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Whitehead's Interpretation of Zeno

Gottfried Heinemann

1. Zeno of Elea

This presocratic philosopher (5th cent. BCE),¹ inventor of dialectic (i.e. the art of refutation) according to Aristotle (fr. 65 Rose, DK 29 A 10), must not be confounded with Zeno of Citium (4-3th cent. BCE), the founder of stoicism. Given the account in Plato's *Parmenides* (127B f.), Zeno of Elea was about two decades older than Socrates and, hence, contemporary with such leading figures in post-Parmenidean cosmology as Anaxagoras, Empedocles and (perhaps) Leucippus. The pioneers of the sophistic movement, Gorgias and Protagoras, may have been a decade younger. Being a disciple of Parmenides, Zeno published a series of arguments which, according to Plato (*Parmenides* 127E ff.), were designed to indirectly support the Parmenidean claim that Being is one by deriving contradictory conclusions from the assumption that there are many things. Other Zenonian arguments, such as the paradoxes of motion reported by Aristotle, may also have been designed to indirectly attack that assumption.

Verbatim quotations have only survived (via Simplicius, 6th cent. CE) of two *arguments on plurality*, demonstrating that, "if there are many things", they must be

(N) both "limited" and "unlimited" in number (DK 29 B 3),

and

(M) "both small and large; so small as not to have magnitude, so large as to be unlimited" (DK 29 B 1-2),

respectively. Only secondary reports are available in all other cases, including the *paradoxes of motion* which, following Aristotle, may be referred to and summarized as follows.

(D) *Dichotomy* (or *Stadium*). There is no motion since "before reaching the goal" the runner "must arrive at the half-way point", and so forth ad infinitum.²

¹ The standard edition of the evidence concerning Zeno is Diels-Kranz (DK), ch. 29 (A: testimonia, B: fragments). More comprehensive editions, with translation and commentary, are Lee 1936 and Caveign 1982. Both Kirk et al. 1983 and Mansfeld 1983-86 have good selections of, and introductions to, the evidence. None of the many survey articles available goes without any complaint, but Vlastos 1969 and Makin 1998 are nevertheless outstanding. – But see now Rapp 2013.

² Aristotle, *Phys.* VI 9, 239b11-13, cf. *Top.* VIII 8, 160b8-9 and *Phys.* VI 2, 233a21-23.

Aristotle first comments that there is no question of passing through an infinity in finite time since the infinities involved are the same concerning space and time (*Phys.* VI 2, 233a24-31). Later, he adds that it makes a difference whether the divisions in question are taken as potential or actual: difficulties arise only in the latter case which, however, requires that the movement in question be interrupted whenever a division takes place (*Phys.* VIII 8, 263a15-b9).

Two variants of this argument must be distinguished (see *ibid.* a4-11).

(DG) In *Dichotomy_G*, infinite division takes place towards the goal of the race-course. The runner first traverses half of the race-course, then another quarter, and so forth, thus (if the race is from 0 to 1) successively being (in the case of 0) or arriving at

0, 1/2, 3/4, 7/8,

(DS) In *Dichotomy_S*, by contrast, infinite division takes place towards the starting point of the race-course. Before having traversed the whole course, the runner must have traversed its first half, and before that its first quarter, and so forth, thus successively arriving at

... 1/8, 1/4, 1/2, 1 .

In both cases, an infinity of actions are exhibited which the runner must perform. *Dichotomy_G* makes it hard to see how the overall task can be completed. There is nothing to be done to complete it in addition to successively arriving at 1/2, 3/4, 7/8, and so forth. This must be sufficient for arriving at 1. But why? – *Dichotomy_S*, by contrast, makes it hard to see how the task of moving from 0 to 1 can be taken up at all since there is nothing to be done first. In a sense, therefore, *Dichotomy_S* is particularly puzzling.

(AC) *Achilles*. "In a race, the slowest is never caught up by the quickest" since "the pursuer must first reach the point where the pursued started, so that the slower must always hold a lead" (*Phys.* VI 9, 239b14-18).

Aristotle remarks (*ibid.* b18-29), and it is generally agreed, that this is a mere restatement of *Dichotomy_G*.

(AR) *Arrow*. Assuming that (i) "everything either is at rest or moves whenever it occupies a position equal to itself" and (ii) "the moving thing is always in the now", the flying arrow is (iii) "motionless" (*Phys.* VI 9, 239b5-7) and, therefore, (iv) "stands still" (*ibid.* b30).³

This argument is based on the observation that instantaneous motion is a contradiction in terms and, hence, (v) "nothing moves in the now" (*Phys.* VI 3, 234a24). (ii) and (v) entail that (iii) the arrow is always "motionless" (and evidently occupies a space equal to itself). Taken together with (i), (iii) entails that (iv) the arrow "stands still".

³ Modern interpreters usually follow Zeller (1876, 547n1) in deleting from (i) the clause "or moves" (*ê kineitai*, b6). But this clause makes perfectly sense, and is in the transmitted text.

