Mathematical Thinking and Learning

The Relationship Between Children’s Familiarity with Numbers and Their Performance in Bounded and Unbounded Number Line Estimations

Mirjam Ebersbach\textsuperscript{a}, Koen Luwel\textsuperscript{b} & Lieven Verschaffel\textsuperscript{c}

\textsuperscript{a} University of Kassel  \\
\textsuperscript{b} Hogeschool-Universiteit Brussel and Katholieke Universiteit Leuven  \\
\textsuperscript{c} Katholieke Universiteit Leuven

Published online: 07 May 2015.

To cite this article: Mirjam Ebersbach, Koen Luwel & Lieven Verschaffel (2015) The Relationship Between Children’s Familiarity with Numbers and Their Performance in Bounded and Unbounded Number Line Estimations, Mathematical Thinking and Learning, 17:2-3, 136-154, DOI: 10.1080/10986065.2015.1016813

To link to this article: http://dx.doi.org/10.1080/10986065.2015.1016813

PLEASE SCROLL DOWN FOR ARTICLE

Taylor & Francis makes every effort to ensure the accuracy of all the information (the “Content”) contained in the publications on our platform. However, Taylor & Francis, our agents, and our licensors make no representations or warranties whatsoever as to the accuracy, completeness, or suitability for any purpose of the Content. Any opinions and views expressed in this publication are the opinions and views of the authors, and are not the views of or endorsed by Taylor & Francis. The accuracy of the Content should not be relied upon and should be independently verified with primary sources of information. Taylor and Francis shall not be liable for any losses, actions, claims, proceedings, demands, costs, expenses, damages, and other liabilities whatsoever or howsoever caused arising directly or indirectly in connection with, in relation to or arising out of the use of the Content.

This article may be used for research, teaching, and private study purposes. Any substantial or systematic reproduction, redistribution, reselling, loan, sub-licensing, systematic supply, or distribution in any form to anyone is expressly forbidden. Terms &
The Relationship Between Children’s Familiarity with Numbers and Their Performance in Bounded and Unbounded Number Line Estimations

Mirjam Ebersbach  
University of Kassel

Koen Luwel  
Hogeschool-Universiteit Brussel and Katholieke Universiteit Leuven

Lieven Verschaffel  
Katholieke Universiteit Leuven

Children’s estimation skills on a bounded and unbounded number line task were assessed in the light of their familiarity with numbers. Kindergartners, first graders, and second graders ($N = 120$) estimated the position of numbers on a 1–100 number line, marked with either two reference points (i.e., 1 and 10: unbounded condition) or three reference points (i.e., 1, 10, 100: bounded condition). Estimations were more accurate and less variable in older compared to younger children and, beyond age, in children who were familiar with larger numbers. The number of reference points yielded neither a main effect nor an interaction effect with familiarity. Our results underline that children’s familiarity with numbers is a pivotal factor for the quality of number line estimations that might even obliterate potential effects of additional reference points.

Numerical estimations involve the approximate assignment of numbers to magnitudes (or vice versa) and are a fundamental ability required in everyday life as well as in mathematical contexts (e.g., Booth & Siegler, 2006; LeFevre, Greenham, & Waheed, 1993) that predicts children’s mathematical achievement in school (e.g., Geary, 2013; Gilmore, McCarthy, & Spelke, 2010; Sasanguie, Van den Bussche, & Reynvoet, 2012). It has also been assumed that performance in numerical estimations reflect the characteristics and development of the mental representation of numbers—also called the mental number line (e.g., Dehaene, Izard, Spelke, & Pica, 2008; Siegler & Opfer, 2003). For this purpose, different mathematical models (e.g., a linear or logarithmic function) were fitted to the estimation patterns and their fits were compared to identify the model that describes the relationship between approximate estimations and actual numbers best. However, studies in this field provided conflicting results and so were the conclusions drawn on...
FAMILIARITY WITH NUMBERS

the characteristics of the mental number line (e.g., Bouwmeester & Verkoeijen, 2012; Ebersbach, Luwel, Frick, Ongena, & Verschaffel, 2008; Moeller, Pixner, Kaufmann, & Nuerk, 2009; Siegler & Opfer, 2003; Simms, Muldoon, & Towse, 2013; Slusser, Santiago, & Barth, 2012). This might be due to the fact that the properties of numerical estimation tasks often were not comparable across studies (cf. Ebersbach, Luwel, & Verschaffel, 2013). Furthermore, many studies focusing on the development of estimation skills did not assess explicitly children’s knowledge of numbers. It can therefore not ultimately be concluded whether children’s estimation performance in fact mirrors their mental representation of numbers or whether estimations are affected by children’s limited knowledge of numbers when solving estimation tasks.

