Integration of Voltage Dependent Power Injections of Distributed Generators into the Power Flow by using a Damped Newton Method

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Abstract

Voltage dependent active and reactive power injections are used to improve the grid integration of distributed generators, such as photovoltaic systems. In state-of-the-art literature these power injections are considered during power flow calculations via an external loop, which requires to perform multiple (up to ten or more) power flow calculations in order to obtain one static operation point. In this paper it is shown how to integrate voltage dependent power injections into the nonlinear grid equations in order to solve for a static working point by means of performing one Newton power flow calculation. That makes the external loop unnecessary and save computational time. Furthermore, it is shown for a two-bus system that the integration of voltage dependent power injections into the nonlinear grid equations can exhibit ill-conditioned power mismatch functions. Therefore, a damped Newton method is used in order to avoid numerical oscillations. Simulations of the IEEE 118-bus test system and a real German 234-bus test system show that the damped method converges faster and is computationally more efficient than the external approach currently used in literature and state-of-the-art simulation tools in power systems. In addition, the implemented power flow algorithm is validated in the laboratory for a two-bus system and a numerical example of a low-voltage feeder is given.

Keywords: Distributed generation, ill-conditioned system, local voltage control, power flow, damped Newton’s method

1. Introduction

In recent years, the penetration of medium and low voltage grids with inverter coupled distributed generators (DGs) has increased significantly and is expected to keep growing [1, 2]. The intermittency and high amount of active power infeed of DGs can cause voltage fluctuations [3]. In order to adhere to the respective voltage quality standards like IEC 60038 [4] or EN 50160 [5] DGs can be equipped with voltage dependent active and reactive power injections, also called Volt/VAR control [6], volt/watt control or local voltage control [7, 8]. The introduction of voltage dependent power injections is, amongst others, motivated by the standard IEEE 1547 [9] in order to mitigate voltage rises caused by DGs [10, 11, 12, 13]. In order to conduct realistic power system studies like grid planning it is necessary to take voltage dependent power injections of DGs into account during the calculation of static operation points, namely during the power flow.

The state-of-the-art method (to take voltage dependent power injections of DGs into account during the calculation of a static operation point) consists of performing multiple power flows. This is done by updating the injected active and reactive power after each power flow iteratively until a convergence is reached [10, 11, 14]. We refer to this solution method as external algorithm in order to simplify further explanations given in this paper.

The goal of this paper is to show how voltage dependent power injections of DGs can be integrated into the Newton (or Newton-Raphson) power flow algorithm [15]. This is done by incorporating the voltage dependent power injections into the nonlinear grid equations and using a damped Newton method presented in [16]. We refer to this solution method as internal algorithm in order to distinguish between the proposed solution method and the state-of-the-art method. The damped (underrelaxed, modified) method used in this paper is presented in [16] under the name bisection method. It is based on famous work done in the 19th and 20th century, e.g., [17, 18, 19]. A recent overview can be found in [20]. The main contribution of this paper is the application of the well-known damped Newton method to the problem of incorporating voltage dependent power injections of DGs into the power flow problem. Another contribution of this paper is to show that voltage dependent power injections can cause ill-conditioned power mismatch functions, which is not very common for power systems according to [16]. Due to this, the use of the damped Newton method is required in order to obtain a converging power flow. Furthermore, the internal algorithm is compared to the external algorithm. We conduct this comparison to show the benefits of our ap-
proach compared to the state-of-the-art method. Also, this comparison is used to validate the computation results of the \textit{internal algorithm}. Both algorithms are implemented in MATLAB\textsuperscript{\textregistered}, they are stand-alone and can be used independently of each other. We use an ill-conditioned two-bus system to show that the \textit{internal algorithm} can have superior convergence properties compared to the \textit{external algorithm}. Furthermore, the \textit{internal algorithm} is validated for the case of a two-bus system with a laboratory setup. We apply the proposed \textit{internal algorithm} to the IEEE 118-bus test system and a real German 234-bus grid and demonstrate that for these cases, the \textit{internal algorithm} is computationally faster than the \textit{external algorithm}.

The paper is structured as follows: We outline the modeling of voltage dependent power injection strategies in Section\textsuperscript{2}. In Section\textsuperscript{3} we describe the state-of-the-art \textit{external algorithm} and describe the \textit{internal algorithm}. Test cases are presented in Section\textsuperscript{4}. In Section\textsuperscript{5} a numerical example of a 19 node low-voltage test feeder with five photovoltaic systems is presented. In Section\textsuperscript{6} we summarize the findings of the paper and highlight the advantages of the \textit{internal algorithm}.

2. Modeling of Voltage Dependent Power Injection

Power injection methods of DGs can be categorized into two groups, as shown in Table\textsuperscript{1} Group (a) is characterized by a reactive power injection that is either constant or depends on the injected active power. Here, $\varphi$ is the angle of the injected complex power and \cos(\varphi) is called power factor. DGs which employ one of these strategies are treated as constant power loads (i.e. \textit{P,Q} constant) during a power flow. Therefore, no further elaboration is needed for this group from an algorithmic point of view as these are not voltage dependent. Injection methods from control group (a) are practically used by transmission and distribution system operators and are specified in technical standards, such as VDE-AR-N 4120\textsuperscript{22}.

Group (b) is characterized by a voltage dependent injection of active and/or reactive power. To consider this group within a power flow calculation, the voltage dependency has to be taken into account. The voltage dependency can be described with a piecewise function $f(V)$ as shown in Fig. 1. This function represents a unified approach for both voltage dependent active and reactive power injection. The maximum active and reactive power, depicted as $y_3$, is the maximum possible active and reactive power outputs of the DG in the current operating state. The piecewise function $f(V)$ can be described as

\[ f(V) = \begin{cases} 
  y_1, & V \leq V_1 \\
  y_1 + \gamma_1 \cdot (V - V_1), & V_1 < V \leq V_2 \\
  y_2, & V_2 < V \leq V_3 \\
  y_2 + \gamma_2 \cdot (V - V_3), & V_3 < V \leq V_4 \\
  y_3, & V_4 < V 
\end{cases} \]

where the gradients are $\gamma_1 = \frac{y_2 - y_1}{V_2 - V_1}$ and $\gamma_2 = \frac{y_3 - y_2}{V_4 - V_3}$.

