

**Numerical Methods of  
Electromagnetic Field Theory I (NFT I)  
Numerische Methoden der  
Elektromagnetischen Feldtheorie I (NFT I) /**

**5th Lecture / 5. Vorlesung**

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## EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

The first two Maxwell's Equations are: /  
Die ersten beiden Maxwell'schen Gleichungen lauten:

Equations of first order /  
Gleichungen der ersten Ordnung



$$\frac{\partial}{\partial t} \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) = -\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t)$$

$$\frac{\partial}{\partial t} \underline{\mathbf{D}}(\underline{\mathbf{R}}, t) = \nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t)$$



Constitutive Equations for Vacuum /  
Konstituierende Gleichungen  
(Materialgleichungen) für Vakuum

$$\underline{\mathbf{B}}(\underline{\mathbf{R}}, t) = \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}}, t)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}, t) = \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}, t)$$



$$\frac{\partial}{\partial t} \mu \underline{\mathbf{H}}(\underline{\mathbf{R}}, t) = -\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t)$$

$$\frac{\partial}{\partial t} \varepsilon \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) = \nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t)$$

$f(\underline{\mathbf{H}}, \underline{\mathbf{E}})$

Constitutive Equations for Vacuum /  
Konstituierende Gleichungen  
(Materialgleichungen) für Vakuum

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}, t) = \nu_0 \underline{\mathbf{B}}(\underline{\mathbf{R}}, t)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}, t) = \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}, t)$$



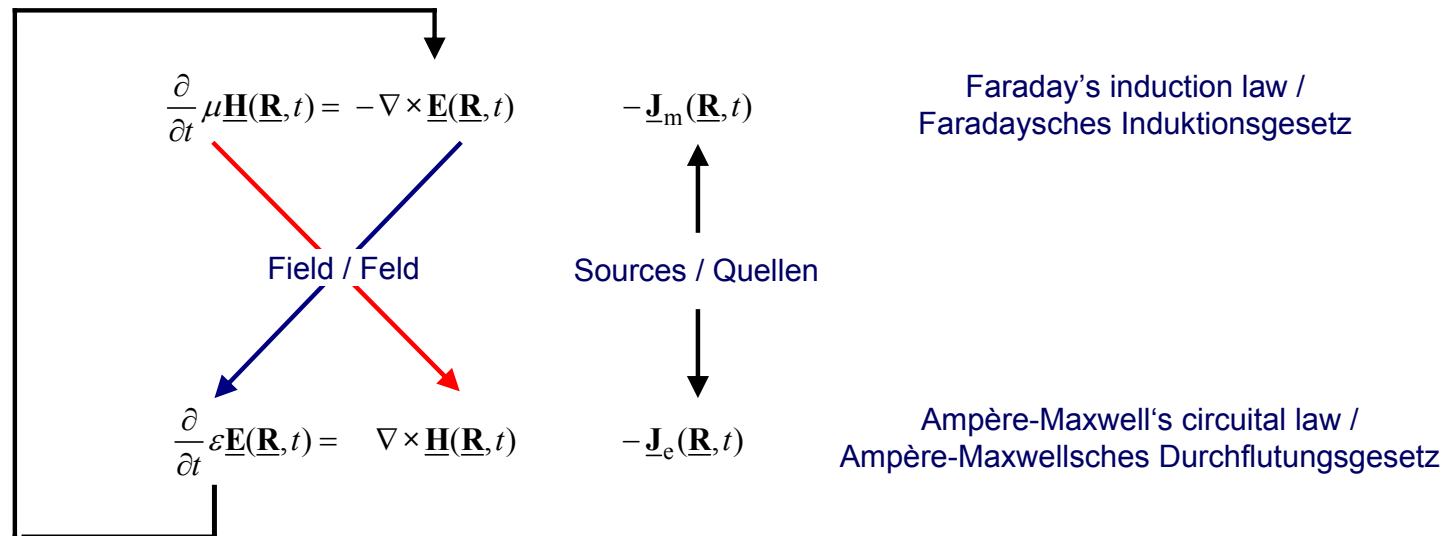
$$\frac{\partial}{\partial t} \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) = -\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t)$$

$$\frac{\partial}{\partial t} [\varepsilon \underline{\mathbf{E}}(\underline{\mathbf{R}}, t)] = \nabla \times [\nu \underline{\mathbf{B}}(\underline{\mathbf{R}}, t)] - \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t)$$

$f(\underline{\mathbf{B}}, \underline{\mathbf{E}})$

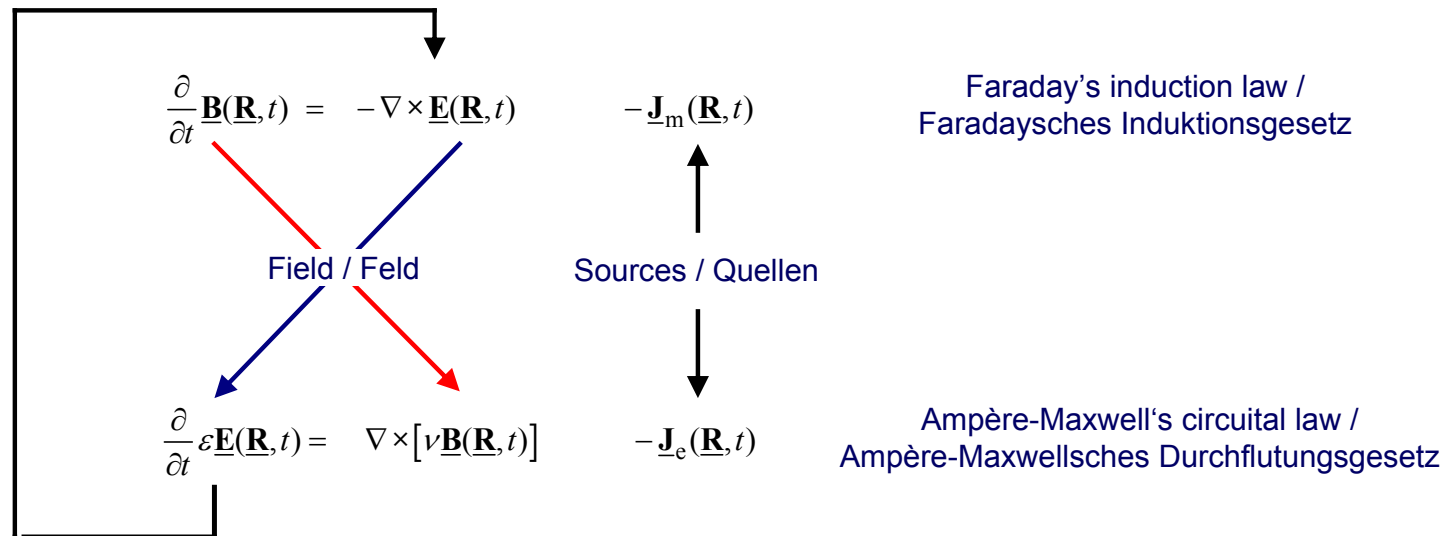
# EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Idea: Outline of a flow chart /  
Idee: Entwurf eines Flussdiagramms



# EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Idea: Outline of a flow chart /  
Idee: Entwurf eines Flussdiagramms



# 1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

The first two Maxwell's Equations are: /  
Die ersten beiden Maxwell'schen Gleichungen lauten:

$$\frac{\partial}{\partial t} \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) = -\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t)$$

$$\frac{\partial}{\partial t} \underline{\mathbf{D}}(\underline{\mathbf{R}}, t) = \nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t)$$

Constitutive Equations for Vacuum /  
Konstituierende Gleichungen  
(Materialgleichungen) für Vakuum

$$\underline{\mathbf{B}}(\underline{\mathbf{R}}, t) = \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}}, t)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}, t) = \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}, t)$$

Ansatz for the electric and magnetic  
field strength /  
Ansatz für die elektrische und  
magnetische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}, t) = E_x(z, t) \underline{\mathbf{e}}_x$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}, t) = H_y(z, t) \underline{\mathbf{e}}_y$$



$$\frac{\partial}{\partial t} H_y(z, t) = -\frac{1}{\mu_0} \frac{\partial}{\partial z} E_x(z, t) - \frac{1}{\mu_0} J_{my}(z, t)$$

$$\frac{\partial}{\partial t} E_x(z, t) = -\frac{1}{\varepsilon_0} \frac{\partial}{\partial z} H_y(z, t) - \frac{1}{\varepsilon_0} J_{ex}(z, t)$$

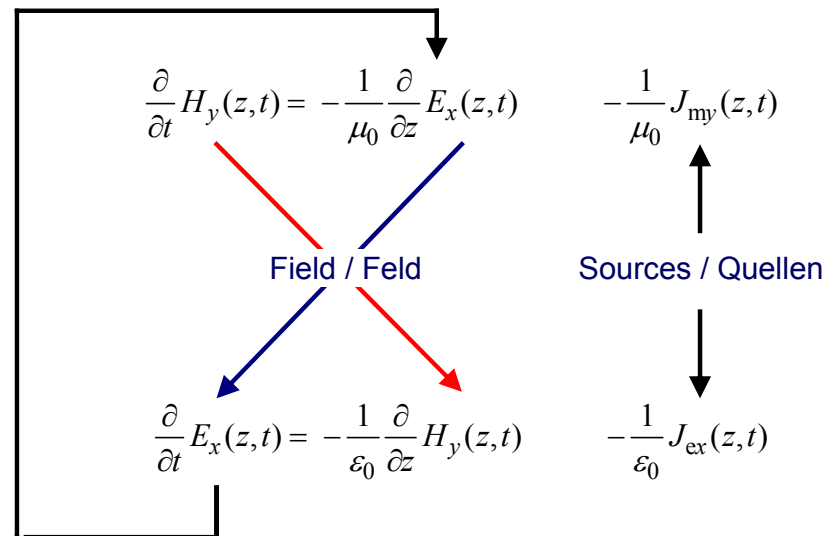


$$\frac{d}{dt} f(t) = \frac{f\left(t + \frac{\Delta t}{2}\right) - f\left(t - \frac{\Delta t}{2}\right)}{\Delta t} + \mathcal{O}[(\Delta t)^2]$$

$$\frac{d}{dz} f(z) = \frac{f\left(z + \frac{\Delta z}{2}\right) - f\left(z - \frac{\Delta z}{2}\right)}{\Delta z} + \mathcal{O}[(\Delta z)^2]$$

# 1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Idea: Outline of a flow chart /  
Idee: Entwurf eines Flussdiagramms



## 1-D EM Wave Propagation – FDTD – Discretization of the 1st Equation / 1D EM Wellenausbreitung – FDTD – Diskretisierung der 1ten Gleichung

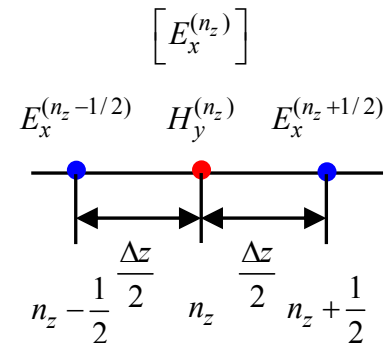
Spatial discretization of the 1st equation /  
Räumliche Diskretisierung der 1ten Gleichung

$$\frac{\partial}{\partial t} H_y(z, t) = -\frac{1}{\mu_0} \frac{\partial}{\partial z} E_x(z, t) - \frac{1}{\mu_0} J_{my}(z, t)$$

$$H_y : z \rightarrow n_z \Delta z, \quad n_z = 1, \dots, N_z$$

$$E_x : z \rightarrow (n_z + 1/2) \Delta z, \quad n_z = 1, \dots, N_z$$

$$\frac{\partial}{\partial z} E_x(z, t) \rightarrow \frac{\partial}{\partial z} E_x(z, t) \Big|_z = \frac{1}{\Delta z} \left[ E_x \left( z + \frac{\Delta z}{2} \right) - E_x \left( z - \frac{\Delta z}{2} \right) \right] + \mathcal{O}[(\Delta z)^2]$$



$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = -\frac{1}{\mu_0} \frac{1}{\Delta z} \left[ E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t) \right] - \frac{1}{\mu_0} J_{my}^{(n_z)}(t)$$

# 1-D EM Wave Propagation – FDTD – Discretization of the 2nd Equation / 1D EM Wellenausbreitung – FDTD – Diskretisierung der 2ten Gleichung

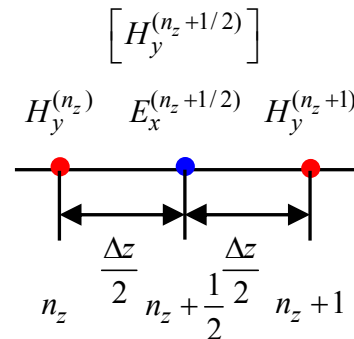
Spatial discretization of the 2nd equation /  
Räumliche Diskretisierung der 2ten Gleichung

$$\frac{\partial}{\partial t} E_x(z, t) = -\frac{1}{\epsilon_0} \frac{\partial}{\partial z} H_y(z, t) - \frac{1}{\epsilon_0} J_{ex}(z, t)$$

$$H_y : z \rightarrow n_z \Delta z, \quad n_z = 1, \dots, N_z$$

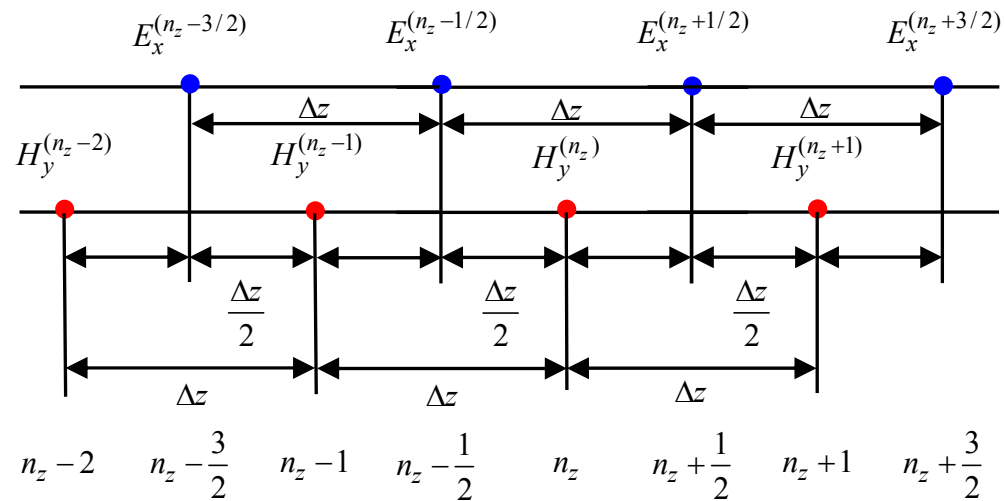
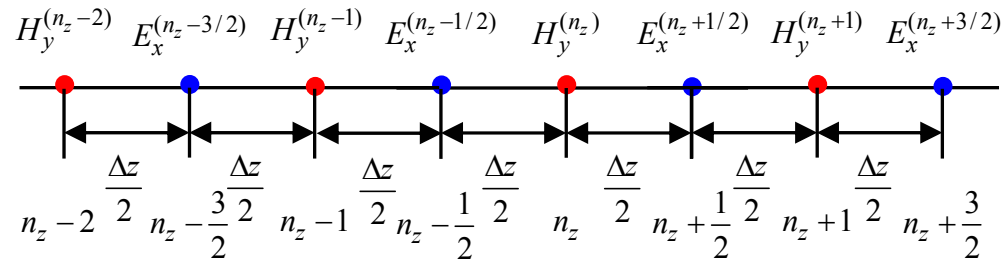
$$E_x : z \rightarrow (n_z + 1/2) \Delta z, \quad n_z = 1, \dots, N_z$$

$$\frac{\partial}{\partial z} H_y(z, t) \rightarrow \frac{\partial}{\partial z} H_y(z, t) \Big|_{z+\frac{\Delta z}{2}} = \frac{1}{\Delta z} [H_y(z + \Delta z) - H_y(z)] + O[(\Delta z)^2]$$