Aristotle's comments that (vi) "time is not composed ofnows" (*Phys.* VI 9, 239b8, b30-31). His point is that, on the one hand, "always" in (ii) and hence in the whole argument (insofar as it is valid) only refers to "nows", i.e. indivisible positions in time. But since, on the other hand, "time is not composed ofnows" nothing follows concerning the extended lapses of time required by motion and rest. In particular, instantaneous rest is as much as instantaneous motion a contradiction in terms. For instance, neither motion nor rest take place in the very moment when something has finished its movement, and will thereupon be at rest (*Phys.* VI 3, 234a31-b9). Since at that moment the thing in question undeniably occupies a space equal to itself, (i) is false and, hence, Zeno's argument is fallacious.

(MR) *Moving rows*. This argument is particularly difficult to reconstruct from Aristotle's discussion (*Phys.* VI 9, 239b33-240a18). It may be dismissed here since it plays no role in Whitehead.

2. Zeno's influence

Reactions to Zeno are already traceable in contemporary cosmology and in the Sophists.⁴ The major part of Plato's *Parmenides* is a dialectical "exercise" formed of a series of Zeno-like arguments. Aristotle's analysis in *Physics* VI of motion and the continuum is evidently designed to avoid the difficulties exhibited by Zeno's paradoxes. Diodorus Cronus, by contrast, is reported to have developed Zeno's arguments and explicitly endorsed the formula "never moves, but has moved" which in Aristotle indicates the absurdity to which the assumption is reduced that time and magnitude are composed of indivisible parts.⁵

Subsequent philosophy was usually aware of Zeno's arguments. In particular, the "new science" of Galileo and his followers required a reconsideration of the infinities involved in continuity. "The whole labyrinth about the composition of the continuum", wrote Leibniz, "must be unraveled."⁶ Kant's antinomies, in his *Kritik der reinen Vernunft* reflect Zeno's *Dichotomy*. Given the contradictions exhibited by "the old dialecticians", Hegel was happy to conclude that "motion is contradiction *in actu*."⁷

Modern scholarship was, on the one hand, deeply influenced by Tannery's claim that Zeno's arguments were not directed against common sense but, rather, against a Pythagorean doctrines describing space and time as composed of indivisible units. Only after the 1950s was this interpretation seen to be ill-founded.⁸ On the other hand, Aristotle's eliminative stratagems against Zeno were successfully resumed. Thus, Russell and, more recently, Grünbaum and others

⁴ Anaxagoras and, particularly, the Atomists. See Kirk et al. 1983, 360 ff., 367, 408 f.

⁵ Diodorus Cronus in Sextus Empiricus, *Adv. math.* 10,85 ff.; Aristotle, *Phys.* VI 1, 232a10 f.

⁶ Loemker's edition, p. 159 (note of February 11, 1676).

⁷ Hegel, *Wissenschaft der Logik* II, in: *Werke*, vol. 6, p. 76: "der daseiende Widerspruch."

⁸ See Vlastos 1967, 366 f.

argued that modern mathematics, based on set theory, provides consistent accounts of continuity and motion, including the infinities involved. But it should be also noted that modern mathematics gives rise to such novel paradoxes as Cantor's proof that the concept of cardinal number does not apply to the universe (i.e., in mathematics, the class of all classes or, more specifically, of all cardinal numbers).⁹ Surprisingly, the similarity between this result and Zeno's paradox of number was rarely observed.¹⁰

In particular, Russell pointed out that Zeno's argument that "there is no such thing as a state of change" (1901, 370) does not prevent a body from being "in one place at one time and in another at another" and, hence, to "move" in the only relevant sense of that term (ibid. 371 f.). Bergson objected that this "cinematographical" description is inevitable in retrospect but fails to account for the unity of the movement which spans a duration of time and is only grasped by "installing oneself in the change" (*L'évolution créative*, p. 307 ff.). For Bergson, Zeno's arguments boil down to rendering absurd the notion of movement being "made of immobilities" (ibid.). Similarly but in a far less sophisticated way, James employed Zeno to confirming his view that, just as perceptual experience "grows by buds or drops", so do time, change, etc.¹¹

3. Zeno in Whitehead

The relevant passages are (in chronological order):

- a section in Whitehead's *Harvard Lectures for 1924-25* (March 31 – April 11; Ford 1984, 275-286),
- *SMW* 124-127,
- *PR* 68 f.¹²

In what follows, I will first examine these passages in their relation to the traditions and topics described above (Section 3.1). In the second place, then, I will describe Whitehead's use of Zeno's

⁹ On Cantor's antinomy, see Dauben 1979, 241 ff.

¹⁰ In short, the common structure of the arguments is this. If there are many things, the question as to *How many?* can be answered by specifying some number x – finite in Zeno's case, finite or transfinite in Cantor's case – such that there are neither more nor less than x things. But assuming that there are no less than x things,

- Zeno's construction demonstrates for finite x that there must be at least $2x-1$ things;
- Cantor's more sophisticated construction demonstrates for both finite and transfinite x that there must be at least 2^x things.

Since $2x-1 > x$ for finite x (such that $x > 1$) and $2^x > x$ for both finite and transfinite x , there are more than x things. Hence, it is not true that the question as to *How many things are there?* can be answered by exhibiting some number x such that there are neither more nor less than x things.

