APPLIED LINEAR AND NONLINEAR SCALAR INVERSE SCATTERING: A COMPARATIVE STUDY

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ABSTRACT

This paper presents a comparative study between different scalar linear and nonlinear inversion algorithms applied to electromagnetic and elastodynamic (ultrasonic) data sets. The inversion schemes are considered as follows: Synthetic Aperture Radar/Synthetic Aperture Focusing Technique, Diffraction Tomography, MUSIC Algorithm, Linear Sampling Method, Modified Gradient in Field Method, Contrast Source Inversion, and Extended Contrast Source Inversion. Reconstructions are presented and compared for an electromagnetic multi-frequency, multi-bistatic data set from the Institute Fresnel, France, and an elastodynamic (ultrasonic) time-domain, multi-bistatic data set from the Fraunhofer Institute for Non-Destructive Testing, Germany. The comparative study focuses on the two-dimensional scalar case.

INTRODUCTION

Scalar (acoustic) wavefields in free space can be backpropagated, either in the time domain via time reversal or in the frequency domain via phase conjugation. If this is performed for sufficiently rich data, frequency or angular diversity mode, wavefield inversion schemes such as Synthetic Aperture Radar or Diffraction Tomography can be formulated. If, and only if, the data match with linear scattering, i. e. satisfying either the Born or the Kirchhoff approximation, the above inversion is quantitative with regard to the object function of the scatterer. Unfortunately, in electromagnetics and elastodynamics (ultrasonic) wave scattering is neither scalar nor linear. Nevertheless, throughout this study we focus on the scalar and even two–dimensional case.

APPLIED LINEAR AND NONLINEAR INVERSION SCHEMES

In the comparative study presented here, we apply different scalar inversion algorithms to electromagnetic and elastodynamic (ultrasonic) data sets as described below. Because of the restricted space we only give some key statements and references.

Synthetic Aperture Radar (SAR) and Synthetic Aperture Focusing Technique (SAFT)

SAR in electromagnetics and SAFT in elastodynamics (ultrasonics) are equivalent linear phenomenological imaging techniques [12].

Diffraction Tomography (DT)

The applied linear scalar DT algorithm solves the linearized Porter–Bojarski integral equation using the first–order Born approximation utilizing K–space calculus. Details of the algorithm can be found e. g. in [12].

MUSIC Algorithm
The MUSIC method from sub–space signal processing was first applied in inverse scattering in [4], where the MUSIC method is used in conjunction with time reversal (TR) imaging (see also [8]). Thus, the MUSIC algorithm applied in inverse scattering requires multi–static data and the multi–static response matrix and the time–reversal matrix and the eigenvalues and eigenvectors of this matrix.

Linear Sampling Method (LSM)
The LSM in linear inverse scattering was first proposed in [2] and has been improved in [3] and [6, 7, 8]. There are different versions of the linear sampling method. We call the LSM algorithm presented in [8] K–LSM. The LSM version using Tikhonov regularization where the regularization parameter is chosen by the Morozov's generalized discrepancy principle we name K–LSM–TI [6].

Modified Gradient in Field (MGF) Method
In the MGF method the inverse problem is posed as an optimization problem where a scalar cost functional is minimized [9]. The unknowns are the total field and the contrast function and the updates of the unknowns are computed simultaneously at each iteration step using two CG methods running concurrently. MGF is designed for dielectric targets, while for metallic (conducting) targets a modified scheme called MGFM is applied as introduced in [10].

Contrast Source Inversion (CSI)
The CSI method proposed in [11], like the MGF method, is an iterative algorithm, where the unknown field quantities are the contrast source and the material contrast. Here, the contrast source is the product of the material contrast and the total field. A variant is CSIW derived by using weighted contrast sources where the contrast source and the total field are weighted by the incident field. It has been observed that the CSIW performs better than the CSI [5].

Extended Contrast Source Inversion (ECSI)
The ECSI works in a similar way as the CSI, except that the contrast is updated using a conjugate gradient iteration at every step instead of the analytical method used in CSI. Additionally, the ECSI uses a total variation (TV) based regularization [13].

