

3.5. Ergänzung zu Zenon von Elea¹

1. Zeno of Elea

This presocratic philosopher (5th cent. BCE),² inventor of dialectic (i.e. the art of refutation) according to Aristotle (fr. 65 Rose, DK 29 A 10), must not be confounded with Zeno of Citium (4-3th cent. BCE), the founder of stoicism. Given the account in Plato's *Parmenides* (127B f.), Zeno of Elea was about two decades older than Socrates and, hence, contemporary with such leading figures in post-Parmenidean cosmology as Anaxagoras, Empedocles and (perhaps) Leucippus. The pioneers of the sophistic movement, Gorgias and Protagoras, may have been a decade younger. Being a disciple of Parmenides, Zeno published a series of arguments which, according to Plato (*Parmenides* 127E ff.), were designed to indirectly support the Parmenidean claim that Being is one by deriving contradictory conclusions from the assumption that there are many things. Other Zenonian arguments, such as the paradoxes of motion reported by Aristotle, may also have been designed to indirectly attack that assumption.

Verbatim quotations have only survived (via Simplicius, 6th cent. CE) of two *arguments on plurality*, demonstrating that, "if there are many things", they must be

(N) both "limited" and "unlimited" in number (DK 29 B 3),

and

(M) "both small and large; so small as not to have magnitude, so large as to be unlimited" (DK 29 B 1-2),

respectively. Only secondary reports are available in all other cases, including the *paradoxes of motion* which, following Aristotle, may be referred to and summarized as follows.

(D) *Dichotomy* (or *Stadium*). There is no motion since "before reaching the goal" the runner "must arrive at the half-way point", and so forth ad infinitum.³

Aristotle first comments that there is no question of passing through an infinity in finite time since the infinities involved are the same concerning space and time (*Phys.* VI 2, 233a24-31).

¹ Auszug aus: G. Heinemann, "Whitehead's Interpretation of Zeno", in: *A Handbook of Whiteheadian Process Thought*, ed. by M. Weber, Ffm: ontos 2007 (nach dem zur Publikation eingereichten Ms., mit redaktionellen Änderungen).

² The standard edition of the evidence concerning Zeno is Diels-Kranz (DK), ch. 29 (A: testimonia, B: fragments). More comprehensive editions, with translation and commentary, are Lee 1936 and Caveing 1982. Both KRS and Mansfeld have good selections of, and introductions to, the evidence. None of the many survey articles available goes without any complaint, but Vlastos 1969 and Makin 1998 are nevertheless outstanding.

- Caveing, Maurice [1982]: *Zénon d'Élée*, Paris: Vrin
- Lee, H.D.P. [1936]: *Zeno of Elea. A Text, with Translation and Notes* (1936), Amsterdam: Hakkert 1967
- Makin, Stephen [1998]: "Zeno of Elea", in: *Routledge Encyclopedia of Philosophy*, ed by E. Craig et al., Vol. 9, London, p. 843-853
- Vlastos, Gregory [1967]: "Zeno of Elea", in: *The Encyclopedia of Philosophy*, ed. by P. Edwards, Vol. VIII, New York-London: Macmillan

³ Aristotle, *Phys.* VI 9, 239b11-13, cf. *Top.* VIII 8, 160b8-9 and *Phys.* VI 2, 233a21-23.

Later, he adds that it makes a difference whether the divisions in question are taken as potential or actual: difficulties arise only in the latter case which, however, requires that the movement in question be interrupted whenever a division takes place (*Phys.* VIII 8, 263a15-b9).

Two variants of this argument must be distinguished (see *ibid.* a4-11).

(DG) In *Dichotomy_G*, infinite division takes place towards the goal of the race-course. The runner first traverses half of the race-course, then another quarter, and so forth, thus (if the race is from 0 to 1) successively being (in the case of 0) or arriving at

$0, 1/2, 3/4, 7/8, \dots$

(DS) In *Dichotomy_S*, by contrast, infinite division takes place towards the starting point of the race-course. Before having traversed the whole course, the runner must have traversed its first half, and before that its first quarter, and so forth, thus successively arriving at

$\dots 1/8, 1/4, 1/2, 1$

In both cases, an infinity of actions are exhibited which the runner must perform. *Dichotomy_G* makes it hard to see how the overall task can be completed. There is nothing to be done to complete it in addition to successively arriving at $1/2, 3/4, 7/8$, and so forth. This must be sufficient for arriving at 1. But why? - *Dichotomy_S*, by contrast, makes it hard to see how the task of moving from 0 to 1 can be taken up at all since there is nothing to be done first. In a sense, therefore, *Dichotomy_S* is particularly puzzling.