One estimation paradigm that has frequently been used to investigate numerical estimation skills is the number line task, in which given numbers are assigned to corresponding positions on an external number line that is usually flanked by a lower and upper anchor (e.g., 0 and 100). It was supposed that the performance in number line tasks reflects the underlying mental representation of numbers (e.g., Dehaene et al., 2008; Siegler & Opfer, 2003; Simms et al., 2013), but this assumption has been challenged (e.g., Moeller & Nuerk, 2011; White & Szucs, 2012 for critical discussions). First, the anchors of the external number line might evoke the application of strategies including the use of those anchors for calibrating and adjusting the estimations. If a participant is presented, for instance, with 100 as the upper anchor of the number line and knows that 50 is half of 100, he or she might place 54 around the center of the number line, which does not necessarily reflect how the number 54 is in fact mentally represented. A second, related issue is the question of whether participants are familiar at all with numbers associated with the anchors. Younger children might not be able to use the upper anchor due to their limited number knowledge, while older children might already have a grasp on the meaning of the anchor’s magnitude. Accordingly, younger children might apply other estimation strategies than older children, which results in estimation patterns that differ between age groups.

The Use of Reference Points in Number Line Estimation Tasks by Children and Adults

Several studies have suggested that children use reference points in number line estimations. Barth and Paladino (2011) presented five and seven-year-olds with a 0–100 number line, on which the positions of different numbers had to be estimated. In a preceding practice trial, the experimenter additionally indicated the location of 50, explaining that 50 is half of 100 and therefore has to be placed in the middle. The relationship between estimations of five-year-olds and the actual locations could be described well by an adapted one-cycle power model resembling an S-shaped curve. This pattern suggests that five-year-olds used the lower and upper reference point as an orientation. Estimations of seven-year-olds, in contrast, were better fit by a two-cycle power model implying that they additionally used the midpoint to calibrate their estimations. The fit of cyclic power models also suggest that estimations might be based on a proportional judgment strategy instead of directly reflecting the characteristics of the mental number line. Thus, the position of a single number is estimated in relation to the whole, that is, the total number range of the task or particular reference points. However, children’s number knowledge was not assessed in this study. Therefore, it remains an open question whether their use of the reference points was dependent on their number knowledge.

White and Szucs (2012) examined children’s use of reference point-based strategies while estimating the positions of numbers on a 0–20 number line (i.e., a number range six- to eight-year-old
participants were supposed to be familiar with). Results suggested that six-year-olds used only the lower reference point as an orientation for their estimations, as indicated by a considerable fit of a logarithmic model with the estimations, which went along with the fact that estimations were most accurate and least variable close to the lower reference point. Seven-year-olds, in contrast, began to use also the upper reference point and to internally create a midpoint, which was derived from more accurate and less variable estimations for numbers also in the middle of the number range, resulting in a better fit of a linear model. This strategy became even more evident in eight-year-olds.

**Estimation Performance in Bounded Versus Unbounded Number Line Estimation Tasks**

Cohen and Blanc-Goldhammer (2011) tested adults with a classical, numerically bounded number line task ranging from 0 to 26 and with another, numerically unbounded number line task, where only 0 and 1 were marked on the left end but no reference point was provided on the right end of the line. Estimation patterns suggested that adults applied a proportional strategy in the bounded condition but a so-called dead reckoning strategy in the unbounded condition. That is, they internally generated multiples of the given unit (0–1) until the requested numerical value was approximately reached.

Link, Huber, Nuerk, and Moeller (2014) used a similar paradigm with children and found that only from third grade on did children show different estimation patterns and error distributions in the unbounded number line task compared to the bounded number line task. Children’s individual familiarity with numbers was not assessed but it was presupposed that they were familiar with the number ranges tested (i.e., first graders: 0–10, second graders: 0–20, third graders: 0–100, etc.). Cohen and Sarnecka (2014), in contrast, observed differences concerning estimation biases between three- and eight-year-old children only in the bounded number line task but not in the unbounded one. This was taken as evidence that even younger children solve bounded and unbounded number line tasks in a different manner. Cohen and Sarnecka hypothesized that children applied a division or subtraction strategy in the bounded number line task—computational skills that would still be improving within the tested age range, yielding the observed performance differences between age groups. In the unbounded number line, they assumed that children may have used repeated addition or multiplication to come to the estimations (cf. Cohen & Blanc-Goldhammer, 2011)—computational skills that the children might have already acquired according to the authors.