$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = -\frac{1}{\epsilon_0 \Delta z} [H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)] - \frac{1}{\epsilon_0} J_{ey}^{(n_z+1/2)}(t)$$

# 1-D EM Wave Propagation – 1-D FDTD – Staggered Grid in Space / 1D EM Wellenausbreitung – 1-D FDTD – Versetztes Gitter im Raum



## 1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

$$\frac{\partial}{\partial t} H_y(z, t) = -\frac{1}{\mu_0} \frac{\partial}{\partial z} E_x(z, t) - \frac{1}{\mu_0} J_{my}(z, t)$$

$$\frac{\partial}{\partial t} E_x(z, t) = -\frac{1}{\varepsilon_0} \frac{\partial}{\partial z} H_y(z, t) - \frac{1}{\varepsilon_0} J_{ex}(z, t)$$



$$\frac{d}{dz} f(z) = \frac{1}{\Delta z} \left[ f\left(z + \frac{\Delta z}{2}\right) - f\left(z - \frac{\Delta z}{2}\right) \right] + O[(\Delta z)^2]$$



$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = -\frac{1}{\mu_0} \frac{1}{\Delta z} \left[ E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t) \right] - \frac{1}{\mu_0} J_{my}^{(n_z)}(t)$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = -\frac{1}{\varepsilon_0 \Delta z} \left[ H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t) \right] - \frac{1}{\varepsilon_0} J_{ex}^{(n_z+1/2)}(t)$$

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = ?$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = ?$$

## 1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = -\frac{1}{\mu_0} \frac{1}{\Delta z} \left[ E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t) \right] - \frac{1}{\mu_0} J_{my}^{(n_z)}(t)$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = -\frac{1}{\varepsilon_0 \Delta z} \left[ H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t) \right] - \frac{1}{\varepsilon_0} J_{ey}^{(n_z+1/2)}(t)$$

$$\frac{d}{dt} f(t) = \frac{1}{\Delta t} \left[ f\left(t + \frac{\Delta t}{2}\right) - f\left(t - \frac{\Delta t}{2}\right) \right] + \mathcal{O}[(\Delta t)^2]$$

Staggered grid in time / Versetztes Gitter in der Zeit

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = \frac{H_y^{(n_z, n_t)} - H_y^{(n_z, n_t-1)}}{\Delta t} + \mathcal{O}[(\Delta t)^2]$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = \frac{E_x^{(n_z+1/2, n_t+1/2)} - E_x^{(n_z+1/2, n_t-1/2)}}{\Delta t} + \mathcal{O}[(\Delta t)^2]$$

$$\frac{H_y^{(n_z, n_t)} - H_y^{(n_z, n_t-1)}}{\Delta t} = -\frac{1}{\mu_0} \frac{1}{\Delta z} \left[ E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t) \right] - \frac{1}{\mu_0} J_{my}^{(n_z)}(t)$$

$$\frac{E_x^{(n_z+1/2, n_t+1/2)} - E_x^{(n_z+1/2, n_t-1/2)}}{\Delta t} = -\frac{1}{\varepsilon_0 \Delta z} \left[ H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t) \right] - \frac{1}{\varepsilon_0} J_{ey}^{(n_z+1/2)}(t)$$

## 1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

$$\frac{H_y^{(n_z, n_t)} - H_y^{(n_z, n_t-1)}}{\Delta t} = -\frac{1}{\mu_0} \frac{1}{\Delta z} \left[ E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t) \right] - \frac{1}{\mu_0} J_{my}^{(n_z)}(t)$$

$$\frac{E_x^{(n_z+1/2, n_t+1/2)} - E_x^{(n_z+1/2, n_t-1/2)}}{\Delta t} = -\frac{1}{\varepsilon_0 \Delta z} \left[ H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t) \right] - \frac{1}{\varepsilon_0} J_{ey}^{(n_z+1/2)}(t)$$

Explicit 1-D FDTD algorithm on a staggered grid in space and time /  
Expliziter 1D-FDTD-Algorithmus auf einem versetzten Gitter im Raum und Zeit

$$H_y^{(n_z, n_t)} = H_y^{(n_z, n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z} \left[ E_x^{(n_z+1/2, n_t-1/2)} - E_x^{(n_z-1/2, n_t-1/2)} \right] - \frac{\Delta t}{\mu_0} J_{my}^{(n_z, n_t-1/2)}$$

$$E_x^{(n_z+1/2, n_t+1/2)} = E_x^{(n_z+1/2, n_t-1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z} \left[ H_y^{(n_z+1, n_t)} - H_y^{(n_z, n_t)} \right] - \frac{\Delta t}{\varepsilon_0} J_{ey}^{(n_z+1/2, n_t)}$$

**FDTD:** Yee, K. S.: Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Transactions on Antennas Propagation*, Vol. AP-14, pp. 302-307, 1966.

## 1-D EM Wave Propagation – 1-D FDTD / 1D EM Wellenausbreitung – 1D FDTD

The first two Maxwell's Equations are: /  
Die ersten beiden Maxwell'schen Gleichungen lauten:

$$\frac{\partial}{\partial t} H_y(z, t) = -\frac{1}{\mu_0} \frac{\partial}{\partial z} E_x(z, t) - \frac{1}{\mu_0} J_{my}(z, t)$$

$$\frac{\partial}{\partial t} E_x(z, t) = -\frac{1}{\varepsilon_0} \frac{\partial}{\partial z} H_y(z, t) - \frac{1}{\varepsilon_0} J_{ex}(z, t)$$

Explicit 1-D FDTD algorithm of leap-frog type on a staggered grid in space and time /  
Expliziter 1D-FDTD-Algorithmus vom „Bocksprung“-Typ auf einem versetzten Gitter im Raum und Zeit

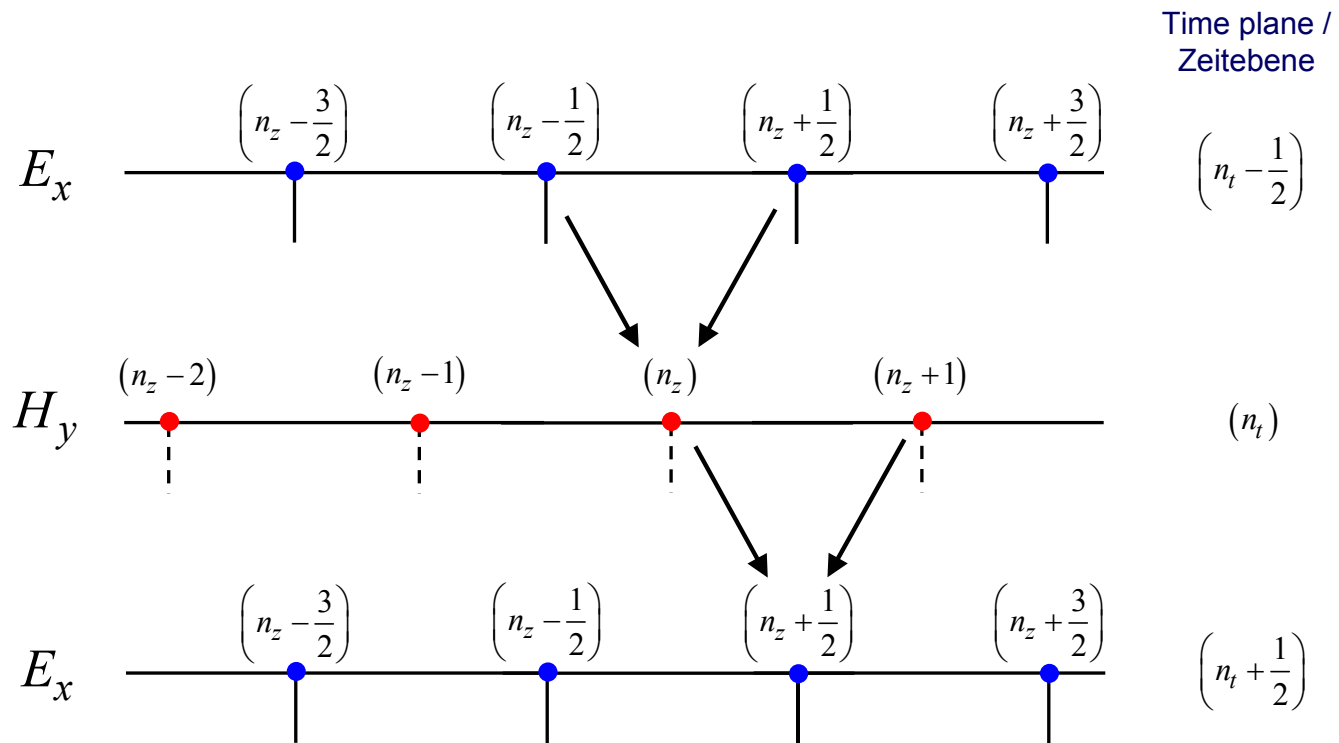
$$H_y^{(n_z, n_t)} = H_y^{(n_z, n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z} \left[ E_x^{(n_z+1/2, n_t-1/2)} - E_x^{(n_z-1/2, n_t-1/2)} \right] - \frac{\Delta t}{\mu_0} J_{my}^{(n_z, n_t-1/2)}$$

$$E_x^{(n_z+1/2, n_t+1/2)} = E_x^{(n_z+1/2, n_t-1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z} \left[ H_y^{(n_z+1, n_t)} - H_y^{(n_z, n_t)} \right] - \frac{\Delta t}{\varepsilon_0} J_{ex}^{(n_z+1/2, n_t)}$$

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## 1-D EM Wave Propagation – 1-D FDTD – Staggered Grid in Space / 1D EM Wellenausbreitung – 1-D FDTD – Versetztes Gitter im Raum

Interleaving of the  $E_x$  and  $H_y$  field components in space and time in the 1-D FDTD formulation /  
Überlappung der  $E_x$ - und  $H_y$ -Feldkomponente in der 1D-FDTD-Formulierung im Raum und in der Zeit



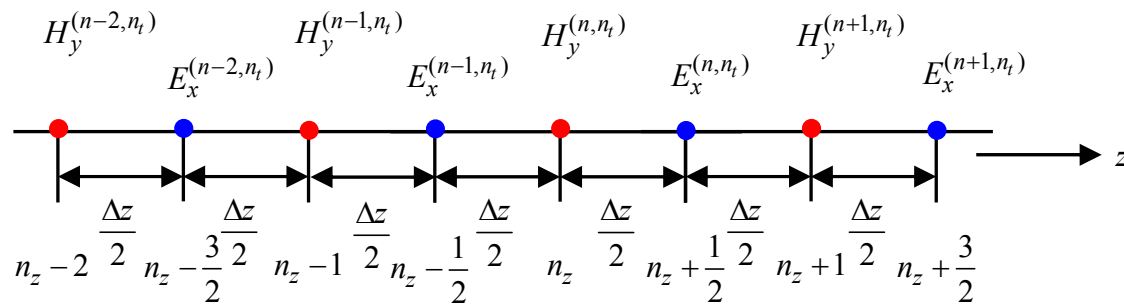
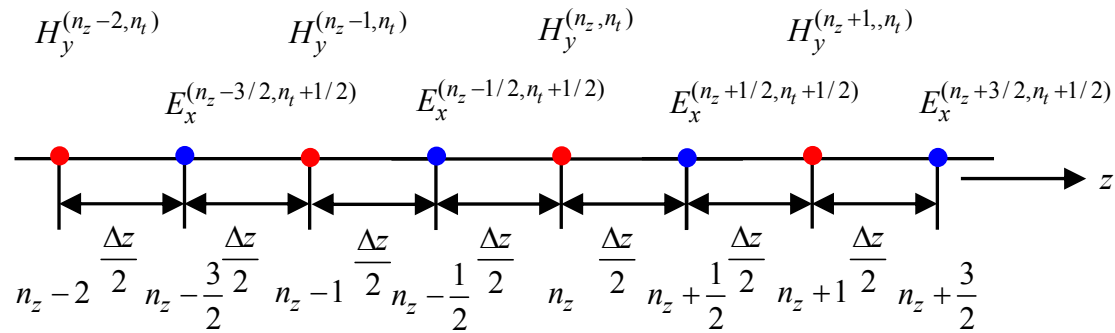
## 1-D EM Wave Propagation – FDTD – Normalization / 1D EM Wellenausbreitung – FDTD – Normierung

$$\begin{aligned}
 H_y^{(n_z, n_t)} &= H_y^{(n_z, n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z} \left[ E_x^{(n_z+1/2, n_t-1/2)} - E_x^{(n_z-1/2, n_t-1/2)} \right] - \frac{\Delta t}{\mu_0} J_{my}^{(n_z, n_t-1/2)} \\
 E_x^{(n_z+1/2, n_t+1/2)} &= E_x^{(n_z+1/2, n_t-1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z} \left[ H_y^{(n_z+1, n_t)} - H_y^{(n_z, n_t)} \right] - \frac{\Delta t}{\varepsilon_0} J_{ex}^{(n_z+1/2, n_t)}
 \end{aligned}$$

