Modelling of Small-Scale Photovoltaic Systems with Active and Reactive Power Control for Dynamic Studies

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Abstract—In this paper an RMS simulation model of a low voltage photovoltaic system for dynamic studies in the range of seconds up to minutes is developed. The model is based on a low voltage photovoltaic system model developed by the Western Electricity Coordinating Council (WECC) Renewable Energy Modeling Task Force. The model is validated with an experimental setup consisting of a photovoltaic emulator, an inverter, a cable and a grid emulator. The laboratory tests show an unexpected dead time of the Q(V) and P(V) responses which the authors could not find in published literature. The novelty, compared to state-of-the-art models, is the introduction of a dead time in order to model the measured behavior correctly. The new model is capable of explaining the grid coupled behavior of the investigated photovoltaic system in retrospective. Therefore, this work suggests more laboratory investigations on the dynamic behavior of photovoltaic systems.

Index Terms—Photovoltaic generation, PV, distributed generation, dynamic modelling, simulation, power system stability, renewable energy.

I. INTRODUCTION

In recent years the complexity of medium and low voltage power grids increased significantly due to a rise of distributed generation such as wind and solar power [1]. Photovoltaic (PV) systems have a large share of the total installed capacity. The worldwide installed PV capacity has reached over 180 GW in 2014. Small residential PV systems represent a major portion of the total installed capacity in Europe [2]. For example, in Germany, about 65% of the installed PV capacity is located in the low voltage grid [3]. The installed PV capacity is expected to increase in the future due to PV cost reduction [2]. An investigation on the improved grid integration of PV systems in Germany can be found in [4]. A techno-economic assessment of Q(V) and P(V) control can be found in [5]. An approach for the static simulation of PV systems can be found in [6].

Because of the increasing presence of PV systems in the low voltage level, it becomes important to study the dynamic behaviour of PV systems at this voltage level. Dedicated PV models are required for dynamic studies. Furthermore, aggregated models of large portions of low voltage grids for the use in stability studies of complex grids are required. The dynamic behavior of the voltage dependent reactive power injection of PV inverters according to different Q(V) characteristics has been thoroughly investigated, e.g., in [7] and [8].

The goal of this paper is to develop a model of a PV inverter suited for stability studies in a time range of several seconds up to minutes. The focus of this work lays not only on the modelling of the dynamic Q(V) behavior, but also on the dynamic P(V) behavior. The investigation is done by comparing a Matlab/Simulink model against an experimental setup with an off-the-shelf inverter. To show the possibilities of the developed model, a dynamic study of a real German low voltage network with several PV systems is performed.

The paper is structured as follows: The modelling is presented in Section II. In Section III the laboratory tests are presented. They contain a steady state and a dynamic investigation. The test case is presented in Section IV and finally a conclusion and an outlook is given in Section V.

II. PV SYSTEM MODELLING

A. Generic PV system model

This paper focuses on the dynamic modelling of grid connected small-scale (residential, low voltage level connected) PV systems. Various PV system models can be found in, e.g., [7], [8] and [9]. The simulation model developed in this paper is based on the generic model for distributed and small PV systems provided by the Western Electricity Coordinating Council (WECC) Renewable Energy Modeling Task Force [10]. The advantages of generic models are: They are independent of the manufacturer and the vendor, they are compatible with grid codes, their model structure is open source and the model itself is simulation platform independent [11]. Note that WECC has also developed a model that is suited for large-scale PV systems. This model is more detailed but due to the complex control not of interest in this paper. A DigSILENT PowerFactory® implementation of a large-scale PV system model can be found in [12].

The model for distributed and small PV systems [10] was specifically developed to represent PV systems which are connected to the distribution grid. Its main features that are of interest in this paper, are:

1) A time constant which specifies the rise time of the output. This time constant is usually in the range of 20 ms for standard inverters.
2) Furthermore, the model contains a voltage dependent reactive power characteristic for volt/var control. An example of such a Q(V) characteristic is shown in Fig. 1. Typical parameters for Q(V) characteristics in...
B. Modified generic PV system model

The small-scale PV model of WECC lacks some functionalities that are of interest in this paper. Therefore, the following capabilities have been added to the model:

1) A ramp rate limiter for the current output is added. In state-of-the-art inverters these rate limiters can be configured via a web interface of the real inverter. Therefore, it is important to include this feature in the model.