INVERSION RESULTS

Electromagnetic Data
Figure 1(a) shows a PEC rectangular metallic cylinder, which is one of the targets of the Fresnel data set measured at the Institut Fresnel, Marseille, France. The experimental setup is described in [1]. Results of DT, TR/MUSIC, K–LSM, K–LSM–TI, MGFM, CSIW, and ECSI are given in Fig. 1(b)–(h). The comparison shows clearly that all algorithms are able to reconstruct the contour of the scatterer. The DT, MUSIC, K–LSM, K–LSM–TI results are comparable, while the K–LSM–TI is better than the DT, MUSIC, and K–LSM results. The MGFM result shows a thin irregular contour, while those of the CSIW and the ECSI are continuous. The image obtained with ECSI seems to be bigger in comparison with the actual dimensions or with the other reconstructions. But the position and shape have been properly reconstructed.

Elastodynamic (Ultrasonic) Data
Figure 2(a) shows the actual profile of two air–filled circular cylinders in an aluminum circular cylinder embedded in water. Multi–bistatic elastodynamic (ultrasonic) data have been obtained by the Fraunhofer Institute for Non–Destructive Testing (IZFP, Saarbrücken, Germany). The SAFT reconstruction in Fig. 2 shows the result for the complete time–domain data which have a bandwidth of 3.5 to 7 MHz. For CSIW and ECSI we take, after a Fourier transform, the data for a single frequency. The best results are obtained for the CSIW at 4.545 MHz, plotted in Fig. 2(c) and for the ECSI at 4.303 MHz as shown in Fig. 2(d). In both cases, the contrast is assumed to be a perfect scatterer and the imaginary contrast is reconstructed and plotted in these images.