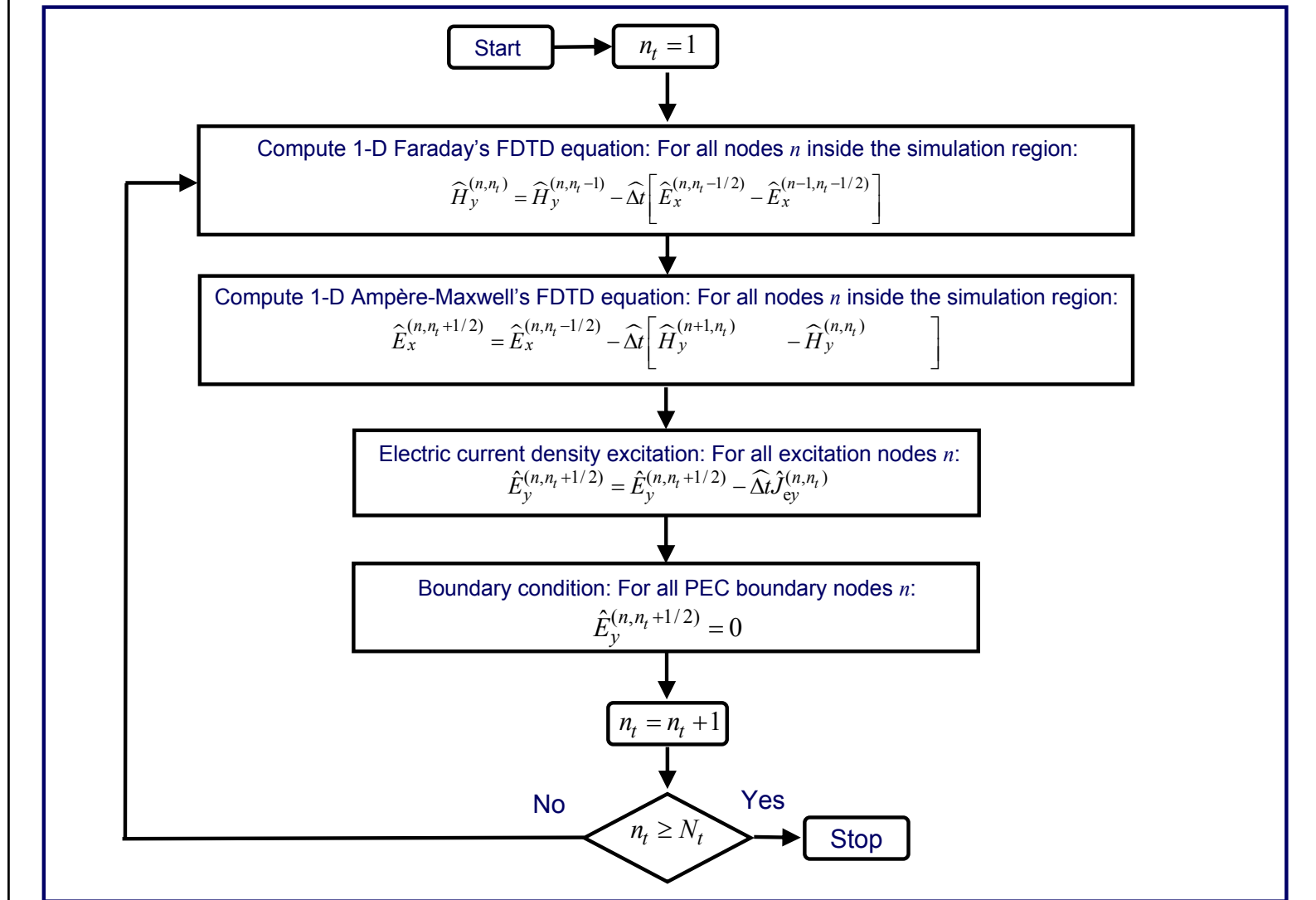
$$\begin{aligned}
 \Delta t &= \Delta t_{\text{ref}} \widehat{\Delta t} & \Delta t_{\text{ref}} &= \frac{\Delta x_{\text{ref}}}{c_{\text{ref}}} & \Delta t &= \frac{\Delta x_{\text{ref}}}{c_{\text{ref}}} \widehat{\Delta t} \\
 \Delta z &= \Delta x_{\text{ref}} \widehat{\Delta z} & c &= c_{\text{ref}} \widehat{c} & \varepsilon &= \varepsilon_{\text{ref}} \widehat{\varepsilon} & \mu &= \mu_{\text{ref}} \widehat{\mu} & \mu_{\text{ref}} &= \mu_0 \\
 E_x &= E_{\text{ref}} \widehat{E}_x \\
 H_y &= H_{\text{ref}} \widehat{H}_y & H_{\text{ref}} &= \frac{E_{\text{ref}}}{c_{\text{ref}} \mu_{\text{ref}}} = \frac{\sqrt{\varepsilon_{\text{ref}} \mu_{\text{ref}}}}{\mu_{\text{ref}}} E_{\text{ref}} = \sqrt{\frac{\varepsilon_{\text{ref}}}{\mu_{\text{ref}}}} E_{\text{ref}} = \frac{E_{\text{ref}}}{Z_{\text{ref}}} \\
 J_{\text{ex}} &= J_{\text{e ref}} \widehat{J}_{\text{ex}} & J_{\text{e ref}} &= \frac{\varepsilon_{\text{ref}}}{\Delta t_{\text{ref}}} E_{\text{ref}} \\
 J_{\text{mx}} &= J_{\text{m ref}} \widehat{J}_{\text{mx}} & J_{\text{m ref}} &= \frac{\mu_{\text{ref}}}{\Delta t_{\text{ref}}} H_{\text{ref}} = \frac{E_{\text{ref}}}{\Delta t_{\text{ref}} c_{\text{ref}}}
 \end{aligned}$$

$$\begin{aligned}
 \widehat{H}_y^{(n_z, n_t)} &= \widehat{H}_y^{(n_z, n_t-1)} - \widehat{\Delta t} \left[ \widehat{E}_x^{(n_z+1/2, n_t-1/2)} - \widehat{E}_x^{(n_z-1/2, n_t-1/2)} \right] - \widehat{\Delta t} \widehat{J}_{my}^{(n_z, n_t-1/2)} \\
 \widehat{E}_x^{(n_z+1/2, n_t+1/2)} &= \widehat{E}_x^{(n_z+1/2, n_t-1/2)} - \widehat{\Delta t} \left[ \widehat{H}_y^{(n_z+1, n_t)} - \widehat{H}_y^{(n_z, n_t)} \right] - \widehat{\Delta t} \widehat{J}_{ex}^{(n_z+1/2, n_t)}
 \end{aligned}$$

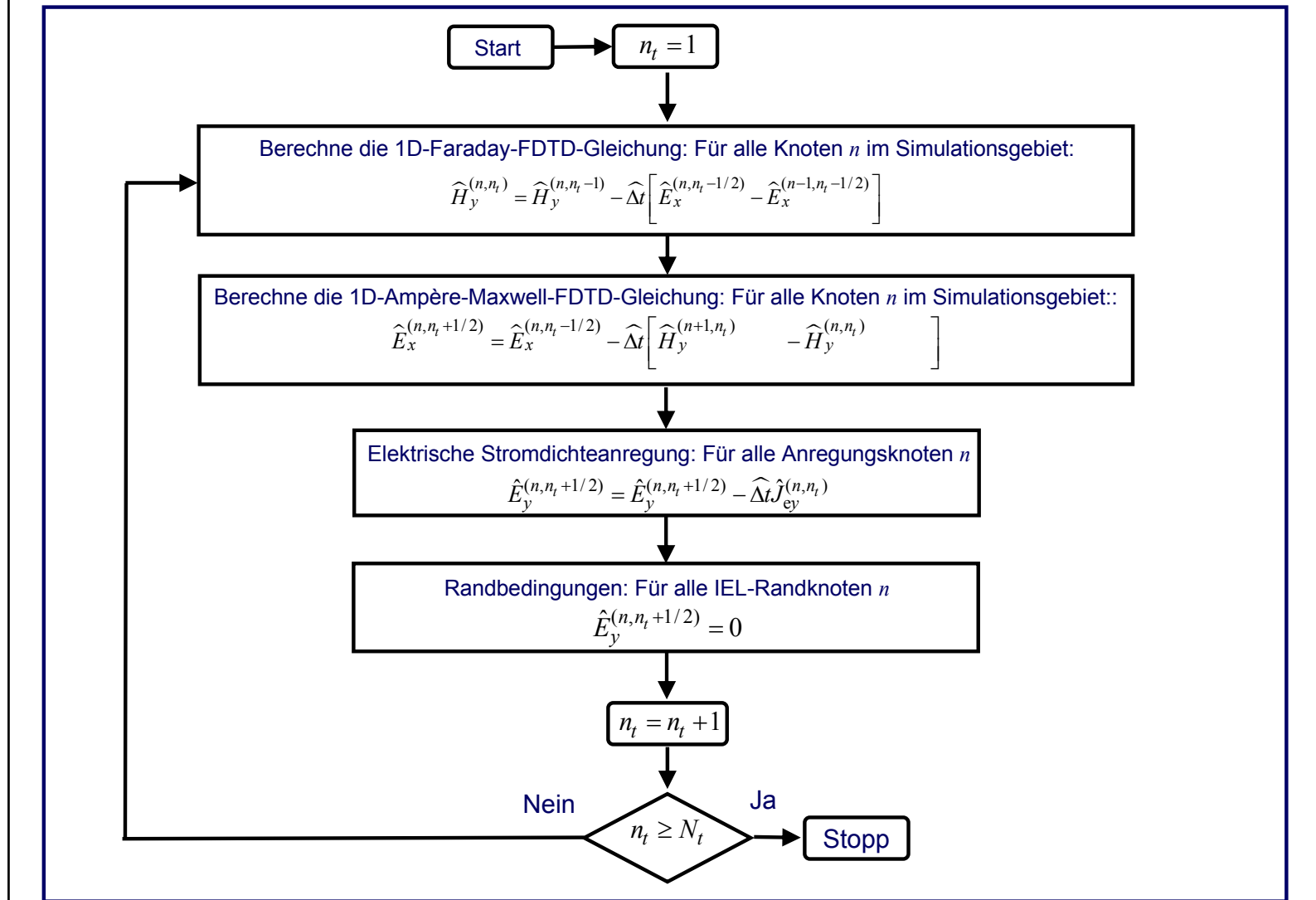
## 1-D FDTD – Staggered Grid in Space – Global Node Numbering / 1D-FDTD – Versetztes Gitter im Raum – Globale Knotennummerierung



## 1-D FDTD Algorithm – Flow Chart / 1D-FDTD-Algorithmus – Flussdiagramm



## 1-D FDTD Algorithm – Flow Chart / 1D-FDTD-Algorithmus – Flussdiagramm



## FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

**Maxwell's equations / Maxwellsche Gleichungen**

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0} \frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{my}(z,t) \quad \text{for / für} \quad \begin{cases} 0 \leq z \leq Z \\ 0 \leq t \leq T \end{cases}$$

$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0} \frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{ex}(z,t)$$

**Initial condition / Anfangsbedingung**

$$\begin{aligned} H_y(z,t) = J_{my}(z,t) &= 0 & t \leq 0 \\ E_x(z,t) = J_{ex}(z,t) &= 0 & t \leq 0 \\ J_{ex}(z,t) = K_{e0}(z_0) \delta(z-z_0) f(t) & & t > 0 \end{aligned}$$

Causality / Kausalität

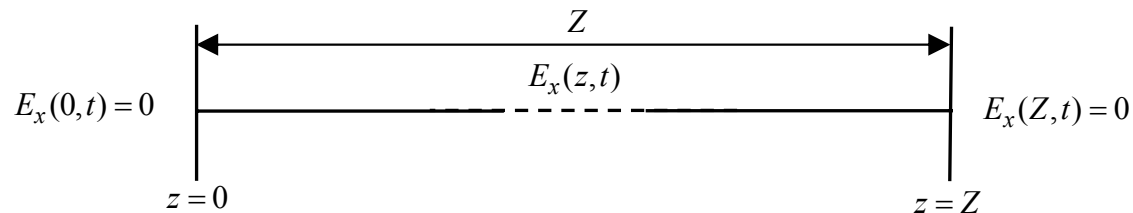
Hyperbolic initial-  
boundary-value problem

/

Hyperbolisches  
Anfangs-Randwert-  
Problem

**Boundary condition for a perfectly electrically conducting (PEC) material /  
Randbedingung für ein ideal elektrisch leitendes Material**

$$\left. \begin{aligned} E_x(0,t) &= 0 \\ E_x(Z,t) &= 0 \end{aligned} \right\} \forall t$$



# FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

Discrete 1-D FDTD equations / Diskrete 1D-FDTD-Gleichungen

$$\begin{aligned} \widehat{H}_y^{(n_z, n_t)} &= \widehat{H}_y^{(n_z, n_t-1)} - \widehat{\Delta t} \left[ \widehat{E}_x^{(n_z+1/2, n_t-1/2)} - \widehat{E}_x^{(n_z-1/2, n_t-1/2)} \right] - \widehat{\Delta t} \widehat{J}_{my}^{(n_z, n_t-1/2)} & \text{for / für } \begin{cases} 1 \leq n_z \leq N_z \\ 1 \leq n_t \leq N_t \end{cases} \\ \widehat{E}_x^{(n_z+1/2, n_t+1/2)} &= \widehat{E}_x^{(n_z+1/2, n_t-1/2)} - \widehat{\Delta t} \left[ \widehat{H}_y^{(n_z+1, n_t)} - \widehat{H}_y^{(n_z, n_t)} \right] - \widehat{\Delta t} \widehat{J}_{ex}^{(n_z+1/2, n_t)} \end{aligned}$$

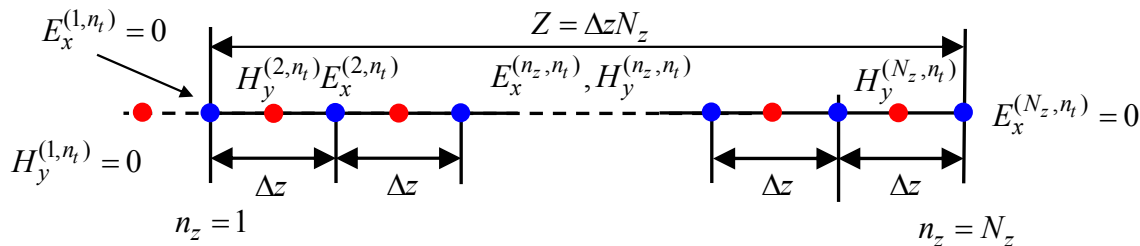
Initial condition / Anfangsbedingung

$$\begin{aligned} H_y^{(n_z, n_t)} &= J_{my}^{(n_z, n_t)} = 0 & n_t \leq 1 \\ E_x^{(n_z, n_t)} &= J_{ex}^{(n_z, n_t)} = 0 & n_t \leq 1 \\ J_{ex}^{(n_z, n_t)} &= K_{ex}^{(n_{z0})} \delta^{(n_z - n_{z0})} f_t^{(n_t)} & n_t > 1 \end{aligned}$$

