Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

4th Lecture / 4. Vorlesung

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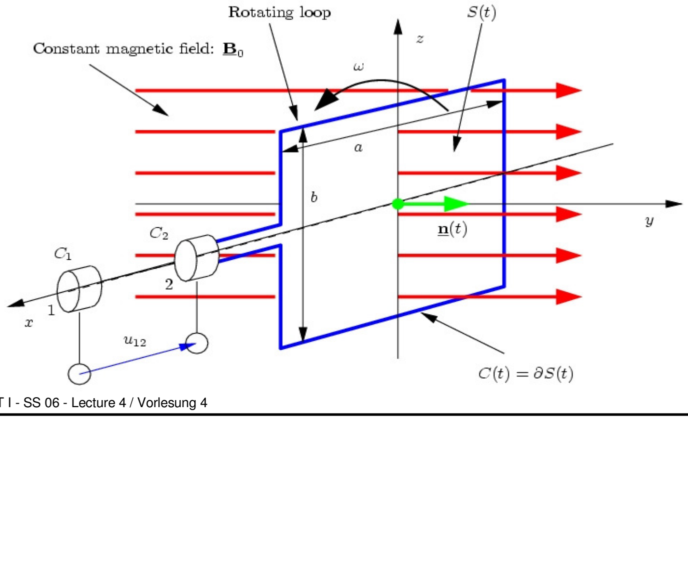
Faraday's Induction Law in Integral Form / Faradaysches Induktionsgesetz in Integralform (1)

Faraday's Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C(t)=\partial S(t)} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{S(t)} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} - \iint_{S(t)} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

Time Dependent Surface / Zeitabhängige Fläche $S(t)$

$C(t) = \partial S(t)$ **Time Dependent Contour / Zeitabhängige Kontur**



Faraday's Induction Law in Integral Form / Faradaysches Induktionsgesetz in Integralform (2)

Faraday's Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C(t)=\partial S(t)} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{S(t)} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} - \iint_{S(t)} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$\oint_{C(t)=\partial S(t)}[\circ]\bullet\underline{\mathbf{dR}}$	[m]	Closed Contour Integral / Geschlossenes Kurvenintegral
$\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$	[V/m]	Electric Field Strength / Elektrische Feldstärke
$\underline{\mathbf{dR}}$	[m]	Vectorial Differential Line Element / Vektorielles differentielles Linienelement
$\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}}$	[V]	Scalar Product of E and dR = tangential projection of E onto dR / Skalarprodukt von E auf dR = Tangentialprojektion von E auf dR

Vectorial Differential Line Element / Vektorielles differentielles Linienelement → $\underline{\mathbf{dR}} = \underline{\mathbf{s}}\ \mathrm{d}R$

Tangential Unit Vector / Tangentialer Einheitsvektor

Scalar Differential Line Element / Skalares differentielles Linienelement

Different Products / Verschiedene Produkte

Scalar Product / Skalarprodukt $C = \underline{\mathbf{A}}\bullet\underline{\mathbf{B}}$

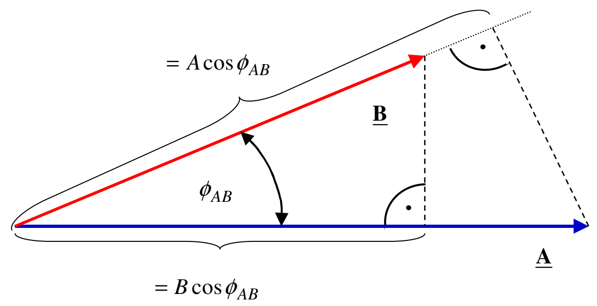
Vector Product / Vektorprodukt $\underline{\mathbf{C}} = \underline{\mathbf{A}}\times\underline{\mathbf{B}}$

Dyadic Product / Dyadisches Produkt $\underline{\underline{\mathbf{C}}} = \underline{\mathbf{A}}\,\underline{\mathbf{B}}$

Scalar Product (Dot or Inner Product) / Skalarprodukt (Punktprodukt oder inneres Produkt) (1)

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = |\underline{\mathbf{A}}||\underline{\mathbf{B}}| \cos \underbrace{\angle(\underline{\mathbf{A}}, \underline{\mathbf{B}})}_{\phi_{AB}}$$
$$= AB \cos \phi_{AB}$$

Enclosed Angle / Eingeschlossener Winkel ϕ_{AB}



$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = \underline{\mathbf{B}} \bullet \underline{\mathbf{A}}$$
$$= BA \cos \phi_{BA}$$
$$= AB \cos \phi_{AB}$$

$$\cos(\phi_{AB}) = \cos(-\phi_{AB})$$

$$\cos \phi_{AB} = \frac{\underline{\mathbf{A}} \bullet \underline{\mathbf{B}}}{|\underline{\mathbf{A}}||\underline{\mathbf{B}}|}$$

$$\phi_{AB} = \arccos\left(\frac{\underline{\mathbf{A}} \bullet \underline{\mathbf{B}}}{|\underline{\mathbf{A}}||\underline{\mathbf{B}}|}\right)$$

Scalar Product (Dot or Inner Product) / Skalarprodukt (Punktprodukt oder inneres Produkt) (2)

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \bullet (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z)$$
$$= A_x B_x \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_x}_{=1} + A_x B_y \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_y}_{=0} + A_x B_z \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_z}_{=0}$$
$$+ A_y B_x \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_x}_{=0} + A_y B_y \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_y}_{=1} + A_y B_z \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_z}_{=0}$$
$$+ A_z B_x \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_x}_{=0} + A_z B_y \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_y}_{=0} + A_z B_z \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_z}_{=1}$$
$$= A_x B_x + A_y B_y + A_z B_z$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_x = 1 \quad \underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_x = 0 \quad \underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_x = 0$$
$$\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_y = 0 \quad \underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_y = 1 \quad \underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_y = 0$$
$$\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_z = 0 \quad \underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_z = 0 \quad \underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_z = 1$$

Cartesian Coordinates / Kartesische Koordinaten

$$x = x_1$$
$$y = x_2$$
$$z = x_3$$

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \bullet (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z)$$
$$= A_x B_x + A_y B_y + A_z B_z$$
$$= (A_{x_1} \underline{\mathbf{e}}_{x_1} + A_{x_2} \underline{\mathbf{e}}_{x_2} + A_{x_3} \underline{\mathbf{e}}_{x_3}) \bullet (B_{x_1} \underline{\mathbf{e}}_{x_1} + B_{x_2} \underline{\mathbf{e}}_{x_2} + B_{x_3} \underline{\mathbf{e}}_{x_3})$$
$$= A_{x_1} B_{x_1} + A_{x_2} B_{x_2} + A_{x_3} B_{x_3}$$
$$= \sum_{i=1}^{3} A_{x_i} B_{x_i}$$

Scalar Product (Dot or Inner Product) / Skalarprodukt (Punktprodukt oder inneres Produkt) (3)

$$
\begin{aligned}
\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} &= (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \bullet (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z) \\
&= \sum_{i=1}^{3} A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet \sum_{j=1}^{3} B_{x_j} \underline{\mathbf{e}}_{x_j} \\
&= \sum_{i=1}^{3} \sum_{j=1}^{3} A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet B_{x_j} \underline{\mathbf{e}}_{x_j} \\
&= \sum_{i=1}^{3} \sum_{j=1}^{3} A_{x_i} B_{x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{=\delta_{ij}} \\
&= A_{x_i} B_{x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{=\delta_{ij}} \\
&= A_{x_i} \underbrace{B_{x_j} \delta_{ij}}_{=B_{x_i}} \quad \left(\text{or/oder} \quad \underbrace{\underbrace{A_{x_i} \delta_{ij}}_{=A_{x_j}} B_{x_j}}_{=A_{x_j} B_{x_j}} \right) \\
&= A_{x_i} B_{x_i}
\end{aligned}
$$

Kronecker Delta / Kronecker-Delta

$$
\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}
$$

with Einstein's Summation Convention / mit Einsteinscher Summationskonvention

***Einstein's Summation Convention*:** **If a index appears two times at one side of an equation (and not at the other side), the index is automatically summed over 1 to 3. /** ***Einsteinsche Summenkonvention*:** **Wenn ein Index auf einer Seite einer Gleichung zweimal vorkommt (und auf der anderen nicht), wird darüber von 1 bis 3 summiert.**

Magnitude of a Vector / Betrag eines Vektors

$$
\begin{aligned}
|\underline{\mathbf{A}}| &= \sqrt{\underline{\mathbf{A}} \bullet \underline{\mathbf{A}}} \\
&= \sqrt{(\mathrm{A}_x \underline{\mathbf{e}}_x + \mathrm{A}_y \underline{\mathbf{e}}_y + \mathrm{A}_z \underline{\mathbf{e}}_z) \bullet (\mathrm{A}_x \underline{\mathbf{e}}_x + \mathrm{A}_y \underline{\mathbf{e}}_y + \mathrm{A}_z \underline{\mathbf{e}}_z)} \\
&= \Big(\mathrm{A}_x \mathrm{A}_x \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_x}_{=1} + \mathrm{A}_x \mathrm{A}_y \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_y}_{=0} + \mathrm{A}_x \mathrm{A}_z \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_z}_{=0} \\
&\quad + \mathrm{A}_y \mathrm{A}_x \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_x}_{=0} + \mathrm{A}_y \mathrm{A}_y \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_y}_{=1} + \mathrm{A}_y \mathrm{A}_z \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_z}_{=0} \\
&\quad + \mathrm{A}_z \mathrm{A}_x \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_x}_{=0} + \mathrm{A}_z \mathrm{A}_y \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_y}_{=0} + \mathrm{A}_z \mathrm{A}_z \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_z}_{=1} \Big)^{\frac{1}{2}} \\
&= \sqrt{\mathrm{A}_x \mathrm{A}_x + \mathrm{A}_y \mathrm{A}_y + \mathrm{A}_z \mathrm{A}_z} \\
&= \sqrt{\mathrm{A}_x^2 + \mathrm{A}_y^2 + \mathrm{A}_z^2} \\
&= \mathrm{A}
\end{aligned}
$$

$$
\begin{aligned}
|\underline{\mathbf{A}}| &= \sqrt{\underline{\mathbf{A}} \bullet \underline{\mathbf{A}}} \\
&= \sqrt{\sum_{i=1}^{3} A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet \sum_{j=1}^{3} B_{x_j} \underline{\mathbf{e}}_{x_j}} \\
&= \sqrt{A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet A_{x_j} \underline{\mathbf{e}}_{x_j}} \\
&= \sqrt{A_{x_i} A_{x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{=\delta_{ij}}} \\
&= \sqrt{A_{x_i}^2}
\end{aligned}
$$