¹¹ James 1911, 154 – The edition of James' *Some Problems ...* as vol. 7 of *The Works of William James*, ed. F.H. Burkhardt et al., Cambridge, Mass. - London 1979, has a new division into chapters. The relevant passages in ch. 10 and 11 (p. 154 ff.) are now in ch. 7 (p. 80 ff.).

¹² Two more mentions of "Zeno's method" in *PR* (35.32 and 307.22) will be discussed in Section 3.4.

arguments, starting in the *Harvard Lectures* (Section 3.2) and successively including *SMW* (Section 3.3), and *PR* (Section 3.4).

3.1. References in *Process and Reality* concerning Zeno are to the relevant chapters in James's *Some Problems of Philosophy* and in *SMW*. Whitehead rightly dismisses James's argument concerning *Achilles* as not "allow[ing] sufficiently for those elements in Zeno's paradoxes which are the product of inadequate mathematical knowledge".¹³ In particular, Zeno's *Achilles* is described by Whitehead as "an invalid argument depending on ignorance of the theory of infinite convergent numerical series".¹⁴ Whitehead's own "consideration of Zeno's arguments" solely relies on *Dichotomy_S*, which he, however, mistakes for the *Arrow*.¹⁵ James mentions the *Arrow* only in passing and amply discusses *Achilles*.¹⁶ One gets the impression that Whitehead's citing the *Arrow* instead of *Dichotomy_S* is a slip that secures him something in James' "argument from Zeno" to "agree with" (if only "in substance").¹⁷

Similarly, only Zeno is cited but something similar with *Dichotomy_S* is employed in *Science and the Modern World* (ch. 7) to exhibit an inconsistency in Kant's attribution to space and time of both extensiveness and continuity. The relevant passages in Kant are quoted by Whitehead as follows.¹⁸

(Ext_K) "I call an extensive quantity that in which the representation of the whole is rendered possible by the representation of its parts, *and therefore necessarily preceded by it*. I cannot represent to myself any line, however small it may be, without drawing it in thought, that is, without producing all its parts one after the other, starting from a given point, and thus, first of all, drawing its intuition. The same applies to every, even the smallest, portion of time. I can only think in it the successive progress from one moment to another, thus producing in the end, by all the portions of time, and their addition, a definite quantity of time." (Kant, *KrV* A162 f. / B203)

(Con_K) "This particular property of quantities that no part of them is the smallest possible part (no part indivisible) is called continuity. Time and space are quanta continua, because there is no part of them that is not enclosed between limits (points and moments), *no part that is not itself again a space or a time. Space consists of spaces only, time of times. Points and moments are only limits, mere places of limitation, and*

¹³ *PR* 68.14-16.

¹⁴ *PR* 69.3-16 (quotation: line 4-5).

¹⁵ *PR* 68.6, 68.18-69.2 and 69.17-26.

¹⁶ James 1911, 157 (*Arrow*); *ibid.* 157 ff., 171, 179 ff. (*Achilles*, unsuccessfully attacking Russell). The corresponding pages in *The Works of William James*, vol. 7 are 81 and 81 f., 87, 91 ff., respectively.

¹⁷ Cf. *PR* 68.13 f.

¹⁸ *SMW* 125 f. (Max Müller's translation with Whitehead's italics).

as places *presupposing always* those intuitions which they are meant to limit or to determine. Mere places or parts that might be given before space and time, could never be compounded into space or time." (Kant, *KrV* A169 / B211)

Whitehead's point is that a Zeno-type argument proves (Ext_K) to be inconsistent with (Con_K) (*SMW* 126). I will return to this later. But it is worth mentioning in advance, firstly, that Whitehead misrepresents Kant by neglecting the fact that (Ext_K) and (Con_K) refer to intuition ("Anschauung") and to reality, respectively,¹⁹ and the inconsistency in question cannot, therefore, be as straightforward as Whitehead suggests. Secondly, Whitehead's strange claim that (Con_K) "is in agreement with Plato as against Aristotle" (*SMW* 127, with a footnote citing "a note on Points" in Heath 1920) illuminates his, to say the least, selective way to deal with classical scholarship and with the relevant documents.

Still earlier than chapter 7 of *SMW* are Whitehead's *Harvard Lectures* of March 31 – April 11, 1925 (Ford 1984, 275-286) where Zeno is credited with "something permanently true" (p. 277) but no particular argument of Zeno's is referred to.²⁰ Whitehead's starting points are relativity theory (p. 275), Bergson's *durée* (p. 276), and Kant's account of extensiveness and continuity (p. 277 and *passim*, with the same quotations as in ch. 7 of *SMW*).²¹ Zeno is claimed to ask: "how is generation possible?" (p. 278). "Aristotle's idea" in dealing with this question (and/or with a "Pythagorean difficulty" concerning "infinity" and "limitations") is described as follows (*ibid.*).

(Ext_A) "We ought to start with points and moments and avoid all these difficulties. Points and moments with external relations."