The method proposed in this paper requires to calculate the derivative of $f(V)$ with respect to the voltage magnitude is a piecewise constant function $g(V)$ with

\[ g(V) = \frac{\partial f}{\partial V} = \begin{cases} 
  0, & V < V_1 \\
  \gamma_1, & V_1 < V < V_2 \\
  0, & V_2 < V < V_3 \\
  \gamma_2, & V_3 < V < V_4 \\
  0, & V_4 < V 
\end{cases} \]

The derivative $g(V)$ is discontinuous and not defined at the points $V_1, V_2, V_3, V_4$. To overcome this issue, a smoothing function will be introduced in subsection\textsuperscript{2.1}.

By choosing appropriate parameters $V_1, V_2, V_3, V_4, y_1, y_2, y_3$ the reactive and active power characteristics can be expressed by different realizations of $f(V)$. It is:

\[ P(V) = f_P(V) \quad (3a) \]

\[ Q(V) = f_Q(V). \quad (3b) \]

![Figure 1: Piecewise function $f(V)$ for $P(V)$ and $Q(V)$ voltage dependent power injection. This is a generalized depiction of a characteristic. Positive and negative slopes are possible.](image-url)
The choice of the parameters is typically done by the grid operator based on internal grid planning and operation guidelines. These guidelines may vary for different operators as well as countries.

2.1. Smoothing function

To avoid the issue of the discontinuity of the derivative \( g(V) \), the following smoothing function for \( f(V) \) is used

\[
\begin{align*}
  f_s(V) &= y_1 \frac{1}{k} \left[ \ln(1 + e^{k(V - V_1)}) - \ln(1 + e^{k(V - V_2)}) \right] \\
  &= y_2 \frac{1}{k} \left[ \ln(1 + e^{k(V - V_3)}) - \ln(1 + e^{k(V - V_4)}) \right]
\end{align*}
\]

where the derivative between the smoothing function and its original function is reduced by increasing values of \( k \). The derivative is

\[
g_s(V) = \gamma_1 \left[ \text{logsig}(k(V - V_1)) - \text{logsig}(k(V - V_2)) \right] + \gamma_2 \left[ \text{logsig}(k(V - V_3)) - \text{logsig}(k(V - V_4)) \right]
\]

with \( \text{logsig}(x) = \frac{1}{1 + e^{-x}} \) being the log-sigmoid function.

Fig. 2 shows the original piecewise linear function \( f \) from (1) and its derivative \( g \) from (2), together with the smoothing functions \( f_s \) from (4) and its derivative \( g_s \) from (5) for different values of \( k \). The parameters chosen are \( y_1 = 0.18, y_2 = 0, y_3 = -0.18, V_1 = 0.93, V_2 = 0.97, V_3 = 1.03, V_4 = 1.07 \) and \( \gamma_1 = \gamma_2 = -4.5 \). By analyzing the curves it can be seen that a choice of \( k = 600 \) results in a reasonable approximation of (1) as is shown in Fig. 2.

3. Integration into the Power Flow

In Section 3.1 the power flow problem is presented, and more details concerning power flow can be found in various literature such as [10, 23, 24]. In Section 3.2 the external algorithm is explained. It is the present state-of-the-art method for considering voltage dependent power injections in a power flow calculation. Section 3.3 presents the proposed novel internal algorithm. Figures 3 and 4 show the details of the implementation of both algorithms.

3.1. Power Flow Formulation

The power flow problem is defined as follows: Assume a power system with \( n \) nodes and \( m \) DGs with a voltage dependent power injection. Without loss of generality it is assumed that the generators are connected to nodes \( 1, \ldots, m \). It is further assumed that, nodes \( m + 1, \ldots, n - 1 \) are connected to constant power loads and thus are modeled as PQ nodes. Node 0 is chosen as slack node. The active and reactive power characteristics of the DG connected to a node \( k \in \{1, \ldots, m\} \) are described by the piecewise function \( P_k = f_{pk}(V_k) \) and \( Q_k = f_{qk}(V_k) \), where \( V_k \) is the voltage magnitude of node \( k \). A combination of a DG and a constant power load at the same node can be implemented by adjusting the parameters \( y_1, y_2, y_3 \) appropriately. If a DG with settings \( y_1, y_2, y_3 \) for a \( P(V) \) characteristic is present at a bus and a constant power load of power \( P_{const} \) is also present at the same bus, then the new setting for the characteristic is \( y_{1new} = y_1 - P_{const}, y_{2new} = y_2 - P_{const} \) and \( y_{3new} = y_3 - P_{const} \). The parameters \( y_1, y_2, y_3 \) denote a generation whereas \( P_{const} \) is a load, therefore they are subtracted from each other.

For a valid steady-state solution of the system the injected power

\[
S_k = P_k + jQ_k
\]

into each node \( k \in \{1, \ldots, n - 1\} \) has to equal the sum of the branch power flows

\[
S_{kt} = P_{kt} + jQ_{kt}
\]

from node \( k \) to all connected nodes \( t \). The power mismatch equation describes the difference between those two quantities. It is \( S_{kt} = 0 \) in case that node \( t \) is not connected to node \( k \). The power mismatch \( E_k \) at node \( k \) is

\[
E_k = S_k - \sum_{\ell = 1}^{n-1} S_{kt}.
\]

As the local voltage characteristic depends on the voltage magnitude, it is practical to use the polar form. The voltage magnitude and angle at node \( k \) are denoted with \( V_k \) and \( \delta_k \). Thus, the state vector is given by

\[
\mathbf{x} = \left[ V_1, \ldots, V_{n-1}, \delta_1, \ldots, \delta_{n-1} \right]^T
\]

where \( \mathbf{x} \in \mathbb{R}^{2n-2} \). The angle difference between the voltage at bus \( k \) and bus \( t \) is defined as

