NDT Related
Quantitative Modeling of Coupled Piezoelectric and Ultrasonic Wave Phenomena

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Dedicated to Professor Dr. H. Wüstenberg
on the Occasion of his 60th Birthday

Abstract. This paper presents an overview of the development of numerical tools for the computational modeling of acoustic (A), elastodynamic (E), and coupled piezoelectric (P) and ultrasonic wave phenomena in the time domain related to nondestructive testing with ultrasound. For this purpose, a numerical method, the Finite Integration Technique (FIT), is directly applied to the set of governing equations of the underlying wave propagation in integral form. Space and time discretization of the governing equations on a staggered grid system results in a set of discrete matrix equations of three explicit time domain modeling codes acronymed PFIT, EFIT, and AFIT. Each of the discrete matrix equations has a one–to–one correspondent in the original set of equations. Analytical properties of the original set of equations are conserved. Mathematical properties like consistency and convergence of the codes are proved and stability conditions are derived. The sparse matrices are highly suitable for high–performance vector computers, workstations, or even for personal computers and state–of–the–art laptops. To handle complex geometries, an IGES interface to commercial CAD systems is implemented, and to solve arbitrary initial–boundary–value problems several kinds of boundary conditions can be defined. The numerical codes PFIT, EFIT, and AFIT have widespread applications in quantitative computational NDT, for example starting from piezoelectric transducer modeling and optimization to ultrasonic wave propagation, diffraction, and scattering in inhomogeneous anisotropic dissipative and even statistically heterogeneous materials. Some of the analytical and experimental validations and applications to real life NDT problems are given in this paper illustrating the potential of these numerical modeling tools.
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1 Introduction

The numerical time domain modeling of transient waves of all types in various disciplines of science and engineering is utilized for increasingly sophisticated applications to communication, remote sensing, medical imaging, and nondestructive testing.

In recent years ultrasonic nondestructive evaluation has been more and more characterized by quantitative methods, which is particularly true for modern diffraction tomographic imaging algorithms. These developments and their appropriate applications to increasingly complex NDT problems — complex relating to materials as well as to geometries — like inhomogeneous anisotropic austenitic welds and fiber-reinforced composites, require a deep and fundamental physical understanding of ultrasonic wave propagation, diffraction and scattering. Consequently, numerical codes to model these phenomena should be as close to real life as possible with only marginal or even no mathematical approximations. Only for canonical problems, where the geometries comply with coordinate systems in which the governing equations are separable, analytical solutions can be constructed, but, for the general case it is necessary to turn to direct numerical methods.

This means that a direct discretization technique has to be applied to the governing equations of the underlying physical phenomena. Well known numerical methods of that kind comprise the

- Finite Difference Method (FDM) [e.g. Alterman, 1968; Bamberger et al., 1980; Chin et al., 1984; Temple, 1988; Bond et al., 1988; Harker, 1988; Harker et al., 1990, 1991],
- Finite Element Method (FEM) [e.g. Ludwig, 1986; Ludwig & Lord, 1988; Ludwig et al., 1989; Lord et al., 1990; Lerch, 1990, 1991; Stam, 1990; Stam & Hoop, 1990; You et al., 1991; Roberts, 1991], and

In this paper, the well–established Finite Integration Technique (FIT) [Weiland, 1977; Fellinger, 1991a; Fellinger et al., 1995; Wolter, 1995b; Marklein, 1998; Bihn, 1998] which is a generalization of the Finite Volume Method (FVM) [e.g. Versteeg & Malalaskera, 1995], is applied to the governing equations in integral form to derive numerical codes to simulate typical ultrasonic NDT situations as shown in Figure 1 including the piezoelectric elements and external electrical circuit elements [Marklein, 1998]. Figure 2 relates the three explicit time domain modeling tools based on FIT [Marklein, 1998]:

- PFIT — Piezoelectric Finite Integration Technique,
- EFIT — Elastodynamic Finite Integration Technique, and
- AFIT — Acoustic Finite Integration Technique.

PFIT is particularly developed for the modeling of the excitation and reception mode of a piezoelectric transducer including an external electrical load. Furthermore, PFIT includes the features of EFIT and AFIT. EFIT and AFIT have been designed to model the elastodynamic wavefields in solids and acoustic wavefields in fluids and gases. Besides that, AFIT can be also used to model scalar approximations of elastic wavefields in solids.
2 Numerical Modeling of Ultrasonic Wave Phenomena with EFIT and AFIT

Historically, the Finite Integration Technique (FIT) has first been applied in electrodynamics to Maxwell’s equations in integral form by Weiland [1977] to model electromagnetic problems [see also Weiland, 1986, 1990; Bartsch et al., 1992]. Based on these developments a commercial software package called MAFIA (MAFIA: solution of MAxwell’s equations using a Finite Integration Algorithm) is today available [CST, 1997].

In the time domain the resulting discrete equations of MAFIA are similar to the equations derived by Yee [1966] starting from the differential form of Maxwell’s equations, which are today known as the Finite-Difference Time-Domain (FD–TD) method [e. g. Taflove, 1995].

Fellinger & Langenberg [1990] [see also Fellinger, 1991a; Fellinger & Marklein, 1991b] transferred the ideas applied in electrodynamics to the linear governing equations of ultrasonic waves in solids, the linear elastodynamic case, i. e. Newton–Cauchy’s equation of motion and the equation of deformation rate for the homogeneous isotropic case and acronymed the numerical procedure Elastodynamic Finite Integration Technique (EFIT). With EFIT, Fellinger has
furnished a tool for the modeling of elastodynamic waves in the time domain, which has been further developed throughout the following years to handle arbitrary inhomogeneous dissipative (viscid) anisotropic materials [Marklein et al., 1994a, 1994b, 1995b; Marklein, 1998] and complex geometries [Langenberg et al., 1996; Marklein et al., 1997]; today it is widely used as a well-accepted quantitative modeling tool in NDT.

AFIT, the acoustic pendant to EFIT, is generally derived by applying FIT to the governing equations of acoustic waves in integral form [Marklein & Fellinger, 1990; Marklein, 1992a, 1998].

Stimulated by the developed AFIT- and EFIT-code, similar algorithms have been implemented in the above mentioned MAFIA package by Wolter & Weiland [1995a], Wolter [1995b] and Bihn & Weiland [1996, 1997], Bihn [1998].

A CAFIT algorithm for ultrasonic NDT problems with cylindrical symmetry has been developed by Peiffer et al. [1997].

