinaten

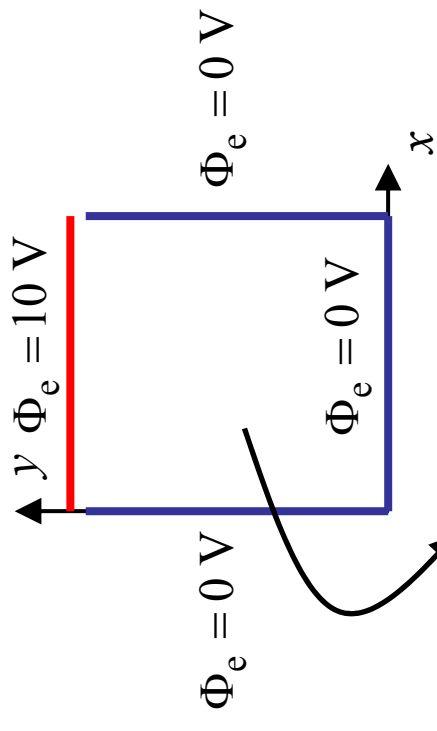
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi_e(x, y, z) = \begin{cases} -\frac{\rho_e(x, y, z)}{\epsilon_0} & \text{for / für } \rho_e(x, y, z) \neq 0 & \text{Poisson Equation / Poisson-Gleichung} \\ 0 & \text{for / für } \rho_e(x, y, z) = 0 & \text{Laplace Equation / Laplace-Gleichung} \end{cases}$$

Example: pn Junction – pn Diode /
Beispiel: pn-Übergang – pn Diode



$$\frac{d^2}{dx^2} \Phi_e(x) = \frac{e}{\epsilon} \begin{cases} -n_e & \text{for / für } -d_n \leq x \leq 0 \\ n_e & \text{for / für } 0 \leq x \leq d_p \end{cases}$$

Example: / Beispiel:



$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Phi_e(x, y) = 0$$

➔ Separation of Variables /
Separation der Variablen !

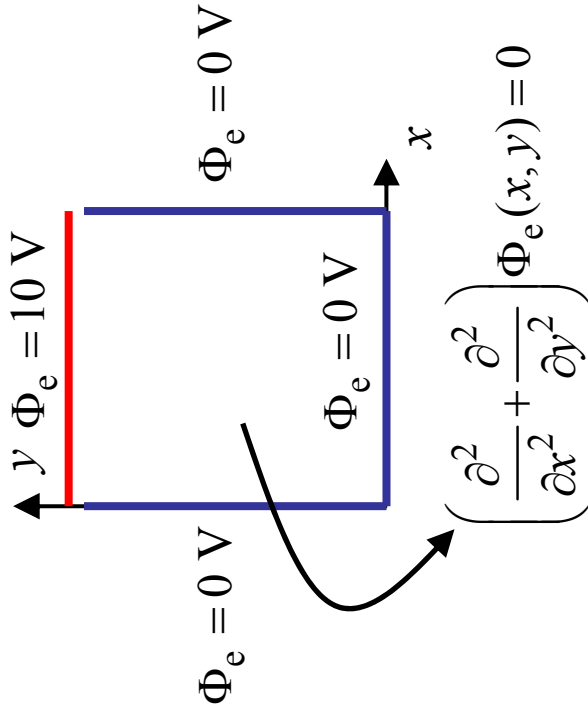
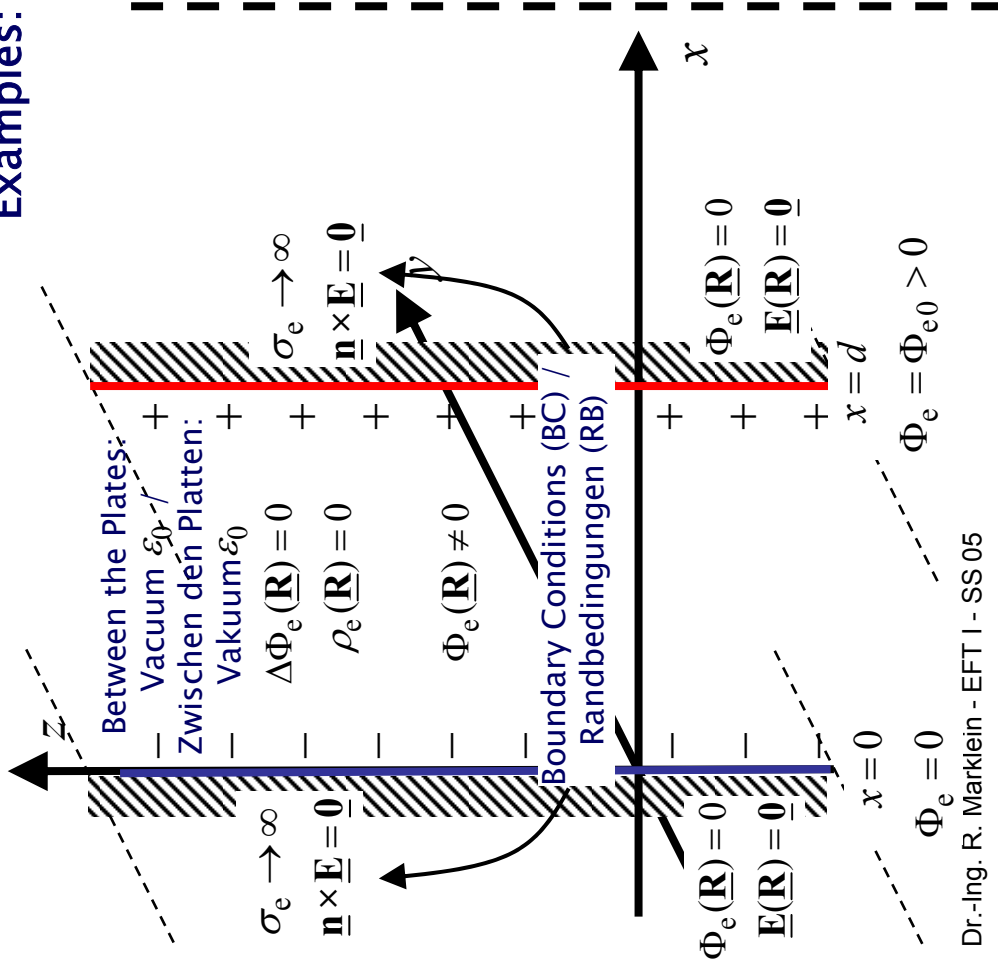
Electrostatic (ES) Fields – Boundary Value Problem (BVP) / Elektrostatistische (ES) Felder – Randwertproblem (RWP)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi_e(x, y, z) = \begin{cases} -\frac{\rho_e(x, y, z)}{\epsilon_0} & \text{for / für } \rho_e(x, y, z) \neq 0 \\ 0 & \text{for / für } \rho_e(x, y, z) = 0 \end{cases}$$