Causality / Kausalität

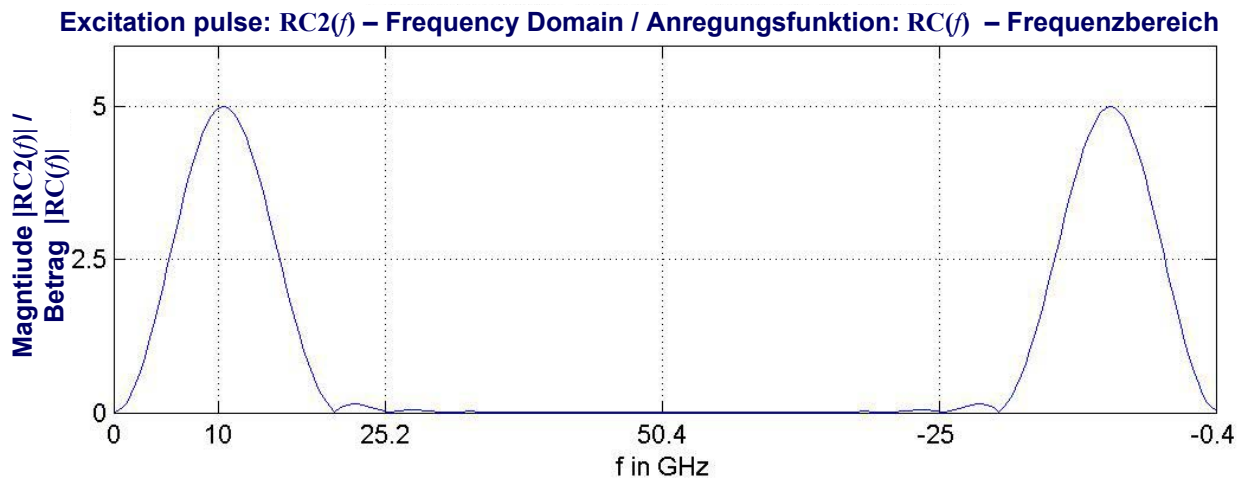
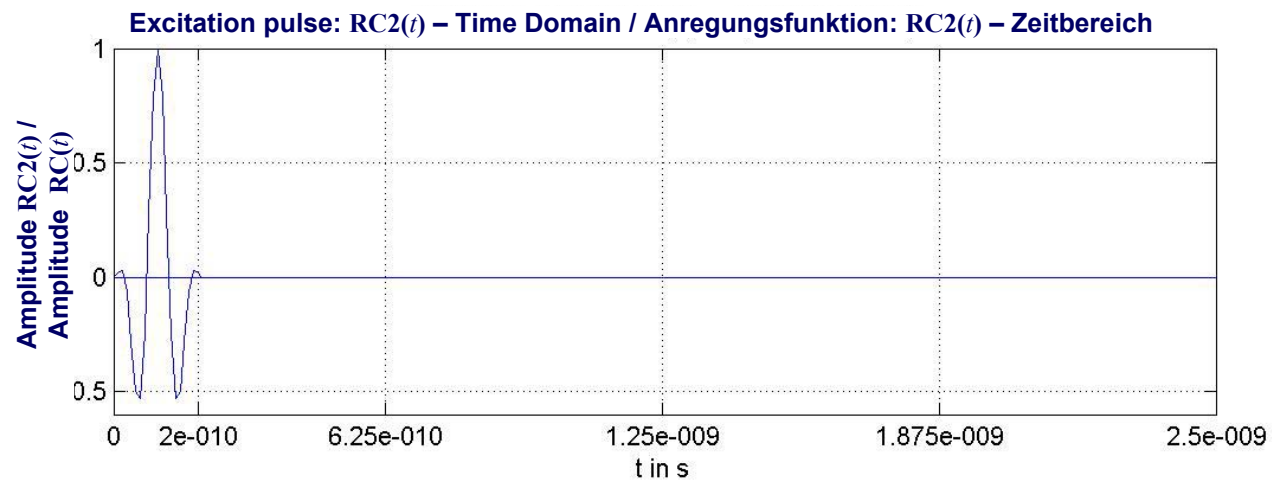
Boundary condition for a perfectly electrically conducting (PEC) material /  
Randbedingung für ein ideal elektrisch leitendes Material

$$\left. \begin{aligned} E_x^{(1, n_t)} &= 0 \\ E_x^{(N_z, n_t)} &= 0 \end{aligned} \right\} 1 \leq n_t \leq N_t$$

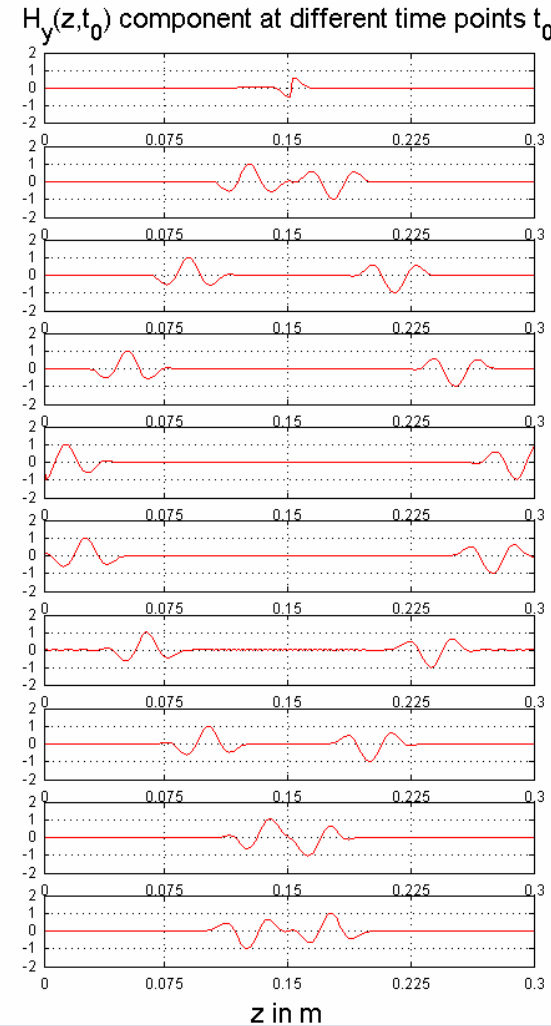
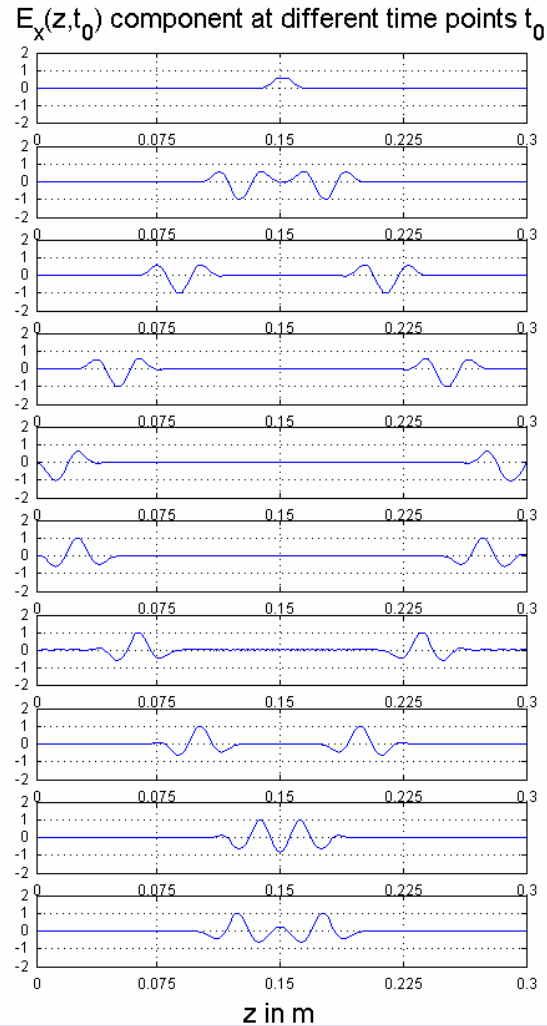


Discrete hyperbolic  
initial-boundary-value  
problem /  
Diskretes  
hyperbolisches  
Anfangs-Randwert-  
Problem

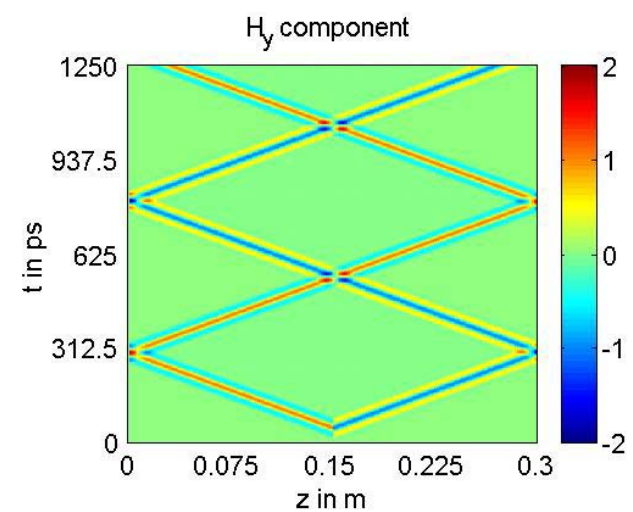
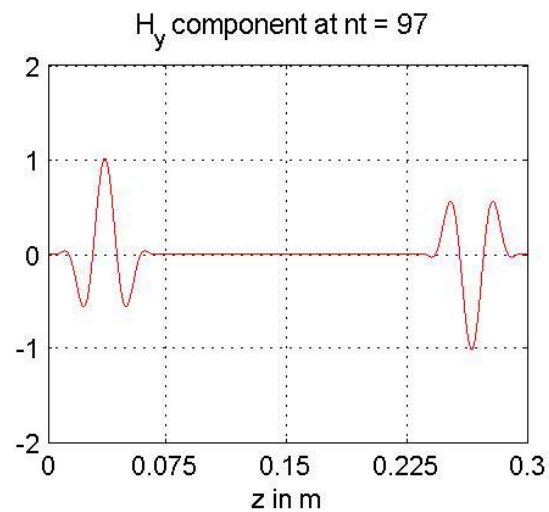
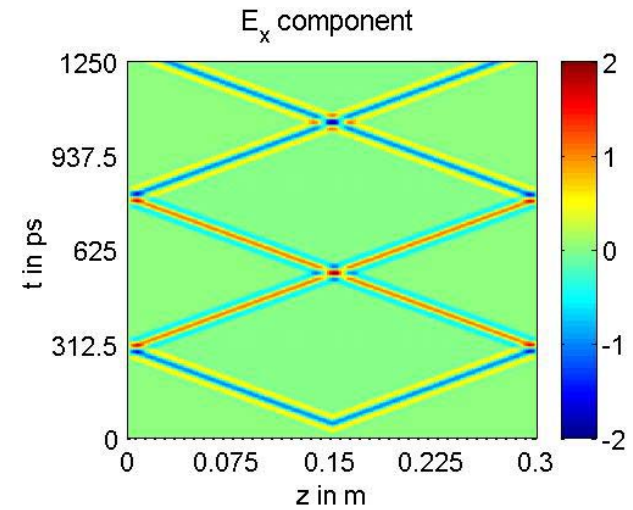
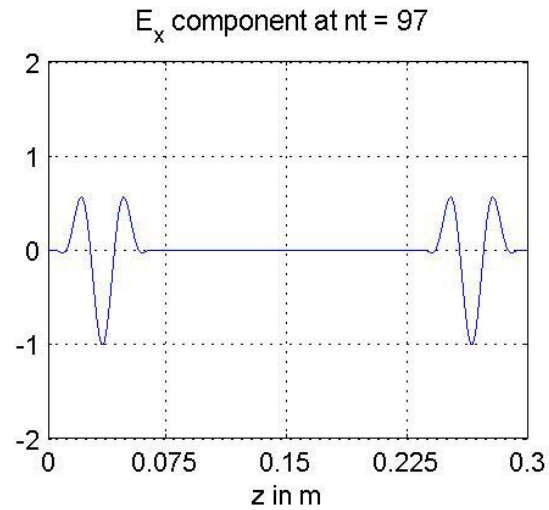
## FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



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## Implementation of Boundary Conditions / Implementierung von Randbedingungen

**Boundary condition for a perfectly electrically conducting (PEC) material /  
Randbedingung für ein ideal elektrisch leitendes Material**

$$\left. \begin{aligned} E_x^{(1,n_t)} &= 0 \\ E_x^{(N_z,n_t)} &= 0 \end{aligned} \right\} 1 \leq n_t \leq N_t$$

**Absorbing/open boundary condition /  
Absorbierende/offene Randbedingung**

Space-time-extrapolation of the first order /  
Raum-Zeit-Extrapolation der ersten Ordnung

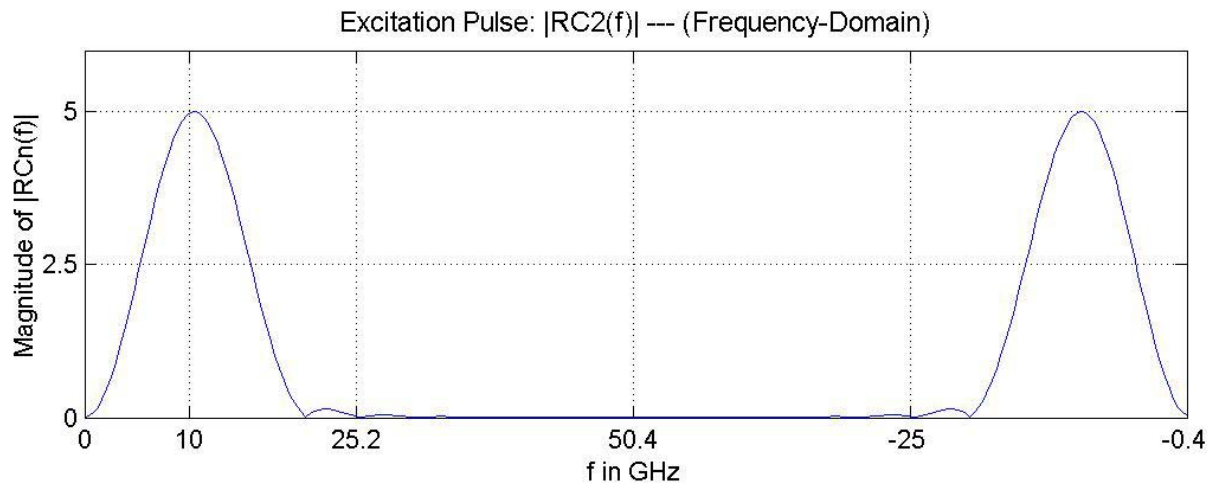
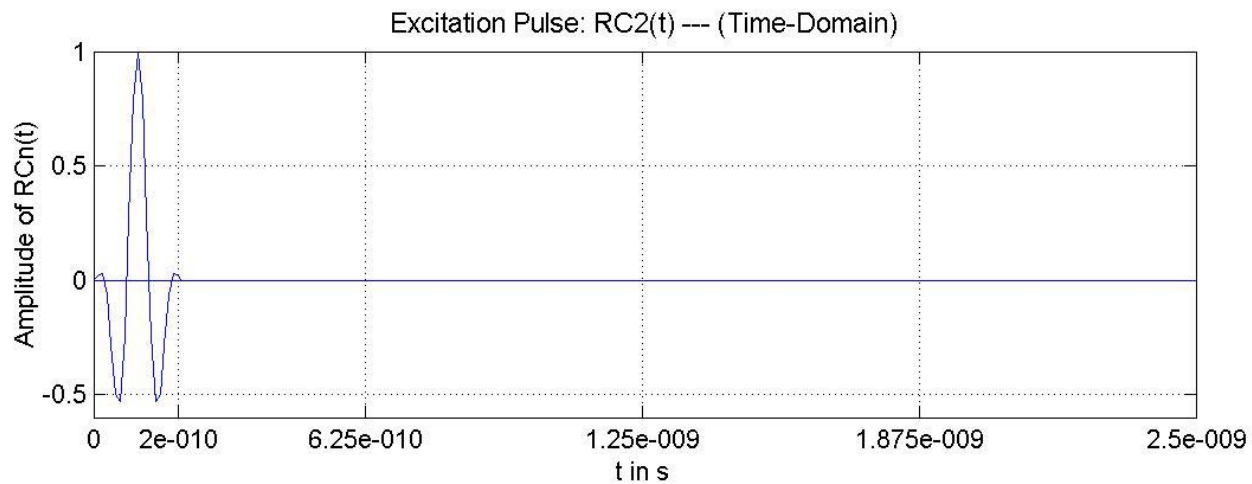
For / Für  $\widehat{\Delta t} = 0.5$

a plane wave needs two time steps,  $2 n_t$ , to travel over one grid cell with the size  $\Delta z$  /  
braucht eine ebene Welle zwei Zeitschritte,  $2 n_t$ , um sich über eine Gitterzelle der Größe  $\Delta z$  auszubreiten