(AC) *Achilles*. "In a race, the slowest is never caught up by the quickest" since "the pursuer must first reach the point where the pursued started, so that the slower must always hold a lead" (*Phys.* VI 9, 239b14-18).

Aristotle remarks (*ibid.* b18-29), and it is generally agreed, that this is a mere restatement of *Dichotomy_G*.

(AR) *Arrow*. Assuming that (i) "everything either is at rest or moves whenever it occupies a position equal to itself" and (ii) "the moving thing is always in the now", the flying arrow is (iii) "motionless" (*Phys.* VI 9, 239b5-7) and, therefore, (iv) "stands still" (*ibid.* b30).⁴

This argument is based on the observation that instantaneous motion is a contradiction in terms and, hence, (v) "nothing moves in the now" (*Phys.* VI 3, 234a24). (ii) and (v) entail that (iii) the arrow is always "motionless" (and evidently occupies a space equal to itself). Taken together with (i), (iii) entails that (iv) the arrow "stands still".

⁴Modern interpreters usually follow Zeller (1876, 547n1) in deleting from (i) the clause "or moves" (*ê kineitai*, b6). But this clause makes perfectly sense, and is in the transmitted text.

• Zeller, Eduard [1876]: *Die Philosophie der Griechen in ihrer geschichtlichen Entwicklung*, 1. Teil: *Allgemeine Einleitung. Vorsokratische Philosophie*, 4th ed. Leipzig: Fues

Aristotles comments that (vi) "time is not composed of nows" (*Phys.* VI 9, 239b8, b30-31). His point is that, on the one hand, "always" in (ii) and hence in the whole argument (insofar as it is valid) only refers to "nows", i.e. indivisible positions in time. But since, on the other hand, "time is not composed of nows" nothing follows concerning the extended lapses of time required by motion and rest. In particular, instantaneous rest is as much as instantaneous motion a contradiction in terms. For instance, neither motion nor rest take place in the very moment when something has finished its movement, and will thereupon be at rest (*Phys.* VI 3, 234a31-b9). Since at that moment the thing in question undeniably occupies a space equal to itself, (i) is false and, hence, Zeno's argument is fallacious.

(MR) *Moving rows*. This argument is particularly difficult to reconstruct from Aristotle's discussion (*Phys.* VI 9, 239b33-240a18). It may be dismissed here since it plays no role in Whitehead.

2. Zeno's influence

Reactions to Zeno are already traceable in contemporary cosmology and in the Sophists.⁵ The major part of Plato's *Parmenides* is a dialectical "exercise" formed of a series of Zeno-like arguments. Aristotle's analysis in *Physics* VI of motion and the continuum is evidently designed to avoid the difficulties exhibited by Zeno's paradoxes. Diodorus Cronus, by contrast, is reported to have developed Zeno's arguments and explicitly endorsed the formula "never moves, but has moved" which in Aristotle indicates the absurdity to which the assumption is reduced that time and magnitude are composed of indivisible parts.⁶

Subsequent philosophy was usually aware of Zeno's arguments. In particular, the "new science" of Galileo and his followers required a reconsideration of the infinities involved in continuity. "The whole labyrinth about the composition of the continuum", wrote Leibniz, "must be unraveled."⁷ Kant's antinomies, in his *Kritik der reinen Vernunft* reflect Zeno's *Dichotomy*. Given the contradictions exhibited by "the old dialecticians", Hegel was happy to conclude that "motion is contradiction *in actu*."⁸

Modern scholarship was, on the one hand, deeply influenced by Tannery's claim that Zeno's arguments were not directed against common sense but, rather, against a Pythagorean doctrine describing space and time as composed of indivisible units. Only after the 1950s was this

⁵ Anaxagoras and, particularly, the Atomists. See KRS, p. 360 ff., 367, 408 f. (KRSd, p. 395 ff., 401 f., 445 f.)