Both in the *Harvard Lectures* and in *SMW*, Whitehead presents himself as being totally unaware of Aristotle's account, in *Phys.* VI, of continuity and, by consequence, of Aristotle's discussion of Zeno's arguments. In particular, Whitehead is unaware of the theorem that

(Con_A) No continuous magnitude such as space, time, or motion is composed of indivisible parts.

which is explicitly employed by Aristotle to refuting the *Arrow-Paradox*.²² Evidently, Kant's (Con_K) is just a restatement of Aristotle's (Con_A). The absurdity in attributing (Ext_A) to Aristotle and in claiming that (Con_K) "is in agreement with Plato as against Aristotle" (*SMW* 127) is not mitigated, but may be rendered comprehensible, by taking Whitehead's use of Heath's *Euclid in*

¹⁹ Compare Kant's statement of the relevant principles (italics mine). *KrV* A162: "Alle Erscheinungen sind ihrer Anschauung nach extensive Größen." *KrV* A166: "In allen Erscheinungen hat die Empfindung, und das Reale, welches ihr an dem Gegenstande entspricht, eine intensive Größe, d.i. einen Grad" (by virtue of which "zwischen Realität und Negation ... ein kontinuierlicher Zusammenhang möglicher Realitäten" is established, A169). – Similarly in the 2nd edition.

²⁰ Only W.E. Hocking's notes of Whitehead's lectures, with scarce verbatim quotations (underlined in the sequel), survived.

²¹ In Hocking's notes, the quotations are incomplete.

²² For the theorem, see *Phys.* VI 1, 231a24, with corollaries *ibid.* b15 f. and 18-20 (similarly, *G.C.* I 2, 316a29-34), restated *Phys.* VI 9, 239b8 f. and b31-33.

Greek into account.²³ When referring to "a note on Points" in this book (*SMW 127n1*), Whitehead had probably Heath's notes on Euclid's definitions I 1 ff. in mind. Passages concerning Plato and Aristotle in the note on Def I 1 – "a point is that which has no part" – include:²⁴

[1] We can only pass "from a dot which has size and position ... to a point which has position but no size ... by abstraction, i.e. by mentally disregarding the magnitude of the dot and regarding the point as indivisible and having position, but possessed of no other attributes. This, in substance, had already been stated ... by Aristotle. Aristotle in fact says [2] that a point is that which is indivisible in respect of quantity and has position, [3] that we can make no distinction between a point and the place where it is, [4] and that, a point being indivisible, no accumulation of points, however far it be carried, can give us anything continuous, such as a line; [5] it is only by motion in space that a point can generate a line." (Heath 1920, 113 f.)

[6] "Plato, we are told, objected to recognising points as a separate class of things at all, and regarded them as a 'geometrical fiction.' He preferred to conceive a point as being merely 'the beginning of a line'; alternatively he spoke of 'indivisible lines.' But, as Aristotle says, even indivisible lines must have extremities: hence an indivisible line (even supposing that there is such a thing) must contain at least two points, and cannot therefore be the same thing as a point." (ibid. p. 115)

In addition to this, a passage in Heath's note on Euclid's Def. I 3 – "the extremities of a line are points" – is remarkable.

[7] "Aristotle alludes to a definition of a point as 'a limit of a line' (*peras grammês*) but objects to it on the ground that it defines what is prior by means of what is posterior, a point being in the order of thought prior to, or more fundamental than, a line, while a line is similarly prior to a surface and a surface to a solid. Aristotle contrasts what is prior in the order of thought with what is prior *relatively to us*. Relatively to us, a solid is prior to a surface, a surface to a line, a line to a point. This is because a solid is nearer to sense than a surface (it is the solid, as Aristotle says, which most of all 'falls under sense,' i.e. is apprehended by sense), similarly a surface is nearer to sense than a line, and a line than a point. [...]" (ibid. p. 117 f.)

Heath's account in these notes lacks all references.²⁵ My relevant findings in Aristotle are:

²³ Heath 1920. I am grateful to Anita Butte, of Kassel University Library, for having made a copy of this rare book available to me.

Heath's *Euclid in Greek* contains a Greek text of *Elements*, Book I, with introduction and notes. It is addressed to a general public (p. vii: "senior boys at school," etc.). In the notes (which are designed "to make the schoolboy ... think," p. viii), much additional material is presented concerning ancient philosophy and mathematics, but the author has found it appropriate to omit (with two insignificant exceptions) all references. Full references are only given in the related passages of Heath's English edition of the *Elements*.

²⁴ This is probably the "note on Points" referred to by Whitehead, *SMW 127n1* (numeration in brackets is mine).

- ad [1] nothing,²⁶
- ad [2] *Met.* V 6, 1016b24-26,
- ad [3] *Phys.* IV 1, 209a11 f.,
- ad [4] *Phys.* VI 1, 231a24 and passim,
- ad [5] *De anima* I 4, 409a4 f.,
- ad [6] *Met.* I 9, 992a19-24,²⁷
- ad [7] *Top.* VI 4, 141b3-14.

[2]-[4] correspond to Aristotle's own teaching; [2] is essentially Euclid's Def. I 1, [4] is Aristotle's non-composition theorem (Con_A).