2) An active power to voltage characteristic, also called $P(V)$ characteristic, is added. In case of a high voltage at the grid coupling point, a $P(V)$ characteristic allows to reduce the injected active power with regard to the terminal voltage. A typical $P(V)$ characteristic can be seen in Fig. 2. The reduction in injected active power is realized in the inverter by leaving the Maximum Power Point (MPP).

3) A representation of the voltage measurement is added. The measurement takes place in the time range of up to 5 cycles.

The resulting, refined PV model is based on the functionalities of the WECC distributed PV system model. It has to be noted that this model is only suited for small-scale PV systems. During the process of the model development, only the listed functionalities of the WECC model were implemented and then combined with the missing functionalities mentioned above.

Fig. 3 shows an overview of the small-scale PV system model. The model was implemented into Matlab/Simulink. It consists of three main blocks: A representation of the voltage measurement and models of the active as well as the reactive power control. These three blocks will be described in detail in the following. For this RMS-model, the active and reactive powers are converted to RMS currents. This is represented by the block on the right in Fig. 3. Since only balanced three-phase systems are considered, this approach is similar to working with DQ0-components.
The time constant $T_Q$ represents the set-up time of the $Q(V)$ characteristics. This time is needed for the inverter to settle the output at the dedicated value. It can be configured in the web interface of the inverter. This parameter determines how fast the inverter changes its reactive power injection after a change of the terminal voltage occurred. The first order time delay constant $T_{Q,PE}$ emulates the physical behavior of the power electronics. As stated, e.g., in [14], this time constant can be estimated to be below 20 ms. In this paper a fast measurement equipment was assumed. Therefore, the value $T_{Q,PE} = 10$ ms was used. The rate limiter $R_Q(u(s))$ limits the positive and negative rate of change of the injected reactive power. Here $u(t)$ is the input to the block. The rate limiter parameters can be adjusted in the inverter’s web interface as well.

The block diagram of the active power control can be seen in Fig. 6. Its structure is identically to the block diagram of the reactive power control. The only difference lies in the parameterisation of the $P(V)$ characteristics and the time constants, which are called $T_P$ and $T_{P,PE}$. $T_P$ represents the set-up time of the injected active power and $T_{P,PE}$ represents the rise time of the power electronic current output. The rate limiter $R_P(u(s))$ limits the positive and negative rate of change of the injected reactive power.

The time constants $T_P$ and $T_{Q,PE}$ that can be manually configured and are typically set in the range of few seconds up to minutes. Therefore, the short time constants of the power electronic ($T_{P,PE}$ and $T_{Q,PE}$) can be neglected for studies in the range of several seconds to minutes. However, the detailed elaboration is important to become aware of the different effects that appear in the inverter.

### III. LABORATORY TESTS

#### A. System overview

The developed PV model was validated by laboratory testing. Fig. 7 shows the general setup. It consists of a programmable AC voltage source, a cable and an inverter connected to two emulators of solar strings. The PV emulators and the programmable AC source are shown in Fig. 8(a) and 8(b). The AC voltage source is capable of performing voltage steps in the half cycle after the command is issued. This can be seen in Fig. 8(c), where a voltage step from $V \approx 247$ V to $V \approx 262$ V is performed at $t = 0$ s. The inverter has a nominal power of 5000 VA. It has a convenient configurable web interface through which various parameters can be configured. The cable, that connects the inverter and the grid emulator, has an inductance of $L = 2.63$ mH and a resistance between $R = 700$ mΩ and $R = 1200$ mΩ depending on its temperature. That corresponds to an $R/X$ ration of about 1.15.

First of all, a steady state validation of the model is conducted. Thereafter, the dynamic behavior of the $Q(V)$ and $P(V)$ characteristics of the inverter are investigated. Therefore, the response of the inverter to voltage steps at the slack bus is investigated. The voltage steps are chosen such that the operating point of the inverter shifts from one section of the $Q(V)$ or $P(V)$ characteristic to another. To refer to the
different sections of the characteristics, they are divided into eight segments as seen in Fig. 9. During the tests, different internal time constants of the inverter are varied.