Poisson Equation /
Poisson-Gleichung

Laplace Equation /
Laplace-Gleichung

Examples: / Beispiele:



**Separation of Variables /
Separation der Variablen !**

ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation /
Randwertproblem (RWP) – Elektrostatistische Poisson-Gleichung

$$\left. \begin{array}{l} = 0 \\ \neq \text{const.} \end{array} \right\} \begin{array}{l} \text{for /} \\ \text{für} \end{array} \begin{array}{l} 0 < x < d \\ x = 0 \\ x = d \end{array}$$

Partial Differential Equation /
Partielle Differentialgleichung $\Delta\Phi_e(\underline{\mathbf{R}})$

$$\Delta\Phi_e(\underline{\mathbf{R}}) \neq 0$$

$$\rho_e(\underline{\mathbf{R}}) \neq 0$$

Between the Plates:
Zwischen den Platten:
Vacuum ϵ_0 /
Vakuum ϵ_0

$$\Delta\Phi_e(\underline{\mathbf{R}}) = 0$$

$$\rho_e(\underline{\mathbf{R}}) = 0$$

$$\Phi_e(\underline{\mathbf{R}}) \neq 0$$

$$\sigma_e \rightarrow \infty$$

$$\underline{\mathbf{n}} \times \underline{\mathbf{E}} = \underline{\mathbf{0}}$$

Boundary Conditions (BC) /
Randbedingungen (RB)

$$\Phi_e(\underline{\mathbf{R}}) = 0$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

$$x = 0$$

$$\Phi_e = 0$$

$$\Phi_e = \Phi_{e0} > 0$$

Boundary Conditions (BC) /
Randbedingungen (RB)

$$x = 0: \quad \Phi_e = 0$$

$$x = d: \quad \Phi_e = \Phi_{e0} > 0$$

Between the Plates Laplace Equation:
Zwischen den Platten: Laplace-Gleichung

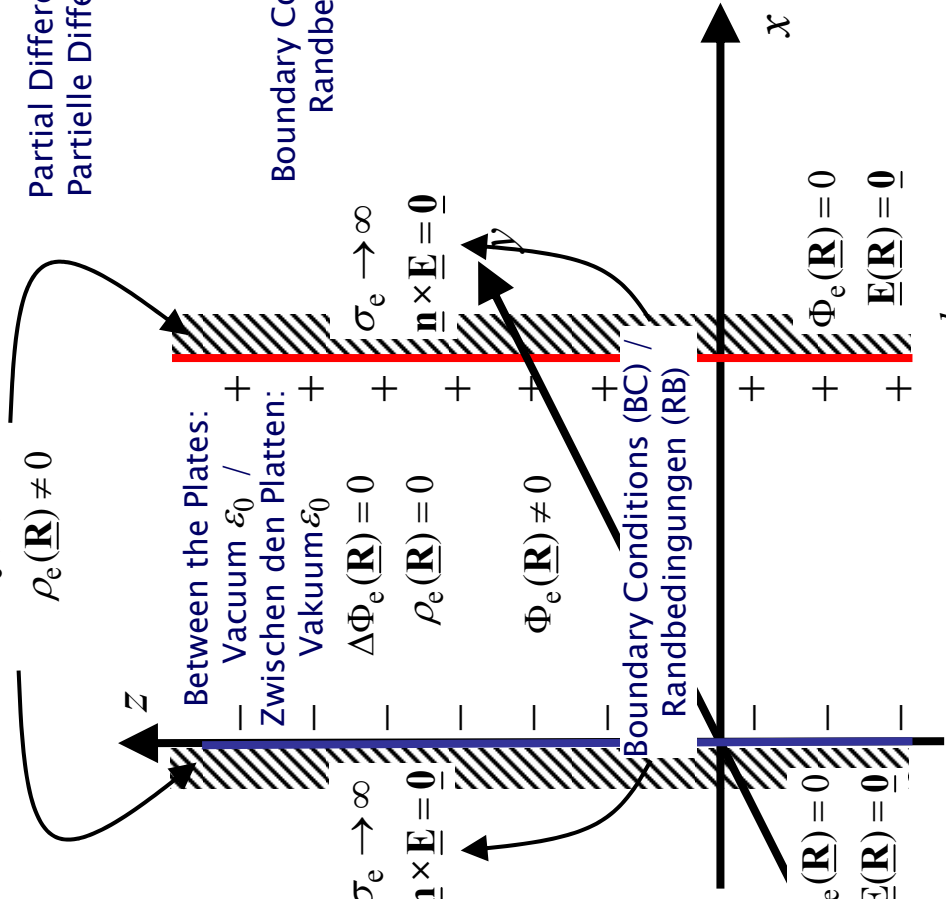
$$\Delta\Phi_e(\underline{\mathbf{R}}) = 0$$