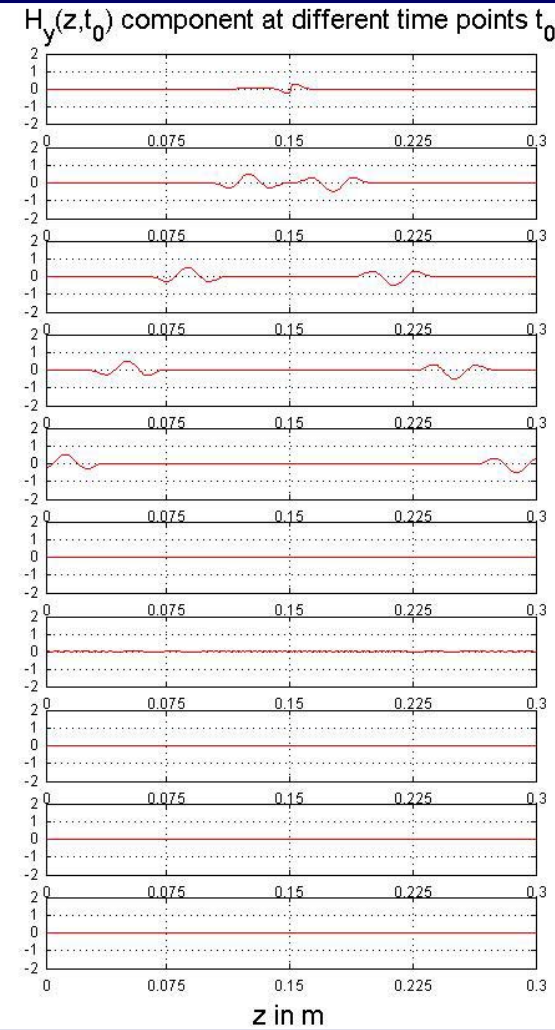
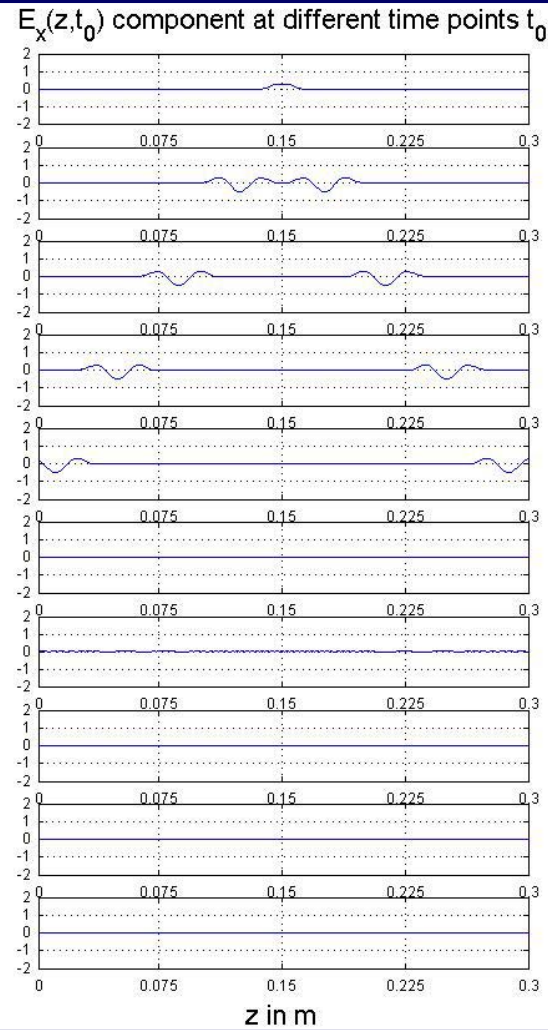
$$\left. \begin{aligned} E_x^{(1,n_t)} &= E_x^{(2,n_t-2)} \\ E_x^{(N_z,n_t)} &= E_x^{(N_z-1,n_t-2)} \end{aligned} \right\} 1 \leq n_t \leq N_t$$

Space-time-extrapolation of the first order /  
Raum-Zeit-Extrapolation der ersten Ordnung

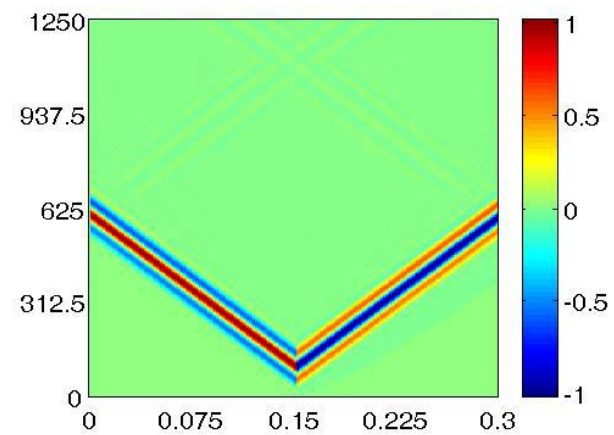
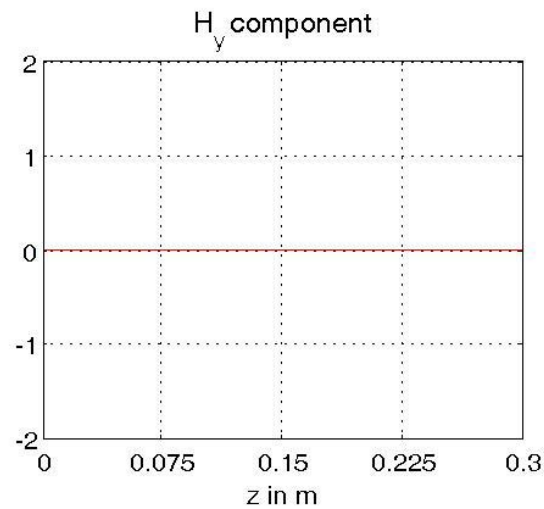
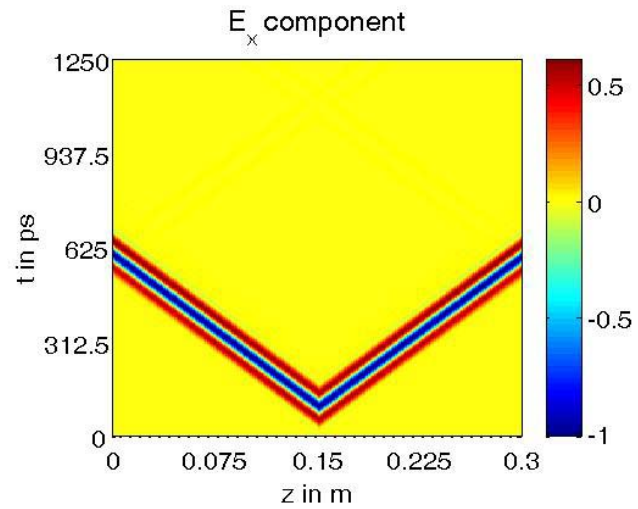
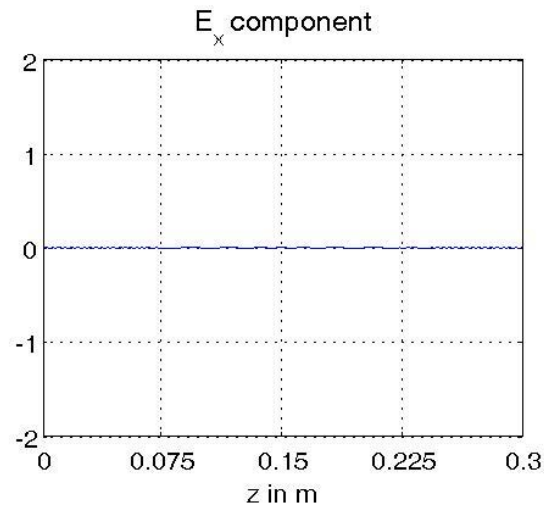
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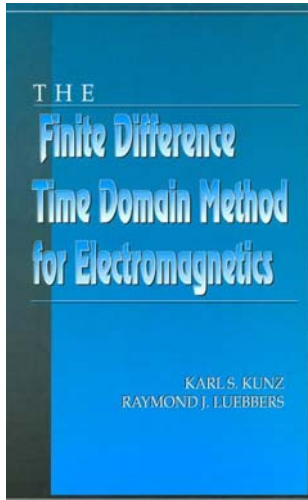
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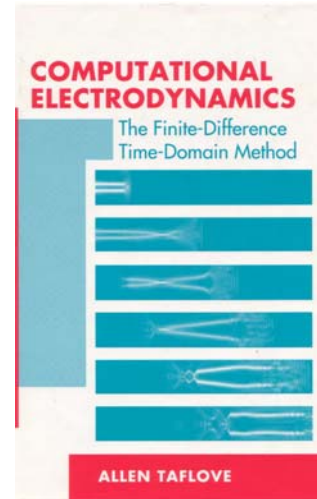
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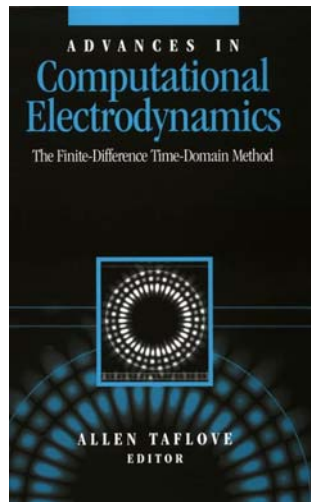
## FDTD Books / FDTD-Bücher



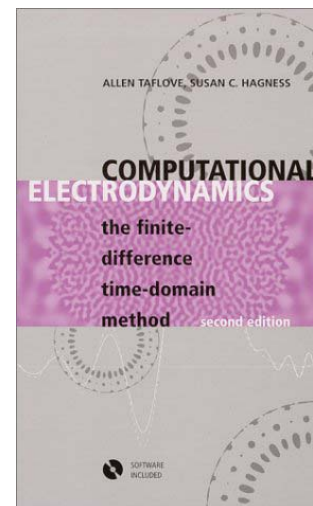
Kunz, K. S., Luebbers, R. J.: *The Finite Difference Time Domain Method for Electromagnetics*. 1993



Taflove, A. (Editor): *Computational Electrodynamics: The Finite-Difference Time-Domain Method*. Artech House, Boston, 1995.

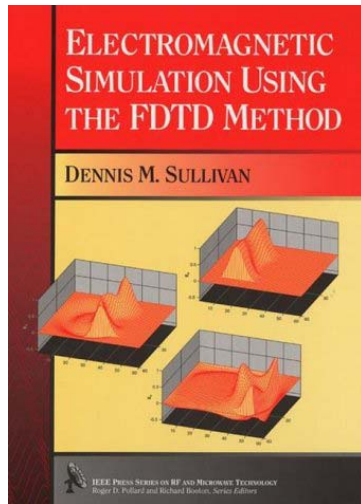


Taflove, A. (Editor): *Advances in Computational Electrodynamics: The Finite-Difference Time-Domain Method*. Artech House, 1998.



Taflove, A. (Editor): *Computational Electrodynamics: The Finite-Difference Time-Domain Method*. 2nd Edition, Artech House, Boston, 2000.

## FDTD Books / FDTD-Bücher



Sullivan, D. M.:  
*Electromagnetic  
Simulation Using the  
FDTD Method.* IEEE  
Press, New York, 2000.

## 3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

The first two Maxwell's Equations are in differential form /  
Die ersten beiden Maxwell'schen Gleichungen lauten in Differentialform:

$$\frac{\partial}{\partial t} \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) = -\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t)$$

$$\frac{\partial}{\partial t} \underline{\mathbf{D}}(\underline{\mathbf{R}}, t) = \nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_c(\underline{\mathbf{R}}, t)$$

In Cartesian Coordinates we find for the Curl operator applied to E and H /  
Im Kartesischen Koordinatensystem finden wir für den Rotationsoperator angewendet auf E und H:

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) = \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(\underline{\mathbf{R}}, t) & E_y(\underline{\mathbf{R}}, t) & E_z(\underline{\mathbf{R}}, t) \end{vmatrix}$$

$$= \left[ \frac{\partial E_z(\underline{\mathbf{R}}, t)}{\partial y} - \frac{\partial E_y(\underline{\mathbf{R}}, t)}{\partial z} \right] \underline{\mathbf{e}}_x + \left[ \frac{\partial E_x(\underline{\mathbf{R}}, t)}{\partial z} - \frac{\partial E_z(\underline{\mathbf{R}}, t)}{\partial x} \right] \underline{\mathbf{e}}_y + \left[ \frac{\partial E_y(\underline{\mathbf{R}}, t)}{\partial x} - \frac{\partial E_x(\underline{\mathbf{R}}, t)}{\partial y} \right] \underline{\mathbf{e}}_z$$

$$\nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}, t) = \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x(\underline{\mathbf{R}}, t) & H_y(\underline{\mathbf{R}}, t) & H_z(\underline{\mathbf{R}}, t) \end{vmatrix}$$

$$= \left[ \frac{\partial H_z(\underline{\mathbf{R}}, t)}{\partial y} - \frac{\partial H_y(\underline{\mathbf{R}}, t)}{\partial z} \right] \underline{\mathbf{e}}_x + \left[ \frac{\partial H_x(\underline{\mathbf{R}}, t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}}, t)}{\partial x} \right] \underline{\mathbf{e}}_y + \left[ \frac{\partial H_y(\underline{\mathbf{R}}, t)}{\partial x} - \frac{\partial H_x(\underline{\mathbf{R}}, t)}{\partial y} \right] \underline{\mathbf{e}}_z$$

## 3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

If we insert the last expressions into the first two Maxwell's equations are in differential form read /  
Wenn wir die letzten Ausdrücke in the ersten beiden Maxwell'schen Gleichungen in Differentialform einsetzen,  
erhalten wir:

$$\frac{\partial}{\partial t} \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) = -\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t)$$

$$\begin{aligned} \frac{\partial}{\partial t} [B_x(\underline{\mathbf{R}}, t)\underline{\mathbf{e}}_x + B_y(\underline{\mathbf{R}}, t)\underline{\mathbf{e}}_y + B_z(\underline{\mathbf{R}}, t)\underline{\mathbf{e}}_z] \\ = - \left\{ \left[ \frac{\partial E_z(\underline{\mathbf{R}}, t)}{\partial y} - \frac{\partial E_y(\underline{\mathbf{R}}, t)}{\partial z} \right] \underline{\mathbf{e}}_x + \left[ \frac{\partial E_x(\underline{\mathbf{R}}, t)}{\partial z} - \frac{\partial E_z(\underline{\mathbf{R}}, t)}{\partial x} \right] \underline{\mathbf{e}}_y + \left[ \frac{\partial E_y(\underline{\mathbf{R}}, t)}{\partial x} - \frac{\partial E_x(\underline{\mathbf{R}}, t)}{\partial y} \right] \underline{\mathbf{e}}_z \right\} \\ - [J_{mx}(\underline{\mathbf{R}}, t)\underline{\mathbf{e}}_x + J_{my}(\underline{\mathbf{R}}, t)\underline{\mathbf{e}}_y + J_{mz}(\underline{\mathbf{R}}, t)\underline{\mathbf{e}}_z] \end{aligned}$$

$$\frac{\partial}{\partial t} \underline{\mathbf{D}}(\underline{\mathbf{R}}, t) = \nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t)$$

$$\begin{aligned} \frac{\partial}{\partial t} [D_x(\underline{\mathbf{R}}, t)\underline{\mathbf{e}}_x + D_y(\underline{\mathbf{R}}, t)\underline{\mathbf{e}}_y + D_z(\underline{\mathbf{R}}, t)\underline{\mathbf{e}}_z] \\ = \left[ \frac{\partial H_z(\underline{\mathbf{R}}, t)}{\partial y} - \frac{\partial H_y(\underline{\mathbf{R}}, t)}{\partial z} \right] \underline{\mathbf{e}}_x + \left[ \frac{\partial H_x(\underline{\mathbf{R}}, t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}}, t)}{\partial x} \right] \underline{\mathbf{e}}_y + \left[ \frac{\partial H_y(\underline{\mathbf{R}}, t)}{\partial x} - \frac{\partial H_x(\underline{\mathbf{R}}, t)}{\partial y} \right] \underline{\mathbf{e}}_z \\ - [J_{ex}(\underline{\mathbf{R}}, t)\underline{\mathbf{e}}_x + J_{ey}(\underline{\mathbf{R}}, t)\underline{\mathbf{e}}_y + J_{ez}(\underline{\mathbf{R}}, t)\underline{\mathbf{e}}_z] \end{aligned}$$