[5] reports a doctrine which can be traced back to Plato (*Laws* X, 894a) but is not at all endorsed by Aristotle.²⁸

Both [6] and [7] refer to pre-Euclidean definitions, endorsed by Plato,²⁹ of 'surface', 'line', and 'point' as boundaries of higher dimensioned objects. In [6], Aristotle suggests that if Plato's objection to points as being "geometrical fiction" is justified at all it should, in view of those definitions, equally apply to lines etc. In the continuation of the passage reported in [7], Aristotle objects that the definitions in question are not given by genus and differentia and, therefore, fail to exhibit the essences of the things defined (*Top.* VI 4, 141b15-28). He seems to tacitly presuppose that points are in the same way prior to lines, etc., as genus and differentia are prior to the species defined (cf. *ibid.* b29-34). But Aristotle never supplies the definitions required.³⁰

As I said, Whitehead is unaware of [4] and of the way this theorem is employed in Aristotle's refutation of Zeno's arrow paradox. His claims about Plato and Aristotle seem to be based upon [6] and [7] only. In particular, those passages may have suggested to Whitehead that his own

²⁵ But see Heath 1926, 155 f.

²⁶ Abstraction, in Aristotle's philosophy of mathematics, accounts for the transition from physical to mathematical, but not from sized to unsized, objects.

²⁷ The last clause in [6], "and cannot therefore ..." is pointless. It has no equivalent in the above-mentioned passage of the *Metaphysics*. The more elaborate version in Heath's notes on his English translation of Euclid's Def. I 1 has instead: "so that the same argument which proves the existence of *lines* can be used to prove that *points* exist" (Heath 1926, 156), which fairly translates *Met.* I 9, 992a23 f.

²⁸ To careless readers, Heath may suggest Aristotle's assent (similarly 1926, 156). But in his posthumous *Mathematics in Aristotle* (Clarendon 1949) the passage is not mentioned at all.

²⁹ See, e.g. the definition of "figure" (*schêma*) as *stereou ... peras* in *Meno* 76a7. This very phrase reappears in the continuation of the passage in Aristotle's *Topics* (141b22) that Heath reports in [7]. – For details, see Waschki 1991

³⁰ The definition that "a line is a breadthless length", discussed at *Top.* VI 6, 143b11 ff. (and reappearing as Euclid's Def. I 2), is given by genus and differentia but does not evidently illustrate the priorities in question. What comes closest is Aristotle's classification of magnitudes as divisible in one, two, or three directions, respectively (*De caelo* I 1, 268a7 f.), which may mean: divisible by a point, a line, or a surface. Similar considerations may apply to the distinction of continuity in one, two, or three directions (*Met.* XI 3, 1061a33 f.; cf. b24 f.), if continuity is defined as coincidence of extremities (*Phys* VI 1, 231a22, cf. V 3, 227a10-12).

mereological approach to continuity (*PNK*, Part III; *PR*, Part IV, ch. ii ff.) reverses Aristotle's priorities but was remotely anticipated by Plato. In [7], the contrast of "what is prior in the order of thought" with "what is prior relatively to us" corresponds to Aristotle's distinction between unqualified cognitive priority and cognitive priority for us.³¹ Aristotle insists, on the one hand, that scientific definitions must observe the former priorities which, therefore, may be also described as priorities in definition.³² On the other hand, priority in definition must be distinguished from, and is in mathematics even reverse to, priority in essence and being.³³ Whitehead was certainly unaware of the latter reversal but would, in view of his ontological principle, have condemned it as inadmissible.

Zeno is never mentioned in Heath's *Euclid in Greek*. Nor could I find any other trace of contemporary scholarship concerning Zeno of Elea in Whitehead. This neglect contrasts conspicuously with Russell's painstaking discussion in ch. 6 of his *Our Knowledge of the External World* (of which, again, no traces are to be found in Whitehead).³⁴ Russell thus presents himself as seriously caring about what the issues raised by Zeno's arguments are. Whitehead's Zeno, by contrast, is mere folklore, and is harnessed for Whitehead's own purpose, i.e. to exhibit "the epochal character of time" (*SMW* 126).

3.2. Three interrelated types of difficulties are described in the first lecture, of March 31, in the relevant part of Whitehead's *Harvard Lectures* for 1924-25. *Firstly*, difficulties in "the idea of alternative time systems" with which relativity theory replaces "the linear idea of becomingness" (Ford 1984, 276). *Secondly*, difficulties in Bergson's claim that "*Durée* is indivisible" (*ibid.*). *Thirdly*, such difficulties in "the old idea of the flux of time" as are exemplified by the above-mentioned quotations from Kant and by Zeno's arguments (p. 276 f.). Whitehead's remark on Kant points to the desideratum of "hav[ing] some theory of the parts and the wholes" (p. 277). A modification of Zeno's arguments is adumbrated by Whitehead's remark that "Zeno made an unfortunate choice in dealing with motion and space – muddling up time and space together" (p. 277). Rather, Zeno should have dealt with time alone (see the corresponding passage in *SMW*, p. 127). Accordingly, the point in Zeno's arguments is represented by Whitehead as follows.

"Zeno: How are you going to move forward into the future? How is process possible?³⁵ If you conceive it under the guise of a temporal transition into the non-existent, you can't get

³¹ Aristotle, *Top.* VI 4, 141b4 f.; cf. b24 f.: *haplôs gnôrimôteron* vs. *hêmin gnôrimôteron*.

³² *Ibid.* b15-28)

³³ See Aristotle, *Met.* XIII 2, 1077b1 f.: *tô_i logô_i* vs. *tê_i ousia_i*, cf. b13: *tô_i einai*.

