Numerical Methods of Electromagnetic Field Theory I (NFT I) Numerische Methoden der Elektromagnetischen Feldtheorie I (NFT I) /

Exercise 1 / Übung 1

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Problem 1 / Aufgabe 1

Given is / Gegeben ist

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = E_x(z,t)\ \underline{\mathbf{e}}_x$$
$$\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = H_y(z,t)\underline{\mathbf{e}}_y$$

Derive the following one-dimensional wave equation for the electric field strength in vacuum / Leite die folgende eindimensionale Wellengleichung für die elektrische Feldstärke in Vakuum her

$$\frac{\partial^2}{\partial z^2}E_x(z,t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_x(z,t) = \begin{cases} -\frac{\partial}{\partial z}J_{\mathrm{my}}(z,t) + \mu_0\frac{\partial}{\partial t}J_{\mathrm{ex}}(z,t) & \text{Inhomogeneous 1-D Wave Equation / Inhomogene 1-D Wellengleichung} \\ 0 & \text{Homogeneous 1-D Wave Equation / Homogene 1-D Wellengleichung} \end{cases}$$

Solution / Lösung:

The first two Maxwell's equations read for vacuum / Die ersten beiden Maxwellschen Gleichungen lauten für Vakuum

$$\frac{\partial}{\partial t}H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z}E_x(z,t) - \frac{1}{\mu_0}J_{\mathrm{my}}(z,t) \qquad (1)$$

$$\frac{\partial}{\partial t}E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z}H_y(z,t) - \frac{1}{\varepsilon_0}J_{\mathrm{ey}}(z,t) \qquad (2)$$

$\frac{\partial}{\partial t}$ of (2) / von (2)

$$\frac{\partial^2}{\partial t^2}E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z}\frac{\partial}{\partial t}H_y(z,t) - \frac{1}{\varepsilon_0}\frac{\partial}{\partial t}J_{\mathrm{ey}}(z,t) \qquad (3)$$

Insert the right-hand side of (1) in (4) / Setze die rechte Seite von (1) in (4) ein

$$\frac{\partial^2}{\partial t^2}E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z}\left[-\frac{1}{\mu_0}\frac{\partial}{\partial z}E_x(z,t) - \frac{1}{\mu_0}J_{\mathrm{my}}(z,t)\right] - \frac{1}{\varepsilon_0}\frac{\partial}{\partial t}J_{\mathrm{ey}}(z,t) \qquad (4)$$

$$\frac{\partial^2}{\partial t^2}E_x(z,t) = \frac{1}{\varepsilon_0\mu_0}\frac{\partial^2}{\partial z^2}E_x(z,t) + \frac{1}{\varepsilon_0\mu_0}\frac{\partial}{\partial z}J_{\mathrm{my}}(z,t) - \frac{1}{\varepsilon_0}\frac{\partial}{\partial t}J_{\mathrm{ey}}(z,t) \qquad (5)$$

Problem 1 / Aufgabe 1 (...)

$$\frac{\partial^2}{\partial z^2}E_x(z,t)-\underbrace{\varepsilon_0\mu_0}_{=1/c_0^2}\frac{\partial^2}{\partial t^2}E_x(z,t)=-\frac{\partial}{\partial z}J_{my}(z,t)+\mu_0\frac{\partial}{\partial t}J_{ey}(z,t) \qquad \textbf{(3)}$$

$$c_0=\frac{1}{\sqrt{\varepsilon_0\mu_0}} \qquad \textbf{(4)}$$

Propagation velocity of an electromagnetic wave (Light) in vacuum /
Ausbreitungsgeschwindigkeit einer elektromagnetische Welle (Licht) in Vakuum

$$\frac{\partial^2}{\partial z^2}E_x(z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_x(z,t)=\begin{cases}-\frac{\partial}{\partial z}J_{my}(z,t)+\mu_0\frac{\partial}{\partial t}J_{ex}(z,t) & \text{Inhomogeneous 1-D Wave Equation / Inhomogene 1-D Wellengleichung}\\ 0 & \text{Homogeneous 1-D Wave Equation / Homogene 1-D Wellengleichung}\end{cases}$$