B. Steady state validation

The \( P(V) \) and \( Q(V) \) characteristics are configured into the inverter. The configured setpoints according to Fig. 1 and Fig. 2 are shown in Table I. In order to validate the steady state behavior of the inverter, the \( P(V) \) and \( Q(V) \) characteristics have been measured and compared to the configured characteristic. Hence, the inverter’s injected active and reactive powers have been measured for various terminal voltages. Fig. 10(a) shows the measured and configured reactive power injection of the inverter against the terminal voltage magnitude. It can be seen that there is a small offset between both curves. As the simulation model needs to emulate the correct steady state behavior the settings for the \( Q(V) \) characteristic were adjusted in the simulation model to match the measured values. Fig. 10(b) shows the adjusted characteristic together with the measured characteristic. It can be seen that they coincide. The same procedure is done for the \( P(V) \) characteristic. Table I shows the adjusted parameters used in the simulation model for both, \( Q(V) \) and \( P(V) \) characteristic.

In the next step, the steady state operation points of the inverter are compared with the simulation model after the \( P(V) \) and \( Q(V) \) characteristics were adjusted according to Table I. Fig. 11 shows a quasi-static time variation of the slack bus voltage and the measured and simulated terminal voltages of the inverter. It can be seen that both, the \( Q(V) \) and \( P(V) \) characteristics are active. Furthermore, the measured and simulated values coincide. The steady state behavior was successfully validated at voltages above 1 pu.

C. Voltage dependent reactive power injection \( Q(V) \)

The \( P(V) \) and \( Q(V) \) characteristics according to Table I are configured in the inverter. Furthermore, a set-up time for each characteristic is configured. For this experiment, set-up times of 5s and 20s are chosen.

For validating the dynamic inverter behavior of the \( Q(V) \) characteristic, a step of the slack bus voltage is applied. Its magnitude is chosen such that the terminal voltage of the inverter jumps from section IV to section V as described in Fig. 9. The relation between slack bus voltage \( V_{\text{slack}} \) and inverter terminal voltage \( V \) can be derived from Fig. 7. It is

\[
V = V_{\text{slack}} + (R + jX) \cdot I
\]

with \( R \) and \( X \) representing the line parameters and \( I \) is the terminal output current of the inverter.

The measurement results can be seen in Fig. 12 and 13. Both figures are divided into two parts. The upper part shows the slack bus voltage \( V_{\text{slack}} \) as well as the measured and simulated terminal voltages \( V_{\text{meas}} \) and \( V_{\text{sim}} \) of the inverter. The lower part shows the measured and simulated injected active and reactive power at the inverter terminal, namely \( P_{\text{meas}}, P_{\text{sim}}, Q_{\text{meas}} \) and \( Q_{\text{sim}} \). In both cases, a voltage step at \( t = 0s \) is performed such that the operating point moves from section IV to section V as described in Fig. 9.

In the measurement results in Fig. 12, it can be seen that the injection of reactive power starts with a short time delay of about 0.4 s after the voltage step occurs. Furthermore, it can be seen that the set-up time, which the inverter needs to impose the final reactive power value, is much less than the
set-up time of 5 s configured in the inverter’s web interface.

In the measurement results in Fig. 13 it can be seen that, similar to Fig. 12, the injection of reactive power starts with a time delay after the voltage step occurs. The delay is approximately 1.4 s. Furthermore, the set-up time which the inverter needs to impose the final reactive power value is much less than the set-up time of 20 s configured in the web interface of the inverter.

To model the delay that occurs, after the voltage step took place, a dead time $T_{D,Q}$ is included after the $Q(V)$ characteristic in the reactive power control section of the PV model. This can be seen in Fig. 14. To make the simulation results match the measurement results, the dead time $T_{D,Q}$ and the time constant $T_Q$ are adjusted manually until sufficient overlapping is achieved.

D. Voltage dependent active power injection $P(V)$

The investigation of the dynamic $P(V)$ behavior is done similar to the investigation of the $Q(V)$ behavior presented in Sec. III-C. The $P(V)$ and $Q(V)$ characteristics were configured according to Table I and the set-up times were configured to 5 s and 20 s in the inverter’s web interface.

The magnitude of the voltage step is chosen such that the terminal voltage of the inverter jumps from section VI to section VII as described in Fig. 9. The measurement results can be seen in Fig. 15 and 16. The graphs are structured identically to Fig. 12 and 13. For a detailed description of the structure, see Sec. III-C.

