Hierarchical distributed scenario-based model predictive control of interconnected microgrids

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Electric power networks are changing

Today

Future


Energy systems are changing

- Increasing amount of renewable generators
- Transition
  - from a small number of large-scale units
  - to a large number of small-scale units
- Uncertainty in generation will increase with installed renewable units

⇒ Need to cope with fluctuations and changing structure

Sources: [Kohleausstiegsgesetz, 2020, Erneuerbare-Energien-Gesetz, 2021]
Future power networks

Conventional RES Storage Load

MG 1 MG 2

MG 4 MG 3

Grid coupling

MG 1 → MG 2 → MG 3 → MG 4
Existing certainty equivalence hierarchical distributed MPC [Hans et al., 2019]
Novel scenario-based hierarchical distributed MPC [Schenck and Hans, 2024]

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Modelling of uncertainties

\[ \begin{align*}
&\text{– Probabilities of tree (by design)} \\
&\quad \text{• Probabilities of stage } j \in \mathbb{N}_{[0,J]} \\
&\quad \quad \sum_{m \in \text{nodes}_i(j)} \pi^{(m)} = 1 \quad (1a) \\
&\quad \text{• Probabilities of child nodes of ancestor } m \in \mathcal{M}_i \setminus \text{nodes}_i(J) \\
&\quad \quad \sum_{m_+ \in \text{child}_i(m)} \pi^{(m_+)} = \pi^{(m)} \quad (1b) \\
&\text{– Nonanticipativity constraint} \\
&\quad v_i^{(m)} = v_i^{(n)} \quad \forall n \in \text{child}_i(\text{anc}_i(m)). \quad (2)
\end{align*} \]
Power- and setpoint-related constraints

- Renewable energy sources
  \[
  p_{r,i}^{\text{min}} \leq u_{r,i}^{(m)} \leq p_{r,i}^{\text{max}}, \quad (3a)
  \]
  \[
  p_{r,i}^{\text{min}} \leq p_{r,i}^{(m)} \leq p_{r,i}^{\text{max}}. \quad (3b)
  \]

- Storage units
  \[
  p_{s,i}^{\text{min}} \leq u_{s,i}^{(m)} \leq p_{s,i}^{\text{max}}, \quad (4a)
  \]
  \[
  p_{s,i}^{\text{min}} \leq p_{s,i}^{(m)} \leq p_{s,i}^{\text{max}}. \quad (4b)
  \]

- Conventional rotating units
  \[
  \text{diag}(p_{t,i}^{\text{min}}) \delta_{t,i}^{(m)} \leq u_{t,i}^{(m)} \leq \text{diag}(p_{t,i}^{\text{max}}) \delta_{t,i}^{(m)}, \quad (5a)
  \]
  \[
  \text{diag}(p_{t,i}^{\text{min}}) \delta_{t,i}^{(m)} \leq p_{t,i}^{(m)} \leq \text{diag}(p_{t,i}^{\text{max}}) \delta_{t,i}^{(m)}. \quad (5b)
  \]

- Point of common coupling (PCC)
  \[
  p_{g,i}^{\text{min}} \leq p_{g,i}(j) \leq p_{g}^{\text{max}}. \quad (6)
  \]
Modelling of dynamical system behaviour

- Storage dynamics and limits
  \[ x_i^{(m)} = x_i^{(m-)} - T_s p_{s,i}^{(m)} \]
  \[ x_i^{\min} - \sigma_i^{(m)} \leq x_i^{(m)} \leq x_i^{\max} + \sigma_i^{(m)} \]