... Cartesian Coordinates /
... Kartesische Koordinaten

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi_e(x, y, z) = 0$$

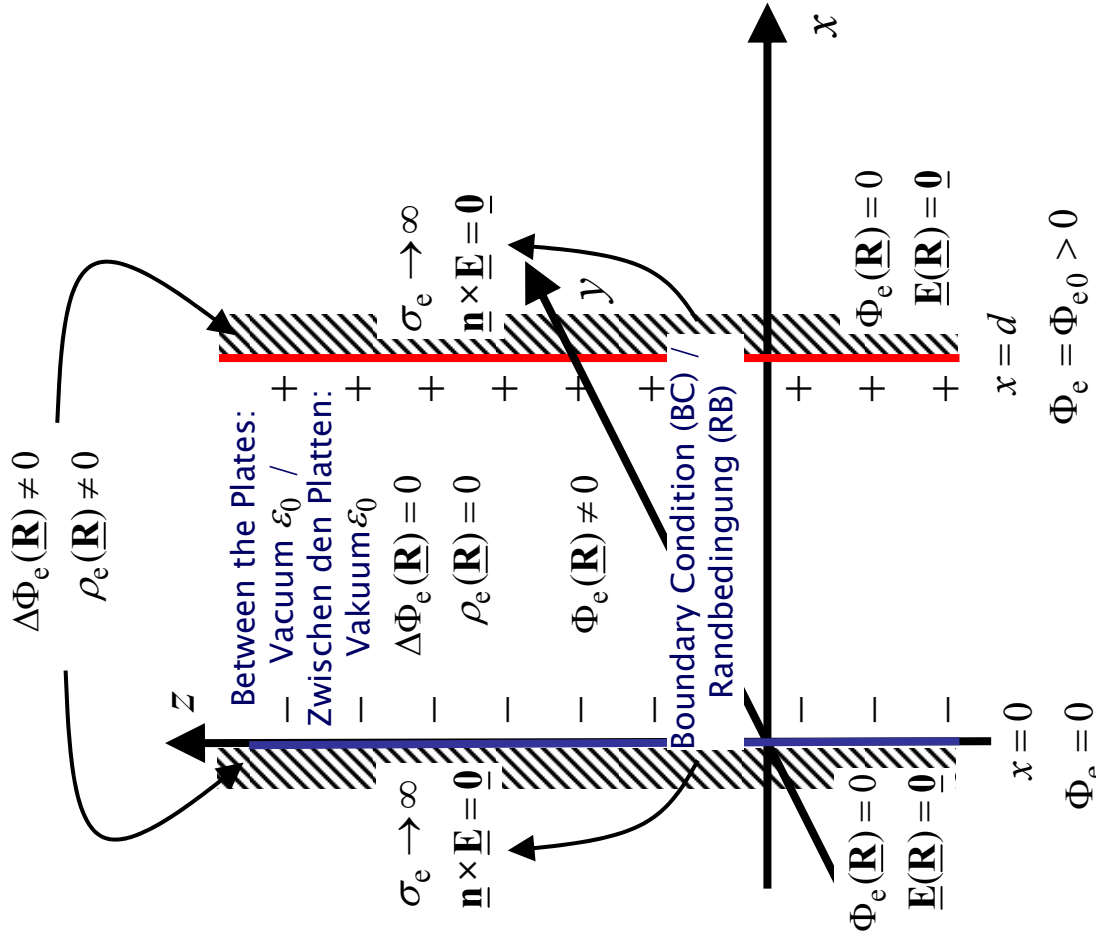
... Because of the Symmetry /
... wegen der Symmetrie

$$\frac{d^2}{dx^2} \Phi_e(x) = 0$$



ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation /
Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



$$\frac{d^2}{dx^2} \Phi_e(x) = 0 \quad 0 < x < d$$

Integrating once / Integriere einmal

$$\int \frac{d^2}{dx^2} \Phi_e(x) dx = \left[\frac{d}{dx} \Phi_e(x) \right] = \text{const} = a$$