⁶ Diodorus Cronus in Sextus Empiricus, *Adv. math.* 10,85 ff.; Aristotle, *Phys.* VI 1, 232a10 f.

⁷ Loemker's edition, p. 159 (note of February 11, 1676).

- Leibniz, Gottfried Wilhelm: *Philosophical Papers and Letters. A selection*, tr. and ed. by L.E. Loemker, 2nd. ed. Dordrecht - Boston: Reidel 1969, repr. 1976

⁸ Hegel, *Wissenschaft der Logik* II 76: "der daseiende Widerspruch."

- Hegel, G.W.F.: *Wissenschaft der Logik*, in: *Werke*, ed. by E. Moldenhauer and K.M. Michel, vol. 5-6, Frankfurt: Suhrkamp 1969

interpretation seen to be ill-founded.⁹ On the other hand, Aristotle's eliminative stratagems against Zeno were successfully resumed. Thus, Russell and, more recently, Grünbaum and others argued that modern mathematics, based on set theory, provides consistent accounts of continuity and motion, including the infinities involved. But it should be also noted that modern mathematics gives rise to such novel paradoxes as Cantor's proof that the concept of cardinal number does not apply to the universe (i.e., in mathematics, the class of all classes or, more specifically, of all cardinal numbers).¹⁰ Surprisingly, the similarity between this result and Zeno's paradox of number was rarely observed.¹¹

In particular, Russell pointed out that Zeno's argument that "there is no such thing as a state of change" (1901, 370) does not prevent a body from being "in one place at one time and in another at another" and, hence, to "move" in the only relevant sense of that term (ibid. 371 f.).¹² Bergson objected that this "cinematographical" description is inevitable in retrospect but fails to account for the unity of the movement which spans a duration of time and is only grasped by "installing oneself in the change" (*L'évolution créatrice*, p. 307 ff.). For Bergson, Zeno's arguments boil down to rendering absurd the notion of movement being "made of immobilities" (ibid.).¹³ Similarly but in a far less sophisticated way, James employed Zeno to confirming his view that, just as perceptual experience "grows by buds or drops", so do time, change, etc.¹⁴

⁹See Vlastos 1967, 366 f.

¹⁰On Cantor's antinomy, see Dauben 1979, 241 ff.

• Dauben, Joseph W. [1979]: *Georg Cantor: His Mathematics and Philosophy of the Infinite*. Cambridge, Mass. - London: Harvard U. Pr.

¹¹In short, the common structure of the arguments is this. If there are many things, the question as to *How many?* can be answered by specifying some number x -- finite in Zeno's case, finite or transfinite in Cantor's case -- such that there are neither more nor less than x things. But assuming that there are no less than x things, Zeno's construction demonstrates for finite x that there must be at least $2x-1$ things; Cantor's more sophisticated construction demonstrates for both finite and transfinite x that there must be at least 2^x things. Since $2x-1 > x$ for finite x (such that $x > 1$) and $2^x > x$ for both finite and transfinite x , there are more than x things. Hence, it is not true that the question as to *How many things are there?* can be answered by exhibiting some number x such that there are neither more nor less than x things.

¹²My quotations are from:

• Russell, Bertrand [1901]: "Recent Work on the Principles of Mathematics", in: *The Collected Papers of Bertrand Russell*. Vol. 3. *Toward the "Principles of Mathematics": 1900-02*, ed. by Gregory H. Moore, London: Routledge 1993, p. 366-379 [reprinted as "Mathematics and the Metaphysicians", in Russell's *Mysticism and Logic* (1918), p. 74-94]

• Similarly in the same author's *The Principles of Mathematics* [1903], repr. London: Allen & Unwin 1972.

• But see also Russell's far more elaborate discussion in his *Our Knowledge of the External World*, 2nd ed. 1926, repr. London: Allen & Unwin 1961

¹³Bergson, Henri: *L'évolution créatrice*, 155^e ed., Paris: PUF 1983

¹⁴James 1911, 154 -- The edition of James' *Some Problems ...* as vol. 7 of *The Works of William James*, ed. F.H. Burkhardt et al., Cambridge, Mass. - London 1979, has a new division into chapters. The relevant passages in ch. 10 and 11 (p. 154 ff.) are now in ch. 7 (p. 80 ff.).

• James, William [1911]: *Some Problems in Philosophy*, repr. New York: Longmans, Green & Co. 1968