- Steady-state approximations of lower control layers
  - Power limit of renewable energy sources
    \[ p_{r,i}^{(m)} = \min(u_{r,i}^{(m)}, w_{r,i}^{(m)}) \]
  - Power sharing of grid-forming storage & conventional
    \[ \text{diag}(\mathcal{X}_{i,1}, \ldots, \mathcal{X}_{i,T_i})^{-1}(p_{r,i}^{(m)} - u_{r,i}^{(m)}) = \rho_i^{(m)} \delta_{t,i}^{(m)} \]
    \[ \text{diag}(\mathcal{X}_{i,(T_i+1)}, \ldots, \mathcal{X}_{i,(T_i+S_i)})^{-1}(p_{s,i}^{(m)} - u_{s,i}^{(m)}) = \rho_i^{(m)} \mathbf{1}_{S_i} \]
  - Power controller at the PCC
    \[ p_{g,i}(\text{stage}_{i}^{(m)}) = -(1_{R_i}^T p_{r,i}^{(m)} + 1_{T_i}^T p_{r,i}^{(m)} + 1_{S_i}^T p_{s,i}^{(m)} + 1_{D_i}^T w_{d,i}^{(m)}) \]

Decision variables of MG \( i \in \mathbb{I} \)

<table>
<thead>
<tr>
<th>Control input</th>
<th>( \mathbf{v}<em>i = [u</em>{t,i}^T, u_{s,i}^T, u_{r,i}^T, \delta_{t,i}^T]^T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>( \mathbf{p}<em>i = [p</em>{t,i}^T, p_{s,i}^T, p_{r,i}^T, p_{g,i}]^T )</td>
</tr>
<tr>
<td>Stored energy</td>
<td>( x_i )</td>
</tr>
<tr>
<td>Uncertain input</td>
<td>( w_i = [w_{d,i}^T, w_{r,i}^T]^T )</td>
</tr>
</tbody>
</table>
Modelling of interconnecting power lines

- DC power flow approximations for AC grids [Purchala et al., 2005].

\[
\begin{bmatrix}
    p_{e,1}(j) \\
    p_{e,2}(j) \\
    p_{e,3}(j) \\
    p_{e,4}(j)
\end{bmatrix}
= 
\begin{bmatrix}
    0 & 1 & 0 & 0 \\
    0 & 0 & 2/3 & 1/3 \\
    0 & 0 & 1/3 & 2/3 \\
    0 & 0 & -1/3 & 1/3
\end{bmatrix}
\begin{bmatrix}
    p_{g,1}(j) \\
    p_{g,2}(j) \\
    p_{g,3}(j) \\
    p_{g,4}(j)
\end{bmatrix}
\]

(12a)

\[0 = 1_i^T p_g(j)\]  
(12b)

- Line power limit

\[p_e^{\text{min}} \leq p_e(j) \leq p_e^{\text{max}}.\]  
(12c)
Operating costs

- Costs of individual MGs

\[
\ell_i = \sum_{m \in I_i} \pi_j^{(m)} (\ell_{r,i}^{(m)} + \ell_{s,i}^{(m)} + \ell_{t,i}^{(m)} + \ell_{sw,i}^{(m)} + \ell_{g,i}^{(m)}) \gamma^{\text{stage}_i(m)}. \tag{13}
\]

- Cost for power transmission

\[
\ell_e = \sum_{j=1}^{J} p_{e}(j) C_{e} p_{e}(j) \cdot \gamma^{j}. \tag{14}
\]
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Problem 1 (Central mixed-integer MPC)

minimize $\ell_e + \sum_{i \in \mathbb{I}} \ell_i$

subject to
eqs. (2) to (12) for all $m \in \mathbb{I}_i$
as well as initial conditions $x_{i}^{(0)} = x_i(k)$, $\delta_{t,i}^{(0)} = \delta_{t,i}(k)$
for all $i \in \mathbb{I}$.

Does not scale well

Mean solver time for simple example grid already $> 2$ minutes
Problem 2 (Central relaxed problem MPC)

\[
\begin{align*}
\text{minimize} & \quad \ell_e + \sum_{i \in I} \ell_i \\
\text{subject to} & \quad \text{eqs. (2) to (12) for all } m \in \mathbb{L}_i \\
& \quad \text{as well as initial conditions } x_i^{(0)} = x_i(k), \, \delta_{t,i}^{(0)} = \delta_{t,i}(k) \\
& \quad \text{with } \delta_{t,i}^{(m)} \in [0, 1]^{T_i} \text{ and } \delta_{r,i}^{(m)} \in [0, 1]^{R_i} \text{ for all } m \in \mathbb{L}_i \\
& \quad \text{and all } i \in I.
\end{align*}
\]
Algorithm 1 (Hierarchical distributed MPC)