The underlying governing equations of elastodynamic waves rely on the following two fundamental physical laws [Achenbach, 1973; Nelson, 1979; Ben-Menahem & Singh, 1981; Auld, 1990; Hoop, 1995]

\[
(\text{Newton’s 2nd law}) \quad \text{force} = \text{mass} \times \text{acceleration} \implies f_{\text{tot}}(\mathbf{R}, t) = \rho_0(\mathbf{R}) \frac{\partial^2}{\partial t^2} \mathbf{u}(\mathbf{R}, t) \quad (1)
\]

\[
(\text{Hooke’s law}) \quad \text{stress} = \text{stiffness} \times \text{deformation} \implies \mathbf{T}(\mathbf{R}, t) = c(\mathbf{R}) : \mathbf{S}(\mathbf{R}, t) \quad (2)
\]

with the total force density vector $f_{\text{tot}}$, the mass density at rest $\rho_0$, the particle displacement vector $\mathbf{u}$, Cauchy’s stress tensor of second rank $\mathbf{T}$, the stiffness tensor of fourth rank $c$, and the deformation tensor of second rank $\mathbf{S}$. All field quantities are functions of the position vector $\mathbf{R}$ and time $t$ and all material parameters are assumed as time independent. The colon denotes the double-scalar product with the property $\mathbf{a} \cdot \mathbf{b} : \mathbf{c} \cdot \mathbf{d} = (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$, the centered dot represents the scalar product.

With the linear momentum vector $\mathbf{j}$ and the particle velocity vector $\mathbf{v}$ given by

\[
\mathbf{j}(\mathbf{R}, t) = \rho_0(\mathbf{R}) \mathbf{v}(\mathbf{R}, t) \quad (3)
\]

\[
\mathbf{v}(\mathbf{R}, t) = \frac{\partial}{\partial t} \mathbf{u}(\mathbf{R}, t)
\]

Eq. (1) reads

\[
f_{\text{tot}}(\mathbf{R}, t) = \frac{\partial}{\partial t} \mathbf{j}(\mathbf{R}, t).
\]

Then, Eq. (2) and Eq. (3) are the constitutive equations of a linear inhomogeneous anisotropic nondissipative elastic solid.

Introduction of Cauchy’s equation for the total force density acting on an elastic solid particle

\[
f_{\text{tot}}(\mathbf{R}, t) = \nabla \cdot \mathbf{T}(\mathbf{R}, t) + \mathbf{f}(\mathbf{R}, t)
\]

with $\mathbf{f}(\mathbf{R}, t)$ accounting for a given or impressed force density, yields the Newton–Cauchy’s equation of motion, which is in integral form for a volume $V$ with the closed surface $S = \partial V$ given by

\[
\iiint \rho_0(\mathbf{R}) \frac{\partial}{\partial t} \mathbf{v}(\mathbf{R}, t) dV = \iint_S \mathbf{n} \cdot \mathbf{T}(\mathbf{R}, t) dS + \iiint_V \mathbf{f}(\mathbf{R}, t) dV \quad (4)
\]
with \( \mathbf{n} \) being the outward normal vector of the closed surface \( S \).

According to the law of deformation a particle flux results in a time variation of the deformation volume. The equation of deformation rate reads in integral form for a volume \( V \) with the closed surface \( S = \partial V \)
\[
\iiint_V s(R) \cdot \frac{\partial}{\partial t} T(R, t) dV = \iint_S \text{sym} \{ \mathbf{n} \mathbf{v}(R, t) \} dS + \iiint_V h(R, t) dV \tag{5}
\]
with \( \mathbf{n} \) being the outward normal vector of the closed surface \( S \) and the compliance tensor of fourth rank \( s \) being the inverse of the stiffness tensor, which yields Hooke’s law in the form
\[
S(R, t) = s(R) : T(R, t) .
\]
Like \( \mathbf{f}, h \) is a given or impressed source term, the so-called source of deformation rate accomplishing the symmetry of both equations. The operator \( \text{sym} \{ \mathbf{n} \mathbf{v}(R, t) \} \) gives the symmetric part of the dyadic \( \mathbf{n} \mathbf{v}(R, t) \).

A given ultrasonic NDT situation is generally described by a given material distribution, for instance a homogeneous isotropic nondissipative block of steel. Discretization of the “continuous“ material by elementary cubic material cells \( m^{(n)} \in M \) with the volume \( V = \Delta x^3 \), \( n \) being the global cell or node number (see Fig. 3), yields a regular material grid \( M \), which defines the grid \( G \) (see Fig. 3 and Fig. 4), whereas a dual grid \( \tilde{G} \) is staggered to both (see Fig. 4). The material parameters are assumed constant in each elementary material cell, but each material cell can be filled with different material parameters.

Figure 5 shows the EFIT grids \( M, G, \tilde{G}, \) and the allocation of the discrete elastodynamic field components assigned to the node \( n \), allowing a consistent implementation of the continuity and boundary conditions.

For a source-free interface between two solid materials, medium 1 and medium 2, the continuity conditions are
\[
\mathbf{v}^{(2)}(R, t) - \mathbf{v}^{(1)}(R, t) = 0
\]
\[
\mathbf{n} \cdot \left[ T^{(2)}(R, t) - T^{(1)}(R, t) \right] = 0
\]
with \( \mathbf{n} \) being the interface normal pointing into medium 2. \( \mathbf{v}^{(i)} \) and \( \mathbf{T}^{(i)} \) are the field contributions at the endface on side of medium \( i \) with \( i = 1, 2 \).

Replacing medium 1 by vacuum and suppressing the index 2 the (homogeneous) boundary conditions for \( \mathbf{R} \in \text{boundary and } \forall t \) are

\[
\mathbf{v}(\mathbf{R}, t) = \mathbf{0} \quad \text{motion-free boundary condition}
\]

or

\[
\mathbf{n} \cdot \mathbf{T}(\mathbf{R}, t) = \mathbf{0} \quad \text{stress-free boundary condition}
\]

with \( \mathbf{n} \) being the surface normal pointing into the medium.

Generally, each governing equation is solved for each discrete field component of \( \mathbf{v} = v_i \mathbf{e}_i \), \( i = 1, 2, 3 \) and \( \mathbf{T} = T_{ij} \mathbf{e}_i \mathbf{e}_j \), \( i, j = 1, 2, 3 \) on the left–hand side of Eq. (4) and Eq. (5) for a cell volume \( V = \Delta x^3 \). For example, the first Newton–Cauchy grid equation for the \( v_1 \) component is (see Fig. 6)

\[
\mathbf{e}_1 \cdot \left[ \iiint_V \rho_{\phi(0)}(\mathbf{R}) \frac{\partial}{\partial t} \mathbf{v}(\mathbf{R}, t) dV \right] = \oint_S \mathbf{n} \cdot \mathbf{T}(\mathbf{R}, t) dS + \iiint_V \mathbf{f}(\mathbf{R}, t) dV
\]

\[
\rho d_0 v_1^{(m)}(t)(\Delta x)^3 = \left[ T_{11}^{(r)}(t) - T_{11}^{(l)}(t) + T_{12}^{(f)}(t) - T_{12}^{(b)}(t) + T_{13}^{(d)}(t) - T_{13}^{(u)}(t) \right] (\Delta x)^2 + f_1^{(m)}(t)(\Delta x)^3
\]

with \( m = \text{middle, } r = \text{right, } l = \text{left, } f = \text{front, } b = \text{back, } d = \text{down, and } u = \text{up} \). The dot indicates the first time derivative.
In a global node formulation this grid equation reads

\[
\frac{\rho_{e0}^{(n)}}{2} \frac{\rho_{e0}^{(n+M_1)}}{2} \nabla_1^{(n)}(t)(\Delta x)^3 = \\
\left[ T_{11}^{(n+M_1)}(t) - T_{11}^{(n)}(t) + T_{12}^{(n)}(t) - T_{12}^{(n-M_2)}(t) + T_{13}^{(n)}(t) - T_{13}^{(n-M_3)}(t) \right](\Delta x)^2 + f_1^{(n)}(t)(\Delta x)^3
\]

with the global grid node \( n = n_S(\vec{n}_1, \vec{n}_2, \vec{n}_3) = 1 + M_1(\vec{n}_1 - 1) + M_2(\vec{n}_2 - 1) + M_3(\vec{n}_3 - 1), \quad n \in \{1, 2, \ldots, N\}, \)

\( N = N_1N_2N_3, \vec{n}_i \in \{1, 2, \ldots, N_i\}, \) and the step counters

- \( x_1 \) direction: \( M_1 = 1 \)
- \( x_2 \) direction: \( M_2 = N_1 \)
- \( x_3 \) direction: \( M_3 = N_1N_2 \).