Example: Position Vector and Electric Field Strength Vector / Beispiel: Ortsvektor und elektrischer Feldstärkevektor

Cartesian Coordinate System / Kartesisches Koordinatensystem

Position Vector / Ortsvektor

$$\underline{\mathbf{R}}(x,y,z) = \mathrm{R}_x(x,y,z)\underline{\mathbf{e}}_x + \mathrm{R}_y(x,y,z)\underline{\mathbf{e}}_y + \mathrm{R}_z(x,y,z)\underline{\mathbf{e}}_z = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$

Electric Field Strength Vector / Elektrische Feldstärkevektor

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{E}}(x,y,z,t) = \mathrm{E}_x(x,y,z,t)\underline{\mathbf{e}}_x + \mathrm{E}_y(x,y,z,t)\underline{\mathbf{e}}_y + \mathrm{E}_z(x,y,z,t)\underline{\mathbf{e}}_z$$

Magnitude of the Position Vector (Distance) / Betrag des Ortsvektor (Abstand)

$$|\underline{\mathbf{R}}(x,y,z)| = \sqrt{\underline{\mathbf{R}}(x,y,z)\bullet\underline{\mathbf{R}}(x,y,z)} = \sqrt{\left(x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z\right)\bullet\left(x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z\right)} = \sqrt{x^2+y^2+z^2}$$

Magnitude of the Electric Field Strength Vector (Strength) / Betrag des elektrische Feldstärkevektors (Stärke)

$$|\underline{\mathbf{E}}(x,y,z)| = \sqrt{\underline{\mathbf{E}}(x,y,z)\bullet\underline{\mathbf{E}}(x,y,z)} = \sqrt{\left(\mathrm{E}_x\underline{\mathbf{e}}_x + \mathrm{E}_y\underline{\mathbf{e}}_y + \mathrm{E}_z\underline{\mathbf{e}}_z\right)\bullet\left(\mathrm{E}_x\underline{\mathbf{e}}_x + \mathrm{E}_y\underline{\mathbf{e}}_y + \mathrm{E}_z\underline{\mathbf{e}}_z\right)} = \sqrt{\mathrm{E}_x^2+\mathrm{E}_y^2+\mathrm{E}_z^2}$$

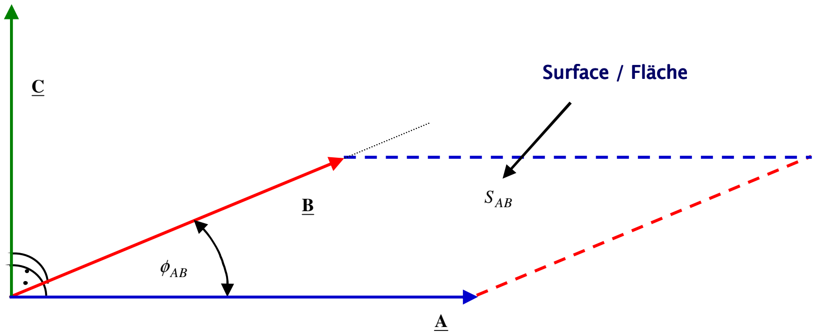
Position Unit Vector (Direction) / Ortseinheitsvektor (Richtung)

$$\hat{\underline{\mathbf{R}}}(x,y,z) = \frac{\underline{\mathbf{R}}(x,y,z)}{|\underline{\mathbf{R}}(x,y,z)|} = \frac{x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z}{\sqrt{x^2+y^2+z^2}}$$

Electric Field Strength Unit Vector (Direction) / Elektrische Feldstärkeeinheitsvektor (Richtung)

$$\hat{\underline{\mathbf{E}}}(x,y,z) = \frac{\underline{\mathbf{E}}(x,y,z)}{|\underline{\mathbf{E}}(x,y,z)|} = \frac{\mathrm{E}_x\underline{\mathbf{e}}_x + \mathrm{E}_y\underline{\mathbf{e}}_y + \mathrm{E}_z\underline{\mathbf{e}}_z}{\sqrt{\mathrm{E}_x^2+\mathrm{E}_y^2+\mathrm{E}_z^2}}$$

Vector Product (Cross or Outer Product) / Vektorprodukt (Kreuzprodukt oder äußeres Produkt) (1)



$$\underline{\mathbf{C}} = \underline{\mathbf{A}} \times \underline{\mathbf{B}}$$

$$C = |\underline{\mathbf{A}}||\underline{\mathbf{B}}|\sin \underbrace{\angle(\underline{\mathbf{A}},\underline{\mathbf{B}})}_{\phi_{AB}} = AB\sin\phi_{AB} = S_{AB}$$

$\underline{\mathbf{C}} \perp \underline{\mathbf{A}}$ and / und $\underline{\mathbf{C}} \perp \underline{\mathbf{B}}$

Vector Product (Cross or Outer Product) / Vektorprodukt (Kreuzprodukt oder äußeres Produkt) (2)

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_x = \underline{\mathbf{0}} \quad \underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y = \underline{\mathbf{e}}_z \quad \underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_z = -\underline{\mathbf{e}}_y$$

$$\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_x = -\underline{\mathbf{e}}_z \quad \underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_y = \underline{\mathbf{0}} \quad \underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z = \underline{\mathbf{e}}_x$$

$$\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x = \underline{\mathbf{e}}_y \quad \underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_y = -\underline{\mathbf{e}}_x \quad \underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_z = \underline{\mathbf{0}}$$

$$\begin{aligned}
\underline{\mathbf{A}} \times \underline{\mathbf{B}} &= (A_x\underline{\mathbf{e}}_x + A_y\underline{\mathbf{e}}_y + A_z\underline{\mathbf{e}}_z) \times (B_x\underline{\mathbf{e}}_x + B_y\underline{\mathbf{e}}_y + B_z\underline{\mathbf{e}}_z) \\
&= A_xB_x \underbrace{\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_x}_{=0} + A_xB_y \underbrace{\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y}_{=\underline{\mathbf{e}}_z} + A_xB_z \underbrace{\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_z}_{=-\underline{\mathbf{e}}_y} \\
&\quad + A_yB_x \underbrace{\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_x}_{=-\underline{\mathbf{e}}_z} + A_yB_y \underbrace{\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_y}_{=0} + A_yB_z \underbrace{\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z}_{=\underline{\mathbf{e}}_x} \\
&\quad + A_zB_x \underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x}_{=\underline{\mathbf{e}}_y} + A_zB_y \underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_y}_{=-\underline{\mathbf{e}}_x} + A_zB_z \underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_z}_{=0} \\
&= (A_yB_z\underline{\mathbf{e}}_x - A_zB_y)\underline{\mathbf{e}}_x + (A_zB_x - A_xB_z)\underline{\mathbf{e}}_y + (A_xB_y - A_yB_x)\underline{\mathbf{e}}_z
\end{aligned}$$

$$\underline{\mathbf{A}} \times \underline{\mathbf{B}} = -\underline{\mathbf{B}} \times \underline{\mathbf{A}} \qquad \underline{\mathbf{A}} \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$

Vector Product (Cross or Outer Product) / Vektorprodukt (Kreuzprodukt oder äußeres Produkt) (3)

$$\underline{\mathbf{A}} \times \underline{\mathbf{B}} = \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Add the first two Columns / Addiere die beiden ersten Spalten

$$= \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \begin{matrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y \\ A_x & A_y \\ B_x & B_y \end{matrix}$$

$$\begin{aligned}
&= (A_yB_z - A_zB_y)\underline{\mathbf{e}}_x \\
&\quad + (A_zB_x - A_xB_z)\underline{\mathbf{e}}_y \\
&\quad + (A_xB_y - A_yB_x)\underline{\mathbf{e}}_z
\end{aligned}$$

Sarrus Law / Regel von Sarrus

[Pierre Frédéric Sarrus, 1831]