1. **Initialize:** At time $k$, $\forall i \in \mathbb{I}$, measure $x_i(k), \delta_{t,i}(k)$ and obtain scenario tree.
2. **ADMM loop:** for $l = 0, \ldots, l_{\text{max}} \in \mathbb{N}$:
   
   (i) For all MGs $i \in \mathbb{I}$ (in parallel):
   
   $\Rightarrow$ Solve Problem 3 in parallel to obtain $P_{g,i}^{l+1}$.
   
   $\Rightarrow$ Send $P_{g,i}^{l+1}$ to central entity.

   (ii) Central entity:
   
   $\Rightarrow$ Solve Problem 4 to obtain $\hat{P}_{g,i}^{l+1}$.
   
   $\Rightarrow$ Update Lagrange multipliers:
   
   $\Lambda^{l+1}_i = \Lambda^l_i + \kappa (P_{g,i}^{l+1} - \hat{P}_{g,i}^{l+1})$.
   
   $\Rightarrow$ Communicate $\hat{P}_{g,i}^{l+1}$ and $\Lambda^{l+1}_i$ to all MGs $i \in \mathbb{I}$.
   
   $\Rightarrow$ Check termination criterion:
   
   if $(|\Lambda^l_i - \Lambda^{l+1}_i| < \epsilon$ and $|P_{g,i}^{l+1} - P_{g,i}^{l+1}| < \epsilon$ and $|P_{g,i}^{l+1} - \hat{P}_{g,i}^{l+1}| < \epsilon)$ or $l = l_{\text{max}}$,
   
   then set $P_{g,i}^* = \hat{P}_{g,i}^{l+1}$ and go to 3.

3. **Mixed-integer update:** For all microgrid (MG) $i \in \mathbb{I}$ (in parallel):
   
   - Solve Problem 5.
Algorithm 1 (Hierarchical distributed MPC)

1. **Initialize:** At time $k$, $\forall i \in \mathbb{I}$, measure $x_i(k)$, $\delta_{t,i}(k)$ and obtain scenario tree.

2. **ADMM loop:** for $l = 0, \ldots, l_{\text{max}} \in \mathbb{N}$:
   (i) For all MGs $i \in \mathbb{I}$ (in parallel):
       » Solve Problem 3 in parallel to obtain $\hat{P}_{g,i}^{l+1}$.
       » Send $\hat{P}_{g,i}^{l+1}$ to central entity.
   (ii) Central entity:
       » Solve Problem 4 to obtain $\hat{P}_{g}^{l+1}$.
       » Update Lagrange multipliers:
         \[ \Lambda_{l+1}^{i} = \Lambda_{l}^{i} + \kappa \left( \hat{P}_{g,i}^{l+1} - \hat{P}_{g,i}^{l+1} \right) \]
         
       » Communicate $\hat{P}_{g,i}^{l+1}$ and $\Lambda_{l+1}^{i}$ to all MGs $i \in \mathbb{I}$.
       » Check termination criterion:
         \[ \left| \Lambda_{l}^{i} - \Lambda_{l+1}^{i} \right| < \epsilon \]
         \[ \left| \hat{P}_{g,i}^{l+1} - \hat{P}_{g,i}^{l+1} \right| < \epsilon \]
         \[ \left| \hat{P}_{g,i}^{l+1} - \hat{P}_{g,i}^{l+1} \right| < \epsilon \]
         
         if $l = l_{\text{max}}$

3. **Mixed-integer update:** For all MG $i \in \mathbb{I}$ (in parallel):
   • Solve Problem 5.
Algorithm 1 (Hierarchical distributed MPC)

1. **Initialize:** At time $k$, $\forall i \in \mathbb{I}$, measure $x_i(k)$, $\delta_{t,i}(k)$ and obtain scenario tree.