Performing this procedure for all components of \( \mathbf{v} \) and \( \mathbf{T} \) and formulating the grid equations in matrix form yields the following Newton–Cauchy’s grid equation of motion and the grid equation of stress rate, the discrete Elastic Grid Equations (EGE) of EFIT

\[
\{ \mathbf{v} \}^{(z-\frac{1}{2})} = [\tilde{\rho}_{e0}]^{-1}[\nabla \mathbf{V}][\tilde{\mathbf{R}}_1^{(z)}]^{-1}[\mathbf{A}_1^{(z)}]^{(z-\frac{1}{2})} + [\tilde{\rho}_{e0}]^{-1}\{ \mathbf{f} \}^{(z-\frac{1}{2})}
\]

\[
\{ \mathbf{T} \}^{(z)} = [\mathbf{c}][\mathbf{R}_y^{(z)}]^{-1}[\text{GRAD}][\mathbf{A}_y^{(z)}]^{(z)} + \{ \mathbf{g} \}^{(z)}
\]

with the algebraic field vectors \( \{ \mathbf{f} \}, \{ \mathbf{g} \}, \{ \mathbf{v} \}, \{ \mathbf{T} \} \), the material matrices \( [\tilde{\rho}_{e0}], [\mathbf{c}] \), the topological operators \( [\nabla \mathbf{V}], [\text{GRAD}] \) containing only the values \(-1, 0, \) and \(1\), the matrices \( [\tilde{\mathbf{R}}_1], [\mathbf{R}_y] \) containing the line elements, and the averaging matrices \( [\mathbf{A}_1^{(z)}], [\mathbf{A}_y^{(z)}] \).
The staggered grid allocation of the discrete field components also requires a staggered allocation in time in full and half time steps $\Delta t$ according to the integration schemes (see Fig. 7)

\[
\begin{align*}
\{v\}^{(2)} &= \{v\}^{(z-1)} + \Delta t\{\dot{v}\}^{(z-\frac{1}{2})} \\
\{T\}^{(z+\frac{1}{2})} &= \{T\}^{(z-\frac{1}{2})} + \Delta t\{\dot{T}\}^{(z)}.
\end{align*}
\]

Scalarization of the elastodynamic case yields the acoustic case, therefore the hydrodynamic pressure $p$ is introduced

\[
\mathbf{T}(\mathbf{R}, t) = -p(\mathbf{R}, t) \mathbf{I}
\]

with the unit dyadic $\mathbf{I} = \delta_{ij}\mathbf{e}_i\mathbf{e}_j$; $\delta_{ij}$ is the Kronecker symbol being equal to 1 for $i = j$ and equal 0 for $i \neq j$.

Furthermore, Hooke’s law (2) reduces to a scalar formulation according to

\[
\text{(Hooke’s law)} \quad \text{stress} = \text{stiffness} \times \text{deformation} \Rightarrow -p(\mathbf{R}, t) = \frac{1}{\kappa(\mathbf{R})} S(\mathbf{R}, t)
\]

with the compressibility $\kappa$ and the scalar deformation $S$.

Generally, AFIT is derived by applying FIT to the governing equations of acoustic waves in integral form [Marklein & Fellinger, 1990; Marklein, 1992a, 1998].

The governing equations of linear acoustic waves for nondissipative materials are Newton’s equation of motion and the scalar deformation rate equation

\[
\begin{align*}
\iiint_V \rho(\mathbf{R}) \frac{\partial}{\partial t} \mathbf{v}(\mathbf{R}, t) dV &= -\iint_S \mathbf{n} p(\mathbf{R}, t) dS + \iiint_V \mathbf{f}(\mathbf{R}, t) dV \\
\iiint_V \kappa(\mathbf{R}) \frac{\partial}{\partial t} p(\mathbf{R}, t) dV &= -\iint_S \mathbf{n} \cdot \mathbf{v}(\mathbf{R}, t) dS - \iiint_S h(\mathbf{R}, t) dV
\end{align*}
\]

with the mass density at rest $\rho_0$, and the given or impressed scalar source of deformation rate $h$.

Figure 8 illustrates the AFIT grids $M$, $G$, $\tilde{G}$, and the allocation of the discrete acoustic field components assigned to the node $n$, allowing a consistent implementation of the continuity conditions, which are for a source–free interface between two acoustic media, medium 1 and medium 2, for $\mathbf{R} \in$ interface and $\forall t$,

\[
\mathbf{n} \cdot [\mathbf{v}^{(2)}(\mathbf{R}, t) - \mathbf{v}^{(1)}(\mathbf{R}, t)] = 0
\]

\[
p^{(2)}(\mathbf{R}, t) - p^{(1)}(\mathbf{R}, t) = 0
\]

with $\mathbf{n}$ being the interface normal pointing into medium 2. $\mathbf{v}^{(i)}$ and $p^{(i)}$ are the field contributions at the endface on side of medium $i$ with $i = 1, 2$. 

9
Replacing medium 1 by vacuum and suppressing the index 2 the (homogeneous) boundary conditions are for $\mathbf{R} \in$ interface and $\forall t$

$$\mathbf{n} \cdot \mathbf{v}(\mathbf{R}, t) = 0 \quad \text{motion-free boundary condition}$$

or

$$p(\mathbf{R}, t) = 0 \quad \text{pressure-free boundary condition}$$

with $\mathbf{n}$ being the surface normal pointing into the medium.