http://de.wikipedia.org/wiki/Regel_von_Sarrus

Dyadic Product / Dyadisches Produkt

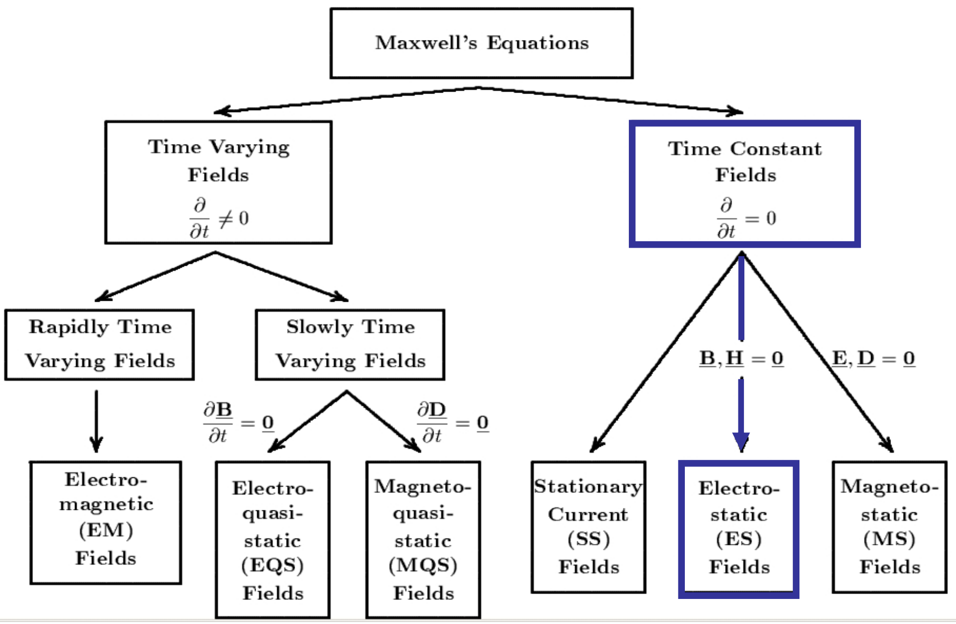
$$
\begin{aligned}
\underline{\mathbf{A}}\,\underline{\mathbf{B}} &= \sum_{i=1}^{3} A_{x_i}\,\underline{\mathbf{e}}_{x_i} \sum_{j=1}^{3} B_{x_j}\,\underline{\mathbf{e}}_{x_j} \\
&= \sum_{i=1}^{3} \sum_{j=1}^{3} A_{x_i}\,\underline{\mathbf{e}}_{x_i}\, B_{x_j}\,\underline{\mathbf{e}}_{x_j} \\
&= A_{x_i}\,\underline{\mathbf{e}}_{x_i}\, B_{x_j}\,\underline{\mathbf{e}}_{x_j} \\
&= \underbrace{A_{x_i}\, B_{x_j}}_{=D_{x_i x_j}}\,\underline{\mathbf{e}}_{x_i}\,\underline{\mathbf{e}}_{x_j} \\
&= D_{x_i x_j}\,\underline{\mathbf{e}}_{x_i}\,\underline{\mathbf{e}}_{x_j} \\
&= \underline{\underline{\mathbf{D}}}
\end{aligned}
$$

$$\underline{\mathbf{B}}\,\underline{\mathbf{A}} \neq \underline{\mathbf{A}}\,\underline{\mathbf{B}}$$

$$\underline{\mathbf{D}} = \underline{\underline{\boldsymbol{\varepsilon}}} \bullet \underline{\mathbf{E}}$$

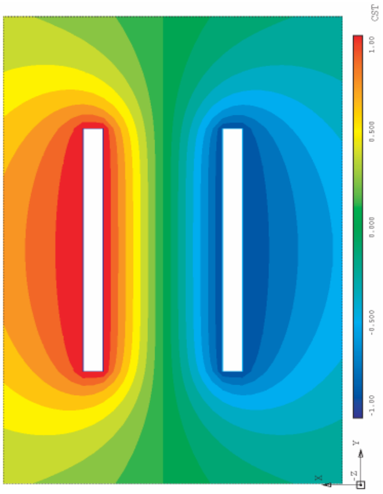
$$\underline{\mathbf{B}} = \underline{\underline{\boldsymbol{\mu}}} \bullet \underline{\mathbf{H}}$$

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

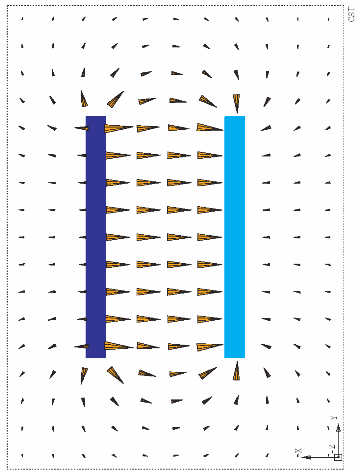


Electrostatic Field Problem – Example: Parallel Plate Capacitor / Elektrostatisches Feldproblem – Beispiel: Paralleler Plattenkondensator

Scalar Field: Electrostatic Potential / Skalarfeld: Elektrostatisches Potenzial



Vector Field: Electrostatic Field Strength / Vektorfeld: Elektrostatische Feldstärke



Electrostatic (ES) Fields – Governing Equations / Elektrostatische (ES) Felder – Grundgleichungen

Electrostatic / Elektrostatik $\frac{\partial}{\partial t} \equiv 0$ No Time Dependence and No Magnetic Field Quantities / Keine Zeitabhängigkeit und keine magnetischen Feldgrößen

$\underline{\mathbf{E}}(\underline{\mathbf{R}})$: Electric Field Strength / Elektrische Feldstärke

$\underline{\mathbf{D}}(\underline{\mathbf{R}})$: Electric Flux Density / Elektrische Flussdichte

$\rho_\mathrm{e}(\underline{\mathbf{R}})$: Electric Charge Density / Elektrische Raumladungsdichte

Integral Form / Integralform	Differential Form / Differentialform
$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{R}} = 0$	$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$
$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{S}} = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V$	$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_\mathrm{e}(\underline{\mathbf{R}})$

Curl-Free $\underline{\mathbf{E}}$ Field / Rotationsfreies $\underline{\mathbf{E}}$ Feld

Divergence of $\underline{\mathbf{D}}$ Represents Electric Charge Density / Quellstärke von $\underline{\mathbf{D}}$ entspricht der elektrischen Raumladungsdichte

Electrostatic (ES) Fields – Governing Equations / Elektrostatische (ES) Felder – Grundgleichungen

Integral Form / Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \cdot d\underline{\mathbf{R}} = 0$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot d\underline{\mathbf{S}} = \iiint_V \rho_e(\underline{\mathbf{R}})\, dV = Q_e$$

$\underline{\mathbf{E}}(\underline{\mathbf{R}})$ [V/m = Newton /Coulomb = N/C]
$\underline{\mathbf{D}}(\underline{\mathbf{R}})$ [As/m^2]
$\rho_e(\underline{\mathbf{R}})$ [As/m^3]

Vacuum / Vakuum

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}})$$

Electric Field Constant / Elektrische Feldkonstante (IEEE, VDE)
Permittivity of Free Space / Permittivität des Freiraumes

Differential Form / Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

$$\nabla \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_e(\underline{\mathbf{R}})$$

Side Remark: In some Cases / Nebenbemerkung: In einigen Fällen

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \varepsilon_r \underline{\mathbf{E}}(\underline{\mathbf{R}})$$

Permittivity / Permittivität

Material	ε_r
Air / Luft	1.006
Paper / Papier	2...4
Wet Earth / Nasse Erde	5...15
Gallium Arsenide / Gallium Arsenid	13
Seawater / Seewasser	70

ES Fields – Electric Points Charge and Electric Field Strength – Coulomb's Law / ES Felder – Elektrische Punktladung und elektrische Feldstärke – Coulombsches Gesetz

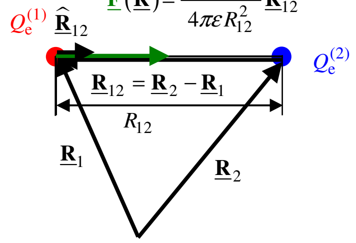
Coulomb's Law / Coulombsches Gesetz

Charles Augustin de Coulomb (1736 – 1806)

$$\underline{\mathbf{F}}(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon} \frac{Q_e^{(1)} Q_e^{(2)}}{R_{12}^2} \hat{\underline{\mathbf{R}}}_{12} \quad [\mathrm{N}]$$

Force / Kraft	$\underline{\mathbf{F}}(\underline{\mathbf{R}})$	[N]
Electric Point Charge / Elektrische Punktladung	$Q_e^{(1)}$	[As]
Electric Point Charge / Elektrische PunktLadung	$Q_e^{(2)}$	[As]
Distance / Abstand	R	[m]
Distance Unit Vector / Abstandseinheitsvektor	$\hat{\underline{\mathbf{R}}}$	[1]
Permittivity of Free-Space / Permittivität des Freiraumes	ε	[As/Vm]

$$\underline{\mathbf{F}}(\underline{\mathbf{R}}) = \frac{Q_e^{(1)} Q_e^{(2)}}{4\pi\varepsilon R_{12}^2} \hat{\underline{\mathbf{R}}}_{12}$$



$$\hat{\underline{\mathbf{R}}} = \frac{\underline{\mathbf{R}}}{|\underline{\mathbf{R}}|} = \frac{\underline{\mathbf{R}}}{R} \quad [1] \qquad R = |\underline{\mathbf{R}}| = \sqrt{\underline{\mathbf{R}} \cdot \underline{\mathbf{R}}} \quad [\mathrm{m}]$$

$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$

$$R = \sqrt{\underline{\mathbf{R}} \cdot \underline{\mathbf{R}}} = \sqrt{x^2 + y^2 + z^2}$$

$$\hat{\underline{\mathbf{R}}} = \frac{x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z}{\sqrt{x^2 + y^2 + z^2}} = \underline{\mathbf{e}}_R(\vartheta, \varphi)$$

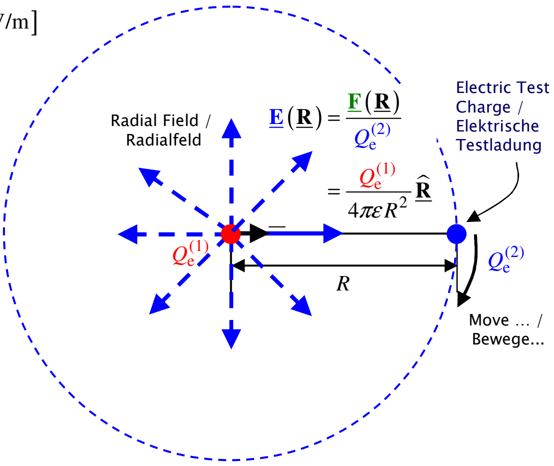
ES Fields – Electric Charge and Electric Field Strength – Coulomb's Law / ES Felder – Elektrische Ladung und elektrische Feldstärke – Coulombsches Gesetz

Electric Field Strength: Force Per Unit Charge / Elektrische Feldstärke: Kraft pro Einheitsladung

$Q_e^{(2)}$ Electric Test Charge / Elektrische Testladung

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{\underline{\mathbf{F}}(\underline{\mathbf{R}})}{Q_e^{(2)}} = \frac{Q_e^{(1)}}{4\pi\varepsilon R^2}\hat{\underline{\mathbf{R}}} \qquad [\text{N/C or V/m}]$$