**Six decoupled scalar equations! /  
Sechs entkoppelte skalare Gleichungen!**

## 3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

If we insert the last expressions into the first two Maxwell's equations are in differential form we read /  
Wenn wir die letzten Ausdrücke in die ersten beiden Maxwell'schen Gleichungen in Differentialform  
einsetzen, erhalten wir:

$$\frac{\partial}{\partial t} B_x(\mathbf{R}, t) = - \left[ \frac{\partial E_z(\mathbf{R}, t)}{\partial y} - \frac{\partial E_y(\mathbf{R}, t)}{\partial z} \right] - J_{mx}(\mathbf{R}, t)$$

$$\frac{\partial}{\partial t} B_y(\mathbf{R}, t) = - \left[ \frac{\partial E_x(\mathbf{R}, t)}{\partial z} - \frac{\partial E_z(\mathbf{R}, t)}{\partial x} \right] - J_{my}(\mathbf{R}, t)$$

$$\frac{\partial}{\partial t} B_z(\mathbf{R}, t) = - \left[ \frac{\partial E_y(\mathbf{R}, t)}{\partial x} - \frac{\partial E_x(\mathbf{R}, t)}{\partial y} \right] - J_{mz}(\mathbf{R}, t)$$

$$\frac{\partial}{\partial t} D_x(\mathbf{R}, t) = \left[ \frac{\partial H_z(\mathbf{R}, t)}{\partial y} - \frac{\partial H_y(\mathbf{R}, t)}{\partial z} \right] - J_{ex}(\mathbf{R}, t)$$

$$\frac{\partial}{\partial t} D_y(\mathbf{R}, t) = \left[ \frac{\partial H_x(\mathbf{R}, t)}{\partial z} - \frac{\partial H_z(\mathbf{R}, t)}{\partial x} \right] - J_{ey}(\mathbf{R}, t)$$

$$\frac{\partial}{\partial t} D_z(\mathbf{R}, t) = \left[ \frac{\partial H_y(\mathbf{R}, t)}{\partial x} - \frac{\partial H_x(\mathbf{R}, t)}{\partial y} \right] - J_{ez}(\mathbf{R}, t)$$

## 3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

**Constitutive equation for homogeneous isotropic materials /  
Konstituierende Gleichungen für homogene isotrope Materialien:**

$$\begin{aligned} B_x(\underline{\mathbf{R}}, t) &= \mu H_x(\underline{\mathbf{R}}, t) & D_x(\underline{\mathbf{R}}, t) &= \mu E_x(\underline{\mathbf{R}}, t) \\ B_y(\underline{\mathbf{R}}, t) &= \mu H_y(\underline{\mathbf{R}}, t) & D_y(\underline{\mathbf{R}}, t) &= \mu E_y(\underline{\mathbf{R}}, t) \\ B_z(\underline{\mathbf{R}}, t) &= \mu H_z(\underline{\mathbf{R}}, t) & D_z(\underline{\mathbf{R}}, t) &= \mu E_z(\underline{\mathbf{R}}, t) \end{aligned}$$

$$\frac{\partial}{\partial t} \mu H_x(\underline{\mathbf{R}}, t) = - \left[ \frac{\partial E_z(\underline{\mathbf{R}}, t)}{\partial y} - \frac{\partial E_y(\underline{\mathbf{R}}, t)}{\partial z} \right] - J_{mx}(\underline{\mathbf{R}}, t)$$

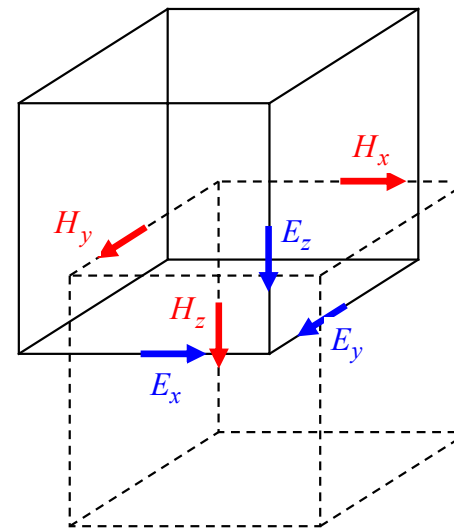
$$\frac{\partial}{\partial t} \mu H_y(\underline{\mathbf{R}}, t) = - \left[ \frac{\partial E_x(\underline{\mathbf{R}}, t)}{\partial z} - \frac{\partial E_z(\underline{\mathbf{R}}, t)}{\partial x} \right] - J_{my}(\underline{\mathbf{R}}, t)$$

$$\frac{\partial}{\partial t} \mu H_z(\underline{\mathbf{R}}, t) = - \left[ \frac{\partial E_y(\underline{\mathbf{R}}, t)}{\partial x} - \frac{\partial E_x(\underline{\mathbf{R}}, t)}{\partial y} \right] - J_{mz}(\underline{\mathbf{R}}, t)$$

$$\frac{\partial}{\partial t} \varepsilon E_x(\underline{\mathbf{R}}, t) = \left[ \frac{\partial H_z(\underline{\mathbf{R}}, t)}{\partial y} - \frac{\partial H_y(\underline{\mathbf{R}}, t)}{\partial z} \right] - J_{ex}(\underline{\mathbf{R}}, t)$$

$$\frac{\partial}{\partial t} \varepsilon E_y(\underline{\mathbf{R}}, t) = \left[ \frac{\partial H_x(\underline{\mathbf{R}}, t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}}, t)}{\partial x} \right] - J_{ey}(\underline{\mathbf{R}}, t)$$

$$\frac{\partial}{\partial t} \varepsilon E_z(\underline{\mathbf{R}}, t) = \left[ \frac{\partial H_y(\underline{\mathbf{R}}, t)}{\partial x} - \frac{\partial H_x(\underline{\mathbf{R}}, t)}{\partial y} \right] - J_{ez}(\underline{\mathbf{R}}, t)$$



$$H_{x_i} = J_{mx_i}, i = 1, 2, 3$$

$$E_{x_i} = J_{ex_i}, i = 1, 2, 3$$

### 3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

$$\frac{\partial}{\partial t} H_x(\mathbf{R}, t) = \dot{H}_x(\mathbf{R}, t)$$

$$\mu \dot{H}_x(\mathbf{R}, t) = - \left[ \frac{\partial E_z(\mathbf{R}, t)}{\partial y} - \frac{\partial E_y(\mathbf{R}, t)}{\partial z} \right] - J_{mx}(\mathbf{R}, t)$$

$$\mu \dot{H}_x(\mathbf{R}, t) = \dot{H}_x^{(m)}(t)$$

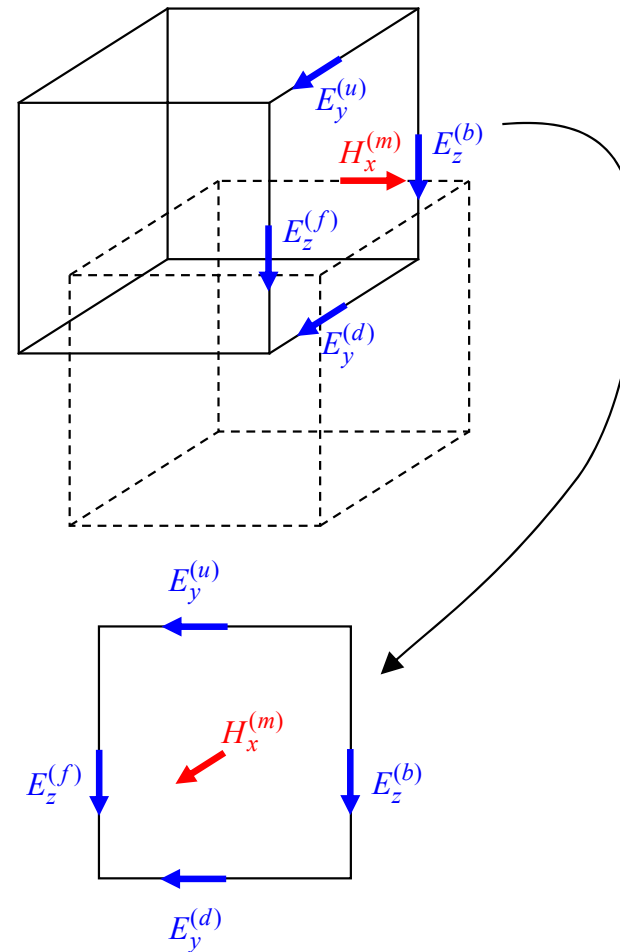
$$J_{mx}(\mathbf{R}, t) = J_{mx}^{(m)}(t)$$

$$\frac{\partial E_z(\mathbf{R}, t)}{\partial y} = \frac{E_z^{(f)}(t) - E_z^{(b)}(t)}{\Delta y} + \mathcal{O}[(\Delta y)^2]$$

$$\frac{\partial E_y(\mathbf{R}, t)}{\partial z} = \frac{E_y^{(d)}(t) - E_y^{(u)}(t)}{\Delta z} + \mathcal{O}[(\Delta z)^2]$$

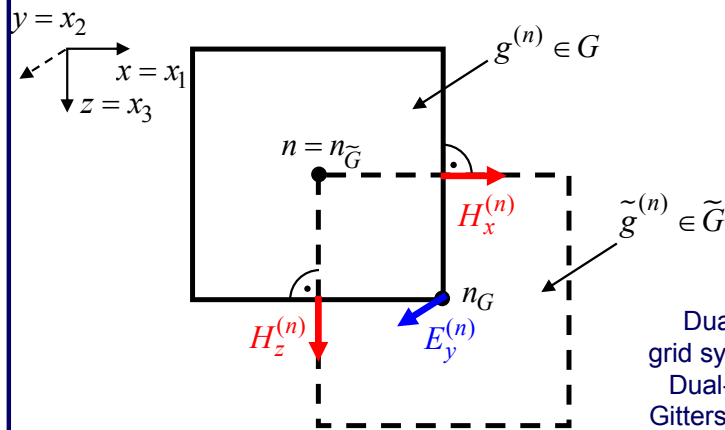
$$\mu \dot{H}_x^{(m)}(t) = - \underbrace{\frac{E_z^{(f)}(t) - E_z^{(b)}(t)}{\Delta y} + \frac{E_y^{(d)}(t) - E_y^{(u)}(t)}{\Delta z}} - J_{mx}^{(m)}(t)$$

A part of the discrete curl operator /  
Ein Teil des diskreten Rotationsoperators

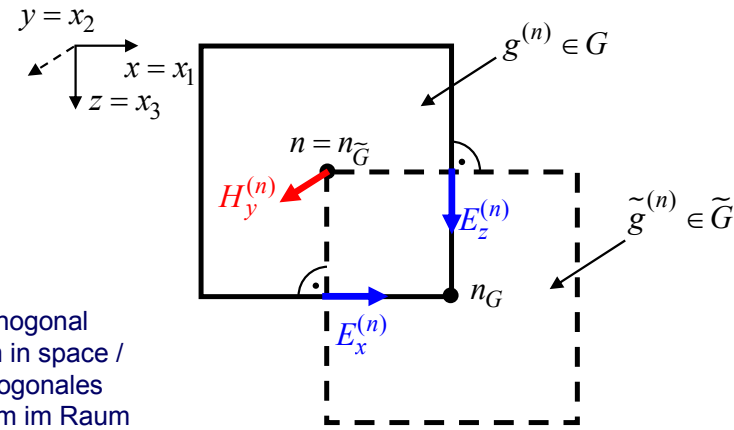


## 2-D EM Wave Propagation – 2-D FDTD – TM and TE Case / 2D EM Wellenausbreitung – 2D-FDTD – TM- und TE-Fall

2-D TM Case / 2D-TM-Fall



2-D TE Case / 2D-TE-Fall



Dual orthogonal  
grid system in space /  
Dual-orthogonales  
Gittersystem im Raum

$$G \perp \tilde{G}$$

$$\frac{\partial}{\partial t} \mu H_x(\mathbf{R}, t) = \frac{\partial E_y(\mathbf{R}, t)}{\partial z} - J_{mx}(\mathbf{R}, t)$$

$$\frac{\partial}{\partial t} \mu H_z(\mathbf{R}, t) = -\frac{\partial E_y(\mathbf{R}, t)}{\partial x} - J_{mz}(\mathbf{R}, t)$$

$$\frac{\partial}{\partial t} \varepsilon E_y(\mathbf{R}, t) = \left[ \frac{\partial H_x(\mathbf{R}, t)}{\partial z} - \frac{\partial H_z(\mathbf{R}, t)}{\partial x} \right] - J_{ey}(\mathbf{R}, t)$$

$$\mathbf{R} = x\mathbf{e}_x + z\mathbf{e}_z$$

$$\frac{\partial}{\partial t} \mu H_y(\mathbf{R}, t) = -\left[ \frac{\partial E_x(\mathbf{R}, t)}{\partial z} - \frac{\partial E_z(\mathbf{R}, t)}{\partial x} \right] - J_{my}(\mathbf{R}, t)$$