For instance, the first Newton grid equation for the $v_1$ component is (see Fig. 9)

$$\varepsilon_1 \cdot \left[ \iiint_V \rho_{\delta_0}(\mathbf{R}) \frac{\partial}{\partial t} \mathbf{v}(\mathbf{R}, t) dV - \iint_S \mathbf{n} p(\mathbf{R}, t) dS + \iiint_V \mathbf{f}(\mathbf{R}, t) dV \right]$$

$$\Downarrow$$

$$\rho_{\delta_0}(m) \dot{v}_1(m)(t)(\Delta x)^3 = \left[ p(r)(t) - p(t)(t) \right] (\Delta x)^2 + f_1(m)(t)(\Delta x)^3.$$

The further application of FIT yields the discrete Acoustic Grid Equations (AGE) of AFIT, Newton’s grid equation of motion, and the scalar grid equation of pressure rate in the form

$$\{\mathbf{v}\}^{z-\frac{1}{2}} = -[\rho_{\delta_0}]^{-1}[\mathbf{R}^{-1} \widetilde{\nabla} \mathbf{p}]^{z-\frac{1}{2}} + [\rho_{\delta_0}]^{-1}\{\mathbf{f}\}^{z-\frac{1}{2}}$$

$$\{p\}^{z} = -[\kappa]^{-1}[\nabla][\mathbf{R}^{-1}\{\mathbf{v}\}^{z} - [\kappa]^{-1}\{\mathbf{h}\}^{z}]$$

with the algebraic field vectors $\{\mathbf{f}\}$, $\{\mathbf{h}\}$, $\{\mathbf{v}\}$, $\{\mathbf{p}\}$, the material matrices $[\rho_{\delta_0}]$, $[\kappa]$, the matrices containing the line elements $[\mathbf{R}]$, $[\mathbf{R}]$, and the topological operators $[\nabla]$, $[\nabla]$ containing only the values $-1$, $0$, and $1$.

Then, the time integration scheme (see Fig. 10)

$$\{\mathbf{v}\}^{z} = \{\mathbf{v}\}^{z-1} + \Delta t\{\mathbf{v}\}^{z-\frac{1}{2}}$$

$$\{p\}^{z+\frac{1}{2}} = \{p\}^{z-\frac{1}{2}} + \Delta t\{p\}^{z}$$
The EFIT– and AFIT–codes are explicit time domain algorithms of leapfrog–type and of 2nd order in space and time. Consistency has been proven for both codes [Marklein, 1998]. Stability for a regular grid \((M, G, \bar{G})\) is given by the multidimensional Courant–Friedrichs–Levy (CFL) condition in \(n\)–dimensions

\[
\Delta t \leq \Delta t_{\text{max}} = \frac{1}{\sqrt{n}} \frac{\Delta x}{c_{\text{max}}}
\]

with the spatial dimension \(n\) and the maximal energy velocity \(c_{\text{max}}\). If consistency of the grid equations has been proven and the stability condition is satisfied, then convergence is ensured by the Lax equivalence theorem [Richtmyer & Morton, 1990].

To model an ideal scatterer or an infinite modeling domain several boundary conditions have been implemented (see Table 1).

<table>
<thead>
<tr>
<th>Boundary condition</th>
<th>EFIT</th>
<th>AFIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress–free</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Pressure–free</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Motion–free</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Open</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Plane wave</td>
<td>•</td>
<td>•</td>
</tr>
</tbody>
</table>

Table 1: Implemented boundary condition in EFIT and AFIT

3 Numerical Results obtained with EFIT and AFIT

3.1 Analytical and Numerical Validation of EFIT

Once a numerical method is applied to compute a physical solution the results, and there- 
with the numerical code, have to be validated either against analytical solutions or numerical 
references.

For a first analytical and numerical validation of the 2–D EFIT–code a strip–like piston 
transducer radiating into a 2–D homogeneous nondissipative isotropic half–space has been 
chosen (see Fig. 11). Figure 12 shows the comparison of the time history of the horizontal

Figure 11: Strip–like piston transducer with uniform normal stress load on a homogeneous 

isotropic stress–free elastic half–space: 2–D EFIT grid with two selected observation points: $P_1 = (0, 12)$ mm and $P_2 = (12, 12)$ mm. For the 
EFIT modeling a stress–free boundary condition has been applied on all four boundaries, for the 
EFEM modeling a stress–free boundary condition has been applied on the top, left, and right side, and a displacement–free boundary condition has 
been chosen on the bottom side.

Figure 12: EFIT (—), EFEM (—–), and SZEW (—–) comparison for a strip–like piston transducer: 
time history of the particle displacement components $u_1(t)$ and $u_3(t)$ at $P_1$ (left) and $P_2$ (right)
and vertical particle displacement with the spectral decomposition method (SZEW: Spektrale Zerlegung nach ebenen Wellen) [Fellinger & Langenberg, 1990; Fellinger, 1991a] and an elastodynamic FEM–code (EFEM) developed by Ludwig [1986, 1988, 1989] [Marklein, 1991; Fellinger & Marklein, 1991b]. Both results are documenting a very good agreement between all three methods.

### 3.2 Experimental Validation of EFIT

The first experimental validation of EFIT has been performed for the insonification of a backwall breaking crack by a 45° shear wave probe (see Fig. 13). The numerical results obtained with the 2–D EFIT–code are compared to those of an accompanying experiment. This problem also has been modeled with 2–D AFIT in order point out explicitly the difference between acoustic and elastodynamic waves. The time history of the displacement as well as the amplitude of the normal traction load within the finite transducer aperture was carefully measured with an electrodynamic probe. The elastic material steel is characterized by the velocities $c_p = 5.9$ mm/μs and $c_s = 3.2$ mm/μs. In order to compare the acoustic and elastodynamic case for the shear wave an acoustic material with $c_p = 3.2$ mm/μs has been chosen. The backwall breaking crack is modeled as an ideal crack with plane surface and stress–free/pressure–free boundary condition. The crack is illuminated with a 45° shear wave angle probe within a prescribed synthetic aperture. Within the transducer aperture — the black bar in Fig. 13 — the elastic/acoustic surface is excited by a normal traction distribution weighted by the measured sole function. The time history of the normal traction is prescribed as a 4 cycle raised cosine pulse with a center frequency of 2 MHz (see Fig. 14).

Time domain snapshots of EFIT and AFIT are given in Figure 14, which are showing explicitly the difference between the elastodynamic case and the scalar approximation.