2. **ADMM loop:** for $l = 0, \ldots, l_{\text{max}} \in \mathbb{N}$:
   
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   - Send $P_{g,i}^{l+1}$ to central entity.

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   - Solve Problem 4 to obtain $\hat{P}_{g,i}^{l+1}$.
   - Update Lagrange multipliers:
     $$\Lambda_{i}^{l+1} = \Lambda_{i}^{l} + \kappa (P_{g,i}^{l+1} - \hat{P}_{g,i}^{l+1}).$$
   - Communicate $\hat{P}_{g,i}^{l+1}$ and $\Lambda_{i}^{l+1}$ to all MGs $i \in \mathbb{I}$.
   - Check termination criterion:
     
     if $|\Lambda_{i}^{l} - \Lambda_{i}^{l+1}| < \epsilon$ and $|P_{g,i}^{l} - P_{g,i}^{l+1}| < \epsilon$ and $|P_{g,i}^{l+1} - \hat{P}_{g,i}^{l+1}| < \epsilon$ or $l = l_{\text{max}}$,
     
     then set $P_{g,i}^{*} = \hat{P}_{g,i}^{l+1}$ and go to 3.

3. **Mixed-integer update:** For all MG $i \in \mathbb{I}$ (in parallel):
   
   - Solve Problem 5.

---

**Problem 5 (Mixed-integer update at MG $i \in \mathbb{I}$)**

$$\minimize_{\ell_{i}} \ell_{i}$$

subject to
eqs. (2) to (5) and (7) to (11) for all $m \in \mathbb{I}_{i}$, as well as initial conditions $x_{i}^{(0)} = x_{i}(k)$, $\delta_{t,i}^{(0)} = \delta_{t,i}(k)$ with fixed $P_{g,i}^{*} = P_{g,i}^{x}$. 
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Closed-loop simulation results of novel approach

(a) MG 1
(b) MG 2
(c) MG 4
(d) MG 3
(e) Stored energy
(f) Grid power

Power in pu
Time in d
Energy in pu h
Stored energy out of desired area
### Numerical comparison

<table>
<thead>
<tr>
<th></th>
<th>Certainty equival.</th>
<th>Stochastic (Alg. 1)</th>
<th>Prescient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Renewable energy in pu h</td>
<td>318.1</td>
<td>333.8</td>
<td>334.0</td>
</tr>
<tr>
<td>Conventional energy in pu h</td>
<td>61.0</td>
<td>45.2</td>
<td>45.9</td>
</tr>
<tr>
<td>No. of switching actions</td>
<td>46</td>
<td>40</td>
<td>29</td>
</tr>
<tr>
<td>Costs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MG 1</td>
<td>1 652.8</td>
<td>1 005.9</td>
<td>786.8</td>
</tr>
<tr>
<td>MG 2</td>
<td>1 366.1</td>
<td>774.2</td>
<td>607.7</td>
</tr>
<tr>
<td>MG 3</td>
<td>2 000.2</td>
<td>1 086.1</td>
<td>1 003.8</td>
</tr>
<tr>
<td>MG 4</td>
<td>1 755.3</td>
<td>1 154.3</td>
<td>1 109.9</td>
</tr>
<tr>
<td>Transmission</td>
<td>21.7</td>
<td>19.0</td>
<td>18.7</td>
</tr>
<tr>
<td>Sum</td>
<td>6 796.1</td>
<td>4 039.5</td>
<td>3 527.0</td>
</tr>
</tbody>
</table>
Numerical properties

Algorithm 1

- Only 0.3% higher costs compared to Problem 1
- Solve time:
  - Mean: 9 s (vs. 127 s of Problem 1)
  - Maximum: 223 s
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Conclusions

– Scenario-based stochastic MPC scheme for the operation of interconnected MGs
– Distributed algorithm that reflects the hierarchical power system structure
  • local controllers are in charge of individual MGs
  • central entity is in charge of the transmission grid
– Better than certainty equivalence MPC concerning number of constraint violations and costs
– Sufficiently fast convergence

Next steps:
– Scalability
– Suboptimality
– Persistent feasibility
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