Electric Field Strength / Elektische Feldstärke	$\underline{\mathbf{E}}(\underline{\mathbf{R}})$	[V/m]
Force / Kraft	$\underline{\mathbf{F}}(\underline{\mathbf{R}})$	[N]
Electric Charge / Elektrische Ladung	$Q_e^{(1)}$	[As]
Electric Test Charge / Elektrische Testladung	$Q_e^{(2)}$	[As]
Distance / Abstand	R	[m]
Distance Unit Vector / Abstandseinheitsvektor	$\hat{\underline{\mathbf{R}}}$	[1]
Permittivity of Free-Space / Permittivität des Freiraumes	ε	[As/Vm]



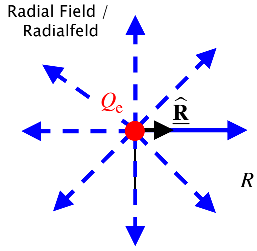
ES Fields – Electric Charge and Electric Field Strength – Coulomb's Law / ES Felder – Elektrische Ladung und elektrische Feldstärke – Coulombsches Gesetz

Electric Field Strength: Force Per Unit Charge / Elektrische Feldstärke: Kraft pro Einheitsladung

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon R^2}\hat{\underline{\mathbf{R}}} = \frac{Q_e}{4\pi\varepsilon R}\underline{\mathbf{R}} \qquad [\text{V/m}]$$

$$\underline{\mathbf{R}} = R\hat{\underline{\mathbf{R}}}$$

Electric Field Strength / Elektische Feldstärke	$\underline{\mathbf{E}}(\underline{\mathbf{R}})$	[V/m]
Electric Charge / Elektrische Ladung	Q_e	[As]
Distance / Abstand	R	[m]
Distance Unit Vector / Abstandseinheitsvektor	$\hat{\underline{\mathbf{R}}}$	[1]
Permittivity of Free-Space / Permittivität des Freiraumes	ε	[As/Vm]



$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon R^2}\hat{\underline{\mathbf{R}}} = \frac{Q_e}{4\pi\varepsilon R}\underline{\mathbf{R}}$$

Electrostatic (ES) Fields – Governing Equations / Elektrostatische (ES) Felder – Grundgleichungen

Electrostatic / Elektrostatik $\frac{\partial}{\partial t} \equiv 0$ — No Time Dependence and No Magnetic Field Quantities / Keine Zeitabhängigkeit und keine magnetischen Feldgrößen

$\underline{\mathbf{E}}(\underline{\mathbf{R}})$: Electric Field Strength / Elektrische Feldstärke
$\underline{\mathbf{D}}(\underline{\mathbf{R}})$: Electric Flux Density / Elektrische Flussdichte
$\rho_e(\underline{\mathbf{R}})$: Electric Charge Density / Elektrische Raumladungsdichte

Integral Form / Integralform	Differential Form / Differentialform
$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \cdot \mathbf{d}\underline{\mathbf{R}} = 0$	$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$
$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \mathbf{d}\underline{\mathbf{S}} = \iiint_V \rho_e(\underline{\mathbf{R}})\,\mathrm{d}V$	$\nabla \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_e(\underline{\mathbf{R}})$

Curl-Free E-Field / Rotationsfreies E-Feld

Divergence of D Represents Electric Charge Density / Quellstärke von D entspricht der elektrischen Raumladungsdichte

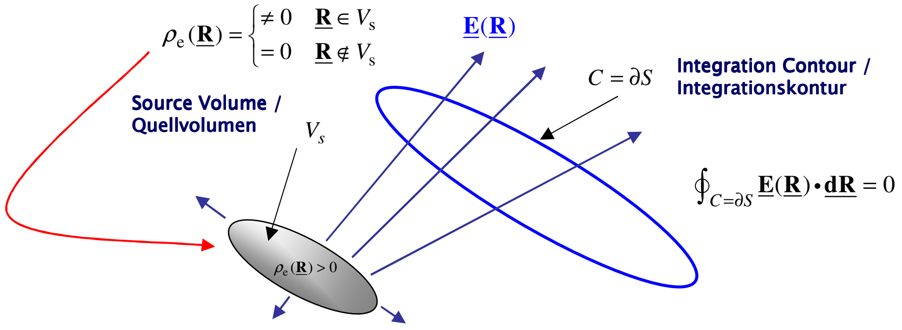
⬇

Method of Gauss' Electric Law / Methode des Gaußschen elektrischen Gesetzes

ES Fields – Method of Electric Gauss' Law / ES-Felder – Methode des elektrischen Gaußschen Gesetzes

Source Distribution / Quellverteilung

$$\rho_e(\underline{\mathbf{R}}) = \begin{cases} \neq 0 & \underline{\mathbf{R}} \in V_s \\ = 0 & \underline{\mathbf{R}} \notin V_s \end{cases}$$

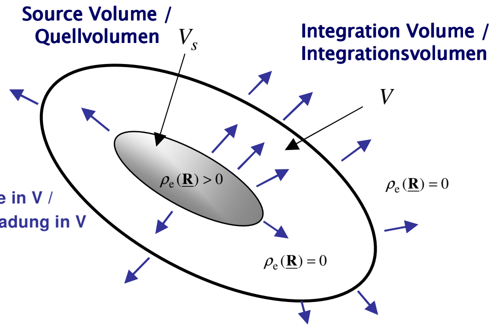


$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \cdot \mathbf{d}\underline{\mathbf{R}} = 0$$

ES Fields – Method of Electric Gauss' Law / ES-Felder – Methode des elektrischen Gaußschen Gesetzes

Source Distribution / Quellverteilung

$$\rho_e(\underline{\mathbf{R}}) = \begin{cases} \neq 0 & \underline{\mathbf{R}} \in V_s \\ = 0 & \underline{\mathbf{R}} \notin V_s \end{cases}$$



$$\psi_e = \oint\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}} = \iiint_V \rho_e(\underline{\mathbf{R}})\, \mathrm{d}V$$

$$= Q_e$$ **Total Electric Charge in V / Elektrische Gesamtladung in V**

⇩

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}} = \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{n}}}_{D_n(\underline{\mathbf{R}})}\, \mathrm{d}S$$

$$\underbrace{\underbrace{\oint\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}}}_{\oint\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{n}}}_{D_n(\underline{\mathbf{R}})} \mathrm{d}S}}_{\text{Summation of all } D_n = \underline{\mathbf{n}} \cdot \underline{\mathbf{D}} \text{ Contributions / Summation aller } D_n = \underline{\mathbf{n}} \cdot \underline{\mathbf{D}}\text{-Beiträge}} = \underbrace{\underbrace{\iiint_V \rho_e(\underline{\mathbf{R}})\, \mathrm{d}V}_{Q_e}}_{\text{Total electric charge inside the volume } V \text{ with the closed surface } S=\partial V}$$

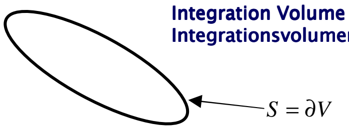
Summation of all $D_n = \underline{\mathbf{n}} \cdot \underline{\mathbf{D}}$ Contributions / Summation aller $D_n = \underline{\mathbf{n}} \cdot \underline{\mathbf{D}}$-Beiträge

Total electric charge inside the volume V with the closed surface $S=\partial V$ / Gesamte elektrische Ladung im Volumen V mit der geschlossenen Oberfläche $S=\partial V$

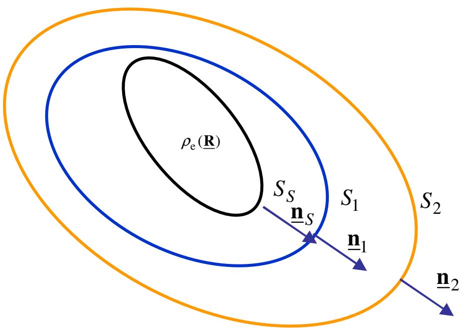
> Flux of $\underline{\mathbf{D}}$ through $S = Q_e$ in V /
> Fluss von $\underline{\mathbf{D}}$ durch $S = Q_e$ in V

ES Fields – Method of Electric Gauss' Law / ES-Felder – Methode des elektrischen Gaußschen Gesetzes

$$\oint\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}} = \iiint_V \rho_e(\underline{\mathbf{R}})\, \mathrm{d}V$$
$$= Q_e$$

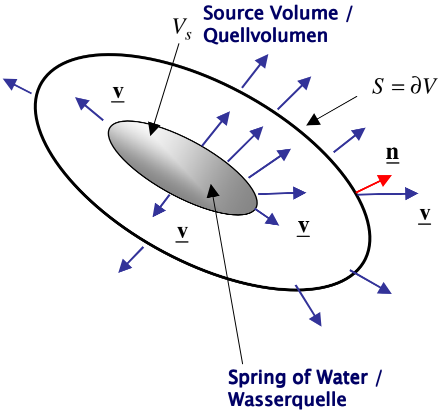


$$\oint\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}} \begin{cases} = 0 & \text{source-free / quellenfrei} \\ > 0 & \text{Source / Quelle} \\ < 0 & \text{Sink / Senke} \end{cases}$$



$$\oint\!\!\oint_{S_S=\partial V_S} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{n}}_S\, \mathrm{d}S$$
$$= \oint\!\!\oint_{S_1=\partial V_1} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{n}}_1\, \mathrm{d}S$$
$$= \oint\!\!\oint_{S_2=\partial V_2} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{n}}_2\, \mathrm{d}S$$
$$= Q_e$$

Example: Fluid Mechanics – Spring of Water / Beispiel: Strömungsmechanik – Wasserquelle



Integration Surface (Closed Surface) / Integrationsfläche (geschlossene Oberfläche)