³⁴ In his earlier work on Zeno, by contrast, Russell relied on only one article grasped from the debate among French scholars (i.e. Noel 1893, cf. Russell 1903, 348n).

³⁵ Zeno is also represented as asking: "how is generation possible?" (April 2; p. 278). The context, however, is different.

going. There is nothing you can point to into which there is a transition, or is there and then created." (p. 277)

In the lecture of April 2, Whitehead observes that "Kant's statement that the parts are antecedent to the whole", i.e. (Ext_K), taken together with the claim that "every part itself is a whole", i.e. (Con_K) leads into a "vicious regress" since every part of time is again "a whole with antecedent parts." (p. 277; *SMW* 126 attributes this objection to Zeno). The rest of the lecture is mainly devoted to Whitehead's claim that temporal relations are "internal" (p. 278) and, accordingly, "that moment" is duly equated with "that particular concrete relatedness of that past to that future" (p. 279).

The next lecture, of April 4, presents itself as a series of historical remarks which do not seem to pertain to the present topic (p. 279-281).

In the lecture of April 7, "an atomic theory of time" (p. 281) is presented.³⁶ Whitehead's starting point is the "distinction between temporality and extensiveness" (ibid.) brought out by the observation that "the idea of extension doesn't include time-direction" (p. 281 – remember that relativity theory "presents us with the notion of alternative progressions in time", p. 275). Extension is only temporalized

"via realization of the potential", i.e. "the individualization of each event into a peculiar togetherness. ... An event as present is real for itself. It is this becoming real which is temporalization." (p. 281 f.)

Here, Whitehead continues, "we bump up against the atomic view of things, also the subjective view" (p. 282). A "subject" is,

on the one hand, "a parallelogram" in the extensive structure described by relativity theory.

On the other hand, "its reality is the realization of something as entering into its own being. The pulling together of a duration from its own viewpoint, i.e. as entering into its own essence. ... The subject is what that grasping together is." (ibid.)

The clue to "atomicity", then, is this. "The becoming real is not the production via the parts of the duration – contradicting Kant" (ibid.); that is, contradicting Kant's description in (Ext_K) of time as extensive, with successive parts "antecedent to the whole" (p. 277). The inconsistency of (Ext_K) with (Con_K) is stated without proof (but Whitehead may have referred to the regress argument presented in the lecture of April 2), and is thus commented on: "if you throw over the first [i.e. (Ext_K)], you get your idea of atomic quantity" (p. 283). "The time transition", Whitehead continues, must not be conceived as a succession in becoming but, rather, as "a transition within what is already there. ... There is no relation between something and nothing." (ibid.) – Zeno is not mentioned in this lecture. But the result just stated corresponds to the principle, attributed to

³⁶ See also Ford's summary (1984, 54).

Zeno by Whitehead, that "process" must not be conceived "under the guise of a temporal transition into the non-existent" (p. 277)

The impact of Zeno's arguments is only adumbrated by a remark in the lecture of April 9 which summarizes the previous lectures as follows.³⁷

"Starting with events, and bringing the future and past into it, didn't give enough differentiation. Had to introduce 'reality' as 'real togetherness', bringing in the time-idea. If you take time as merely generating the event, Zeno gets at you. There is no such thing as a moment. What must be real is the togetherness of the content of the event." (p. 283)

3.3. How and why does Zeno "get at you"? – The extant lecture notes are silent about this. In the corresponding passage in *SMW*,³⁸ just after having quoted Kant's (Ext_K) and (Con_K), Whitehead claims that

"Zeno would object that a vicious infinite regress is involved [sc. in (Ext_K), if (Con_K) is accepted]. Every part of time involves some smaller part of itself, and so on. Also this series regresses backwards ultimately to nothing; since the initial moment is without duration and merely marks the relation of contiguity to an earlier time. Thus time is impossible, if the two extracts are both adhered to. I accept the later, and reject the earlier, passage." (*SMW* 126)

I take it that "Zeno gets at you" in the *Harvard Lecture* of April 9 just in the same way as he gets at Kant in the passage of *SMW* 126 just quoted (which, in turn, echoes the regress argument presented in the *Harvard Lecture* of April 2; see above, 3.2.).³⁹ Accordingly, I suggest that in the *Harvard Lectures*, the description of "time as merely generating the event" (April 9, p. 283) corresponds to the claim, attributed to Kant, that "the becoming real is ... the production via the parts of the duration" (April 7, p. 282).

It should be noted, however, that Whitehead "is in complete agreement with the second extract" only "if 'time and space' is the extensive continuum." (*SMW* 126). The qualification is essential since Whitehead is, of course, not at all willing to equate time with extension. Rather, Whitehead affirms the doctrine of the *Harvard Lectures* (p. 281) that extension is only temporalized "via realization of the potential" as follows.

"Realization is the becoming of time in the field of extension. Extension is the complex of events, *qua* their potentialities. In realization the potentiality becomes actuality. ...

³⁷ The addenda presented in the lectures of April 9 and April 11 (p. 283-286) throw no additional light on Whitehead's interpretation of Zeno.

³⁸ Nothing is added to this by a preliminary mention of Zeno at *SMW* 125 and by the summary at *SMW* 127.