Problem 2 / Aufgabe 2

Calculate the $\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$ and $\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$ for /
Berechnen Sie $\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$ und $\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$ für

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=E_x(z,t)\;\underline{\mathbf{e}}_x$$
$$\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=H_y(z,t)\underline{\mathbf{e}}_y$$

Solution / Lösung:

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\begin{vmatrix}\underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ E_x(z,t) & 0 & 0\end{vmatrix}=\frac{\partial}{\partial z}E_x(z,t)\underline{\mathbf{e}}_y-\underbrace{\frac{\partial}{\partial y}E_x(z,t)}_{=0}\underline{\mathbf{e}}_z=\frac{\partial}{\partial z}E_x(z,t)\underline{\mathbf{e}}_y$$

$$\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=\begin{vmatrix}\underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & H_y(z,t) & 0\end{vmatrix}=\underbrace{\frac{\partial}{\partial x}E_y(z,t)}_{=0}\underline{\mathbf{e}}_y-\frac{\partial}{\partial z}H_y(z,t)\underline{\mathbf{e}}_x=-\frac{\partial}{\partial z}H_y(z,t)\underline{\mathbf{e}}_x$$

Problem 2 / Aufgabe 2 (...)

Compute the Poynting vector / Berechnen Sie den Poynting--Vektor

$$\underline{\mathbf{S}}_{\text{em}}(\underline{\mathbf{R}},t)=\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$

In which direction does the energy flux density propagate? /
In welche Richtung breitet sich die Energieflussdichte aus?

Solution / Lösung:

$$\begin{aligned}\underline{\mathbf{S}}_{\text{em}}(\underline{\mathbf{R}},t)&=\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)\\&=\left|\begin{matrix}\underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z\\ E_x(z,t) & 0 & 0\\ 0 & H_y(z,t) & 0\end{matrix}\right|\begin{matrix}\underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y\\ E_x(z,t) & 0\\ 0 & H_y(z,t)\end{matrix}\\&=\underbrace{E_x(z,t)H_y(z,t)}_{=S_{\text{em}\,z}(z,t)}\underline{\mathbf{e}}_z\\&=S_{\text{em}\,z}(z,t)\underline{\mathbf{e}}_z\qquad\left[\frac{\text{V}\cdot\text{A}}{\text{m}^2}=\frac{\text{W(att)}}{\text{m}^2}\right]\end{aligned}$$

The energy flux density propagates in z direction. /
Die Energieflussdichte breitet sich in z Richtung aus.

Problem 3 / Aufgabe 3

Determine the approximation error $\mathcal{O}(?)$ of the backward FD approximation /
Bestimme den Approximationsfehler $\mathcal{O}(?)$ der Rückwärts--FD--Approximation

$$\frac{d}{dx}f(x)=\frac{f(x+\Delta x)-f(x)}{\Delta x}+\mathcal{O}(?)$$

Solution / Lösung:

$$f(x)=f(x) \qquad \textbf{(1)}$$

$$f(x-\Delta x)=f(x)-\Delta x\frac{df(x)}{dx}+\frac{(\Delta x)^2}{2!}\frac{d^2f(x)}{dx^2}-\frac{(\Delta x)^3}{3!}\frac{d^3f(x)}{dx^3}+\frac{(\Delta x)^4}{4!}\frac{d^4f(x)}{dx^4}+\mathcal{O}[(\Delta x)^5] \qquad \textbf{(2)}$$