$$\left[\frac{d}{dx} \Phi_e(x) \right] = \text{const} = a$$

Integrating twice / Zweifache Integration ergibt

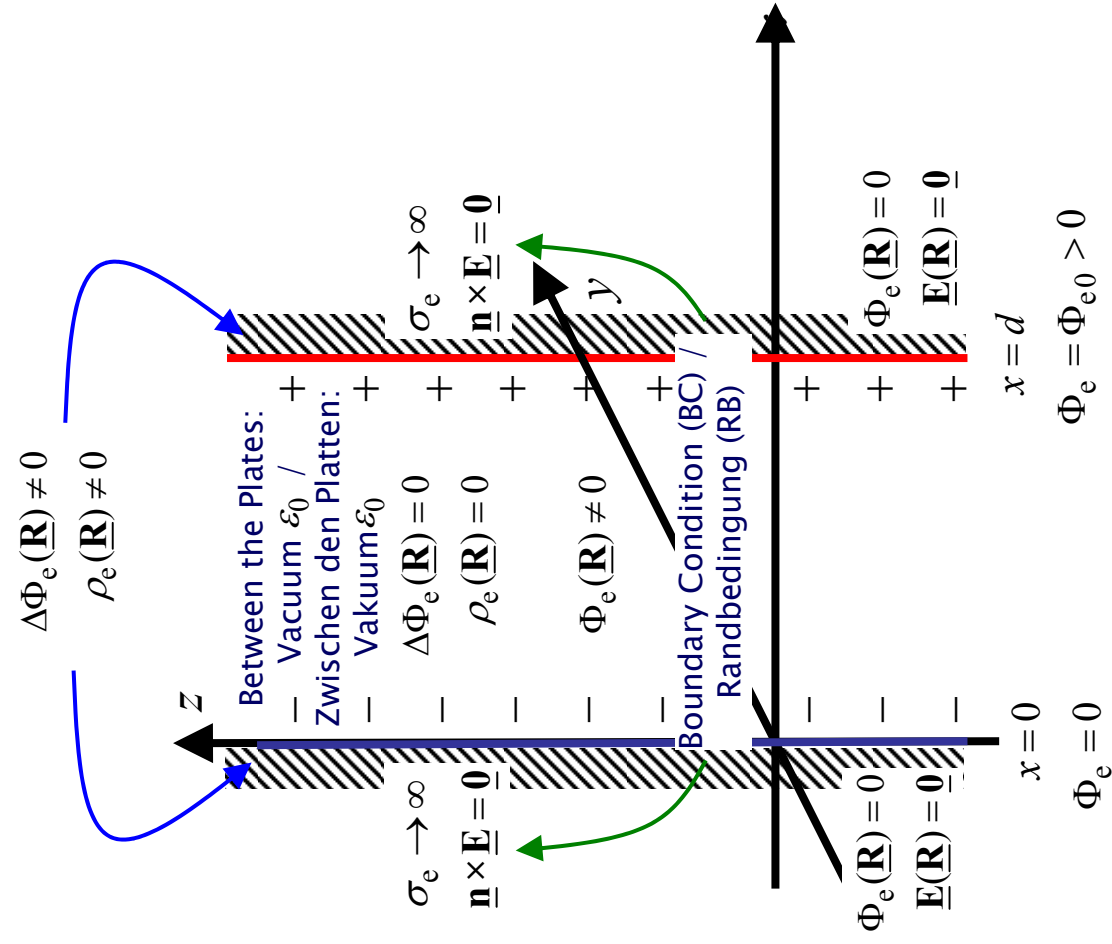
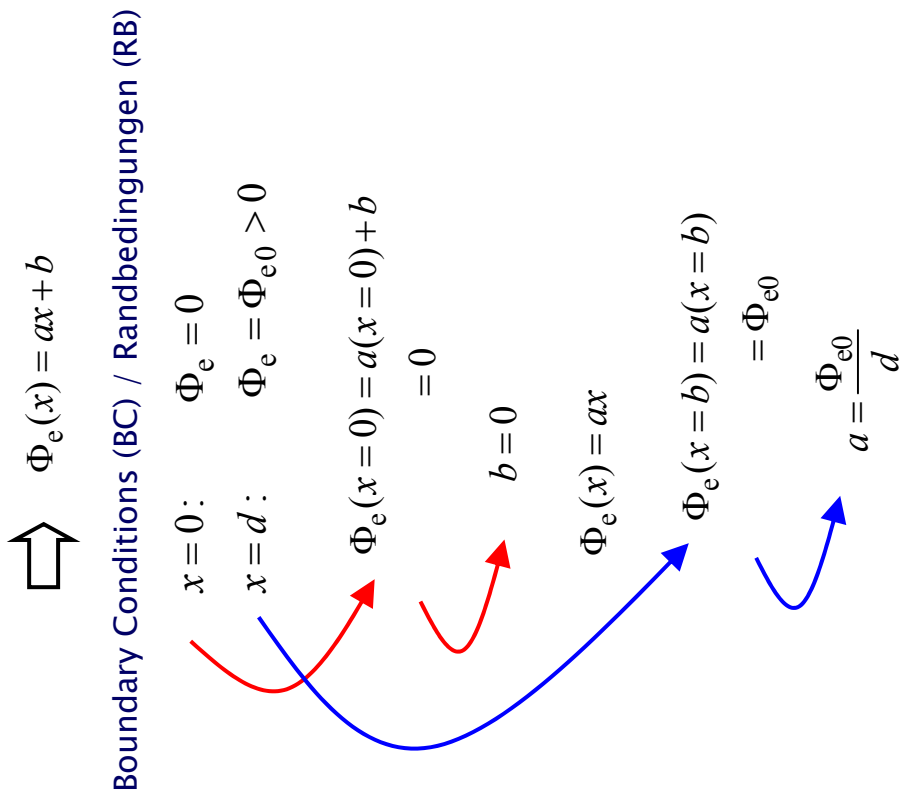
$$\int \left\{ \left[\frac{d}{dx} \Phi_e(x) \right] = \text{const} = a \right\} dx = \Phi_e(x) = ax + b$$