Total Flux through the Closed Surface / Gesamtfluss durch die geschlossene Oberfläche

$$\oiint_{S=\partial V} \underline{\mathbf{v}}(\underline{\mathbf{R}})\cdot \underline{\mathbf{dS}} = \oiint_{S=\partial V} \underbrace{\underline{\mathbf{v}}(\underline{\mathbf{R}})\cdot\underline{\mathbf{n}}}_{=\mathrm{v}_n(\underline{\mathbf{R}})}\,\mathrm{d}S$$

$$= \oiint_{S=\partial V} \mathrm{v}_n(\underline{\mathbf{R}})\,\mathrm{d}S$$

$$= \psi_{\mathrm{v}}$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

Consider the Electrostatic (ES) Case / Betrachte den elektrostatischen (ES) Fall

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}})\cdot \underline{\mathbf{dS}} = \oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\cdot\underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})}\,\mathrm{d}S = \underbrace{\iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}})\,\mathrm{d}V}_{=Q_{\mathrm{e}}}$$

Prescribed: Electric Charge Density / Vorgegeben: Elektrische Raumladungsdichte

Radial Symmetry / Radialsymmetrie **!**

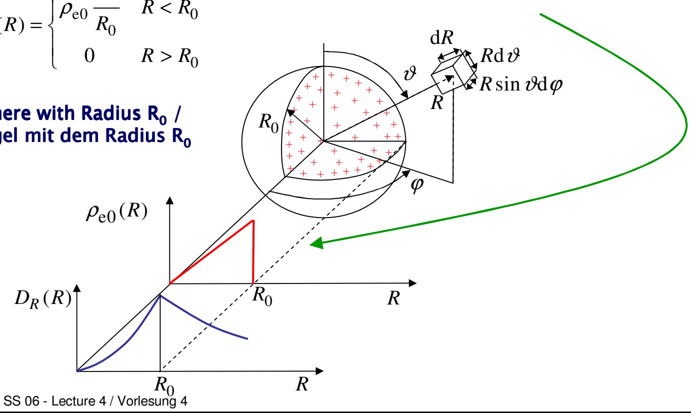
$$\underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\cdot\underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} = \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\cdot\underline{\mathbf{e}}_R}_{=D_R(\underline{\mathbf{R}})}$$

$$D_n(\underline{\mathbf{R}}) = D_R(\underline{\mathbf{R}})$$

$$\rho_{\mathrm{e}}(\underline{\mathbf{R}}) = \rho_{\mathrm{e}}(R) = \begin{cases} \rho_{\mathrm{e}0}\dfrac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases}$$

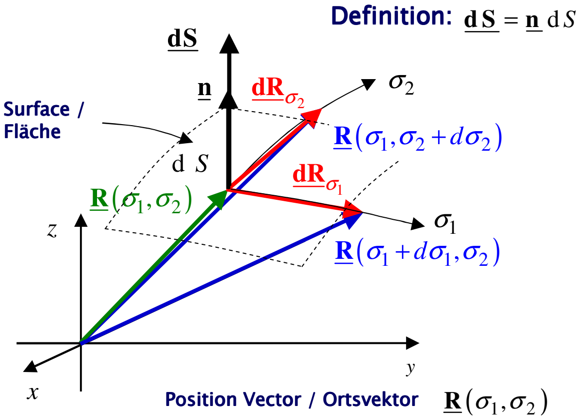
Charged Sphere with Radius R_0 / Geladene Kugel mit dem Radius R_0

Solution for $\mathbf{D}(\underline{\mathbf{R}})$ / Lösung für $\mathbf{D}(\underline{\mathbf{R}})$



Vector Differential Surface Element / Vektorielles differentielles Flächenelement (1)

Definition: $\underline{\mathbf{dS}} = \underline{\mathbf{n}}\, \mathrm{d}S$



σ_1, σ_2 — Surface Parameters / Flächenparameter

$\underline{\mathbf{R}}(\sigma_1,\sigma_2)$ — Position Vector / Ortsvektor

$\underline{\mathbf{R}}(\sigma_1+\mathrm{d}\sigma_1,\sigma_2)$, $\underline{\mathbf{R}}(\sigma_1,\sigma_2+\mathrm{d}\sigma_2)$ — Position Vectors / Ortsvektoren

$\underline{\mathbf{dR}}_{\sigma_1}$, $\underline{\mathbf{dR}}_{\sigma_2}$ — Vector Differential Line Elements / Vektorielle differentielle Linienelemente

Position Vector / Ortsvektor $\underline{\mathbf{R}}(\sigma_1,\sigma_2)$

Tangential Vectors / Tangentialvektoren

$$\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2) = \frac{\partial}{\partial \sigma_1}\underline{\mathbf{R}}(\sigma_1,\sigma_2)$$

$$\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2) = \frac{\partial}{\partial \sigma_2}\underline{\mathbf{R}}(\sigma_1,\sigma_2)$$

Vector Differential Surface Element / Vektorielles differentielles Flächenelement (2)

Vector Differential Line Elements / Vektorielles differentielles Linienelement

$$\underline{\mathbf{dR}}_{\sigma_1} = \underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_1$$

$$\underline{\mathbf{dR}}_{\sigma_2} = \underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_2$$

Scalar Differential Surface Elements / Skalares differentielles Flächenelement

$$\mathrm{d}S = \left|\underline{\mathbf{dR}}_{\sigma_1} \times \underline{\mathbf{dR}}_{\sigma_2}\right| = \left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|\mathrm{d}\sigma_1\mathrm{d}\sigma_2$$

Normal Unit-Vector / Normaleneinheitsvektor

$$\underline{\mathbf{n}} = \frac{\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)}{\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|}$$

Vector Differential Surface Element / Vektorielles differentielles Flächenelement

$$\underline{\mathbf{dS}} = \underline{\mathbf{n}}\,\mathrm{d}S = \frac{\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)}{\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|}\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|\mathrm{d}\sigma_1\mathrm{d}\sigma_2 = \underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_1\mathrm{d}\sigma_2$$

Gauss' Electric Law / Gaußsches elektrisches Gesetz

$$\underbrace{\oint\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}}}_{\text{Closed Surface Integral / Geschlossenes Flächenintegral}} = \underbrace{\oint\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} \mathrm{d}S}_{=\psi_\mathrm{e}} = \underbrace{\iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V}_{=Q_\mathrm{e}}$$

Closed Surface Integral / Geschlossenes Flächenintegral

Summation of all Normal Componentes of $\underline{\mathbf{D}}$ at the Closed Surface $S=\partial V$ of the Volume V / Summation aller Normalkomponenten von $\underline{\mathbf{D}}$ auf der geschlossenen Oberfläche $S=\partial V$ des Volumens V

$=\psi_\mathrm{e}$

Flux Through the Colsed Surface / Fluss durch die geschlossene Oberfläche

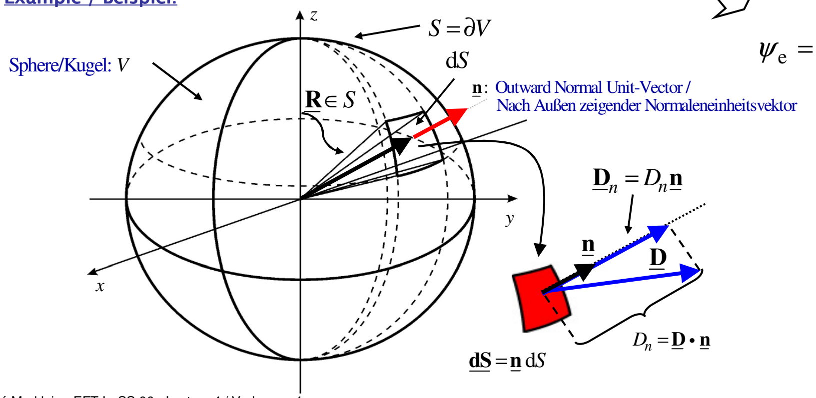
Volume Integral / Volumenintegral

Summation of all charges inside the Volume V / Summation aller Ladungen in dem Volumen V

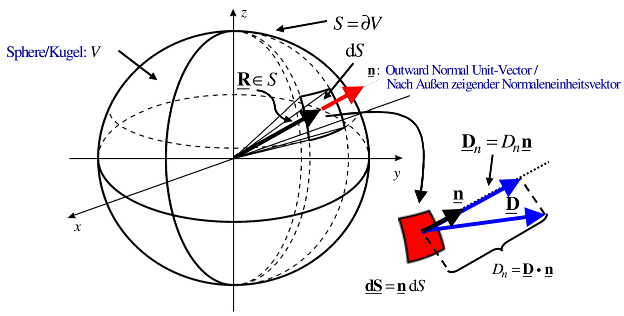
$=Q_\mathrm{e}$

$$\psi_\mathrm{e} = Q_\mathrm{e}$$

Example / Beispiel:



Example: Sphere with Radius *a* / Beispiel: Kugel mit Radius *a* (1)



$$\oint\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} \mathrm{d}S = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V$$

$$\underline{\mathbf{dS}} = \underline{\mathbf{n}}\,\mathrm{d}S \quad (= \underline{\mathbf{n}}_{\vartheta\varphi}\, h_\vartheta\, h_\varphi\, \mathrm{d}\vartheta\, \mathrm{d}\varphi)$$