Compute (1) minus (2) and subsequently divide by Δx /
Berechne (1) minus (2) und dividiere nachfolgend durch Δx

$$\frac{f(x)-f(x-\Delta x)}{\Delta x}=\frac{df(x)}{dx}-\frac{\Delta x}{2!}\frac{d^2f(x)}{dx^2}+\frac{(\Delta x)^2}{3!}\frac{d^3f(x)}{dx^3}-\frac{(\Delta x)^3}{4!}\frac{d^4f(x)}{dx^4}+\mathcal{O}[(\Delta x)^5]$$

$$\frac{d}{dx}f(x)=\frac{f(x)-f(x-\Delta x)}{\Delta x}+\underbrace{\frac{\Delta x}{2!}\frac{d^2f(x)}{dx^2}-\frac{(\Delta x)^2}{3!}\frac{d^3f(x)}{dx^3}+\frac{(\Delta x)^3}{4!}\frac{d^4f(x)}{dx^4}+\mathcal{O}[(\Delta x)^5]}_{=\mathcal{O}(\Delta x)}$$

Problem 3 / Aufgabe 3 (...)

$$O(\Delta x) = \frac{\Delta x}{2!}\frac{d^2 f(x)}{dx^2} \underbrace{- \frac{(\Delta x)^2}{3!}\frac{d^3 f(x)}{dx^3} + \frac{(\Delta x)^3}{4!}\frac{d^4 f(x)}{dx^4} + O[(\Delta x)^5]}_{=O[(\Delta x)^3]}$$

$$= \frac{\Delta x}{2}\frac{d^2 f(x)}{dx^2} + O[(\Delta x)^2]$$

$$\frac{d}{dx}f(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x} + O(\Delta x)$$

Approximation error / Approximationsfehler

Landau symbol "O", big "oh" / Landau-Symbol „O", großes „oh"

Problem 4 / Aufgabe 4

Derive the central finite difference approximation of 4th order for the first--order derivative of the form / Leiten Sie die zentrale FD-Approximation 4. Ordnung für die Ableitung erster Ordnung ab

$$\frac{d}{dx}f(x) = \frac{1}{\Delta x}\left[f\left(x+\frac{\Delta x}{2}\right) - f\left(x-\frac{\Delta x}{2}\right)\right]$$
$$-\frac{1}{24}\frac{1}{\Delta x}\left[f\left(x+\frac{3\Delta x}{2}\right) - 3f\left(x-\frac{\Delta x}{2}\right) + 3f\left(x-\frac{\Delta x}{2}\right) - f\left(x-\frac{3\Delta x}{2}\right)\right] + O\left[(\Delta x)^4\right]$$

Or rearranged / Oder umgestellt

$$\frac{d}{dx}f(x) = \frac{1}{24}\frac{1}{\Delta x}\left[-f\left(x+\frac{3\Delta x}{2}\right) + 27f\left(x-\frac{\Delta x}{2}\right) - 27f\left(x-\frac{\Delta x}{2}\right) + f\left(x-\frac{3\Delta x}{2}\right)\right] + O\left[(\Delta x)^4\right]$$

Solution / Lösung:

$$f(x+\frac{\Delta x}{2}) = f(x) + \frac{\Delta x}{2}\frac{df(x)}{dx} + \frac{(\frac{\Delta x}{2})^2}{2!}\frac{d^2 f(x)}{dx^2} + \frac{(\frac{\Delta x}{2})^3}{3!}\frac{d^3 f(x)}{dx^3} + \frac{(\frac{\Delta x}{2})^4}{4!}\frac{d^4 f(x)}{dx^4} + \frac{(\frac{\Delta x}{2})^5}{5!}\frac{d^5 f(x)}{dx^5} + O[(\Delta x)^6]$$

$$f(x-\frac{\Delta x}{2}) = f(x) - \frac{\Delta x}{2}\frac{df(x)}{dx} + \frac{(\frac{\Delta x}{2})^2}{2!}\frac{d^2 f(x)}{dx^2} - \frac{(\frac{\Delta x}{2})^3}{3!}\frac{d^3 f(x)}{dx^3} + \frac{(\frac{\Delta x}{2})^4}{4!}\frac{d^4 f(x)}{dx^4} - \frac{(\frac{\Delta x}{2})^5}{5!}\frac{d^5 f(x)}{dx^5} + O[(\Delta x)^6]$$