$$\Phi_e(x) = ax + b$$

$$\Rightarrow \Phi_e(x) = ax + b \quad 0 < x < d$$

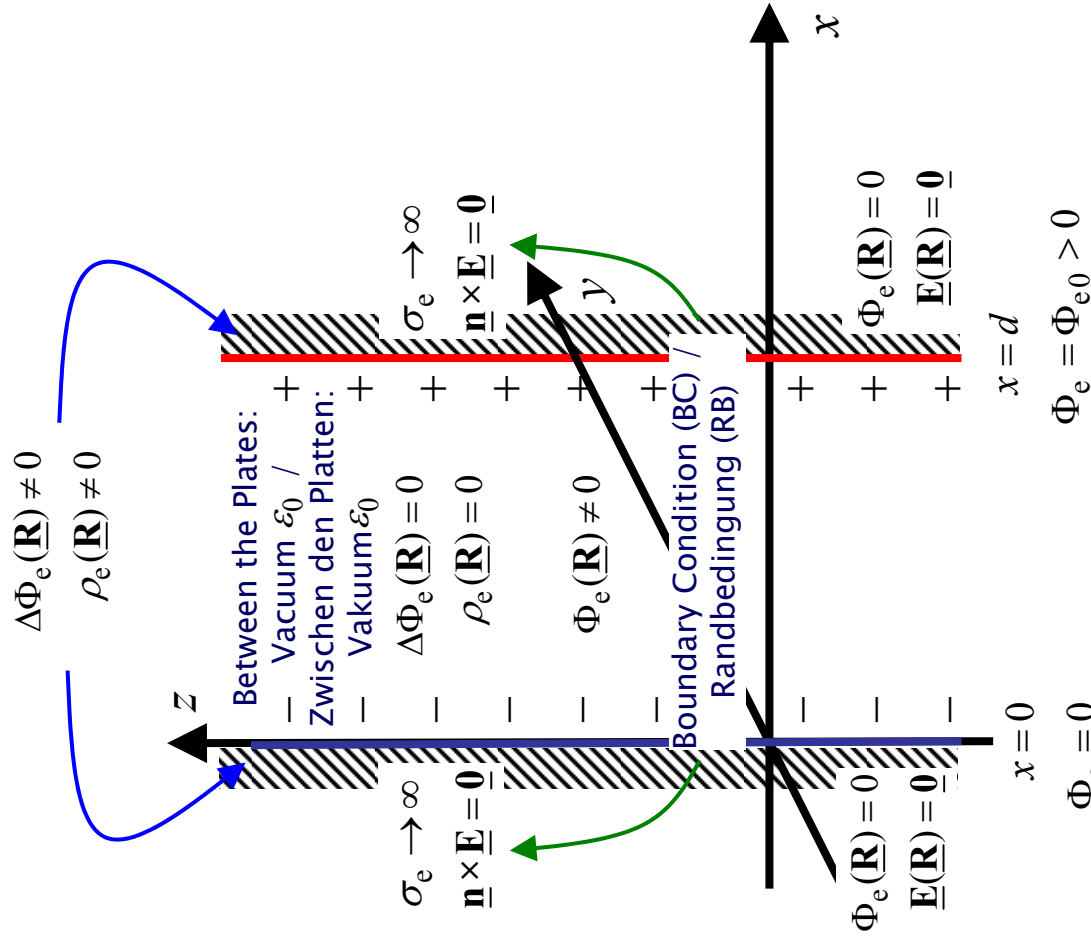
ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation /
Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation /
Randwertproblem (RWP) – Elektrostatische Poisson–Gleichung



Partial Differential Equation (PDE) /
Partielle Differentialgleichung (DGL)

$$\frac{d^2}{dx^2} \Phi_e(x) = 0 \quad 0 < x < d$$

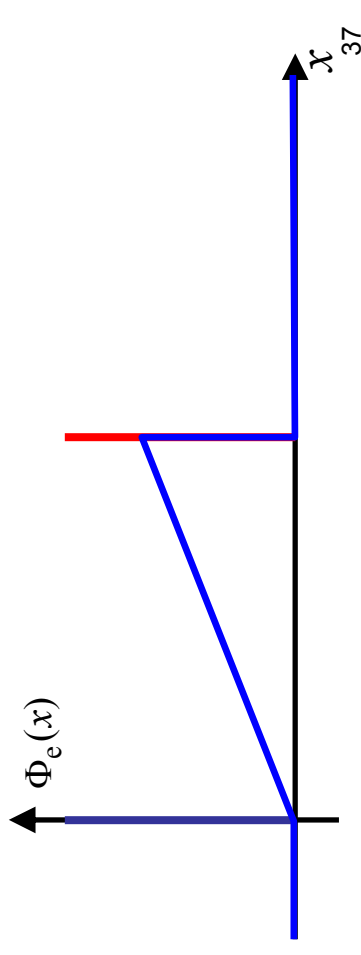
Boundary Conditions (BC) / Randbedingungen (RB)