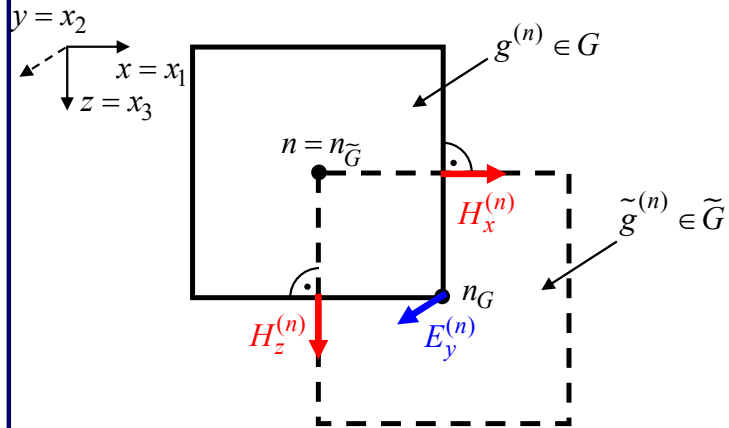
$$\frac{\partial}{\partial t} \varepsilon E_x(\mathbf{R}, t) = -\frac{\partial H_y(\mathbf{R}, t)}{\partial z} - J_{ex}(\mathbf{R}, t)$$

$$\frac{\partial}{\partial t} \varepsilon E_z(\mathbf{R}, t) = \frac{\partial H_y(\mathbf{R}, t)}{\partial x} - J_{ez}(\mathbf{R}, t)$$

$$\mathbf{R} = x\mathbf{e}_x + z\mathbf{e}_z$$

## 2-D EM Wave Propagation – 2-D FDTD – TM Case/ 2D EM Wellenausbreitung – 2D-FDTD – TM-Fall

2-D TM Case / 2D-TM-Fall



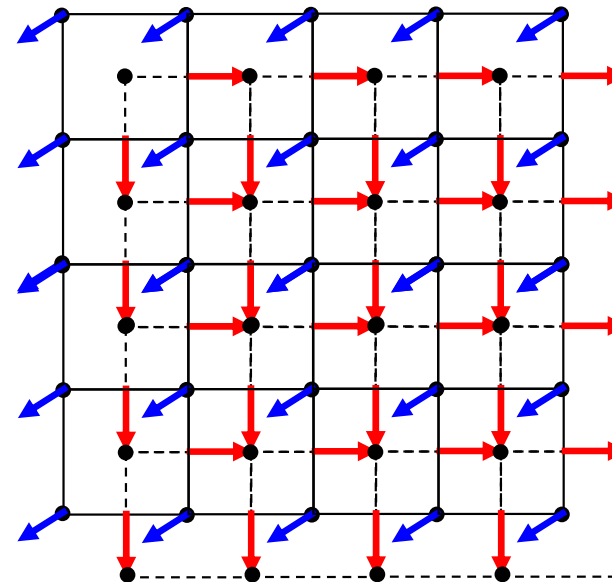
$$\frac{\partial}{\partial t} \mu H_x(\mathbf{R}, t) = \frac{\partial E_y(\mathbf{R}, t)}{\partial z} - J_{mx}(\mathbf{R}, t)$$

$$\frac{\partial}{\partial t} \mu H_z(\mathbf{R}, t) = -\frac{\partial E_y(\mathbf{R}, t)}{\partial x} - J_{mz}(\mathbf{R}, t)$$

$$\frac{\partial}{\partial t} \varepsilon E_y(\mathbf{R}, t) = \left[ \frac{\partial H_x(\mathbf{R}, t)}{\partial z} - \frac{\partial H_z(\mathbf{R}, t)}{\partial x} \right] - J_{ey}(\mathbf{R}, t)$$

$$\mathbf{R} = x\mathbf{e}_x + z\mathbf{e}_z$$

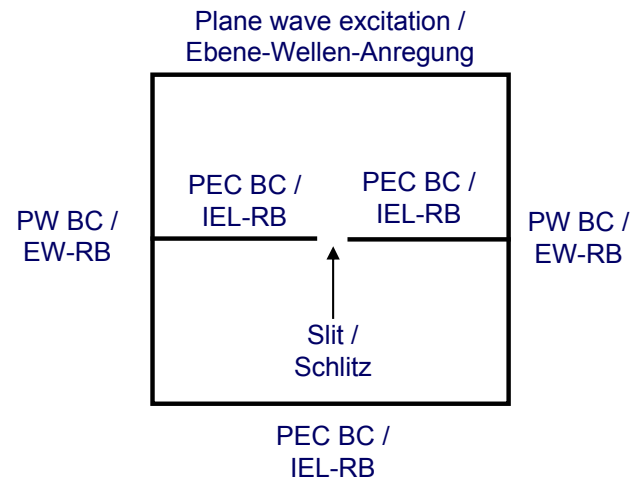
Two-dimensional staggered grid system in the 2-D TM case / Zweidimensionales versetztes Gittersystem im 2D-TM-Fall



## Implementation of Boundary Conditions / Implementierung von Randbedingungen

**Boundary condition for a perfectly electrically conducting (PEC) material /  
Randbedingung für ein ideal elektrisch leitendes Material**

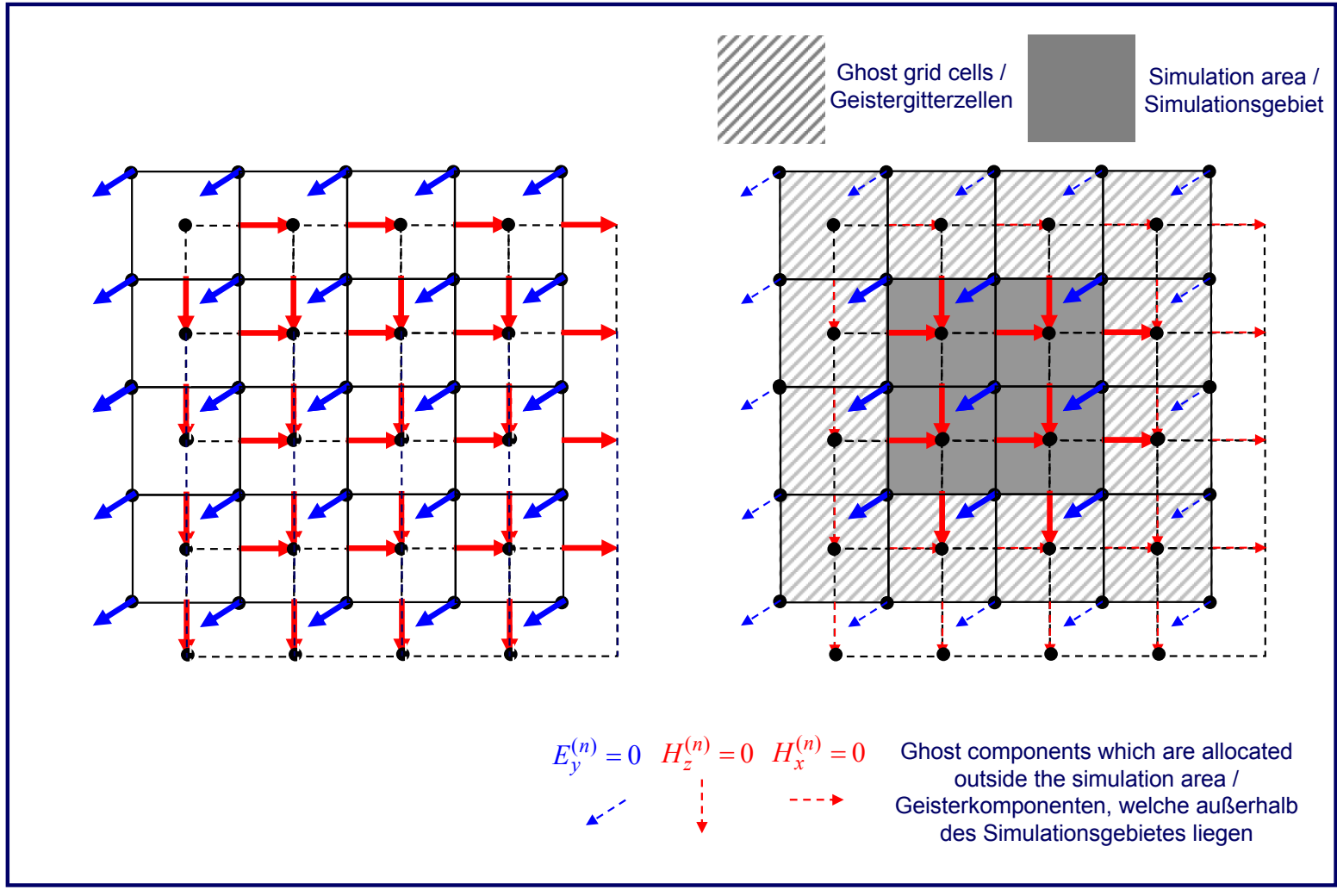
$$\left. \begin{aligned} E_y^{(*,*,n_t)} &= 0 \\ E_y^{(*,*,n_t)} &= 0 \end{aligned} \right\} 1 \leq n_t \leq N_t$$



**Plane wave boundary condition for a vertical incident plane wave /  
Ebene-Wellen-Randbedingung für eine vertikal einfallende ebene Welle**

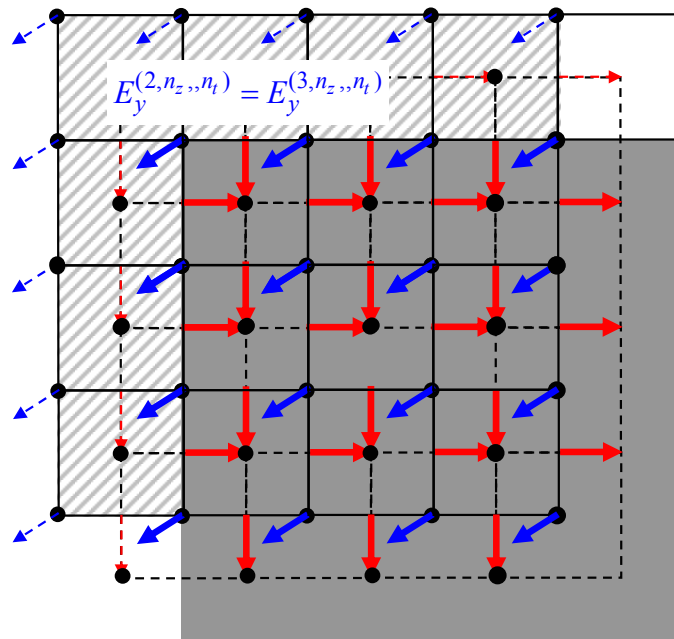
$$\left. \begin{aligned} E_y^{(2,n_z,n_t)} &= E_y^{(3,n_z,n_t)} \\ E_y^{(N_x-1,n_z,n_t)} &= E_y^{(N_x-2,n_z,n_t-2)} \end{aligned} \right\} \begin{aligned} 1 \leq n_z \leq N_z \\ 1 \leq n_t \leq N_t \end{aligned}$$

## 2-D EM Wave Propagation – 2-D FDTD – TM Case/ 2D EM Wellenausbreitung – 2D-FDTD – TM-Fall

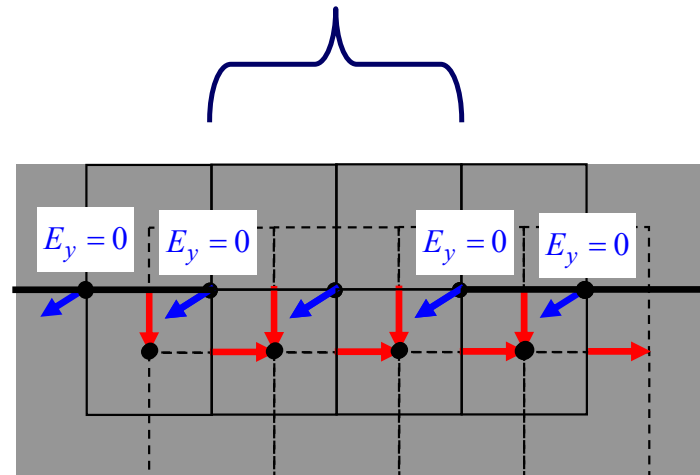


## 2-D EM Wave Propagation – 2-D FDTD – TM Case/ 2D EM Wellenausbreitung – 2D-FDTD – TM-Fall

Plane wave excitation /  
Ebene-Wellen-Anregung



Slit /  
Schlitz

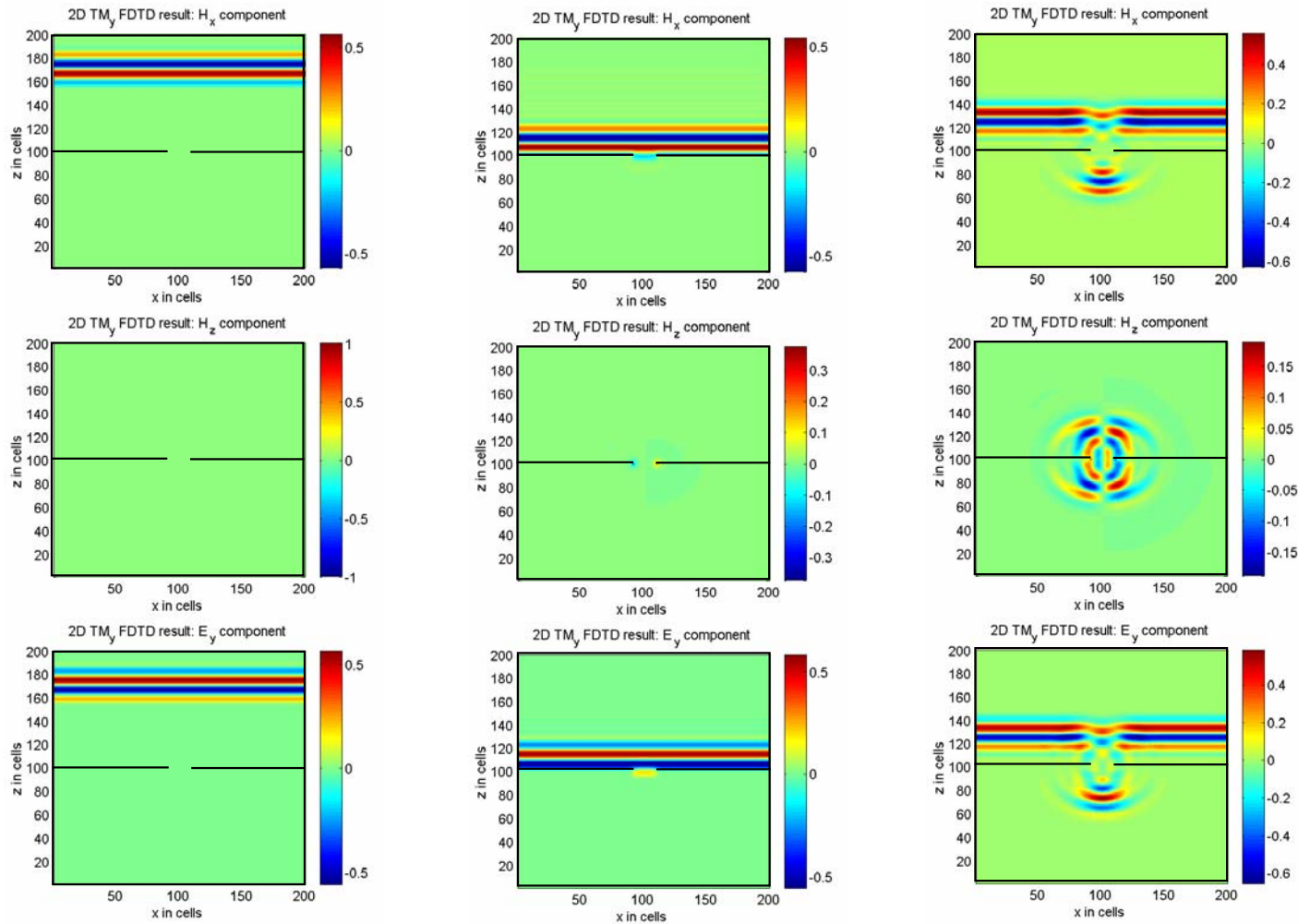


Ghost grid cells /  
Geistergitterzellen



Simulation area /  
Simulationsgebiet

## 2-D TM FDTD – Diffraction on a Single Slit / 2D-TM-FDTD – Beugung an einem Spalt



## 2-D TM FDTD – Diffraction on a Single Slit / 2D-TM-FDTD – Beugung am Spalt

Wave field movie of the  $H_x$  field  
component / Wellenfeldfilm der  
 $H_x$ -Feldkomponente