\[
\delta_{kt} = \delta_k - \delta_t.
\]
The $k\ell$-th element of the bus admittance matrix is defined as
\[ Y_{k\ell} = G_{k\ell} + j B_{k\ell} \tag{11} \]
where $G_{k\ell}$ is the conductance and $B_{k\ell}$ is the susceptance of the branch connecting nodes $k$ and $\ell$. The active and reactive power mismatch at node $k$ is
\[ \text{Re}(F_k) = P_k(V_k) - V_k \sum_{\ell=1}^{n} V_{\ell}(G_{k\ell} \cos \delta_{k\ell} + B_{k\ell} \sin \delta_{k\ell}) \]
\[ \text{Im}(F_k) = Q_k(V_k) - V_k \sum_{\ell=1}^{n} V_{\ell}(G_{k\ell} \sin \delta_{k\ell} - B_{k\ell} \cos \delta_{k\ell}). \tag{12a} \]
\[ \text{Re}(F_k) = 0 \tag{14} \]
\[ \text{Im}(F_k) = 0 \tag{15} \]
where $F(x) \in \mathbb{R}^{2n-2}$. The goal of a power flow calculation is to find the bus voltage magnitudes and angles such that the system of nonlinear power mismatch equations becomes zero. These equations can be written in matrix form as
\[ F(x) = 0. \tag{14} \]
If the nonlinear equation \[(14)\] is well conditioned, it can be solved by Newton’s method. The system is linearized at point $x^\nu$: \[(15)\]
where the superscript $\nu$ denotes the present iteration count of Newton’s method. \[ J^\nu \in \mathbb{R}^{(n-1) \times (n-1)} \] is the Jacobian of $F$ at point $x^\nu$. The change of the state vector is
\[ \Delta x^{\nu+1} = x^{\nu+1} - x^\nu. \tag{16} \]
After the linear problem \[(15)\] is solved, the next approximation point is calculated by
\[ x^{\nu+1} = x^\nu + \Delta x^{\nu+1}. \tag{17} \]
This process is repeated iteratively until the maximum change of the state vector is below the power flow threshold $\varepsilon_{PF}$:
\[ \max|\Delta x^{\nu+1}| < \varepsilon_{PF}. \tag{18} \]
A value of $\varepsilon_{PF} = 10^{-3}$ pu is usually sufficient for power system applications and also chosen in this paper.

3.2. External Algorithm

The *external algorithm*, as used in, e.g., [10] [11] [14], represents the present state-of-the-art to obtain static operation points in an electric grid under the presence of DGs with voltage dependent power injections. To compare the *external* and *internal algorithm*, it is beneficial to look at the *external algorithm* in some more detail. The main idea is to sequentially perform a power flow and to update the injected active and reactive powers according to the voltage characteristics of the DGs until the bus voltages converge. Details of the implementation are shown in Fig.3. The prescript $(i)$ denotes the iteration counter of the external loop, which represents the number of power flows. This avoids confusion with the index $\nu$, which will later denote the counter of the Newton iteration of the *internal algorithm*. For example, $(i)P_k$ is the active power injection at bus $k$ during the $(i)$-th external iteration.

Initially, flat start conditions are chosen, thus the bus voltage magnitudes are set to 1 and the bus voltage angles are set to 0 (line 3 of Fig.3). The active and reactive power injections of all DGs are initially calculated by using the voltage characteristics divided by a filter value $\zeta$ (line 4-7). This avoids large overshoots of the voltages in the first power flow iteration. In practical applications, filter values between 3 and 5 result in a convergence of the power flow. The higher the filter value the more likely the *external algorithm* is to converge. On the other hand, the computational time increases with higher filter values. Thus, the filter value has to be selected manually and adjusted if convergence is not reached. Each power flow is initialized with the previous power flow result (line 10). During the power flow the linear problem \[(15)\] is set up without considering the voltage dependency of the injected powers of the DGs. The DGs are only present at buses $1,..,m$. The other buses are static. Therefore, an iteration across all DGs is started (line 11). The bus voltages are used to update the injected powers of the DGs (lines 12-13). A filter $\zeta$ is applied (lines 14-15). The external loop is repeated until the maximum change of the bus voltages and angles is below a threshold $\varepsilon_{external}$ (line 17).

The overall power flow problem is nonlinear and con-

1: **procedure** External Algorithm
2: \[ (i)x = (1,\ldots,1,0,\ldots,0)^T \]
3: \[ \text{for all } k \in \{ 1, \ldots, m \} \text{ do} \]
4: \[ (i)P_k = \frac{P_{k}(i) V_k}{\zeta} \]
5: \[ (i)Q_k = \frac{Q_{k}(i) V_k}{\zeta} \]
6: \[ \text{end for} \]
7: \[ \text{repeat} \]
8: \[ i = i + 1 \]
9: \[ ((i-1) x) \rightarrow \text{Power Flow} \rightarrow (i) x \]
10: \[ \text{for all } k \in \{ 1, \ldots, m \} \text{ do} \]
11: \[ (i)P_k = f_{P_{k}}(i) V_k \]
12: \[ (i)Q_k = f_{Q_{k}}(i) V_k \]
13: \[ (i)P_k = ((i-1) P_k + (i) P_{k-1})_{\zeta} \]
14: \[ (i)Q_k = ((i-1) Q_k + (i) Q_{k-1})_{\zeta} \]
15: \[ \text{end for} \]
16: \[ \text{until max}((i)x_{- (i-1)x}) < \varepsilon_{external} \]
17: \[ \text{end procedure} \]

Figure 3: Description of the external algorithm.
vergence cannot be guaranteed in all cases. Some power flow problems without a solution exist. For example this is the case when the system is in a point of voltage collapse \[25\].

To gain more insights into the necessity of the filter it is useful to assume an example grid with a noticeable amount of DGs with voltage dependent active power injections. Normally, the power flow calculation starts with the full accessible active power injection of all DGs. That would be the case if lines 4-7 were not included in the algorithm. However, a power flow calculation in which all DGs inject their full active power often leads to very high bus voltages. These numerical voltage overshoots can be avoided if the injected powers of the DGs are not at 100 % during the first power flow. This is implemented by line 4-7, where the active and reactive powers are set to a value which is the injected active and reactive power, according to the \( P(V) \) and \( Q(V) \) characteristic, divided by the filter value. Let us further assume that the bus voltages computed by the first power flow calculation \((i = 1)\) are very low because of the presence of the initial filter (line 4-7). In this case, the \( P(V) \) characteristics in line 12 would result in using the 100 % injected power in the next power flow calculation \((i = 2)\). However, using the full active power might lead to very high bus voltages. This will result in a substantial reduction of injected active power in the subsequent power flow \((i = 3)\), which will then show again high bus voltages and the cycle will start over again. This is denoted as numerical oscillation. The oscillation affects the voltages, the currents and the injected powers.