³⁹ A similar argument, but without mentioning Zeno, reappears in *Time* (Ford 1984, 307 f.; cf. Ford 1999, 12 f.).

Temporalization is realization. Temporalization is not another continuous process. It is an atomic succession." (*SMW* 126)

In order to understand this passage, Whitehead's observation in the *Harvard Lectures* (p. 281) that "the idea of extension doesn't include time-direction" should be adduced. I take it that "time-direction", for Whitehead, must take the form of there being "succession", i.e. of there being earlier and later events. Given Whitehead's claim that extension is only temporalized and, hence, time-direction is only imposed on the extensive continuum "via realization of the potential" (*ibid.*), two candidates present themselves for the succession in question.

- (a) Assuming that "the becoming real is ... the production via the parts of the duration" (*ibid.*, p. 282), a succession of temporal parts might be supposed to correspond to that production.
- (b) Assuming that the "transition" in question "is in the nature of what has become real" but "hasn't become real because of the transition" (*ibid.*), the succession that corresponds to becoming is a transition from one antecedent duration to the duration occupied by "what has [thus] become real".

Since (a) is refuted by Zeno's arguments, "the time transition" (*ibid.*, p. 283) is easily seen to give rise to a succession of durations each of which is atomic.

3.4. In *Process and Reality*, "Zeno's method" is employed "to prove that there can be no continuity of becoming" (*PR* 35.32 f.) and thus to reaffirm the "epochal theory of time" of *SMW*, ch. 7 (*PR* 68.3). The relevant paragraphs at *PR* 68 f. are these (numeration in brackets is mine).

[1] "The argument, so far as it is valid, elicits a contradiction from the two premises: (i) that in a becoming something (*res vera*) becomes, and (ii) that every act of becoming is divisible into earlier and later sections which are themselves acts of becoming. [2] Consider for example, an act of becoming during one second. The act is divisible into two acts, one during the earlier half of the second, the other during the later half of the second. Thus that which becomes during the whole second presupposes that which becomes during the first half-second. Analogously, that which becomes during the first half-second presupposes that which becomes during the first quarter-second, and so on indefinitely. Thus if we consider the process of becoming up to the beginning of the second in question, and ask what then becomes, no answer can be given. For whatever creature we indicate presupposes an earlier creature which became after the beginning of the second and antecedently to the indicated creature.⁴⁰ [3] Therefore there is nothing which becomes, so as to effect a transition into the second in question." (*PR* 68.18-21 / 21-31 / 31-33)

⁴⁰ At *PR* 68.31, the first editions have "earlier" instead of "indicated". See the relevant editors' note (*PR* 397).

In [1], premise (i) may be taken as a truism. Premise (ii) corresponds to the premises of the regress arguments mentioned above.⁴¹ Given (ii), it is easily seen that in a Zeno-type division of an act of becoming none of the parts is earlier than all other parts. In [2], premise (i) is employed to trace back the temporal relation referred to in (ii) of

one being earlier than another

between acts of becoming to the causal, or ontological, relation of

one being presupposed by another

between the relevant creatures. Accordingly, the act of becoming is now divided into a Zeno-type sequence of *res verae* (i.e. actual occasions) such that each element in the sequence presupposes another and, therefore, there is no first element corresponding to the beginning of that act. Evidently, the conclusion stated in [3] that "there is nothing which becomes [sc. first], so as to effect a transition into the second in question" is meant to indicate an absurdity to which premise (ii) is reduced. But that statement does not seem to be absurd in itself. Its refutation requires still another premise which is tacitly presupposed in [3] but may be equated with Whitehead's claim, in the *Harvard Lecture* of April 7, that the transition in question must be "in the nature of what has [thus] become real". In *Process and Reality*, what comes closest to this is "the principle that every act of becoming must have an immediate successor, if we admit that something becomes [sc. "so as to effect a transition into the second" to follow]."⁴² In conclusion, since (ii) is false it is not the case that "every act of becoming is divisible into earlier and later sections which are themselves acts of becoming". Rather, at least some acts of becoming – actual occasions – cannot be divided into earlier and later sections which are themselves acts of becoming. Accordingly, the succession of actual occasions derives from such transitions as are required to take place in [3]. Insofar as time is this succession, Whitehead's claim that time exhibits an "epochal" structure follows immediately.

[4] "The difficulty is not evaded by assuming that something becomes at each non-extensive instant of time. For at the beginning of the second of time there is no next instant at which something can become.

[5] Zeno in his 'Arrow in Its Flight' seems to have had an obscure grasp of this argument. [6] But the introduction of motion brings in irrelevant details. The true difficulty is to understand how the arrow survives the lapse of time. Unfortunately, Descartes' treatment of 'endurance' is very superficial, and subsequent philosophers have followed up his example." (*PR* 68.34-36 / 68.37-69.2)

In [4], the same additional premise as in [3] is tacitly employed. [5] is remarkable for its falsehood. The reference is to [1]-[3] which is a variant of *Dichotomy_S* but has nothing to do with

⁴¹ *Harvard Lecture* of April 2 (Ford 1984, 277) and *SMW* 126, respectively.

⁴² *PR* 69.18 f. – see below, [7].