$$x = 0: \quad \Phi_e = 0$$

$$x = d: \quad \Phi_e = \Phi_{e0} > 0$$

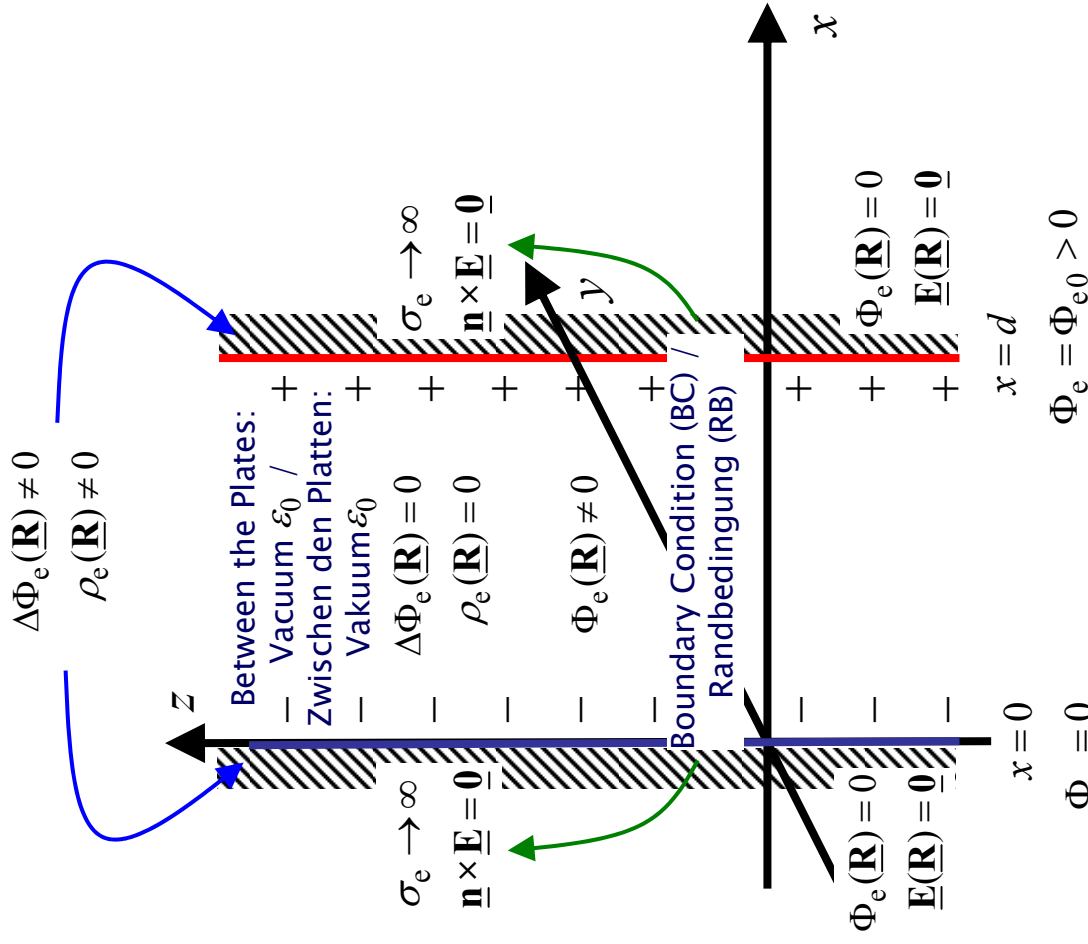
Solution for the Electrostatic Potential /
Lösung für das elektrostatische Potential

$$\Phi_e(x) = \begin{cases} \frac{\Phi_{e0}}{d}x & 0 \leq x \leq d \\ 0 & \text{else / sonst} \end{cases}$$



ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation /
Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



Electrostatic Potential / Elektrostatisches Potential

$$\Phi_e(x) = \begin{cases} \frac{\Phi_{e0}}{d}x & 0 \leq x \leq d \\ 0 & \text{else / sonst} \end{cases}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_e(\underline{\mathbf{R}})$$

$$\underline{\mathbf{E}}(x) = -\frac{d}{dx}\Phi_e(x)\underline{\mathbf{e}}_x \quad -\infty < x < \infty$$

$$= \begin{cases} -\frac{d}{dx}\left(\frac{\Phi_{e0}}{d}x\right)\underline{\mathbf{e}}_x & 0 \leq x \leq d \\ 0 & \text{else / sonst} \end{cases}$$

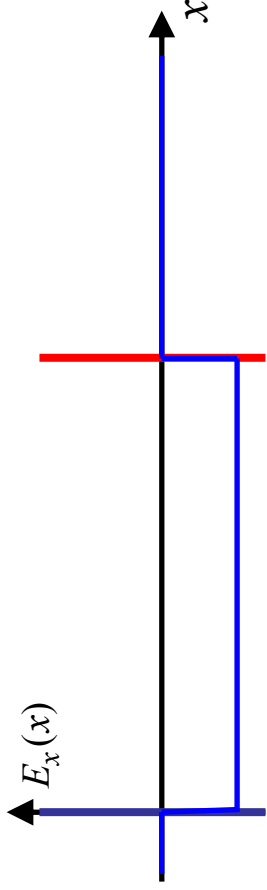
$$= \begin{cases} -\frac{\Phi_{e0}}{d}\underline{\mathbf{e}}_x & 0 \leq x \leq d \\ 0 & \text{else / sonst} \end{cases}$$