**Familiarity with Numbers As One Aspect of Children’s Number Knowledge**

As the use of reference points in estimation tasks requires some basic number knowledge, we will refer to this—often neglected—issue including its assessment in the following. Basic number knowledge may include counting, recognizing numbers, and magnitude understanding (e.g., Whyte & Bull, 2008). Simple counting can be assessed by asking children to count as far as they can. But instead of reflecting their robust knowledge of numbers, this task might evoke a meaningless recitation of number words in young children in particular (Fuson, 1988; Le Corre & Carey, 2007; Wynn, 1990, 1992). A more advanced task is asking children to count from given numbers (e.g., 17, 18) up to the next decade change (i.e., 20; Lipton & Spelke, 2005). This task
requires recognizing the given numbers, activating the decade structure of the number system, and producing the corresponding decade without the previous decade being mentioned explicitly. The skill behind this process has also been referred to as children’s familiarity with numbers (e.g., Ebersbach et al., 2008). Familiarity with numbers thus reflects children’s robust and flexible knowledge of numbers in contrast to simply to count as far as possible. However, simple counting as well as familiarity with numbers do not necessarily reflect children’s grasp of the underlying value of numbers, which refers to an even more proficient magnitude understanding. This skill is usually assessed by means of numerical estimations that involve the mapping of symbolic numbers to nonsymbolic magnitudes and vice versa.

At this point, the question rises how familiarity with numbers is related to children’s performance in numerical estimation tasks in general. Lipton and Spelke (2005), using a numerosity estimation task, found a linear relationship between children’s estimations and the actual nonsymbolic numerosities only if the numerosities were within children’s familiar number range. In contrast, children hardly discriminated between nonsymbolic numerosities that exceeded their familiar number range. In another study, Ebersbach and Erz (2014) showed that kindergartners’ familiarity with numbers was related to the accuracy of their numerosity estimations. However, no such relationship was found in first graders who were largely familiar with the tested number range.

The Role of Familiarity with Numbers in Number Line Estimations

Chesney and Matthews (2013) provided evidence that familiarity with numbers may affect the use of reference points of adults in number line estimations. To artificially reduce familiarity in adults, they presented the numbers to be estimated in an exponential notation (e.g., .006 × 10^{4.5}). As a result, adults’ number line estimations were logarithmically related to the actual magnitudes, even if adults were assumed to possess a linear mental representation of numbers. Usually, most adults are not familiar with such notations and have problems when mapping between these symbols and their underlying values. The logarithmic bias thus occurs due to inadequate mapping rather than due to a distorted mental representation of magnitudes. A similar effect occurred when adults were uncertain about the meaning of large (or fictitious) number words (Rips, 2013). Thus, familiarity with numbers in a more general view appears to affect number line estimations in adults and it is conceivable that the same is true for children. However, many studies that examined children’s number line estimation skills did not assess whether children were in fact familiar with numbers corresponding to the reference points or not (e.g., Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Siegler & Booth, 2004). These and other studies came to the conclusion that young children exhibit a developmental shift from a logarithmic mental number representation to a linear mental representation, but it cannot be ruled out that young children were just unfamiliar with the upper reference point and therefore could not use it for calibrating their estimations compared to older children or adults.

Nonetheless, some studies using the number line task considered children’s familiarity, such as Ebersbach and colleagues (2008), who focused only on the shape of the estimation patterns, but not on the accuracy in number line estimations. They fitted a bilinear model to the data consisting of two linear segments of a regression line with different slopes. They found an association between children’s familiarity with numbers and the number range covered by the two segments—with a better discrimination between numbers in the familiar than in the unfamiliar number range (see also Lipton & Spelke, 2005).
Newman and Berger (1984) assessed children’s familiarity with numbers and the accuracy of their number line estimations. Children from kindergarten to third grade were asked to estimate numbers that corresponded to given positions on a number line ranging from 1 to 23. Children’s familiarity with numbers up to 30 was assessed differently than in subsequent studies (e.g., Lipton & Spelke, 2005), that is, by asking children to count forward and backward in two, three, and five steps within this range. However, this approach still taps children’s flexible and robust number knowledge. Newman and Berger found that the number of correct counts in this assessment was positively related to the use of effective estimation strategies on the number line (i.e., mentally “counting” from the end points of the number line or from internally created reference points), which was in turn positively associated with more accurate estimations. However, children received feedback on their estimations during the experiment in terms of misses and hits and were allowed to repeat their estimate until they met the correct number. This might have elicited learning and calibration effects and makes the study hard to compare with more current studies.