Problem 4 / Aufgabe 4 (...)

$$f(x+\frac{\Delta x}{2})-f(x-\frac{\Delta x}{2})=f(x)-f(x)+\frac{\Delta x}{2}\frac{df(x)}{dx}+\frac{\Delta x}{2}\frac{df(x)}{dx}+\frac{(\frac{\Delta x}{2})^2}{2!}\frac{d^2f(x)}{dx^2}-\frac{(\frac{\Delta x}{2})^2}{2!}\frac{d^2f(x)}{dx^2}$$
$$+\frac{(\frac{\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}+\frac{(\frac{\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}+\frac{(\frac{\Delta x}{2})^4}{4!}\frac{d^4f(x)}{dx^4}-\frac{(\frac{\Delta x}{2})^4}{4!}\frac{d^4f(x)}{dx^4}$$
$$+\frac{(\frac{\Delta x}{2})^5}{5!}\frac{d^5f(x)}{dx^5}+\frac{(\frac{\Delta x}{2})^5}{5!}\frac{d^5f(x)}{dx^5}+O[(\Delta x)^6]$$

$$f(x+\frac{\Delta x}{2})-f(x-\frac{\Delta x}{2})=\Delta x\frac{df(x)}{dx}+2\frac{(\frac{\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}+2\frac{(\frac{\Delta x}{2})^5}{5!}\frac{d^5f(x)}{dx^5}+O[(\Delta x)^7]$$

$$\frac{df(x)}{dx}=\frac{1}{\Delta x}\left[f\left(x+\frac{\Delta x}{2}\right)-f\left(x+\frac{\Delta x}{2}\right)\right]-\frac{(\Delta x)^2}{24}\underbrace{\frac{d^3f(x)}{dx^3}}+O[(\Delta x)^5]$$

Derive an FD approximation for this term /
Leite eine FD-Approximation für diesen Term ab

Problem 4 / Aufgabe 4 (...)

$$f(x+\frac{\Delta x}{2})=f(x)+\frac{\Delta x}{2}\frac{df(x)}{dx}+\frac{(\frac{\Delta x}{2})^2}{2!}\frac{d^2f(x)}{dx^2}+\frac{(\frac{\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}+\frac{(\frac{\Delta x}{2})^4}{4!}\frac{d^4f(x)}{dx^4}+\frac{(\frac{\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}+O[(\Delta x)^6] \quad \textbf{(1)}$$

$$f(x-\frac{\Delta x}{2})=f(x)-\frac{\Delta x}{2}\frac{df(x)}{dx}+\frac{(\frac{\Delta x}{2})^2}{2!}\frac{d^2f(x)}{dx^2}-\frac{(\frac{\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}+\frac{(\frac{\Delta x}{2})^4}{4!}\frac{d^4f(x)}{dx^4}-\frac{(\frac{\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}+O[(\Delta x)^6] \quad \textbf{(2)}$$

$$f(x+\frac{3\Delta x}{2})=f(x)+\frac{3\Delta x}{2}\frac{df(x)}{dx}+\frac{(\frac{3\Delta x}{2})^2}{2!}\frac{d^2f(x)}{dx^2}+\frac{(\frac{3\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}+\frac{(\frac{3\Delta x}{2})^4}{4!}\frac{d^4f(x)}{dx^4}+\frac{(\frac{3\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}+O[(\Delta x)^6] \quad \textbf{(3)}$$

$$f(x-\frac{3\Delta x}{2})=f(x)-\frac{3\Delta x}{2}\frac{df(x)}{dx}+\frac{(\frac{3\Delta x}{2})^2}{2!}\frac{d^2f(x)}{dx^2}-\frac{(\frac{3\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}+\frac{(\frac{3\Delta x}{2})^4}{4!}\frac{d^4f(x)}{dx^4}-\frac{(\frac{3\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}+O[(\Delta x)^6] \quad \textbf{(4)}$$

Now compute (3) - 3 times (1) + 3 times (2) – (4) /
Nun berechne (3) - 3 mal (1) + 3 mal (2) – (4)

Problem 4 / Aufgabe 4 (...)