Figure 15 compares experimental and simulation results for a single A–scan, where reception has been modeled via averaging over the transducer aperture with the same amplitude weighting as applied for transmission. The coincidence is nearly complete, except for the late pulses being associated with Rayleigh waves traveling down and up along the crack faces and being converted into shear waves. The somewhat different amplitudes and the nonresolved time separation of the latter in the experiment may be due to the fact that the realization of the crack was a fatigue crack with a surface roughness, whereas the model crack was perfectly plane and stress–free. Nevertheless, the modeling result obtained with EFIT complies very well with the experiment, giving us some confidence for further applications of this code.
Another way to compare the modeling results with the experiment is to use the output rf-data field as a testbed for an imaging algorithm like the Fourier-Transform Synthetic Aperture Focusing Technique (FT-SAFT) \cite{Mayer1990}, which is the Fourier-Transform-Version of SAFT \cite{Schmitz1995}. The rf-data fields are produced in a pulse-echo experiment with the proper amplitude tapering of the transducer in the transmitting as well as in the receiving mode. Since FT-SAFT "knows" nothing about mode conversion and resonances, each time of flight curve is attributed to a distinct defect, resulting in the image of Fig. 16: the corner between the crack and the backwall is properly imaged from the most pronounced time of flight curve, a weaker image of the crack tip is obtained, and the resonances are focused to a ghost defect. Figure 16 not only presents simulations, but also experimental results, and, as we anticipated, our model is well reproduced. When investigated within the framework of inverse scattering, the derivation of SAFT exhibits that a linearization is involved and, therefore, in the present version SAFT is not bound to handle resonances and mode conversions properly; neither is it designed for inhomogeneous anisotropic materials. A theory of inverse scattering is available which identifies these drawbacks quantitatively \cite{Langenberg1987, Langenberg1993, Langenberg1997a}.
Figure 15: Experimental and 2-D EFIT modeled A-scan for a selected transducer position, where the crack tip echo is more prominent than the corner reflection. The single pulses can be interpreted as follows: SS: Shear–Shear–Reflection (crack tip echo); SPS: Shear–Pressure–Shear–Mode–Conversion; SSS: Shear–Shear–Shear–Reflection (corner reflection); SRS: Shear–Rayleigh–Shear–Mode–Conversion; SRRS: Shear–Rayleigh–Rayleigh–Shear–Mode–Conversion.
Figure 16: FT–SAFT image obtained from the simulated rf-data (a) and SAFT image from the measured experimental rf-data (b)
3.3 AFIT and EFIT Modeling of Ultrasonic Pipeline Inspection

The principle geometry for another example of 2-D EFIT modeling is given in Figure 17. Here, an interface crack at \( x = 40 \text{ mm} \) is considered. Figure 18 shows two 2-D EFIT \(|\{\tilde{v}\}|\)-

![Figure 17: Ultrasonic pipeline inspection: geometry for 2-D EFIT modeling](image)

![Figure 18: 2-D EFIT \(|\{\tilde{v}\}|\)-snapshots of the ultrasonic wavefield](image)

![Figure 19: Top: EFIT modeled A-scan; bottom: comparison between modeled (solid line) and experimental ALOK-scan (dashed line). 1: P-P; 2: P-4S-2S-P; 3: P-4S-4S-P; 4: P-4S-6S-P](image)

snapshots for an insonification angle of \( \alpha = 18^\circ \). The modeled A-scan is displayed at the top of Fig. 19, below, a comparison between the modeled and experimental ALOK-scan is given. Convincing coincidence has been found. The four strongest echo signals can be interpreted via snapshots analysis.
3.4 EFIT Modeling of Ultrasonic NDT of Concrete

Figure 20 shows three typical concrete samples with different parameters. Due to the additives and air inclusions which may occur concrete is a statistically heterogeneous material. Usually concrete consists of cement and several additives like basalt, plaster, and biatitgranite. Different situations can be modeled: concrete without and with air inclusions. Figure 21 displays one of the concrete models under concern. The base material is cement and the additives are basalt, plaster, and biatitgranite with a max. aggregate size of 8 mm: a grading curve B8 is used. The additives are modeled by approximately 60,000 ellipses which are varying statistically in size, orientation, and additives (see Fig. 21). Details can be found in Marklein [1998]; first applications of the EFIT-code and the application of an inverse scattering algorithm to the EFIT-data can be found in Marklein et al. [1996b, 1996c]. Each material cell of the EFIT grid can be filled with different material parameters. From a numerical point of view the total number of the ellipses is only limited by the number of material cells. In this example the cell size is $\Delta x = 250 \mu m$ and the resulting mesh size is $2,000 \times 2,000$ with $4 \times 10^6$ material cells. The detail drawings at the right side of Figure 21 clearly indicate the statistical distribution of the ellipses. A normal pressure probe is applied in the pulse–echo mode on the top surface which has a diameter of $D = 5 cm$ and a center frequency of $f_c = 80 kHz$. The time history of the probe is modeled by a raised cosine with two cycles. For EFIT modeling a stress–free boundary condition is applied on the top and bottom surface and an open boundary condition [Higdon, 1991, 1992] at the left and right boundary in order to model an infinite concrete structure. Snapshots of the ultrasonic wavefield are shown in Figure 22. With increasing time, the wavefield becomes totally distorted because of multiple reflections and mode conversion effects at the air inclusions. Due to the open boundary condition reflections and mode conversions are suppressed at the vertical boundaries. The backwall echo of the pressure wave shows up in Figure 23 at the point $t = 240 \mu s$. The traveling time of the backwall echo can then be further analyzed, e. g., for thickness determination.

Similar results can be found in Schubert & Köhler [1997] and Burr et al. [1997].

Figure 20: Three typical concrete samples with different grain size, grading curve, and water/air concentration
Figure 21: Concrete model for 2-D EFIT modeling with 6% air concentration; 3% in the grain class 1 and grain class 2

Figure 22: 2-D EFIT $|\hat{\psi}|$— snapshots of the ultrasonic wave propagation, diffraction, and scattering: a) bis d) with 6% air; 3% each in the grain class 1 and 2

Figure 23: 2-D EFIT modeled A-scan: EP = excitation pulse and BE = backwall echo
3.5 Transducer Radiation Modeling with EFIT

This example illustrates what happens if a $45^\circ$ shear wave transducer from the shelf, designed for conventional applications in isotropic materials like ferritic steel, is applied to the surface of a transversely isotropic material like austenitic steel characterized by a grain orientation $\hat{a}$. 

![Figure 24: Superimposed 2–D EFIT $|\{\hat{S}_d\}|$–snapshots of a 2 MHz $45^\circ$ shear wave transducer identify the geometric ultrasonic (GU) beam, according to the geometric optical (GO) beam in electromagnetics: a) isotropic ferritic steel; b) transversely isotropic austenitic steel with $\hat{a} = \cos 60^\circ \hat{e}_1 + \sin 60^\circ \hat{e}_3$.](image)
Figure 24 shows EFIT snapshots of the elastic Poynting vector displaying the rather dramatic effect in the anisotropic case, where the main beam is skewed with regard to the originally intended energy direction. A weld in austenitic steel might not even be „hit“ by the ultrasound. The explanation of Figure 24 is given in Figure 27 using the characteristic slowness and group velocity diagrams (see Fig. 25 and 26).

A geometric construction of the above skewing effects can be obtained via Huygens’ principle (envelope construction) applied to group velocity diagrams; this is done in Fig. 27 for the considered 2–D case. Figure 27a shows the properly adjusted linear time delay resulting in the tilted beam as desired in the isotropic case. Figure 27b displays the isotropic case and Figure 27c the anisotropic case, where two time frames of the wavefronts, accounting for a broadband signal, emanating from the left and right corner of the aperture are superimposed. The geometric ultrasonic (GU) beam, according to the geometric optical (GO) beam in electromagnetics, is the tangent of the left and right wavefront representing all radiating line sources — „wavelets“ — inside the finite transducer aperture (indicated as a black bar). Tracing the amplitude along a wavefront an EFIT snapshot can also be utilized to calculate an appropriate radiation pattern.