### 3.3. Internal Algorithm

In this section, it is shown how to include the \( Q(V) \) and \( P(V) \) characteristics directly into the Newton power flow algorithm. During each Newton iteration \( \nu \) the injected power into bus \( k \in \{1, ..., m\} \) is updated according to

\[
P_k^\nu = f_{pk}(V_k^\nu) \quad \text{and} \quad Q_k^\nu = f_{qk}(V_k^\nu) \tag{19a-b}
\]

As can be seen in [25], the power mismatch consists of two voltage dependent terms, namely the injected powers \( S_k \) and the sum of the branch power flows \( \sum_{\ell=1, \ell \neq k}^{n} S_{k\ell} \). Therefore, the overall Jacobian is a linear combination of the Jacobians of these two terms and can be written as

\[
J^\nu = J_a^\nu + J_b^\nu \tag{20} \]

\( J_b \) is the Jacobian of the sum of the branch power flows \( S_{k\ell} \). Its construction can be found in, e.g., [24]. \( J_b \) is the Jacobian of the voltage dependent power injection characteristics of the injected powers \( S_k \). If, besides DGs with voltage dependent power injection, no other voltage dependent load is present, \( J_b \) only exists for the internal algorithm and is zero in case of the external algorithm. As the injected powers are voltage dependent, the Jacobian \( J_b \) may contain nonzero entries. It contains the derivatives of the voltage characteristics:

\[
J_b = \begin{pmatrix} \frac{\partial P}{\partial V} & \frac{\partial P}{\partial Q} \\ \frac{\partial Q}{\partial V} & \frac{\partial Q}{\partial Q} \end{pmatrix} = \begin{pmatrix} J_{b11} & J_{b12} \\ J_{b21} & J_{b22} \end{pmatrix} \tag{21}
\]

where \( J_{b11}, J_{b12}, J_{b21}, J_{b22} \in \mathbb{R}^{(n-1) \times (n-1)} \). The active and reactive power vectors are described by

\[
P = (P_1^\nu, ..., P_{n-1}^\nu)^T \tag{22a}
Q = (Q_1^\nu, ..., Q_{n-1}^\nu)^T \tag{22b}
\]

where \( P, Q \in \mathbb{R}^{n-1} \). Let \( g_{Pk}(V_k^\nu) \) and \( g_{Qk}(V_k^\nu) \) be the derivative of the active and reactive power characteristics of the DG connected to bus \( k \) according to [5], which is a smoothing function of [2]. The injected active and reactive powers do not depend on the bus voltage angle. It is

\[
J_{b11} = \begin{pmatrix} g_{p1}(V_1^\nu) & \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & g_{pm}(V_m^\nu) \end{pmatrix} \tag{23a}
J_{b12} = \begin{pmatrix} g_{q1}(V_1^\nu) & \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & g_{qm}(V_m^\nu) \end{pmatrix} \tag{23b}
\]

\[
J_{b12} = J_{b22} = 0. \tag{23c}
\]

If only \( P(V) \) (or \( Q(V) \)) injections are present, the matrix \( J_{b21} \) (or \( J_{b11} \)) becomes zero. If both injections are active, both matrices can have nonzero entries.

According to Section 4.1 the nonlinear voltage characteristic can lead to ill-conditioned cases. An explanation of ill-conditioned power flow cases is presented in [16]. For solving those cases robust techniques of the basic Newton’s method have been introduced in literature, e.g., [25, 27, 28]. These techniques are mainly based on modifying \([15]\). In this paper, we use a modified (damped) Newton method, described as bisection method in [16]. Following the nomenclature of [16], this type of modified (damped) Newton method will be called bisection Newton method in this paper. After the new state vector \( \Delta x^{\nu+1} \) is calculated, it is tested if the power mismatch \( F(x^{\nu+1}) \) decreased. If not, then the change in the state vector \( \Delta x^{\nu} \) is reduced by a factor \( \alpha \). During the first iteration it is \( \alpha = 1 \). For an unsolvable case, \( \alpha \) will converge towards zero, thus a minimum value of \( \alpha \) has to be defined.

The bisection Newton method is similar to the more sophisticated damped Newton-Raphson method presented
in [29] which works with a damping multiplier that corresponds to the variable \( \alpha \) used in this work. The difference is, that in the damped Newton-Raphson method an optimum value is found for the damping multiplier by approximating the power mismatch function with a quadratic function. This method is suitable for obtaining power flow results in case that the conventional Newton-Raphson is not converging. This is done by obtaining the minimum of the power mismatch. In the modified method used in this paper, \( \alpha \) is simply decreased (divided by 2) during each iteration if the power mismatch function does not decrease.

Details of the implementation of the internal algorithm are shown in Fig. 4. The Newton iteration counter \( \nu \) is initially set to zero (line 2 of Fig. 4) and flat start conditions are chosen (line 3). At the beginning of each Newton iteration the injected powers are updated according to the voltage characteristics of the DGs [19] (lines 5-8). The Jacobian and the power mismatch function are updated (lines 9-10). In the next step, the modified Newton method is applied (lines 11-16) and the internal iteration counter is increased (line 17). This process is repeated until the change in bus voltages is below the threshold \( \epsilon_{\text{internal}} \) (line 18).

4. Test Cases

To investigate the proposed internal algorithm, a conventional Newton power flow without any voltage dependent power injection has been implemented in MATLAB [21]. Subsequently, the internal as well as the external algorithm were implemented on top of this basic power flow routine and they are used for the following test cases. We used MATPOWER [30] to validate our implementation of the conventional Newton power flow. All simulations were performed on an Intel i7-3520M 2.90 GHz processor with 16 GB RAM and on a Windows 7 operating system. The voltage characteristics used in the case studies are based on values used by distribution system operators in practical settings.

4.1. Two-Bus System

A two-bus system, as shown in Fig. 5, is used to compare the convergence properties of the internal and the external algorithm. It consists of a DG connected to an infinite bus via a resistor and an inductor. Furthermore, the simulation results are validated in the laboratory.

![Two-Bus System Diagram]

Figure 5: Two-bus system with a distributed generator connected to bus 1.