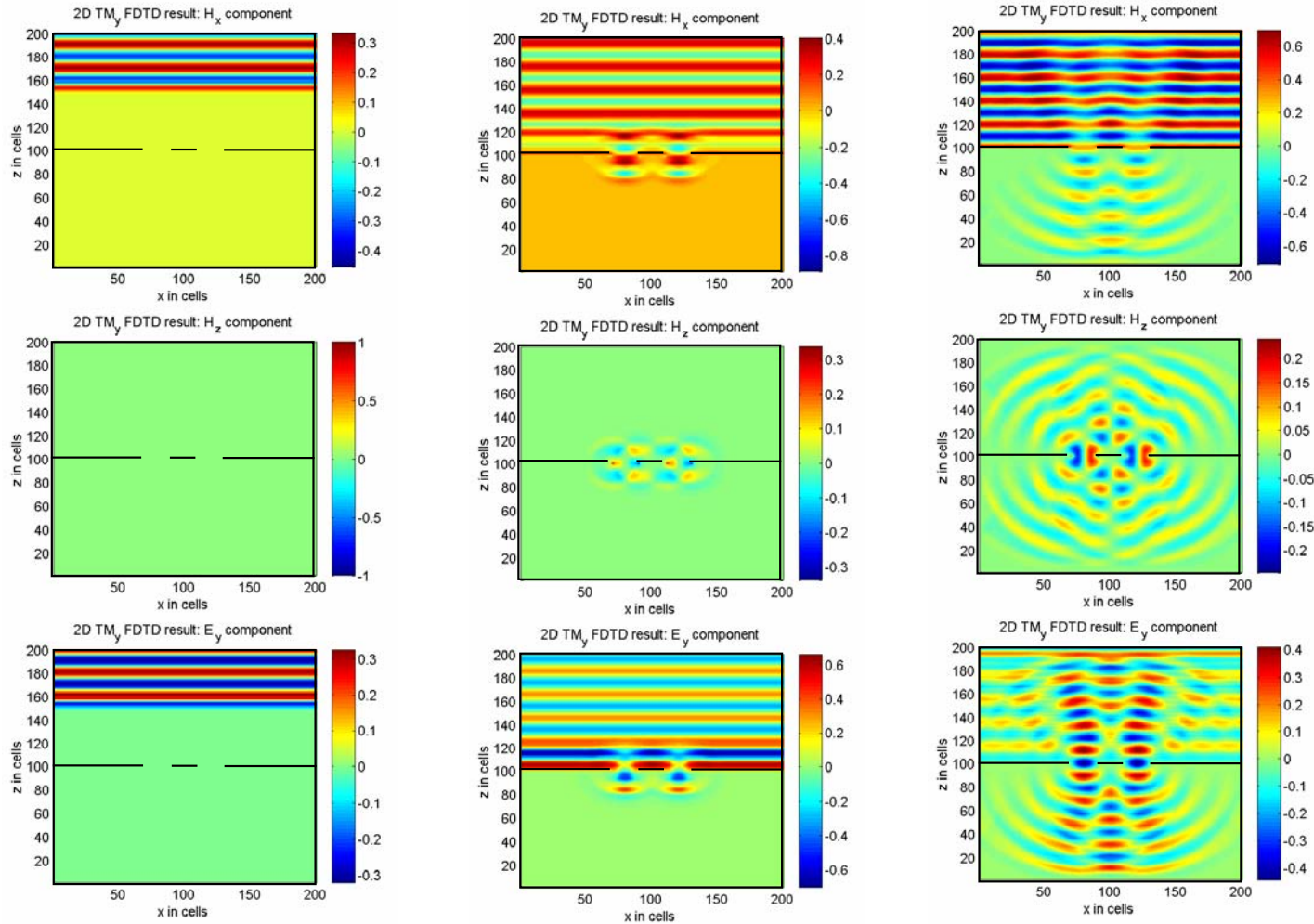
Wave field movie of the  $H_z$  field  
component / Wellenfeldfilm der  
 $H_z$ -Feldkomponente



Wave field movie of the  $E_y$  field  
component / Wellenfeldfilm der  
 $E_y$ -Feldkomponente



## 2-D TM FDTD – Diffraction on a Double Slit / 2D-TM-FDTD – Beugung am Doppelspalt

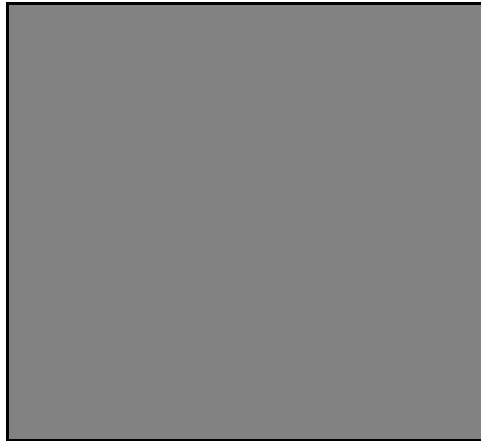


## 2-D TM FDTD – Diffraction on a Double Slit / 2D-TM-FDTD – Beugung am Doppelspalt

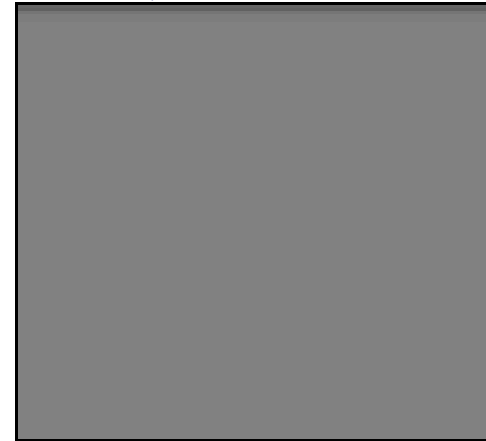
Wave field movie of the  $H_x$  field  
component / Wellenfeldfilm der  
 $H_x$ -Feldkomponente



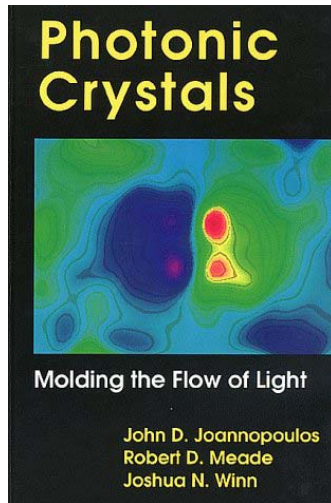
Wave field movie of the  $H_z$  field  
component / Wellenfeldfilm der  
 $H_z$ -Feldkomponente



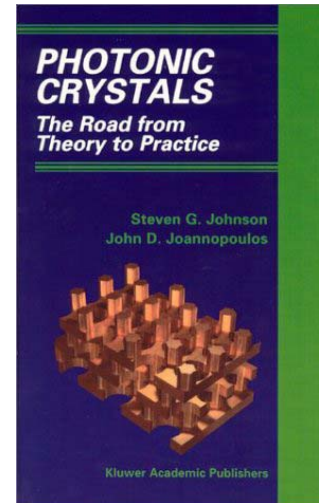
Wave field movie of the  $E_y$  field  
component / Wellenfeldfilm der  
 $E_y$ -Feldkomponente



## Photonic Crystals / Photonische Kristalle



Joannopoulos, J. D.,  
R. D. Meade,  
J. N. Winn:  
*Photonic Crystals –  
Molding the Flow of  
Light.*  
Princeton University  
Press, Princeton, 1995.



Johnson, S. G.:  
*Photonic Crystals: The  
Road from Theory to  
Practice.*  
Kluwer Academic  
Press, 2001.

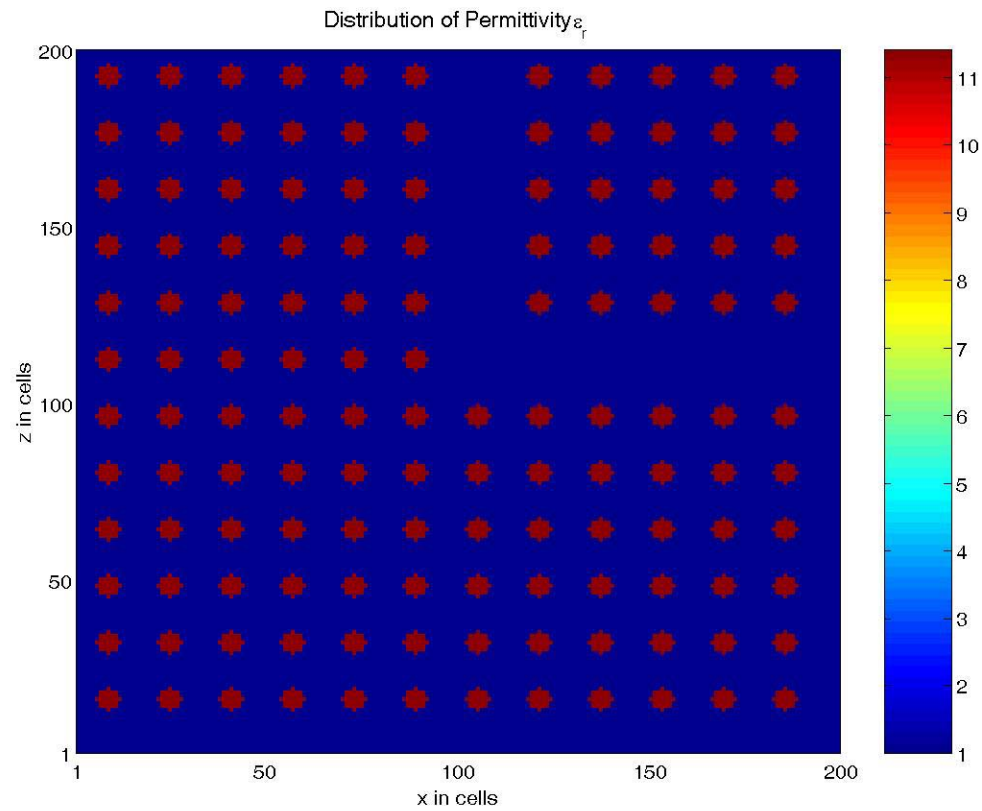
**Links:**

[Photonic Crystals Research at MIT](#)  
[Homepage of Prof. Sajeew John, University of Toronto, Canada](#)

## 2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

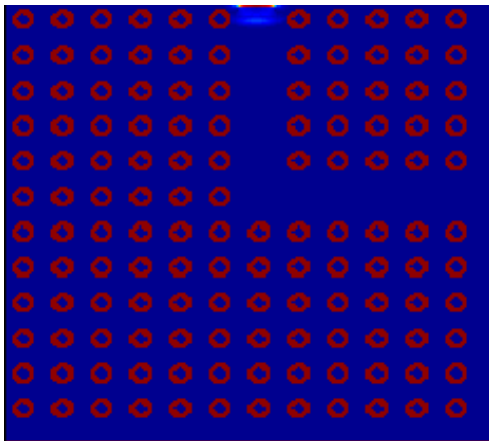
Relative permittivity of the background  $\epsilon_r^{(b)} = 1$   
Relative Permittivität des Hintergrundes

Relative permittivity of the rods  $\epsilon_r^{(r)} = 11.4$   
Relative Permittivität der Stäbe

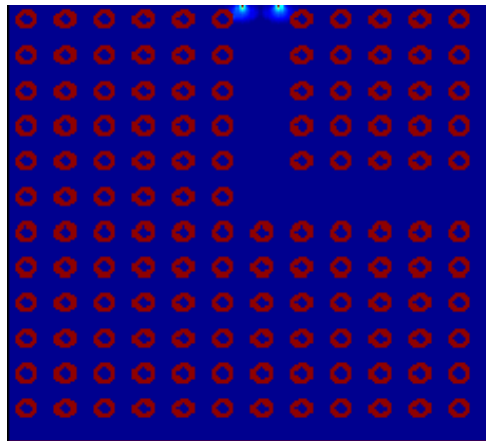


## 2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

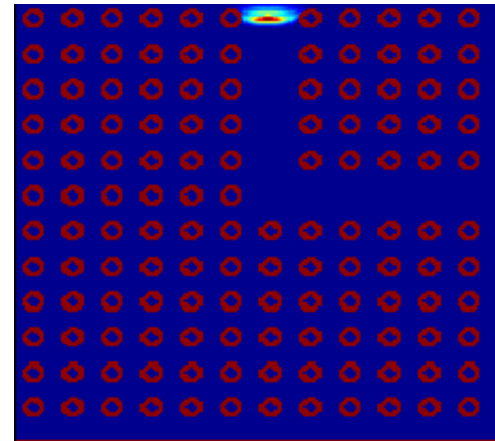
Wave field movie of the  $H_x$  field  
component / Wellenfeldfilm der  
 $H_x$ -Feldkomponente



Wave field movie of the  $H_z$  field  
component / Wellenfeldfilm der  
 $H_z$ -Feldkomponente

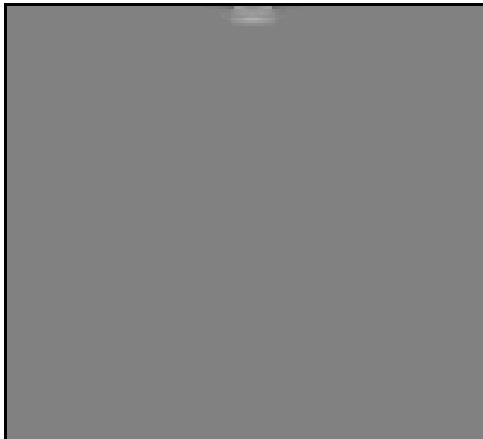


Wave field movie of the  $E_y$  field  
component / Wellenfeldfilm der  
 $E_y$ -Feldkomponente

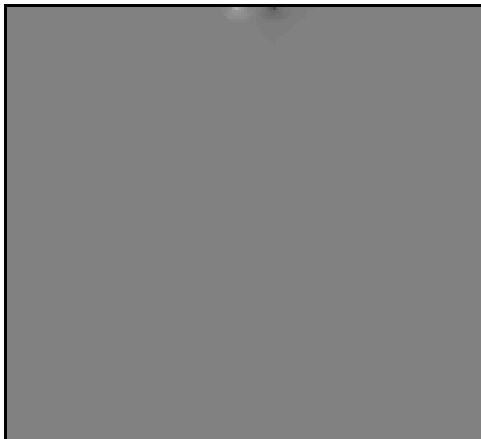


## 2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

Wave field movie of the  $H_x$  field  
component / Wellenfeldfilm der  
 $H_x$ -Feldkomponente



Wave field movie of the  $H_z$  field  
component / Wellenfeldfilm der  
 $H_z$ -Feldkomponente



Wave field movie of the  $E_y$  field  
component / Wellenfeldfilm der  
 $E_y$ -Feldkomponente



**End of Lecture 5 /  
Ende der 5. Vorlesung**