Zeno's *Arrow*.⁴³ Nor has Whitehead's restatement in [6] of his concluding remarks in his *Harvard Lecture* of March 31 (see above 3.2.). The allusion in [6] to the *Arrow* is merely verbal, and there is nothing corresponding to Zeno's crucial premise that "the moving thing is always in the now".⁴⁴ – For the subsequent paragraph (*PR* 69.3-16), concerning Zeno's *Achilles*, see above 3.1.

[7] "The modification of the 'Arrow' paradox, stated above, brings out the principle that every act of becoming must have an immediate successor, if we admit that something becomes. For otherwise we cannot point out what creature becomes as we enter upon the second in question. [8] But we cannot, in the absence of some additional premise, infer that every act of becoming must have had an immediate predecessor." (*PR* 69.17-20 / 20-22)

In [7], the reference is again to [1]-[3].⁴⁵ The principle stated in the first sentence, that

(a) "every act of becoming must have an immediate successor,"

has no obvious equivalent in the "Categoreal Scheme" of *PR*. Whitehead claims that (a) is brought out by [1]-[3]. This is not to say that (a) derives its validity from that argument. Rather, the indispensability of (a) as a principle is exhibited by the observation that the crucial step in the argument depends upon (a). In the sequel, the transitional clause "for otherwise ..." suggests that (a) is only substantiated by another principle which secures that

(b) "we can[...] point out what creature becomes as we enter upon the second in question,"
i.e., "enter upon" the duration required by the act of becoming considered in [2].

It is not easy to state the principle required in an appropriate way. But it should be noted that the transition in question must exemplify the Principle of Creativity, that

(c) "the many, which are the universe disjunctively, become the one actual occasion, which is the universe conjunctively." (*PR* 21.18-20)

Whitehead may have thought of (a) and (b) as obvious specifications of (c). If so, the asymmetry in (c) – "the many become one, and are increased by one," (*PR* 21.35), but no one becomes many – may also account for the asymmetry that is emphasized by Whitehead in [8].

[9] "The conclusion is that in every act of becoming there is the becoming of something with temporal extension; but that the act itself is not extensive, in the sense that it is divisible into earlier and later acts of becoming which correspond to the extensive divisibility of what has become." (*PR* 69.23-26)

⁴³ Cf. Chappell 1962, 72. – In a previous essay, I considered Whitehead's error as unbelievable and, therefore, suggested that the reference in [5] was to [6], cf. Heinemann 1990a, 104n12.

⁴⁴ Aristotle, *Phys.* VI 9, 239b6 f.

⁴⁵ In the essay mentioned (Heinemann 1990a, 144n23), I equated "the modification of the 'Arrow' paradox, stated above" with [3]. But [3] corresponds to just the crucial step in *Dichotomy_S* (see above, Section 1). No purpose is served by assuming that Whitehead confused the *Arrow* with that step in *Dichotomy_S* (instead of simply mistaking *Dichotomy_S* for the *Arrow*).

Two more passages in *PR* may be adduced that illustrate this conclusion.

[10] "... the extensive continuity of the physical universe has usually been construed to mean that there is a continuity of becoming. But, if we admit that 'something becomes,' it is easy, by employing Zeno's method, to prove that there can be no continuity of becoming.⁴⁶ There is a becoming of continuity but no continuity of becoming. The actual occasions are the creatures which become, and they constitute a continuously extensive world. In other words, extensiveness becomes, but 'becoming' is not itself extensive." (*PR* 35.30-37)

[11] "So long as the atomic character of actual occasions is unrecognized, the application of Zeno's method makes it difficult to understand the notion of continuous transmission which reigns in physical science." (*PR* 307.31-34)

Evidently, [9] is meant to correspond to the announcement in [10], with an insignificant switch in terminology. It is claimed in [9] that, in spite of "the extensive divisibility of what has become," the relevant "act of becoming" is indivisible.⁴⁷ A contradiction seems to be implied in this since "what has become" is just the same actual occasion as the relevant "act of becoming". The contradiction is only resolved by taking the diversity of the relevant temporal perspectives, which give rise to diverse meanings of 'divisibility', into account (and thus, in a sense, by following Bergson's suggestion).⁴⁸ The doctrine that "the atomic character of actual occasions [i.e., of acts of becoming]" is compatible with "the extensive divisibility of what has become" is also presupposed in [11]; but the issue raised in the context is far beyond the scope of the present essay.

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⁴⁶ The footnote refers the reader both to Part II, Ch.II, Sect. II (i.e. *PR* 68 f.) and to *SMW* ch. 7.

⁴⁷ Whitehead's announcement at *PR* 68.3 that the "epochal theory of time" is at issue is, therefore, misleading. As Ford pointed out, what is indivisible is only in *SMW* equated with time (but with "supersession" in *Time* and with the "act of becoming" in *PR*), see Ford 1999, 47. Ford rightly insists (also against my own criticism in Heinemann 1990a) that it is not Whitehead's aim at *PR* 68 f. to establish atomism but to "locate" atomicity in such a way that "the sense[s] in which an occasion [sc. is, or] is not divisible" are "carefully spelled out" (*ibid.*).

⁴⁸ For a similar argument, see Heinemann 1990b, 122 f. (concerning Bergson, see the final paragraph in Section 2, above).

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