Table 2: Parameters of the simulated two-bus system

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base apparent power</td>
<td>( S_{\text{base}} )</td>
<td>100</td>
<td>kVA</td>
</tr>
<tr>
<td>Base voltage</td>
<td>( V_{\text{base}} )</td>
<td>230</td>
<td>[V]</td>
</tr>
<tr>
<td>Line resistance</td>
<td>( R )</td>
<td>0.383</td>
<td>[pu]</td>
</tr>
<tr>
<td>Line reactance</td>
<td>( X )</td>
<td>0.190</td>
<td>[pu]</td>
</tr>
</tbody>
</table>

Table 3: Voltage characteristic of the simulated two-bus system

<table>
<thead>
<tr>
<th>Injection</th>
<th>([V_1, V_2, V_3, V_4])</th>
<th>([y_1, y_2, y_3])</th>
<th>method</th>
<th>([\text{pu}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(V) )</td>
<td>([1.08, 1.08, 1.08, 1.09] )</td>
<td>([0.35, 0.35, 0.20] )</td>
<td>([\text{pu}])</td>
<td></td>
</tr>
<tr>
<td>( Q(V) )</td>
<td>([0.93, 0.97, 1.03, 1.07] )</td>
<td>([0.18, 0.0, -0.18] )</td>
<td>([\text{pu}])</td>
<td></td>
</tr>
</tbody>
</table>
the strategy $P(V) \& Q(V)$ the DG feeds in reactive power and reduces its active power in order to reduce the voltage at bus 1. Active and reactive power injection takes place depending on the voltage at bus 1 according to the voltage characteristic given in Table 4.

A more detailed investigation of the convergence behavior of the external and internal algorithm is presented for the case of $P(V)$ injection. It is assumed that no reactive power is injected. Fig. 6 shows the maximum power mismatch and the voltage magnitude after each iteration of the external algorithm for three different filter values.

For a filter value of $\zeta = 2$ the voltage magnitude shows an oscillatory behavior and does not converge. The reason is that the injected active power keeps on jumping from its highest ($y_1$) to its lowest value ($y_3$). A more detailed explanation and a plot of the injected power can be found in [14]. For $\zeta = 3$ and $\zeta = 4$ the voltage magnitude converge towards the same final value and thus both power mismatches converge towards zero. For $\zeta = 3$ the overall convergence speed is lower and the numerical oscillations are higher than for $\zeta = 4$.

The convergence behavior of the internal algorithm is shown in Fig. 7 for both Newton’s method and bisection Newton’s method. The maximum power mismatch (top) and the voltage magnitude (bottom) are shown for each Newton iteration $\nu$. It can be observed that the bisection method converges and the normal Newton method without bisection algorithm does not converge. It has to be pointed out that the power mismatch is in the order of $10^{-6}$ after eight Newton iterations. For reaching the same level of power mismatch the external algorithm requires about 14 external iterations which corresponds to 30 Newton iterations. Thus, the internal algorithm required much less Newton iterations than the external algorithm (8 instead of 30) in this case. Furthermore, the filter value needs to be set in the external algorithm, which is a crucial disadvantage that our proposed internal algorithm does not suffer from. The speedup in terms of actual computational time is not very significant for the two-bus system. The external algorithm needs about 13 ms and the internal algorithm about 5 ms of computational time. The reason is, that for the two-bus system, the Newton step is actually less time consuming than the other parts of the algorithm like, for example, the initialization. As the matrices involved are only of size $[2 \times 2]$ for a two-bus system, the benefit in computational time becomes more evident in case of larger grids. This is shown in Section 4.2.

To illustrate the ill-conditioned power mismatch function, Fig. 8 (top) shows the $P(V)$ characteristic and the inverse nose curve. The latter is derived in the Appendix.

<table>
<thead>
<tr>
<th>Injection strategy</th>
<th>Voltage magnitude [pu]</th>
<th>Voltage angle [degree]</th>
<th>Active power [pu]</th>
<th>Reactive power [pu]</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>1.118</td>
<td>3.392</td>
<td>0.35</td>
<td>0.00</td>
</tr>
<tr>
<td>$Q(V)$</td>
<td>1.085</td>
<td>7.161</td>
<td>0.35</td>
<td>-0.18</td>
</tr>
<tr>
<td>$P(V)$</td>
<td>1.087</td>
<td>2.477</td>
<td>0.25</td>
<td>0.00</td>
</tr>
<tr>
<td>$P(V) &amp; Q(V)$</td>
<td>1.081</td>
<td>7.062</td>
<td>0.34</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

Figure 6: Active power mismatch (top) and bus voltage magnitude (bottom) against the power flow iteration $i$ of the external algorithm with $P(V)$ injection for different filter values $\zeta$ for the simulated two-bus system.

Figure 7: Active power mismatch (top) and bus voltage magnitude (bottom) for the internal algorithm of the Newton method and the bisection Newton method for the simulated two-bus system.
for zero injected reactive power. The nose curve describes the active power injection depending on the voltage magnitude $V$. The operating point is the intersection between the two curves. The intersection coincides with the value in Table 4 for the injection strategy $P(V)$. The power mismatch function is the difference between the inverse nose curve and the $P(V)$ characteristic and is shown in Fig. 5 (bottom). It can be seen that the power mismatch function is ill-conditioned. The reason is the piecewise linear definition of the $P(V)$ characteristic and its steep decrease relative to the inverse nose curve in the area of $1.08 < V < 1.09$. Such a system can only be solved by the conventional Newton method in case the initial value for the voltage $V$ is located in the area of $1.08 < V < 1.09$. A more detailed explanation on ill-conditioned power flow cases can be found in, e.g., [10].

To guarantee convergence of the internal algorithm in those ill-conditioned cases the bisection method as explained in Section 3.3 is used.

The iterative process of the internal and external algorithm is now addressed in detail. Table 5 shows the interim results for the external algorithm for a filter value of $\zeta = 4$. Note that $(0) P_1$ is the injected power of the DG into bus 1. During initialization ($i = 0$) the injected power of the DG is set to $(0) P_1 = 0.35/4 = 0.087$ pu. The first power flow calculation ($i = 1$) results in a voltage of $1.0324 \pm 0.918^\circ$. With this voltage the new injected power is $f_{P_1}(1.0324) = 0.35$. The application of the filter results in $(1) P_1 = 0.087 + (0.35 - 0.087)/4 = 0.153$ pu. This injected power leads to a voltage of $1.0553 \pm 1.572^\circ$ in the second iteration ($i = 2$). The process is repeated until a steady state is reached in the 12th external iteration.