Figure 25: Slowness diagrams: a): isotropic ferritic case, b): anisotropic austenitic case

Figure 26: Group velocity diagrams: a): isotropic ferritic case, b): anisotropic austenitic case
Figure 27: Huygens construction of transducer soundfields: a) linear time retardation as a function of the local transducer aperture coordinate; b) Huygens construction for isotropic ferritic steel; c) Huygens construction for transversely isotropic austenitic steel.
3.6 EFIT Modeling of the Ultrasonic NDT of Fiber–Reinforced Double T–Stringer

In the construction of aircraft wings fiber–reinforced graphite epoxy material is a new–fangled material. It has a transverse isotropy characterized by the fiber direction $\hat{a}$.

For instance, the interior of an aircraft wing consists of multiple double T–stringers which are made of fiber–reinforced composites and which form a honeycomb structure. At the I–section interface the double T–stringer has a very complicated variation of the fiber direction, but in the EFIT model an orthogonal fiber direction at this intersection is presumed. Therefore, in the upper and lower part we have a horizontal and in the I–section a vertical fiber direction. Selected EFIT–$|\{\hat{V}\}|$–snapshots of the elastic wavefield of a 5 MHz normal pressure probe are given in Figure 29b. At the sole of the transmitter and receiver we applied an absorbing boundary condition (ABC) [Higdon, 1991, 1992] in order to take into account the energy absorption of the mounted probe. The transmitter signal is shown in Figure 29a.

The time history of the probe is modeled by an RC2 pulse. The receiver signal is displayed in Figure 29d. This signal represents an echo sequence which is due to the wave guide behavior of the I–section. In a second EFIT modeling we half surrounded the I–section with water in order to model a half filled tank. Figure 29c shows the according 2–D EFIT $|\{\hat{V}\}|$–snapshots.

Obviously, when the Lamb wave arrives at the water/I-section interface leaky waves appear. Further, when the Lamb wave hits the lower corners of the I–section, Scholte waves are generated which are traveling up along the water/I-section interface. The received signal is given in Figure 29e. Because of the leakage effect the amplitude of the received echo decreases and the wave guiding effect appears weaker. Due to this fact only one single transmitted echo, not a whole echo sequence as shown in Figure 29d, is received.
Figure 29: 2-D EFIT modeling of a fiber-reinforced double T-stringer of transversely isotropic graphite epoxy
3.7 EFIT Modeling of Ultrasonic Wave Propagation in an Austenitic V–Butt Weld and Notch Scattering

Figure 30 shows another example geometry of an inhomogeneous anisotropic NDT situation: an austenitic V–butt weld with a backwall breaking notch modeled with stress–free boundary condition. A sketch of the two grain orientations under concern is given in Figure 31.

Simulated A–scans and 2–D EFIT {$\mathbf{\{v\}}$}–snapshots obtained in pulse–echo mode are given for both grain orientations in Figure 32 and Figure 33, respectively. The time domain snapshots are displaying the excitation and propagation of the ultrasonic wavefield inside the weld comprising the notch scattering, and the reception of the characteristic echo signals. It has to be pointed out, that in the case of perpendicular grain structure, no notch tip echo is received (see Fig. 32, left). For an interpretation of the obtained results using slowness and group velocity diagrams the reader is refered to Hannemann et al. [1998].

![Figure 30: Austenitic V–butt weld with backwall breaking notch embedded in isotropic ferritic steel; $\alpha$ is the inclination angle of the interface between the V–butt weld and the isotropic ferritic steel](image)

Figure 30: Considered grain orientation characterized by $\mathbf{\hat{a}}$: a) perpendicular grain orientation and b) herringbone grain orientation

![Figure 31: Considered grain orientation characterized by $\mathbf{\hat{a}}$: a) perpendicular grain orientation and b) herringbone grain orientation](image)

Figure 32: Austenitic V–butt weld with backwall breaking notch and an inclination of the interface of $\alpha = 15^\circ$: 2–D EFIT modeled A–scans of the perpendicular grain orientation (left) and herringbone grain orientation (right)
Figure 33: Austenitic V–butt weld with backwall breaking notch and an inclination of the interface of $\alpha = 15^\circ$: 2–D EFIT $\{\vec{V}\}$–snapshots of the perpendicular grain orientation (left) and herringbone grain orientation (right)
3.8 ULIAS — EFIT Modeling Module

Within a joint European research project involving several industrial partners an ultrasonic modeling and imaging tool acronymed Ultrasonic Inspection Applying Simulation (ULIAS) has been developed [Langenberg et al., 1996; Marklein et al., 1997a].

ULIAS is a module based modeling, imaging, and visualization system, which also comprises a selection of data processing modules. The data exchange between different modules relies on a unified ultrasonic data file in Ultrasonic Data Exchange Format (UDEF). This paper focuses on the EFIT modeling module, the UDEF File, the UDEF Editor, the FT-SAFT imaging module, and 1-D/2-D/3-D visualization capabilities (VISLIB). The 3-D graphic features are based on the graphic libraries PEX, GL, or OpenGL. For instance, the VISLIB allows the 3-D visualization of defect images within the 3-D test piece geometry as provided by a CAD file in IGES format. ULIAS has been originally designed to run on UNIX based computers and is supported for the following computer platforms: IBM RISC System/6000, HP, SUN, and SGI. The user interface is based on X-Windows and OSF/Motif. The general programming language is ANSI C. All ULIAS modules have read/write access to a selected UDEF File using the so-called I/O functions of the Low Level Library (LLL). The UDEF File is the „center“ of the ULIAS, it contains all parameters needed to build a realistic computer model of the underlying NDT situation. It is a binary file containing several NDT related structures, e. g., Block, Flaw(s), Channel(s), Frame(s), Material(s), Transducer(s) (see Fig. 34). The structures Block and Flaw have a reference to a CAD file in IGES format containing the 2-D/3-D geometry of the test specimen and the flaw. The EFIT Modeling Module requires a voxel grid representation of the simulation domain with material information. In order to generate such a voxel grid for a general geometry from a CAD file in IGES format, a Voxel Grid Generator (VGG) has been developed, which is written in C++. Presently, we are working on a LINUX version of ULIAS.

Fig. 35 illustrates a basic configuration of the ULIAS package, which comprises the following parts:

- the UDEF File containing NDT related structures, e. g., Block, Flaw(s), Channel(s), Frame(s), Material(s), Transducer(s);
- the EFIT Modeling Module simulating the given NDT situation exactly, i. e., arbitrary NDT situations and flaw geometries including mode conversion effects and surface wave phenomena [Marklein, 1998]. Primary results are snapshots of the ultrasonic wavefield for

Figure 34: A typical NDT situation: block with backwall breaking flaw and a scan path with two channels, a multi-static channel (Channel 1) and a monostatic channel (Channel 2)
selected time frames and recorded A-scan, which are written to a selected UDEF File. EFIT generates also 2-D/3-D ultrasonic rf-datafields, which can be used as a testbed for the ULIAS Imaging Modules:

- the FT-SAFT (Fourier Transform-SAFT) Imaging Module, which is a scalar algorithmic version of Synthetic Aperture Focusing Technique (SAFT) using only Fourier transforms [Mayer et al., 1990]. This results in a fast reconstruction scheme especially in 3-D. A 2-D/3-D algorithm is implemented, which supports a linear/plane measurement surface. Furthermore, signal analysis and signal processing techniques like filtering, correlation, convolution, and deconvolution are implemented;

- the UDEF Editor which is a special editor for UDEF Files.