Now compute (3) - 3 times (1) + 3 times (2) – (4) /
Nun berechne (3) - 3 mal (1) + 3 mal (2) – (4)

$$f(x+\frac{3\Delta x}{2})-3f(x+\frac{\Delta x}{2})+3f(x-\frac{\Delta x}{2})-f(x-\frac{3\Delta x}{2})=\ldots$$

$$+f(x+\frac{3\Delta x}{2})=+f(x)+\frac{3\Delta x}{2}\frac{df(x)}{dx}+\frac{(\frac{3\Delta x}{2})^2}{2!}\frac{d^2f(x)}{dx^2}+\frac{(\frac{3\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}+\frac{(\frac{3\Delta x}{2})^4}{4!}\frac{d^4f(x)}{dx^4}+\frac{(\frac{3\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}+O[(\Delta x)^6]$$

$$-3f(x+\frac{\Delta x}{2})=-3f(x)-3\frac{\Delta x}{2}\frac{df(x)}{dx}-3\frac{(\frac{\Delta x}{2})^2}{2!}\frac{d^2f(x)}{dx^2}-3\frac{(\frac{\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}-3\frac{(\frac{\Delta x}{2})^4}{4!}\frac{d^4f(x)}{dx^4}-3\frac{(\frac{\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}+O[(\Delta x)^6]$$

$$+3f(x-\frac{\Delta x}{2})=+3f(x)-3\frac{\Delta x}{2}\frac{df(x)}{dx}+3\frac{(\frac{\Delta x}{2})^2}{2!}\frac{d^2f(x)}{dx^2}-3\frac{(\frac{\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}+3\frac{(\frac{\Delta x}{2})^4}{4!}\frac{d^4f(x)}{dx^4}-3\frac{(\frac{\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}+O[(\Delta x)^6]$$

$$-f(x-\frac{3\Delta x}{2})=-f(x)+\frac{3\Delta x}{2}\frac{df(x)}{dx}-\frac{(\frac{3\Delta x}{2})^2}{2!}\frac{d^2f(x)}{dx^2}+\frac{(\frac{3\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}-\frac{(\frac{3\Delta x}{2})^4}{4!}\frac{d^4f(x)}{dx^4}+\frac{(\frac{3\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}+O[(\Delta x)^6]$$

Problem 4 / Aufgabe 4 (...)

$$\begin{aligned}
f(x+\frac{3\Delta x}{2})-3f(x+\frac{\Delta x}{2})+3f(x-\frac{\Delta x}{2})-f(x-\frac{3\Delta x}{2}) &= \frac{(\frac{3\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}+\frac{(\frac{3\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}\\
&-3\frac{(\frac{\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}-3\frac{(\frac{\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}\\
&-3\frac{(\frac{\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}-3\frac{(\frac{\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}\\
&+\frac{(\frac{3\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}+\frac{(\frac{3\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}+O[(\Delta x)^6]\\
&=\left[\frac{(\frac{3\Delta x}{2})^3}{3!}-3\frac{(\frac{\Delta x}{2})^3}{3!}-3\frac{(\frac{\Delta x}{2})^3}{3!}+\frac{(\frac{3\Delta x}{2})^3}{3!}\right]\frac{d^3f(x)}{dx^3}\\
&+\left[\frac{(\frac{3\Delta x}{2})^5}{4!}-3\frac{(\frac{\Delta x}{2})^5}{4!}-3\frac{(\frac{\Delta x}{2})^5}{4!}+\frac{(\frac{3\Delta x}{2})^5}{4!}\right]\frac{d^5f(x)}{dx^5}+O[(\Delta x)^6]
\end{aligned}$$

Problem 4 / Aufgabe 4 (...)