Table 5: Interim results for each external iteration $i$ of the external algorithm for $P(V)$ characteristic of the simulated 2-bus system

<table>
<thead>
<tr>
<th>$i$</th>
<th>$V_1$ [pu]</th>
<th>$(1) P_1$ [pu]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$120^\circ$</td>
<td>0.087</td>
</tr>
<tr>
<td>1</td>
<td>1.0324±0.918$^\circ$</td>
<td>0.153</td>
</tr>
<tr>
<td>2</td>
<td>1.0553±1.572$^\circ$</td>
<td>0.202</td>
</tr>
<tr>
<td>3</td>
<td>1.0718±2.045$^\circ$</td>
<td>0.239</td>
</tr>
<tr>
<td>4</td>
<td>1.0836±2.392$^\circ$</td>
<td>0.253</td>
</tr>
<tr>
<td>5</td>
<td>1.0881±2.516$^\circ$</td>
<td>0.247</td>
</tr>
<tr>
<td>6</td>
<td>1.0862±2.4609$^\circ$</td>
<td>0.249</td>
</tr>
<tr>
<td>7</td>
<td>1.0870±2.485$^\circ$</td>
<td>0.248</td>
</tr>
<tr>
<td>8</td>
<td>1.0866±2.474$^\circ$</td>
<td>0.249</td>
</tr>
<tr>
<td>9</td>
<td>1.0868±2.479$^\circ$</td>
<td>0.248</td>
</tr>
<tr>
<td>10</td>
<td>1.0867±2.477$^\circ$</td>
<td>0.249</td>
</tr>
<tr>
<td>11</td>
<td>1.0868±2.478$^\circ$</td>
<td>0.248</td>
</tr>
<tr>
<td>12</td>
<td>1.0868±2.477$^\circ$</td>
<td>0.249</td>
</tr>
</tbody>
</table>

Table 6 shows the iteration results of the internal algorithm. $P_1^\nu$ is the injected power of the DG into bus 1 and $g_{P_1}(V_1^\nu)$ is the slope of the $P(V)$ characteristic at $V_1^\nu$. During initialization ($\nu = 0$) the injected power is set to $P_1^0 = 0.350$ which is the total power accessible. In the first Newton iteration ($\nu = 1$), the resulting voltage is $1.1342\angle 3.791^\circ$ which is on the far right of the $P(V)$ characteristic and thus the injected power is $P_1^1 = 0.200$. During the next Newton iteration ($\nu = 2$) the voltage drops again to $1.0746\angle 2.108^\circ$ which is left of the point $V_3$ of the $P(V)$ characteristic according to Table 3. Thus the injected power is increased to its maximum value of $P_1^2 = 0.35$. During the third iteration ($\nu = 3$) the voltage is $1.0974\angle 2.774^\circ$ and thus the injected power is set to its minimum value of $P_1^3 = h_{ln0.2}$. During these first 3 iterations, the Jacobian $J_1$ is always zero because the voltage was not on the slope of the $P(V)$ characteristic. In the fourth iteration ($\nu = 4$) the voltage is $1.0845\angle 2.407^\circ$ and for the first time, the voltage is such that the operating point is on the slope of the $P(V)$ characteristic. Therefore the injected power is set to $P_1^4 = 0.282$. The Jacobian is thus nonzero. It is

Table 6: Interim results for each internal iteration $\nu$ of the internal algorithm for $P(V)$ characteristic of the simulated 2-bus system

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$V_1^\nu$</th>
<th>$P_1^\nu$</th>
<th>$g_{P_1}(V_1^\nu)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$120^\circ$</td>
<td>0.0350</td>
<td>0.0</td>
</tr>
<tr>
<td>1</td>
<td>1.1342±3.791$^\circ$</td>
<td>0.0200</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>1.0746±2.108$^\circ$</td>
<td>0.0350</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>1.0974±2.774$^\circ$</td>
<td>0.0200</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>1.0845±2.407$^\circ$</td>
<td>0.0282</td>
<td>-1.50</td>
</tr>
<tr>
<td>5</td>
<td>1.0869±2.482$^\circ$</td>
<td>0.0246</td>
<td>-1.50</td>
</tr>
<tr>
<td>6</td>
<td>1.0868±2.477$^\circ$</td>
<td>0.0249</td>
<td>-1.50</td>
</tr>
</tbody>
</table>

Figure 8: Inverse nose curve and $P(V)$ characteristic (top) and resulting active power mismatch function (bottom) for the simulated two-bus system.
Inverter
DC Source
Cable

\[
J_{b11} = \begin{pmatrix} -1.5 & 0 \\ 0 & 0 \end{pmatrix},
\]
\[
J_{b21} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.
\]

Finally, the Jacobian due to the DG is

\[
J_b' = \begin{pmatrix} -1.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.
\]

The Jacobian \( J_a \) of the system is computed as usually during a power flow algorithm. The process is repeated until sufficient accuracy is reached.

4.1.2. Experimental Validation

The algorithm was verified with an experimental setup of the two-bus system as shown in Fig. 9. A DC source which mimics the properties of a string of solar panels was connected to a 4.6 kW SMA Sunny Boy inverter. The inverter was connected to an AMETEK grid emulator via a cable. The resistance and inductance of the cable was measured as \( R = 0.41 \Omega \) and \( L = 672 \mu \Omega \).

The voltage \( V_1 \) was varied. Due to this variation, the \( Q(V) \) and \( P(V) \) injection of the inverter was activated. The measured \( Q(V) \) and \( P(V) \) characteristics of the inverter are shown in Fig. 10. Fig. 11 shows the measurement values and the simulated values of \( V_1 \) and \( V_2 \). It can be observed that the measured and the simulated values coincide. It is crucial to investigate new inverter capabilities like voltage dependent active and reactive power injection also in a practical lab setup.

4.2. Performance Analysis

A performance analysis of the internal and external algorithm was done with two test systems.