Figure 35: A basic configuration of ULIAS comprising the EFIT Modeling Module, the FT-SAFT Imaging Module, a UDEF File, and the UDEF Editor

Figure 36: ULIAS: EFIT Modeling Module, CAD file in IGES format, and the Voxel Grid Generator (VGG)

All ULIAS modules use the 1-D/2-D/3-D capabilities of the Visualization Library (VISLIB) to display 1-D Plots, 2-D Raster Images, or to display the ultrasonic data within the 3-D geometry given by the CAD file in IGES format. The 3-D window allows for example light transformation, object transformation, volume and surface rendering, and voxel grid slicing. Fig. 36 summarizes the implemented CAD file handling and material coding strategies with the EFIT Modeling Module. The Voxel Grid Generator (VGG) requires a layer (IGES: level) based material coding philosophy. If the used CAD system does not support layers, like the CAD system I-DEAS, the material coding can alternatively be based on colors. Then the
supplied Color–to–Layer Converter must be applied in order to generate a CAD file with layer based material coding. The VGG is contoled by a configuration file and is called via a UNIX system call. Fig. 37 shows the main window of the EFIT Modeling Module displaying the test specimen (Block), the transducer (wedge probe), and the linear scanning path (black line) in solid mode. The specimen consists of two parts, which are screwed together. A 2–D $xz$–plane centered in $y$–direction has been selected for a 2–D EFIT modeling session in the background mode. Fig. 38 displays a selected snapshot of the ultrasonic wavefield within the 3–D geometry.

Figure 37: EFIT Modeling Module: Main window with CAD file in IGES format

Here, a cross-section plane has been moved through the 3–D geometry in order to visualize the interior of the block, the ultrasonic wavefield. The recorded A–scan is displayed in the 1–D A–Scan Window. The 3–D Main Window and 1–D A–Scan Window are synchronized in two ways: (1) the displayed A–scan is synchronized with the given receiver position, and (2) the time point of the displayed snapshot is marked by a cross in the 1–D A–Scan Window. This example gives you a first idea of the potential of ULIAS.
Figure 38: EFIT modeling module: Main window with CAD file in IGES format and superimposed wavefront snapshot with the synchronized A–scan. The time point of the displayed snapshot is marked in the A–scan by a cross, which moves if the time point of the displayed snapshot is changing.
Numerical Modeling of Coupled Piezoelectric and Ultrasonic Wave Phenomena with the Piezoelectric Finite Integration Technique — PFIT

In order to model coupled piezoelectric and ultrasonic phenomena, like the excitation and reception of an ultrasonic wavefield by a piezoelectric transducer, which converts an electrical signal to an ultrasonic signal and vice-versa, the piezoelectric effect has to be taken into account. For linear inhomogeneous nondissipative piezoelectric materials the constitutive equations are [Nelson, 1979; Auld, 1990]

\[
\begin{align*}
\mathbf{D}(\mathbf{E}, \mathbf{S}) &= \varepsilon^S \cdot \mathbf{E} + \varepsilon : \mathbf{S} \\
\mathbf{S}(\mathbf{T}, \mathbf{E}) &= \mathbf{s}^E : \mathbf{T} + d^{231} : \mathbf{E}
\end{align*}
\]

with the electric flux density vector \( \mathbf{D} \), the electric field strength vector \( \mathbf{E} \), the permittivity tensor of second rank \( \varepsilon^S \) measured at \( \mathbf{S} = \text{const.} \), the piezoelectric coupling tensor of third rank \( \varepsilon \), the compliance tensor of forth rank \( \mathbf{s}^E \) measured at \( \mathbf{E} = \text{const.} \), and the piezoelectric coupling tensor of third rank \( d \). The upper indicial notation \( d^{231} = (d_{ijk} e_i e_j e_k)^{231} = d_{ijk} e_j e_k e_i \).

In general, the typical dimension of the piezoelectric element is small compared to the wavelength of the electromagnetic wave inside the piezoelectric element. Therefore it is convenient to introduce the electroquasistatic (EQS) approximation in Maxwell’s equations

\[
\mathbf{E}(\mathbf{R}, t) = -\nabla \Phi(\mathbf{R}, t)
\]

with the electroquasistatic scalar potential \( \Phi(\mathbf{R}, t) \) [e.g., Nelson, 1979; Haus & Melcher, 1989; Auld, 1990]. For instance, this holds for a frequency \( f < 1 \text{ GHz} \) for a typical dimension of \( X_{\text{max}} = 1 \text{ cm} \). Within the EQS approximation all magnetic effects are neglected.

Then, the governing equations of piezoelectric waves are Newton–Cauchy’s equation of motion, Poisson’s equation, and the equation of deformation rate, which read in integral form for a volume \( V \) with the closed surface \( S = \partial V \)

\[
\begin{align*}
\iiint_V \rho_{p0}(\mathbf{R}) \mathbf{v}(\mathbf{R}, t) dV &= \iint_S n \cdot \mathbf{T}(\mathbf{R}, t) dS + \iiint_V f(\mathbf{R}, t) dV \\
\iint_S n \cdot \varepsilon^S(\mathbf{R}) \cdot \nabla \Phi(\mathbf{R}, t) dS &= \iint_S n \cdot \varepsilon^S(\mathbf{R}) : \text{sym} \{ \nabla \mathbf{u}(\mathbf{R}, t) \} \\
&\quad - \iiint_V \varrho(\mathbf{R}, t) dV - \int_S \eta(\mathbf{R}, t) dS \\
\iiint_V \varepsilon^E(\mathbf{R}) : \mathbf{T}(\mathbf{R}, t) dV &= \iint_S \text{sym} \{ n \mathbf{v}(\mathbf{R}, t) \} dS + \iiint_V d^{231}(\mathbf{R}) \cdot \nabla \Phi(\mathbf{R}, t) dV \\
&\quad + \iiint_V \mathbf{h}(\mathbf{R}, t) dV
\end{align*}
\]

with the mass density at rest \( \rho_{p0} \), the electrical charge density \( \varrho \), and the electrical surface charge density \( \eta \), which is in general unequal to zero at a perfectly conducting boundary.

Introducing the staggered grid system \((G, \tilde{G})\) as shown in Figure 39 with the grid spacing \( \Delta x \) and applying FIT to the governing equation yields a set of consistent matrix equations of an explicit elliptic–hyperbolic time domain algorithm called PFIT [Marklein et al. 1997b; Marklein, 1998]. Two versions of the PFIT-code have been developed, a voltage or charge
driven and a current algorithm (U–PFIT and I–PFIT).