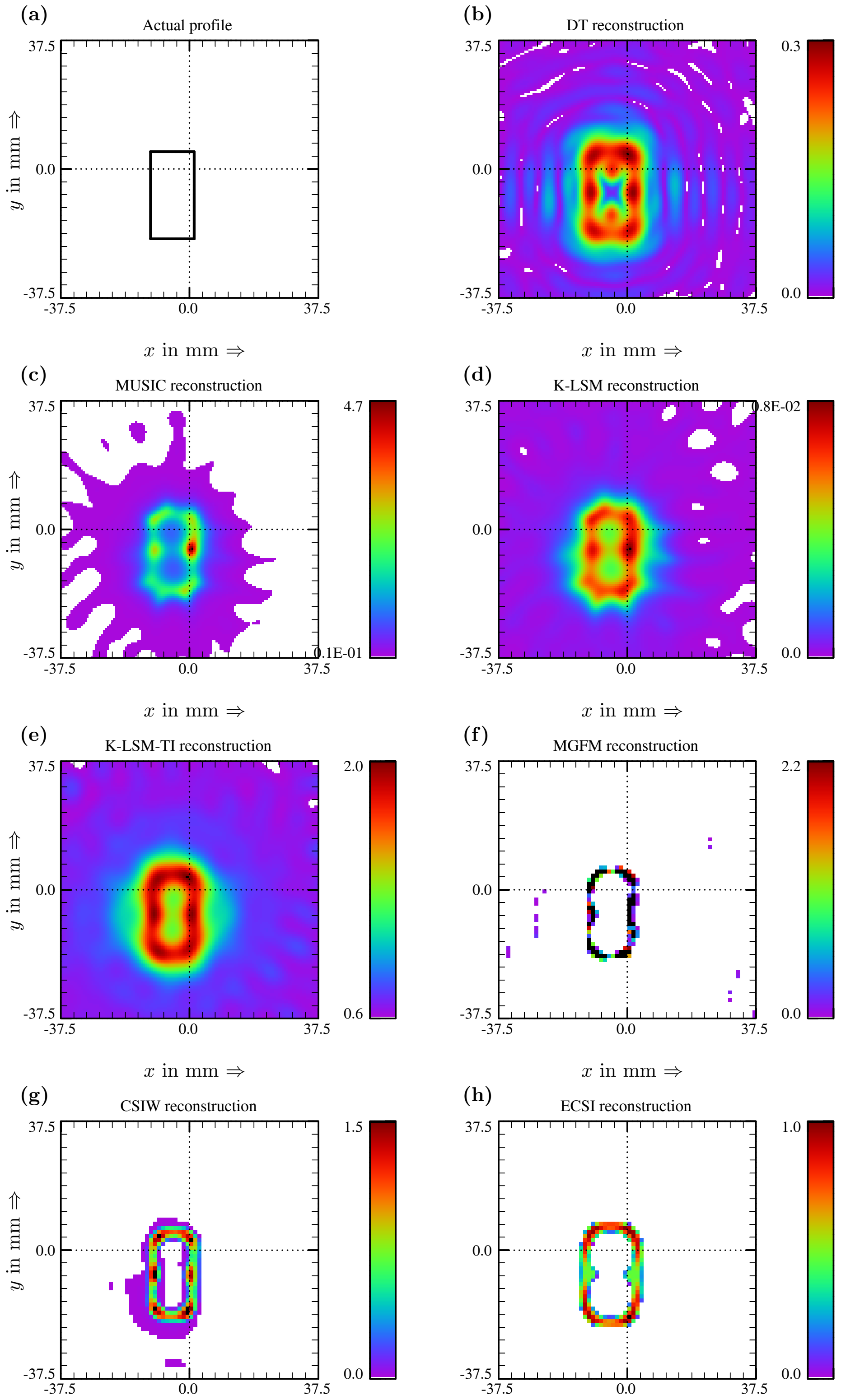

Fig. 1. Reconstruction results for a centered metallic rectangular cylinder (PEC) target of electromagnetic Fresnel data with TM polarization. **(a)** Actual shape and position; **(b)** DT reconstruction; **(c)** MUSIC reconstruction; **(d)** K–LSM reconstruction; **(e)** K–LSM–TI reconstruction; **(f)** MGFM reconstruction; **(g)** CSIW reconstruction; **(h)** ECSI reconstruction. The results in **(b)**–**(e)** were obtained using the 16 GHz data. The results in **(f)**–**(h)** were obtained using the frequency–hopping approach with the data at 4, 8, 12 and 16 GHz.

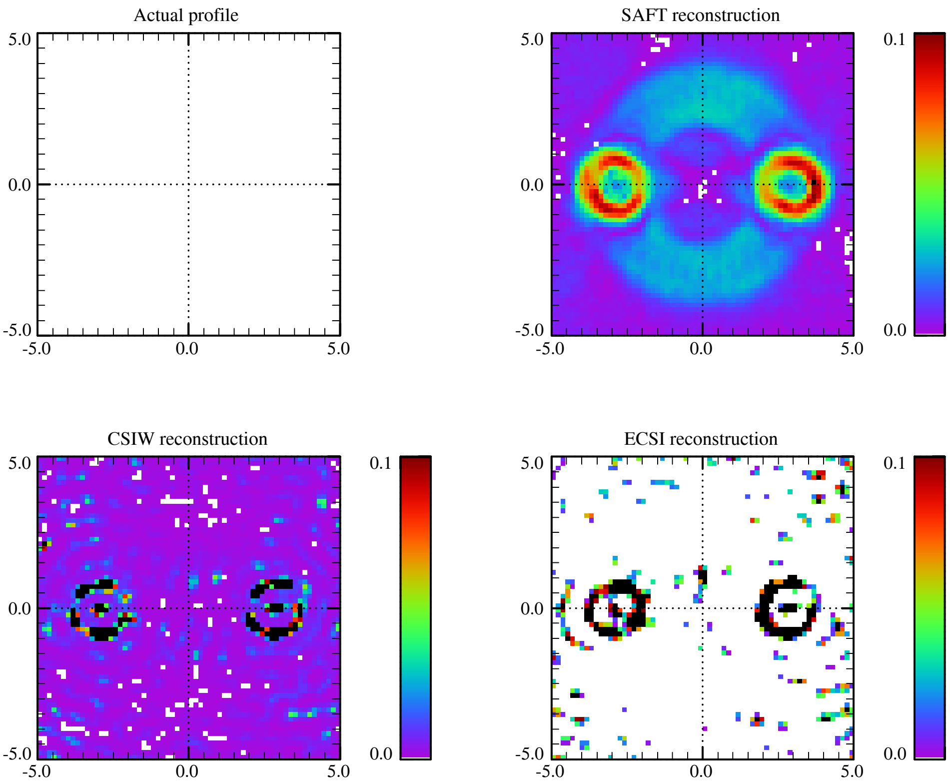

Fig. 2. Reconstructions for two air–filled circular cylinders in an aluminum block embedded in water. Actual profile **(a)**, SAFT **(b)**, CSIW **(c)** and ECSI **(d)** reconstruction.

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