The first test system is the IEEE 118-bus system which represents a portion of the American Electric Power System. The data of this test system is given in [31]. In this system, 54 synchronous generators with power and voltage control are present. As the goal of the investigation is to test voltage dependent power injections with \( P(V) \) and \( Q(V) \) characteristics, all synchronous generators were removed and replaced with constant power injections. In the next step, 50 DGs with \( P(V) \) and \( Q(V) \) characteristics were placed randomly in the grid. The total nominal power of the DGs was chosen to be 10% of the total power of the original generators. This value was chosen in order to obtain significantly high voltages in the range of 1.03 to 1.1 pu, where the voltage characteristics of the DGs have an effect. The description of the voltage characteristics is given in Table 7. The table contains the \( P(V) \) and \( Q(V) \) characteristics for all generators where \( P_n \) is the nominal power of the DG and \( d \) as well as \( e \) are parameters that are specified for each case. By varying these parameters, different characteristics are obtained. The simulations were performed in two setups S1 and S2. In setup S1, all DGs were given the same \( P(V) \) and \( Q(V) \) characteristics. Specifically we chose \( P_n = 0.0875 \text{pu} \), \( d = 0 \) and \( e = 0 \). In setup S2 all characteristics are chosen differently in order to investigate the influence of different characteristics on the runtime. We have set \( d = 0.02/50 \cdot (i - 1) \) and \( e = 0.01/50 \cdot (25.5 - i) \) with \( i = 1, \ldots, 50 \) being the index of the DG.

The second test system is a real German 234-bus grid. In this system, 108 DGs with \( P(V) \) and \( Q(V) \) characteristics are included. Similar to the test with the IEEE 118-bus system, two setups S1 and S2 are investigated. For both setups we have set \( P_n = 0.0153 \text{pu} \). For setup S1, all generators were given the same characteristics, thus \( d = 0 \) and \( e = 0 \). For setup S2, different characteristics were selected for all DGs, specifically \( d = 0.02/108 \cdot (i - 1) \) and \( e = 0.01/108 \cdot (54.5 - i) \) with \( i = 1, \ldots, 108 \) being the index of the DG. The values chosen are similar to practi-
Simulations of each setup where performed for 40 different random locations of DGs. For each setting, ten simulation runs were performed. The average values and variances of all simulation run times were computed. The results for the complex bus voltages of the internal variances of all simulation run times were computed. The simulation runs were performed. The average values and different random locations of DGs. For each setting, ten values and variances.

Table 8: Comparison of external and internal algorithm (average iteration which took an average of 6 Newton iterations. The advantage of the internal algorithm can be found in

iterative algorithm. The filter value was chosen such that it was the minimal value for which all simulations still converged. Lower filter values resulted in a significant amount of simulations that did not converge. Higher filter values only increased the computational time. Therefore, the chosen filter value was the best possible choice concerning computational speed and convergence. The results for the run time are displayed in Table 8. It shows the average computation time, the total number of Newton iterations and the number of power flows for each algorithm. For example in case of the IEEE 118-bus system the external algorithm requires an average number of 44 power flows to converge. This results in an average total number of 102 Newton iterations. This means that on average, a power flow calculation needed 102/44 = 2.3 iterations to converge. The internal algorithm needed only one power flow iteration which took an average of 6 Newton iterations. The advantage of the internal algorithm can be found in both the run time and the average total number of Newton iterations. The internal algorithm is between 8 and 14 times faster than the external algorithm and the external algorithm needs between 11 to 17 times more Newton iterations.

5. Numerical Example

In this section a numerical example of a low-voltage feeder with photovoltaic systems is presented. The feeder topology can be seen in Fig. 12. It consists of 19 nodes and 5 PV systems. The branch data is given in Table 10 in per unit. The base power is 1 MV A and the base voltage is 230 V. The slack bus is located at bus 1 and has a fixed voltage of 1.0420 pu. A total of seven loads, each having a power of 3 kW are connected to the feeder. This is also listed in Table 10. The PV systems are located at buses 9, 11, 13, 15, 17, having an available active power infeed between 30 and 38 kW. Their voltage dependent active and reactive power characteristics are depicted in Table 14. A power flow is conducted for three cases: In the first case, all photovoltaic systems are disconnected from the feeder. This cases serves as a reference scenario in order to verify the bus voltages for the case in which only the loads are connected. In the second case the photovoltaic systems feed in their full active power without voltage dependent power injection. In the third case all photovoltaic systems are connected and the voltage dependent active and reactive power injection is active.

The results for the bus voltages for each case can be seen in Table 11. The injected active powers are depicted in Table 12 and the injected reactive power are depicted in Table 13. The example is designed such that voltages

<table>
<thead>
<tr>
<th>Injection</th>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_3$</th>
<th>$V_4$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>method</td>
<td>[pu]</td>
<td>[pu]</td>
<td>[pu]</td>
<td>[pu]</td>
<td>[pu]</td>
<td>[pu]</td>
<td>[pu]</td>
</tr>
<tr>
<td>$P(V)$</td>
<td>1.08 $-d$</td>
<td>1.08 $-d$</td>
<td>1.08 $-d$</td>
<td>1.09 $-d$</td>
<td>$P_n + e$</td>
<td>$P_n + e$</td>
<td>($P_n + e$) · 0.2</td>
</tr>
<tr>
<td>$Q(V)$</td>
<td>0.92 $-d$</td>
<td>0.97 $-d$</td>
<td>1.03 $-d$</td>
<td>1.08 $-d$</td>
<td>($P_n + e$) · 0.5</td>
<td>0.000</td>
<td>($P_n + e$) · 0.5</td>
</tr>
</tbody>
</table>

Table 7: Voltage characteristics for the performance test of the IEEE 118-bus test system and the real German 234-bus grid.

![Figure 12: Topology of the example Grid. Photovoltaic systems are highlighted in red.](image)
The method was named internal algorithm in order to distinguish it from the state-of-the-art method, which is called external algorithm in this work. We have compared the internal algorithm with the commonly used external algorithm. The comparison was done with respect to convergence properties for an ill-conditioned two-bus system and with respect to computational complexity for the IEEE 118-bus test system and for a real German 234-

Table 11: Bus voltages in per unit for the three cases of the example.