For instance, the current driven version of the PFIT–code has in the nondissipative case to following form

\[
\{\dot{\mathbf{v}}\}^{\frac{z-\frac{1}{2}}{2}} = [\tilde{\rho}_{\text{po}}]^{-1}[\text{DIV}][\tilde{\mathbf{R}}_\xi^T]^{-1}[\mathbf{A}_\xi^T]\{\mathbf{T}\}^{\frac{z-\frac{1}{2}}{2}} + [\tilde{\rho}_{\text{po}}]^{-1}\{\mathbf{f}\}^{\frac{z-\frac{1}{2}}{2}}
\]

\[
\{\mathbf{v}\}^{(z)} = \{\mathbf{v}\}^{(z-1)} + \Delta t\{\dot{\mathbf{v}}\}^{\frac{z-\frac{1}{2}}{2}}
\]

\[
[\text{div}][\tilde{\mathbf{S}}][\tilde{\mathbf{e}}^S][\tilde{\mathbf{R}}]^{-1}[\text{grad}][\mathbf{\Phi}]^{(z)} = [\text{div}][\tilde{\mathbf{S}}][\tilde{\mathbf{e}}][\tilde{\mathbf{R}}_\Phi^T]^{-1}[\text{GRAD}][\mathbf{A}_\Phi^T]\{\mathbf{v}\}^{(z)}
\]

\[
- [\tilde{\mathbf{V}}_\Phi^T]\{\hat{\mathbf{q}}\}^{(z)} - [\tilde{\mathbf{S}}_\Phi^T]\{\hat{\mathbf{\eta}}\}^{(z)}
\]

\[
\{\dot{\mathbf{T}}\}^{(z)} = [\mathbf{c}^E][\mathbf{R}_E^\Phi]^{-1}[\text{GRAD}][\mathbf{A}_E^\Phi]\{\mathbf{v}\}^{(z)}
\]

\[
+ [\mathbf{e}^T][\mathbf{R}_E^\Phi]^{-1}[\mathbf{A}_E^\Phi][\text{grad}][\mathbf{\Phi}]^{(z)} + \{\mathbf{g}\}^{(z)}
\]

\[
\{\mathbf{T}\}^{\frac{z+\frac{1}{2}}{2}} = \{\mathbf{T}\}^{\frac{z-\frac{1}{2}}{2}} + \Delta t\{\dot{\mathbf{T}}\}^{(z)}.
\]

In order to take into account an external electrical load the PFIT algorithm, the I–PFIT algorithm has been combined with a 1–D network algorithm.

Iterative algorithms like the symmetric successive overrelaxation (SSOR) method and the conjugate gradient (CG) method are implemented in the PFIT–code to solve Poisson’s grid equation. In the following examples a CG algorithm with diagonal scaling (DCG) is used.

PFIT allows the modeling of a piezoelectric transducer as a transformer of electrical quantities, for example voltage and current, into elastodynamic quantities, for instance displacement and stress.

Details can be found in Marklein [1998].
5 Experimental Validation and Numerical Results of PFIT

5.1 Piezoelectric Pz27 Disk Transducer on a Brass Cylinder

Figure 40 shows a photograph of the sample and the 2-D geometry for the 2-D PFIT modeling. The sample consists of a disk of piezoelectric ceramics (Pz27) which is adhesively bonded on a brass cylinder. All materials are considered nondissipative in this example. A comparison between results of a 1-D lattice model [Hayward & Jackson, 1986], the 1-D PFIT modeling, and the 2-D PFIT modeling against measurement is given in Figure 41.

Only the 2-D PFIT modeling result gives a comparable amplitude of the backwall echoes (BE\(_n\)) and the secondary echoes [Krautkämper & Krautkämper, 1983; Langenberg et al., 1994], like the first secondary echo (SE1) in Figure 41c.

2-D PFIT snapshots are shown in Figure 42 especially demonstrating the generation of the first and second backwall echo and the first secondary echo.

5.2 Piezoelectric Pz27 Disk Transducer on a Brass Cylinder with Backwall Breaking Notch

Another example, which is of considerable interest in ultrasonic NDT, is given in Figure 43. The sample has a backwall breaking notch. This problem can not be treated with a 1-D model like the 1-D lattice model by Hayward & Jackson [1986]. A comparison between the modeled and experimental voltage observed at the Pz27 disk is given in Fig. 44, which validates the numerical results. The dominant signals are the excitation pulse (EP), the notch echo (NE), and the 1st backwall echo (BE1). Because of the 2-D modeling the experimental backwall echo has a smaller amplitude. A sequence of 2-D PFIT \(\{\overline{V}\}\)--snapshots is displayed in Fig. 45 showing the excited ultrasonic waves and the generation of the notch echo (NE) and backwall echo (BE1).

Further analytical and experimental validations and examples of the PFIT–code can be found in Marklein et al. [1997b] and Marklein [1998].
Figure 40: Pz27 disk on a brass cylinder: a) photograph and b) 2-D geometry for the 2-D PFIT modeling.

Figure 41: Pz27 disk on a brass cylinder: comparison between modeled and experimental voltage at the Pz27 disk (A-scan) for $R_g = 50 \, \Omega$ and a sine pulse excitation with $u_0 = 10 \, V$, $n = 2$ cycles, and $f_c = 2 \, MHz$. EP: excitation pulse; BE$n$: $n$-th backwall echo; SE$n$: $n$-th secondary echo.
Figure 42: Pz27 disk on a brass cylinder: 2-D PFIT |(\vec{v})|-snapshots. **BE**\(n\)-th backwall echo; **SE**\(n\)-th secondary echo
Figure 43: Pz27 disk on a brass cylinder with a backwall breaking notch: a) photograph and b) 2-D geometry for the 2-D PFIT modeling.

Figure 44: Pz27 disk on a brass cylinder with a backwall breaking notch: comparison between the modeled (a) and experimental (b) voltage at the Pz27 disk (A-scan) for $R_g = 50 \, \Omega$ and a sine pulse excitation with $u_0 = 10 \, V$, $n = 2$ cycles, and $f_c = 2 \, MHz$
Figure 45: Pz27 disk on a brass cylinder with a backwall breaking notch: 2-D PFIT $|\{\vec{v}\}|$-snapshots
Concluding Remarks

In this paper an overview on the numerical modeling tools PFIT, EFIT, and AFIT has been given starting from the application of the Finite Integration Technique (FIT) to the governing equations of elastodynamics and acoustics, the derivation of the algorithms EFIT and AFIT, the validation of the codes with analytical and numerical references, and the application to ultrasonic NDT real life situations. In the last section the recently developed PFIT algorithm has been presented, which allows the computer simulation of a typical ultrasonic NDT problem including the piezoelectric element and an external electrical load. As an example, the validation of the PFIT-code with experimental data has been given. All presented results are documenting the power of the established numerical tools and give trust for further industrial applications:

- to help to understand ultrasonic wave propagation in complex materials and complex geometries,
- to optimize well–accepted NDT techniques, or
- to develop new and more efficient NDT techniques.

From a numerical point of view, some of the open problems, which are presently subject of the continuing work, are for example

- 3–D modeling of real life ultrasonic NDT situations,
- CAD/IGES interface for inhomogeneous anisotropic materials (e. g., austenitic V–butt weld)
- irregular and triangular (Voronoi grid, Delaunay tessellation), and
- angled piezoelectric excitation.

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