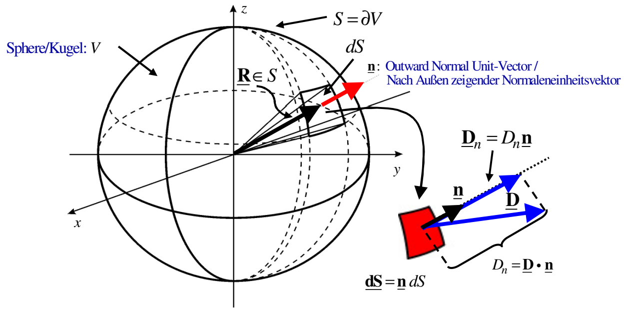
$$0 \le \vartheta \le \pi$$
$$0 \le \varphi < 2\pi$$

$$= \underbrace{\underline{\mathbf{e}}_R(\vartheta,\varphi)}_{\underline{\mathbf{n}}} \underbrace{R^2 \sin\vartheta\, \mathrm{d}\vartheta\, \mathrm{d}\varphi}_{\mathrm{d}S}\Big|_{R=a} = \underbrace{\underline{\mathbf{e}}_R(\vartheta,\varphi)}_{\underline{\mathbf{n}}} \underbrace{a^2 \sin\vartheta\, \mathrm{d}\vartheta\, \mathrm{d}\varphi}_{\mathrm{d}S}$$

$$\oint\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \oint\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} \mathrm{d}S = \int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi} \underbrace{\underline{\mathbf{D}}[\underline{\mathbf{R}}(R=a,\vartheta,\varphi)] \bullet \underline{\mathbf{e}}_R(\vartheta,\varphi)}_{\substack{=D_R[\underline{\mathbf{R}}(R=a,\vartheta,\varphi)] \\ =D_n[\underline{\mathbf{R}}(R=a,\vartheta,\varphi)]}} a^2 \sin\vartheta\, \mathrm{d}\vartheta\, \mathrm{d}\varphi$$

$$= \psi_\mathrm{e}$$

Example: Sphere with Radius *a* / Beispiel: Kugel mit Radius *a* (2)



$$\oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{D_n(\underline{\mathbf{R}})} \, \mathrm{d}S = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}}) \, \mathrm{d}V$$

$$\mathrm{d}V = R^2 \sin\vartheta \, \mathrm{d}R \, \mathrm{d}\vartheta \, \mathrm{d}\varphi \ \left(= h_R h_\vartheta h_\varphi \, \mathrm{d}R \, \mathrm{d}\vartheta \, \mathrm{d}\varphi \right)$$

$$\iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}}) \, \mathrm{d}V = \int_{\varphi=0}^{2\pi} \int_{\vartheta=0}^{\pi} \int_{R=0}^{a} \rho_\mathrm{e}[\underline{\mathbf{R}}(R,\vartheta,\varphi)] \, R^2 \sin\vartheta \, \mathrm{d}R \, \mathrm{d}\vartheta \, \mathrm{d}\varphi$$
$$= Q_\mathrm{e}$$

$0 \le R \le a$
$0 \le \vartheta \le \pi$
$0 \le \varphi < 2\pi$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

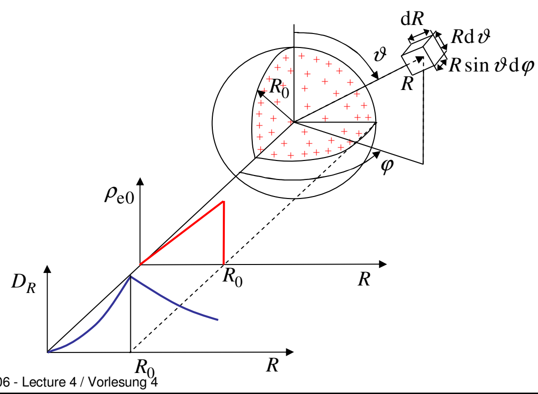
Consider the Electrostatic (ES) Case / Betrachte den elektrostatischen Fall

$$\oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} \, \mathrm{d}S = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}}) \, \mathrm{d}V$$

Electric Charge Density / Elektrische Raumladungsdichte

$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = \begin{cases} \rho_{\mathrm{e}0} \dfrac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases}$$

Radial Symmetry / Radialsymmetrisch **!**



Metric Coefficients / Metrische Koeffizienten

Cartesian Coordinates / Kartesische Koordinaten

$x, y, z = x_1, x_2, x_3$

$\underline{\mathbf{e}}_x, \underline{\mathbf{e}}_y, \underline{\mathbf{e}}_z = \underline{\mathbf{e}}_{x_1}, \underline{\mathbf{e}}_{x_2}, \underline{\mathbf{e}}_{x_3}$

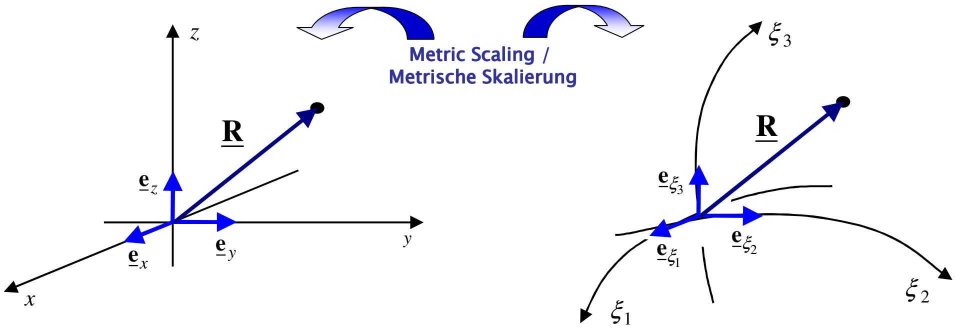
$\underline{\mathbf{e}}_x, \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z \; ; \; \underline{\mathbf{e}}_{x_1} \perp \underline{\mathbf{e}}_{x_2} \perp \underline{\mathbf{e}}_{x_3}$

Orthogonal Curvilinear Coordinates / Orthogonale Krummlinige Koordinaten

ξ_1, ξ_2, ξ_3

$\underline{\mathbf{e}}_{\xi_1}, \underline{\mathbf{e}}_{\xi_2}, \underline{\mathbf{e}}_{\xi_3}$

$\underline{\mathbf{e}}_{\xi_1} \perp \underline{\mathbf{e}}_{\xi_2} \perp \underline{\mathbf{e}}_{\xi_3}$



Cartesian Coordinates / Kartesische Koordinaten

$x = x(\xi_1, \xi_2, \xi_3)$

$y = y(\xi_1, \xi_2, \xi_3)$

$z = z(\xi_1, \xi_2, \xi_3)$

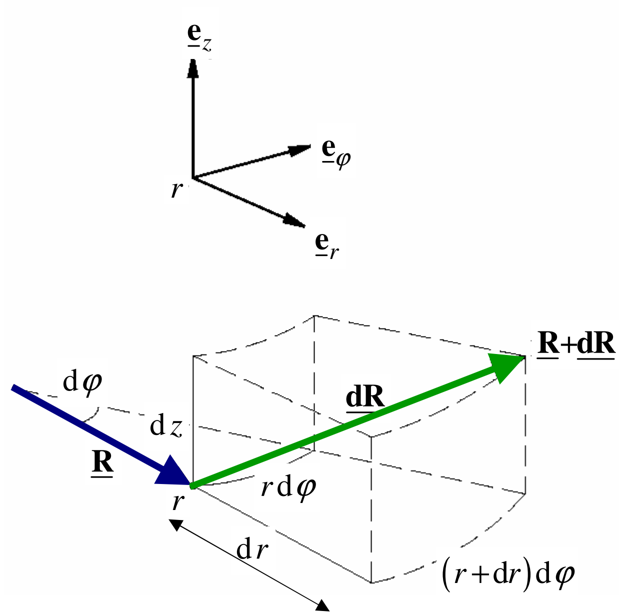
$\longleftrightarrow$

$\xi_1 = \xi_1(x, y, z)$

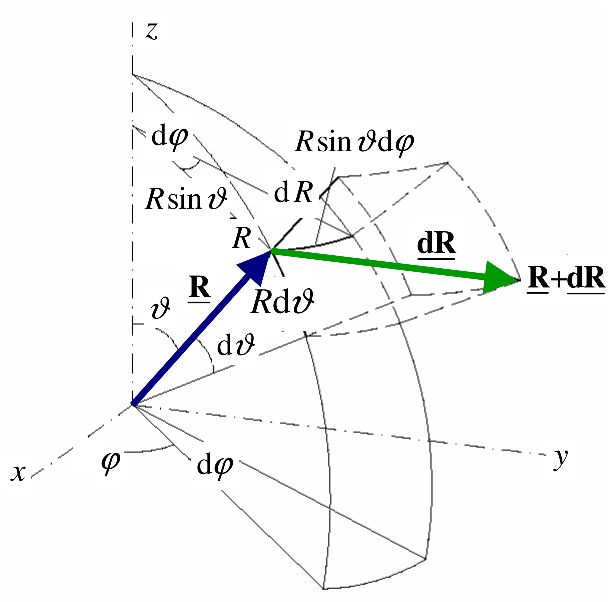
$\xi_2 = \xi_2(x, y, z)$

$\xi_3 = \xi_3(x, y, z)$

Metric Coefficients – Cylindrical Coordinate System / Metrische Koeffizienten – Zylinderkoordinatensystem



Metric Coefficients – Spherical Coordinate System / Metrische Koeffizienten – Kugelkoordinatensystem



Metric Coefficients / Metrische Koeffizienten

Cartesian Coordinates / Kartesische Koordinaten

$$x, y, z = x_1, x_2, x_3 = \xi_1, \xi_2, \xi_3$$
$$x = x(\xi_1, \xi_2, \xi_3)$$
$$y = y(\xi_1, \xi_2, \xi_3)$$
$$z = z(\xi_1, \xi_2, \xi_3)$$

Orthogonal Curvilinear Coordinates / Orthogonale Krummlinige Koordinaten

$$\xi_1, \xi_2, \xi_3$$
$$\xi_1 = \xi_1(x, y, z)$$
$$\xi_2 = \xi_2(x, y, z)$$
$$\xi_3 = \xi_3(x, y, z)$$

$$\underline{\mathbf{R}} = x(\xi_1,\xi_2,\xi_3)\underline{\mathbf{e}}_x + y(\xi_1,\xi_2,\xi_3)\underline{\mathbf{e}}_y + z(\xi_1,\xi_2,\xi_3)\underline{\mathbf{e}}_z$$

$$\frac{\partial \underline{\mathbf{R}}}{\partial \xi_i} = \frac{\partial x(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\underline{\mathbf{e}}_x + \frac{\partial y(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\underline{\mathbf{e}}_y + \frac{\partial z(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\underline{\mathbf{e}}_z$$
$$i = 1,2,3$$

$$\frac{\partial \underline{\mathbf{R}}}{\partial \xi_i} = \underbrace{\underbrace{\left|\frac{\partial \underline{\mathbf{R}}(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\right|}_{\text{Magnitude / Betrag}}}_{=h_{\xi_i}} \underbrace{\underbrace{\frac{\dfrac{\partial \underline{\mathbf{R}}(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}}{\left|\dfrac{\partial \underline{\mathbf{R}}(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\right|}}_{\text{Direction / Richtung}}}_{=\underline{\mathbf{e}}_{\xi_i}}, \quad i = 1,2,3$$