<table>
<thead>
<tr>
<th>Bus</th>
<th>Case 1 (no PV)</th>
<th>Case 2 (Full)</th>
<th>Case 3 (P(V) and Q(V))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.040∠0°</td>
<td>1.040∠0°</td>
<td>1.040∠0°</td>
</tr>
<tr>
<td>2</td>
<td>1.040∠0°</td>
<td>1.042∠0.7°</td>
<td>1.038∠0.4°</td>
</tr>
<tr>
<td>3</td>
<td>1.031∠0°</td>
<td>1.098∠1.2°</td>
<td>1.066∠1.6°</td>
</tr>
<tr>
<td>4</td>
<td>1.031∠0°</td>
<td>1.098∠1.2°</td>
<td>1.066∠1.6°</td>
</tr>
<tr>
<td>5</td>
<td>1.027∠0°</td>
<td>1.128∠1.4°</td>
<td>1.081∠2.2°</td>
</tr>
<tr>
<td>6</td>
<td>1.027∠0°</td>
<td>1.131∠1.5°</td>
<td>1.082∠2.2°</td>
</tr>
<tr>
<td>7</td>
<td>1.026∠0°</td>
<td>1.131∠1.5°</td>
<td>1.084∠2.3°</td>
</tr>
<tr>
<td>8</td>
<td>1.026∠0°</td>
<td>1.133∠1.5°</td>
<td>1.085∠2.3°</td>
</tr>
<tr>
<td>9</td>
<td>1.026∠0°</td>
<td>1.135∠1.5°</td>
<td>1.084∠2.3°</td>
</tr>
<tr>
<td>10</td>
<td>1.026∠0°</td>
<td>1.137∠1.6°</td>
<td>1.085∠2.4°</td>
</tr>
<tr>
<td>11</td>
<td>1.026∠0°</td>
<td>1.137∠1.6°</td>
<td>1.085∠2.4°</td>
</tr>
<tr>
<td>12</td>
<td>1.026∠0°</td>
<td>1.138∠1.6°</td>
<td>1.086∠2.4°</td>
</tr>
<tr>
<td>13</td>
<td>1.026∠0°</td>
<td>1.138∠1.6°</td>
<td>1.086∠2.4°</td>
</tr>
<tr>
<td>14</td>
<td>1.026∠0°</td>
<td>1.140∠1.6°</td>
<td>1.086∠2.4°</td>
</tr>
<tr>
<td>15</td>
<td>1.026∠0°</td>
<td>1.138∠1.6°</td>
<td>1.086∠2.4°</td>
</tr>
<tr>
<td>16</td>
<td>1.026∠0°</td>
<td>1.140∠1.6°</td>
<td>1.087∠2.4°</td>
</tr>
<tr>
<td>17</td>
<td>1.026∠0°</td>
<td>1.138∠1.6°</td>
<td>1.086∠2.4°</td>
</tr>
</tbody>
</table>

Table 12: Injected active power of the photovoltaic systems for the three cases of numerical example.

<table>
<thead>
<tr>
<th>Bus</th>
<th>Case 1 (no PV)</th>
<th>Case 2 (Full)</th>
<th>Case 3 (P(V) and Q(V))</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0</td>
<td>30</td>
<td>21.0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>32</td>
<td>20.2</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>34</td>
<td>19.7</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>36</td>
<td>19.4</td>
</tr>
<tr>
<td>17</td>
<td>0</td>
<td>38</td>
<td>19.4</td>
</tr>
</tbody>
</table>

Table 13: Injected reactive power of the photovoltaic systems for the three cases of numerical example.

<table>
<thead>
<tr>
<th>Bus</th>
<th>Case 1 (no PV)</th>
<th>Case 2 (Full)</th>
<th>Case 3 (P(V) and Q(V))</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>8.0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>8.0</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
<td>8.0</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0</td>
<td>8.0</td>
</tr>
<tr>
<td>17</td>
<td>0</td>
<td>0</td>
<td>8.0</td>
</tr>
</tbody>
</table>

6. Conclusion and Outlook

In this paper, we presented a method to integrate voltage dependent power injections of DGs into the power flow. The method was named internal algorithm in order to distinguish it from the state-of-the-art method, which is called external algorithm in this work. We have compared the internal algorithm with the commonly used external algorithm. The comparison was done with respect to convergence properties for an ill-conditioned two-bus system and with respect to computational complexity for the IEEE 118-bus test system and for a real German 234-
bus network. Furthermore, we performed a laboratory experiment to verify the internal algorithm and provided a numerical example.

The main contribution of this paper is the integration of voltage dependent power characteristics into the power flow. From a mathematical point, this approach is obvious. However, to the knowledge of the authors, this has not been reported in literature before. Furthermore, this work shows that the integration of voltage dependent power injections into the power flow can lead to ill-conditioned power mismatch functions. Therefore, a damped Newton method is used in order to solve the power flow problem.

In the paper it is demonstrated that the incorporation of voltage dependent power injections into the power flow has major advantages over the external algorithm. First of all, it allows integration of voltage dependent power injections with minimal effort in analyses where the Jacobian matrix is required, e.g., voltage stability problems. A further advantage is the improved robustness compared to the external algorithm. The filter \( \zeta \) of the external algorithm needs to be tuned and it is a trade-off between speed and robustness. In case of the internal algorithm, ill-conditioned cases can be treated with robust power flow methods and no manual tuning is required. For this, the bisection Newton method was used. Another advantage is the improved computational efficiency. This allows significant speed-ups in cases where thousands of power flows have to be solved, e.g., in power system planning and operation.

### Appendix

Derivation of inverse nose curve: The single phase complex power consumption at bus 1 in Fig. 5 is

\[
\tilde{S} = P + jQ = \Delta V^2. \tag{27}
\]

The current can be calculated as \( I = (V - V_{\text{slack}})V = (V - 1) V \) with \( V = \sqrt{(R + jX)^{-1} = G + jB} \) being the line admittance. The capacitance of the line is neglected. Taking real and imaginary part of (27) results in active and reactive powers. After reordering we obtain:

\[
\begin{align*}
P &= GV^2 - GV \cos(\delta) - BV \sin(\delta) \quad \text{(28a)} \\
Q &= BV^2 + BV \cos(\delta) - GV \sin(\delta). \quad \text{(28b)}
\end{align*}
\]

Taking the square of (28a) and (28b) and summing them up removes the angle dependency. Solving it for the active power finally leads to the inverse nose curve

\[
P(V) = GV^2 \pm \sqrt{(G^2 + B^2 - 2BQ)V^2 - B^2V^4 - Q^2}, \tag{29}
\]

which is approximately linear in the range where it is plotted in Fig. 8.

### Acknowledgement

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### References


[5] Voltage characteristics of electricity supplied by public distribution systems, European Standard EN 50160, CENELEC.


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Table 14: Voltage dependent active and reactive power characteristics for the photovoltaic systems connected to the example low voltage feeder.


[22] VDE-AR-N 4120:2015-01 , Technical requirements for the connection and operation of customer installations to the high-voltage network (TCC High-Voltage), VDE Standards.