⇒ The Electrostatic Potential and Electrostatic Field Strength are Discontinuous at the Plates /
Das elektrostatische Potential und die elektrostatische Feldstärke sind unstetig an den Platten

ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

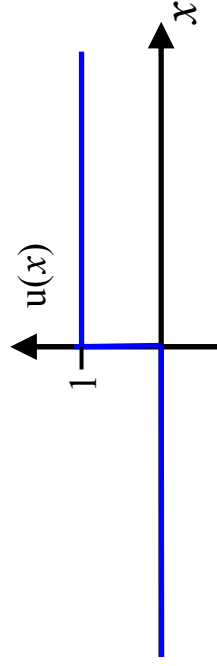
Boundary Value Problem (BVP) – Electrostatic Poisson Equation /
Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung

$$\underline{\mathbf{E}}(x) = \begin{cases} -\frac{\Phi_{e0}}{d} \mathbf{e}_x & 0 \leq x \leq d \\ 0 & \text{else / sonst} \end{cases}$$

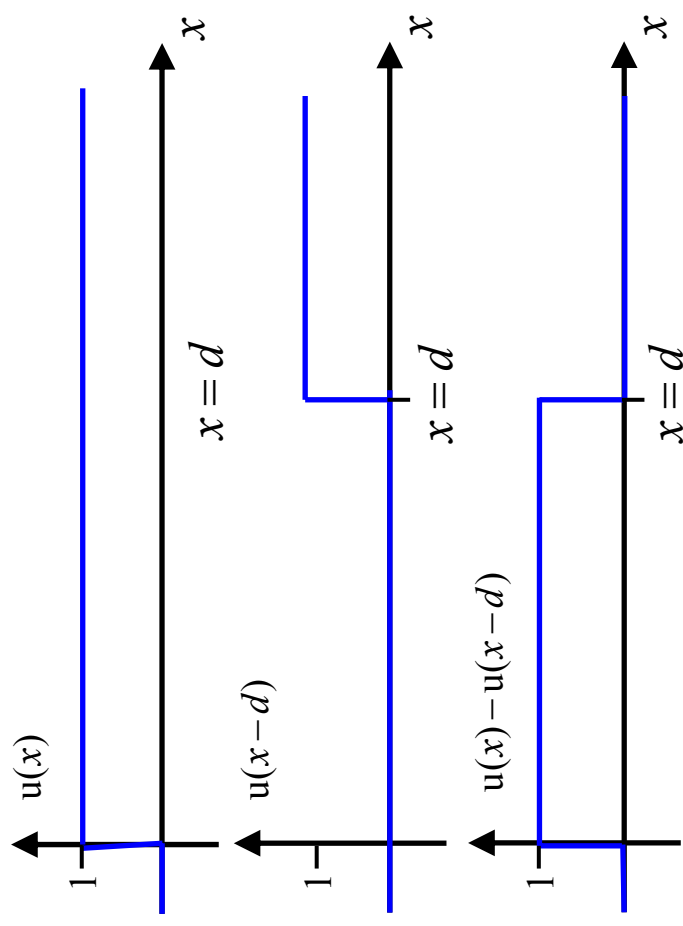


Step Functions / Einheitssprungfunktionen

$$u(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$



Representation of the Electrostatic Field Strength
using the Unit Step Functions: /
Darstellung der elektrostatischen Feldstärke
durch Einheitssprungfunktionen:



$$\underline{\mathbf{E}}(x) = -\frac{\Phi_{e0}}{d} [u(x) - u(x-d)] \mathbf{e}_x \quad -\infty < x < \infty$$

ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation /
Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung

$$\nabla \cdot \underline{\mathbf{D}}(\mathbf{R}) = \rho_e(\mathbf{R})$$

$$\frac{d}{dx} D_x(x) = \rho_e(x)$$

$$\varepsilon_0 \frac{d}{dx} E_x(x) = \rho_e(x)$$

$$\frac{d}{dx} E_x(x) = \frac{\rho_e(x)}{\varepsilon_0}$$

$$\begin{aligned} \frac{d}{dx} E_x(x) &= -\frac{\Phi_{e0}}{d} \frac{d}{dx} [u(x) - u(x-d)] \\ &= -\frac{\Phi_{e0}}{d} \left[\underbrace{\frac{d}{dx} u(x)}_{\delta(x)} - \underbrace{\frac{d}{dx} u(x-d)}_{\delta(x-d)} \right] \\ &= -\frac{\Phi_{e0}}{d} [\delta(x) - \delta(x-d)] \\ &= \frac{\rho_e(x)}{\varepsilon_0} \end{aligned}$$

$$\frac{d}{dx} u(x) \underset{=u'(x)}{=} \delta(x)$$

$$\int_{-\infty}^{\infty} u'(x) f(x) dx = u(x) f(x) \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} u(x) f'(x) dx$$

$$= \left[\underbrace{u(\infty)}_{=1} \underbrace{f(\infty)}_{=f(\infty)} - \underbrace{u(-\infty)}_0 \underbrace{f(-\infty)}_{=0} \right] - \int_0^{\infty} f'(x) dx$$