$$f(x+\frac{3\Delta x}{2})-3f(x+\frac{\Delta x}{2})+3f(x-\frac{\Delta x}{2})-f(x-\frac{3\Delta x}{2})$$

$$=\left[\frac{27(\frac{\Delta x}{2})^3}{3!}-3\frac{(\frac{\Delta x}{2})^3}{3!}-3\frac{(\frac{\Delta x}{2})^3}{3!}+\frac{27(\frac{\Delta x}{2})^3}{3!}\right]\frac{d^3f(x)}{dx^3}$$

$$+\left[\frac{243(\frac{\Delta x}{2})^5}{4!}-3\frac{(\frac{\Delta x}{2})^5}{4!}-3\frac{(\frac{\Delta x}{2})^5}{4!}+\frac{243(\frac{\Delta x}{2})^5}{4!}\right]\frac{d^5f(x)}{dx^5}+\bigcirc[(\Delta x)^6]$$

$$=\underbrace{\left[27-3-3+27\right]}_{48}\frac{(\frac{\Delta x}{2})^3}{3!}\frac{d^3f(x)}{dx^3}+\underbrace{\left[243-3-3+243\right]}_{480}\frac{(\frac{\Delta x}{2})^5}{4!}\frac{d^5f(x)}{dx^5}+\bigcirc[(\Delta x)^6]$$

$$=\underbrace{\frac{48}{6}\frac{1}{8}}_{=1}(\Delta x)^3\frac{d^3f(x)}{dx^3}+\underbrace{\frac{480}{24}\frac{1}{32}}_{=1/8}(\Delta x)^5\frac{d^5f(x)}{dx^5}+\bigcirc[(\Delta x)^6]$$

$$=(\Delta x)^3\frac{d^3f(x)}{dx^3}+\frac{1}{8}(\Delta x)^5\frac{d^5f(x)}{dx^5}+\bigcirc[(\Delta x)^6]$$

$$=(\Delta x)^3\frac{d^3f(x)}{dx^3}+\bigcirc[(\Delta x)^5]$$

Problem 4 / Aufgabe 4 (...)

$$f(x+\frac{3\Delta x}{2})-3f(x+\frac{\Delta x}{2})+3f(x-\frac{\Delta x}{2})-f(x-\frac{3\Delta x}{2})=(\Delta x)^3\frac{d^3f(x)}{dx^3}+\bigcirc[(\Delta x)^5]$$

$$\frac{d^3f(x)}{dx^3}=\frac{1}{(\Delta x)^3}\left[f(x+\frac{3\Delta x}{2})-3f(x+\frac{\Delta x}{2})+3f(x-\frac{\Delta x}{2})-f(x-\frac{3\Delta x}{2})]\right]+\bigcirc[(\Delta x)^2$$

$$\frac{df(x)}{dx}=\frac{1}{\Delta x}\left[f\left(x+\frac{\Delta x}{2}\right)-f\left(x+\frac{\Delta x}{2}\right)\right]-\frac{(\Delta x)^2}{24}\frac{d^3f(x)}{dx^3}+\bigcirc[(\Delta x)^5]$$

$$\frac{(\Delta x)^2}{24}\frac{d^3f(x)}{dx^3}=\frac{1}{24}\frac{f(x+\frac{3\Delta x}{2})-3f(x+\frac{\Delta x}{2})+3f(x-\frac{\Delta x}{2})-f(x-\frac{3\Delta x}{2})}{\Delta x}+\bigcirc[(\Delta x)^4]$$

$$\frac{d}{dx}f(x)=\frac{1}{\Delta x}\left[f\left(x+\frac{\Delta x}{2}\right)-f\left(x-\frac{\Delta x}{2}\right)\right]$$

$$-\frac{(\Delta x)^2}{24}\frac{1}{(\Delta x)^3}\left[f\left(x+\frac{3\Delta x}{2}\right)-3f\left(x-\frac{\Delta x}{2}\right)+3f\left(x-\frac{\Delta x}{2}\right)-f\left(x-\frac{3\Delta x}{2}\right)\right]+\bigcirc\left[(\Delta x)^4\right]$$