Simms and associates (2013) examined the relationship between the accuracy of four to seven year olds in number line estimations and their simple counting skills (but not their familiarity with numbers, as defined earlier), assessed by asking children to count as high as they could and also to count up to 20 objects. Simms and associates reported a negative relationship between the highest number children could count to and the absolute error rate in number line estimations. Moreover, they fitted a bilinear model to the data (cf. Ebersbach et al., 2008). This analysis suggested that children discriminated successfully between smaller numbers by means of their estimations but rather guessed for larger numbers as inferred by a slope of zero for the second regression line. The counterintuitive finding that the meeting point of the two regression lines was still within children’s counting range contradicted the assumption of Ebersbach and colleagues (2008) that the children’s familiarity with numbers is reflected by the meeting point of the two regression lines in the bilinear model and led Simms and colleagues to assume that simple counting may not be the only indicator of children’s estimation skills.

Muldoon, Towse, Simms, Perra, and Menzies (2013) conducted a longitudinal study across a period of 30 weeks with five year olds on the relationship between simple counting skills and the shape of their estimation pattern—but not the accuracy of the estimations. Better counting skills were related to more linear estimations in the number line task. Again, a bilinear model was fitted to the estimations. The meeting point of the two linear regression lines remained constant across time, while children’s counting skills improved. This finding, too, appears to contradict the assumption that children’s counting skills are directly reflected by the shape (i.e., the meeting point in the bilinear model) of their estimations. However, children were provided not only with a lower and an upper reference point of the number line but also with the midpoint, which might have served for additional orientation and calibration (cf. Izard & Dehaene, 2008), contributing to a relatively stable meeting point of the regression lines.

Finally, LeFevre and colleagues (2013) showed in a longitudinal study that children’s knowledge of symbolic numbers including their ordering and understanding of the place value concept was predictive of their number line performance, that is, the slope of a linear regression model indicating estimation accuracy.

Taken together, children’s number knowledge—and their familiarity with numbers in particular—might play an important role in number line estimations and potentially affects not only the accuracy of estimations but also the use of given reference points.
The Current Study

The current study aimed at examining whether children’s familiarity with numbers affects their use of reference points in number line estimations. Children of three age groups were tested in two reference point conditions with a 1–100 number line task. In the unbounded condition, solely the positions of 1 and 10 were marked as reference points on the number line while in the bounded condition the position of 100 was marked as an upper reference point in addition to the positions of 1 and 10. The accuracy, variability, and shape of the estimations were considered. “One” was chosen as lower reference point (cf. Berteletti et al., 2010) as young children often have no reliable grasp of zero (Wellman & Miller, 1986; cf. Newman & Berger, 1984) because they usually start to count from one and acquire the concept of zero only later (Butterworth, 1999).

Two main hypotheses were tested: (1) that the accuracy of the estimations would be higher and their variability lower in older compared to younger children, in the bounded compared to the unbounded condition, and in children who were familiar with a larger number range compared to children who were familiar with a smaller number range, and (2) that mainly children who were familiar with the numerical value of the upper reference point (i.e., 100) would profit from it in terms of a better performance in the bounded condition.

METHOD

Participants

Participants were 40 kindergartners (20 boys, 20 girls; mean age: $M = 5.88$ years, $SD = 0.38$ years), 40 first graders (21 boys, 19 girls; mean age: $M = 6.54$ years, $SD = 0.51$ years), and 40 second graders (20 boys, 20 girls; mean age: $M = 8.03$ years, $SD = 0.41$ years), recruited from kindergartens and elementary schools located in the same middle-class neighborhoods of a medium-sized town. These age groups were chosen to cover a broad range of children’s familiarity with numbers in the tested range up to 100. Kindergartners in Germany, where the study was conducted, usually receive no structured instruction on numbers. Accordingly, their knowledge is quite variable ranging from children who do not even know all numbers up to 10 to some children who can already count to 100 (cf. Ebersbach, 2015). First graders usually learn to count and calculate with numbers in the range up to 20 and also acquire the decades up to 100, while second graders are acquainted with the full number range up to 100 (Ebersbach et al., 2008). Children participated voluntarily and with informed consent of their parents.