$$= f(\infty) - f(x) \Big|_0^{\infty}$$

$$= f(\infty) - [f(\infty) - f(0)]$$

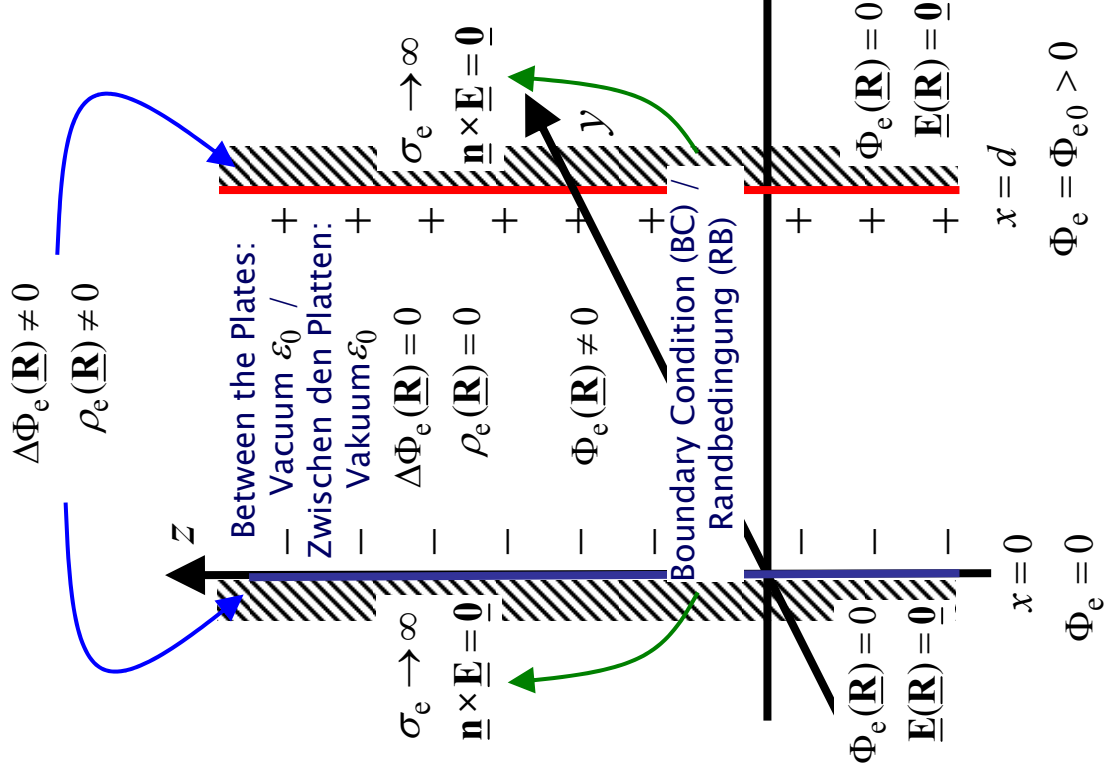
$$= f(0)$$

$$\int_{-\infty}^{\infty} \underbrace{u'(x)}_{=\delta(x)} f(x) dx = f(0)$$

$$u'(x) = \delta(x)$$

ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

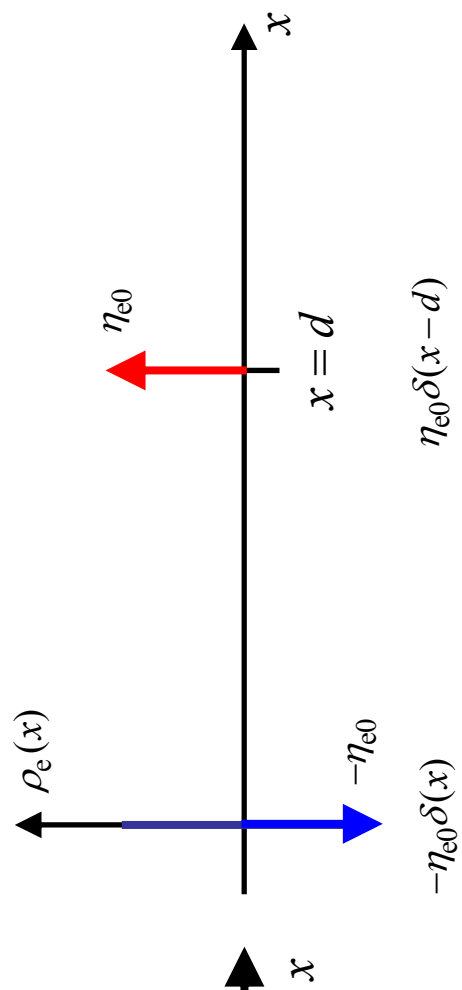
Boundary Value Problem (BVP) – Electrostatic Poisson Equation /
Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



$$\rho_e(x) = \epsilon_0 \underbrace{\frac{\Phi_{e0}}{d}}_{=\eta_{e0}} [-\delta(x) + \delta(x-d)]$$

Electric Surface Charge Density /
Elektrische Flächenladungsdichte

$$= -\eta_{e0}\delta(x) + \eta_{e0}\delta(x-d)$$



End of Lecture 6 / Ende der 6. Vorlesung