Metric Coefficients / Metrische Koeffizienten

$$h_{\xi_i} = \left|\frac{\partial \underline{\mathbf{R}}(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\right|$$
$$= \sqrt{\frac{\partial \underline{\mathbf{R}}(\xi_1,\xi_2,\xi_3)}{\partial \xi_i} \bullet \frac{\partial \underline{\mathbf{R}}(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}}$$
$$= \sqrt{\left(\frac{\partial x(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\right)^2 + \left(\frac{\partial y(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\right)^2 + \left(\frac{\partial z(\xi_1,\xi_2,\xi_3)}{\partial \xi_i}\right)^2}$$
$$i = 1,2,3$$

Example: Metric Coefficients of the Cartesian Coordinate System / Beispiel: Metrische Koeffizienten des Kartesischen Koordinatensystems

$\xi_1 = x; \xi_2 = y; \xi_3 = z$

$$h_x = \left|\frac{\partial \underline{\mathbf{R}}(x,y,z)}{\partial x}\right| = \sqrt{\frac{\partial \underline{\mathbf{R}}(x,y,z)}{\partial x} \bullet \frac{\partial \underline{\mathbf{R}}(x,y,z)}{\partial x}}$$
$$= \sqrt{\left(\frac{\partial x(x,y,z)}{\partial x}\right)^2 + \left(\frac{\partial y(x,y,z)}{\partial x}\right)^2 + \left(\frac{\partial z(x,y,z)}{\partial x}\right)^2}$$
$$= \sqrt{\underbrace{\left(\frac{\partial x}{\partial x}\right)^2}_{=1} + \underbrace{\left(\frac{\partial y}{\partial x}\right)^2}_{=0} + \underbrace{\left(\frac{\partial z}{\partial x}\right)^2}_{=0}} = 1$$

$$h_z = \left|\frac{\partial \underline{\mathbf{R}}(x,y,z)}{\partial z}\right| = \sqrt{\frac{\partial \underline{\mathbf{R}}(x,y,z)}{\partial z} \bullet \frac{\partial \underline{\mathbf{R}}(x,y,z)}{\partial z}}$$
$$= \sqrt{\left(\frac{\partial x(x,y,z)}{\partial z}\right)^2 + \left(\frac{\partial y(x,y,z)}{\partial z}\right)^2 + \left(\frac{\partial z(x,y,z)}{\partial z}\right)^2}$$
$$= \sqrt{\underbrace{\left(\frac{\partial x}{\partial z}\right)^2}_{=0} + \underbrace{\left(\frac{\partial y}{\partial z}\right)^2}_{=0} + \underbrace{\left(\frac{\partial z}{\partial z}\right)^2}_{=1}} = 1$$

$$h_y = \left|\frac{\partial \underline{\mathbf{R}}(x,y,z)}{\partial y}\right| = \sqrt{\frac{\partial \underline{\mathbf{R}}(x,y,z)}{\partial y} \bullet \frac{\partial \underline{\mathbf{R}}(x,y,z)}{\partial y}}$$
$$= \sqrt{\left(\frac{\partial x(x,y,z)}{\partial y}\right)^2 + \left(\frac{\partial y(x,y,z)}{\partial y}\right)^2 + \left(\frac{\partial z(x,y,z)}{\partial y}\right)^2}$$
$$= \sqrt{\underbrace{\left(\frac{\partial x}{\partial y}\right)^2}_{=0} + \underbrace{\left(\frac{\partial y}{\partial y}\right)^2}_{=1} + \underbrace{\left(\frac{\partial z}{\partial y}\right)^2}_{=0}} = 1$$

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$h_x = 1$$
$$h_y = 1$$
$$h_z = 1$$

Metric Coefficients / Metrische Koeffizienten

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$h_x = 1, \quad h_y = 1, \quad h_z = 1$$

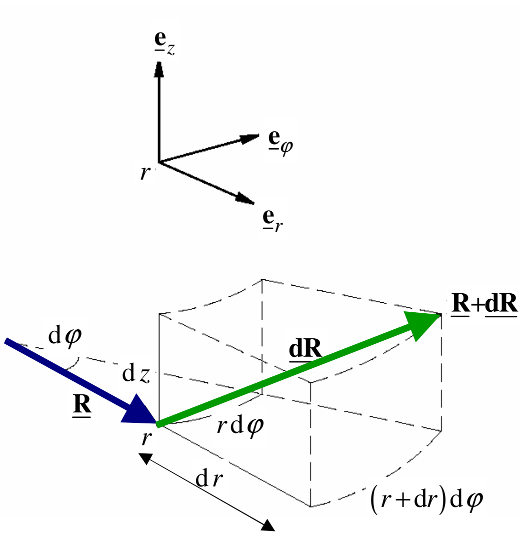
Cylindrical Coordinate System / Zylinderkoordinatensystem

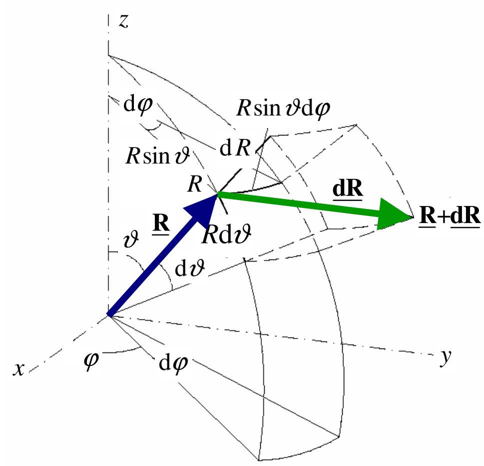
$$h_r = 1, \quad h_\varphi = r, \quad h_z = 1$$

Spherical Coordinate System / Kugelkoordinatensystem

$$h_R = 1, \quad h_\vartheta = R, \quad h_\varphi = R \sin\vartheta$$

Metric Coefficients – Cylindrical and Spherical Coordinate System / Metrische Koeffizienten – Zylinder- und Kugelkoordinatensystem





Metric Coefficients and Vector Differential Line Elements / Metrische Koeffizienten und vektorielle differentielle Linienelemente

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$h_x = 1, \quad h_y = 1, \quad h_z = 1$$

$$\underline{\mathbf{dR}}_x = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_x\,h_x\,\mathrm{d}x = \underline{\mathbf{e}}_x\,\mathrm{d}x$$

$$\underline{\mathbf{dR}}_y = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_y\,h_y\,\mathrm{d}y = \underline{\mathbf{e}}_y\,\mathrm{d}y$$

$$\underline{\mathbf{dR}}_z = \underline{\mathbf{n}}\,\mathrm{d}R = \underline{\mathbf{e}}_z\,h_z\,\mathrm{d}z = \underline{\mathbf{e}}_z\,\mathrm{d}z$$

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$h_r = 1, \quad h_\varphi = r, \quad h_z = 1$$

$$\underline{\mathbf{dR}}_r = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_r\,h_r\,\mathrm{d}r = \underline{\mathbf{e}}_r\,\mathrm{d}r$$

$$\underline{\mathbf{dR}}_\varphi = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_\varphi\,h_\varphi\,\mathrm{d}\varphi = \underline{\mathbf{e}}_\varphi\,r\,\mathrm{d}\varphi$$

$$\underline{\mathbf{dR}}_z = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_z\,h_z\,\mathrm{d}z = \underline{\mathbf{e}}_z\,\mathrm{d}z$$

Spherical Coordinate System / Kugelkoordinatensystem

$$h_R = 1, \quad h_\vartheta = R, \quad h_\varphi = R\sin\vartheta$$

$$\underline{\mathbf{dR}}_R = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_R\,h_R\,\mathrm{d}R = \underline{\mathbf{e}}_R\,\mathrm{d}R$$

$$\underline{\mathbf{dR}}_\vartheta = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_\vartheta\,h_\vartheta\,\mathrm{d}\vartheta = \underline{\mathbf{e}}_\vartheta\,R\,\mathrm{d}\vartheta$$

$$\underline{\mathbf{dR}}_\varphi = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_\varphi\,h_\varphi\,\mathrm{d}\varphi = \underline{\mathbf{e}}_\varphi\,R\sin\vartheta\,\mathrm{d}\varphi$$

Metric Coefficients and Differential Volume and Surface Elements / Metrische Koeffizienten und differentielle Volumen- und Flächenelemente

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$h_x = 1, \quad h_y = 1, \quad h_z = 1$$

$$\begin{aligned} \mathrm{d}V &= h_x\,\mathrm{d}x\, h_y\,\mathrm{d}y\, h_z\,\mathrm{d}z \\ &= h_x\, h_y\, h_z\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\ &= \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{yz} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z)\, h_y\, h_z\,\mathrm{d}y\,\mathrm{d}z \\ &= \underline{\mathbf{e}}_x\,\mathrm{d}y\,\mathrm{d}z \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{xz} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x)\, h_x\, h_z\,\mathrm{d}x\,\mathrm{d}z \\ &= \underline{\mathbf{e}}_y\,\mathrm{d}x\,\mathrm{d}z \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{xy} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y)\, h_x\, h_y\,\mathrm{d}x\,\mathrm{d}y \\ &= \underline{\mathbf{e}}_z\,\mathrm{d}x\,\mathrm{d}y \end{aligned}$$