In the measurement results in Fig. 15 it can be seen that the injection of active power starts with a time delay of about 0.5 s after the voltage step occurs. Furthermore, the time that is needed for the reduction of active power injection is less than the setup time of 5 s configured in the inverter’s web interface. In Fig. 16 it can be seen that the injection of active power starts with a time delay of about 2.2 s after the voltage step occurs. Furthermore, the time that...
is needed for the reduction of active power injection is less than the setup time of 5s configured in the inverter’s web interface. As done in Sec. III-C, the model of the active power control is modified in order to simulate the physical behavior correctly. Therefore a dead-time $T_{D,P}$ was included after the $P(V)$ characteristics. The modified model of the active power control can be seen in Fig. 17. In order to make the simulation results match the measurement results, the dead time $T_{D,P}$ and the time constant $T_P$ are adjusted until sufficient overlapping is attained.

### E. Discussion of measurement results

The three main observations from the measurements are: First, the $P(V)$ and $Q(V)$ characteristics configured in the inverter have a slightly offset, compared to the measurements. The offset is smaller than 0.005 pu. Second, the time delay, configured in the inverter, did not correspond to the time constant of the PT1 block, used in the simulation model. Third, the measurements showed a dead time that was not expected. This dead time is not known before the measurement is done and it varies with the set-up time that is configured in the inverter’s web interface. To match the simulation and the measurement results, the characteristics, the time constant and the dead time of the simulation model had to be adapted manually. After adaptation of these parameters, a good match between measurements and simulations was obtained. $P(V)$ and $Q(V)$ controls are not yet part of low voltage grid codes. Therefore the behavior of the inverter in the investigated situations is not standardized yet. Also the internal control loops of the inverter are not known in detail. Therefore, the presented model can only be an approximation of the real inverter behavior.

### IV. Simulation test case

To show a possible application of the investigated PV model, a larger test case was composed in Matlab/Simulink. It consists of a real German 0.4 kV low voltage grid that is connected via a transformer to a 20 kV medium voltage grid. The general setup can be seen in Fig. 18. The LV grid consists of 234 nodes. Altogether 20 PV systems with a total installed apparent power of 250 kVA were included in the system. The $P(V)$ and $Q(V)$ characteristics were implemented for all PV systems. For the investigation, a low load setup was selected, where the total injected PV power is much higher than the total load consumption.

The dynamic behavior of the low voltage grid was investigated for two configurations, as described in Table II. In the first configuration, the PV systems were placed as far away from the slack bus as possible, namely at the end of the feeders. In the second configuration, the PV systems were placed near the slack bus, namely at the beginning of the feeders.

Fig. 19 shows the reverse power flow from the low voltage grid into the medium voltage grid after a voltage step at
TABLE II

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PV systems are far from slack node</td>
</tr>
<tr>
<td>2</td>
<td>PV systems are close to slack node</td>
</tr>
</tbody>
</table>

Slack bus

LV grid

V\textsubscript{slack}

Fig. 18. Setup of the low voltage grid test case.

**Fig. 19.** Step response of the slack bus active and reactive power after a voltage step at the slack bus at \( t = 0 \text{s} \).

In this paper a small-scale RMS model for low voltage PV systems is developed, based on a distributed PV system model proposed by WECC. The main focus of the work is to emulate the dynamic behavior of the voltage dependent active and reactive power injection via \( P(V) \) and \( Q(V) \) characteristics. The developed simulation model is validated against an experimental setup with an off-the-shelf inverter in the power range of 5000 VA.

The new contribution of this paper is to show that there are dead times in the measurement data that change significantly with the set-up time configured in the inverter. However, no connection between these two variables can be found. As far as the authors know, measurements of such dead times have not been reported in literature yet. By including these dead times into the PV model, the simulation and the measurement data match. However, to achieve that, the parameters of the model have to be tuned manually after the measurement was performed. Thus, the behavior of the inverter can not be determined by simulation prior to conducting the experiment. The main outcome of this work is, that the generic models, used as a basis for this work did not give adequate results for the specific inverter used.

As the cause of the dead time is not known by the authors, it will be necessary to conduct future work on validating PV system models. Also it will be beneficial to test more scenarios in the laboratory.

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