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$h_r = 1, \quad h_\varphi = r, \quad h_z = 1$$

$$\begin{aligned} \mathrm{d}V &= h_r\,\mathrm{d}r\, h_\varphi\,\mathrm{d}\varphi\, h_z\,\mathrm{d}z \\ &= h_r\, h_\varphi\, h_z\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}y \\ &= r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{\varphi z} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_\varphi \times \underline{\mathbf{e}}_z)\, h_\varphi\, h_z\,\mathrm{d}\varphi\,\mathrm{d}z \\ &= \underline{\mathbf{e}}_r\, r\,\mathrm{d}y\,\mathrm{d}z \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{rz} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_r)\, h_r\, h_z\,\mathrm{d}r\,\mathrm{d}z \\ &= \underline{\mathbf{e}}_\varphi\,\mathrm{d}r\,\mathrm{d}z \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{r\varphi} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_r \times \underline{\mathbf{e}}_\varphi)\, h_r\, h_\varphi\,\mathrm{d}r\,\mathrm{d}\varphi \\ &= \underline{\mathbf{e}}_z\, r\,\mathrm{d}r\,\mathrm{d}\varphi \end{aligned}$$

Spherical Coordinate System / Kugelkoordinatensystem

$$h_R = 1, \quad h_\vartheta = R, \quad h_\varphi = R\sin\vartheta$$

$$\begin{aligned} \mathrm{d}V &= h_R\,\mathrm{d}R\, h_\vartheta\,\mathrm{d}\vartheta\, h_\varphi\,\mathrm{d}\varphi \\ &= h_R\, h_\vartheta\, h_\varphi\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \\ &= R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{\vartheta\varphi} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_\vartheta \times \underline{\mathbf{e}}_\varphi)\, h_\vartheta\, h_\varphi\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \\ &= \underline{\mathbf{e}}_R\, R^2\sin\vartheta\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{r\varphi} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_\varphi \times \underline{\mathbf{e}}_R)\, h_R\, h_\varphi\,\mathrm{d}R\,\mathrm{d}\varphi \\ &= \underline{\mathbf{e}}_\vartheta\, R\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\varphi \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{R\vartheta} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_R \times \underline{\mathbf{e}}_\vartheta)\, h_R\, h_\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta \\ &= \underline{\mathbf{e}}_\varphi\, R\,\mathrm{d}R\,\mathrm{d}\vartheta \end{aligned}$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

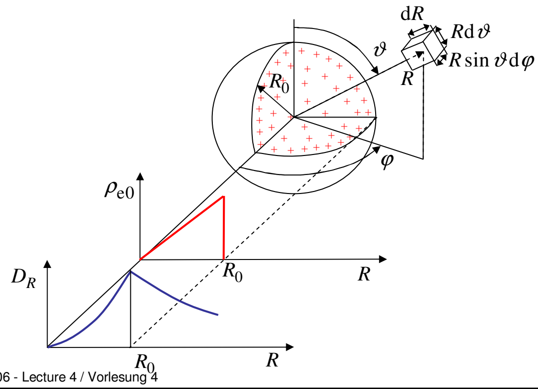
Consider the Electrostatic (ES) Case / Betrachte den elektrostatischen Fall

$$\oiint_{S=\partial V} \underbrace{\underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\cdot\underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})}}_{=D_R(\underline{\mathbf{R}})}\,\mathrm{d}S = \iiint_V \rho_e(\underline{\mathbf{R}})\,\mathrm{d}V$$

Electric Charge Density / Elektrische Raumladungsdichte

$$\rho_e(\underline{\mathbf{R}}) = \begin{cases} \rho_{e0}\dfrac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases}$$

Radial Symmetry / Radialsymmetrisch !



Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

$$\oiint_{S=\partial V} \underbrace{\underbrace{\mathbf{D}(\underline{\mathbf{R}})\bullet\underline{\mathbf{n}}}_{=D_n(R)}}_{=D_R(R)} \mathrm{d}S = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\mathrm{d}V = Q_\mathrm{e}$$

$$\rho_\mathrm{e}(R) = \begin{cases} \rho_{\mathrm{e}0} \dfrac{R}{R_0} & R \le R_0 \\ 0 & R > R_0 \end{cases}$$

2 Cases / 2 Fälle

$0 \le R < R_0$

$$\oiint_{S=\partial V} D_R(R)\mathrm{d}S = D_R(R) \oiint_{S=\partial V} \mathrm{d}S$$
$$= D_R(R) \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} h_\varphi h_\vartheta \,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$
$$= D_R(R) \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} R^2 \sin\vartheta \,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

!

$$\iiint_V \rho_\mathrm{e}(R)\mathrm{d}V = \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} \int_{R=0}^{R} \rho_\mathrm{e}(R) R^2 \sin\vartheta \,\mathrm{d}R\,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

$R > R_0$

$$\oiint_{S=\partial V} D_R(R)\mathrm{d}S = D_R(R) \oiint_{S=\partial V} \mathrm{d}S$$
$$= D_R(R) \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} h_\varphi h_\vartheta \,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$
$$= D_R(R) \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} R^2 \sin\vartheta \,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

!

$$\iiint_V \rho_\mathrm{e}(R)\mathrm{d}V = \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} \int_{R=0}^{R_0} \rho_\mathrm{e}(R) R^2 \sin\vartheta \,\mathrm{d}R\,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

2 Cases / 2 Fälle

$R < R_0$

$$\oiint_{S=\partial V} D_R(R)\mathrm{d}S = D_R(R) \oiint_{S=\partial V} \mathrm{d}S$$
$$= D_R(R) \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} R^2 \sin\vartheta \,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$
$$= D_R(R) R^2 \underbrace{\int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} \sin\vartheta \,\mathrm{d}\varphi\,\mathrm{d}\vartheta}_{=4\pi}$$
$$= D_R(R) 4\pi R^2$$

$R > R_0$

$$\oiint_{S=\partial V} D_R(R)\mathrm{d}S = D_R(R) \oiint_{S=\partial V} \mathrm{d}S$$
$$= D_R(R) \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} R^2 \sin\vartheta \,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$
$$= D_R(R) R^2 \underbrace{\int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} \sin\vartheta \,\mathrm{d}\varphi\,\mathrm{d}\vartheta}_{=4\pi}$$
$$= D_R(R) 4\pi R^2$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

2 Cases / 2 Fälle

$R < R_0$

$$\iiint_V \rho_e(R)\mathrm{d}V = \int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\int_{R=0}^{R} \rho_e(R)\,R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

$$= \rho_{e0}\int_{R=0}^{R}\frac{R}{R_0}R^2\underbrace{\left[\int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\sin\vartheta\,\mathrm{d}\varphi\,\mathrm{d}\vartheta\right]}_{=4\pi}\mathrm{d}R$$

$$= \frac{4\pi\rho_{e0}}{R_0}\int_{R=0}^{R} R^3\,\mathrm{d}R$$

$$= \frac{4\pi\rho_{e0}}{R_0}\left.\frac{R^4}{4}\right|_{R=0}^{R}$$

$$= \frac{4\pi\rho_{e0}}{R_0}\frac{R^4}{4}$$

$$= \pi\rho_{e0}\frac{R^4}{R_0}$$

$R > R_0$

$$\iiint_V \rho_e(R)\mathrm{d}V = \int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\int_{R=0}^{R_0} \rho_e(R)\,R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\varphi\,\mathrm{d}\vartheta$$

$$= \rho_{e0}\int_{R=0}^{R_0}\frac{R}{R_0}R^2\underbrace{\left[\int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\sin\vartheta\,\mathrm{d}\varphi\,\mathrm{d}\vartheta\right]}_{=4\pi}\mathrm{d}R$$

$$= \frac{4\pi\rho_{e0}}{R_0}\int_{R=0}^{R_0} R^3\,\mathrm{d}R$$

$$= \frac{4\pi\rho_{e0}}{R_0}\left.\frac{R^4}{4}\right|_{R=0}^{R_0}$$

$$= \frac{4\pi\rho_{e0}}{R_0}\frac{R_0^4}{4}$$

$$= \pi\rho_{e0}R_0^3$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

2 Cases / 2 Fälle

$R < R_0$

$$\underbrace{\oiint_{S=\partial V} D_R(R)\,\mathrm{d}S}_{=D_R(R)4\pi R^2} = \underbrace{\iiint_V \rho_e(R)\mathrm{d}V}_{=\pi\rho_{e0}\frac{R^4}{R_0}}$$

$$D_R(R)4\pi R^2 = \pi\rho_{e0}\frac{R^4}{R_0}$$

$$D_R(R) = \frac{\pi\rho_{e0}\frac{R^4}{R_0}}{4\pi R^2}$$

$$= \frac{\rho_{e0}}{4}\frac{R^2}{R_0}$$

$R > R_0$

$$\underbrace{\oiint_{S=\partial V} D_R(R)\,\mathrm{d}S}_{=D_R(R)4\pi R^2} = \underbrace{\iiint_V \rho_e(R)\mathrm{d}V}_{=\pi\rho_{e0}R_0^3}$$

$$D_R(R)4\pi R^2 = \pi\rho_{e0}R_0^3$$

$$D_R(R) = \frac{\pi\rho_{e0}R_0^3}{4\pi R^2}$$

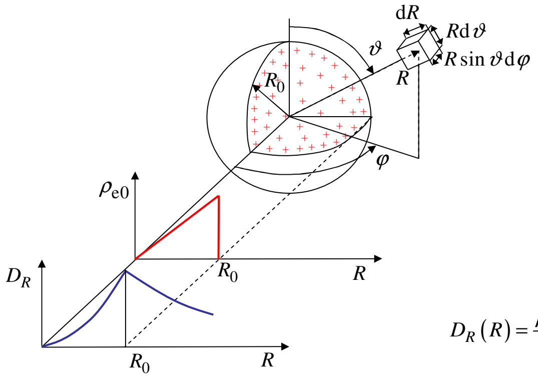
$$= \frac{\rho_{e0}}{4}\frac{R_0^3}{R^2}$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

Electric Charge Density / Elektrische Raumladungsdichte

$$\rho_e(\underline{\mathbf{R}}) = \begin{cases} \rho_{e0} \dfrac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases}$$

Radial Symmetry / Radialsymmetrisch !



$$D_R(R) = \frac{\rho_{e0}}{4} \begin{cases} \dfrac{R^2}{R_0} & R < R_0 \\ \dfrac{R_0^3}{R^2} & R > R_0 \end{cases}$$

End of Lecture 4 / Ende der 4. Vorlesung