Design, Materials, and Procedure

Children estimated the position of given numbers on a number line represented by a 100 cm long, three-dimensional bar. In the unbounded condition, only the positions of 1 and 10—but not of 100—were marked on that number line at the corresponding positions (i.e., at the length of 1 cm and 10 cm) and small cards with the matching Arabic numerals were located beneath these positions. In the bounded condition, the position of 100 was additionally marked as upper reference point on the right end along with the corresponding Arabic numeral. A total length of
100 cm was chosen to allow for a sufficient discrimination between the different locations (see also Ebersbach et al., 2008).

A between-subjects design was used to avoid calibration and training effects. Half of the children of each age group were randomly assigned to the bounded reference point condition and the other half to the unbounded reference point condition. Sample sizes of 20 participants per condition are relatively common in comparable studies (e.g., Barth & Paladino, 2011; Booth & Siegler, 2006; Mejias & Schiltz, 2013).

The target numbers to be estimated were 17, 26, 35, 44, 53, 68, 76, 84, and 92 (i.e., one randomly chosen number per decade, except of multiples of ten). They were presented twice in two separate blocks in random order. Each number was printed as Arabic numeral on a small card, placed in the center above the number line and additionally named by the experimenter to ensure that all children grasped that number. The children were tested individually. They sat in front of the number line in the center so that they could see and reach easily each location on the line. The task was introduced as an estimation task. The experimenter first pointed to the reference points and stated their numbers. She then told the child that he or she would have to indicate the position of other numbers by placing a small pin on the corresponding position on the number line, which was measured by the experimenter via a ruler attached at the back of the bar and only visible to the experimenter. The pin was returned after the estimation to the left end of the line. The experimenter encouraged the child to provide an estimate even if he or she was uncertain about the exact position. During the experiment, the child did not receive feedback on the accuracy of the estimations, but only general motivating comments. After the main test, children’s familiarity with numbers was examined by providing them with a sequence of either two or three successive numbers (e.g., 5, 6, 7) and asking them to count on until the next decade change (cf. Lipton & Spelke, 2005). All decade changes up to 100 were consecutively tested, with the units of the starting numbers being varied. If a child failed to count to the next decade change, the same sequence was repeated once. If the child failed again, the test was terminated. The highest decade change successfully completed by a child was recorded and then averaged across children of each age group.

RESULTS

Preliminary analyses revealed first that children’s familiarity with numbers differed significantly between age groups (see Appendix A). Second, it was analyzed whether children in the two reference point conditions were comparable concerning their sociodemographic variables and familiarity with numbers, which was largely the case, except of first graders who were marginally older in the bounded than in the unbounded condition (see Appendix B). Third, it was tested whether children provided estimations that systematically increased with larger numbers, which was confirmed for the majority of first graders and all second graders (see Appendix C). In addition, three prominent mathematical models (i.e., a linear, a logarithmic, and a one-cycle power model) were fitted to the estimations to check whether the best-fitting model differed between

---

1Even if a larger number of trials would be preferable to enhance the reliability of the data, we decided not to use more trials to maintain children’s attention and motivation (for similar approaches see Berteletti et al., 2010; Siegler & Opfer, 2003).
both reference point conditions as this could indicate the use of different estimation strategies (cf. Cohen & Sarnecka, 2014). However, there were no differences between the two reference point conditions but between age groups with a logarithmic model fitting the estimations of younger children better, while the estimations of older children were better fit by a linear model, and in general a poorer model fit in younger children (see Appendix C).

In the following, our two main hypotheses are tested, namely (1) that there are effects of age, reference point condition, and familiarity with numbers on the accuracy and variability of the estimations, and (2) that only children who were familiar with numbers up to 100 are able to use the upper reference point in the bounded condition. Median estimations are shown in Figure 1.

**Effects of Age, Reference Point Condition, and Familiarity with Numbers on the Accuracy and Variability of the Estimations**

To test our first main hypothesis that age, reference point condition, and familiarity with numbers would affect the accuracy and variability of the estimations, three indices were computed. Accuracy was operationalized by two measures: (1) error rate (i.e., $ER = (\text{estimation} - \text{target number}) / \text{target number}$; cf. Crollen, Castronovo, & Seron, 2011) and (2) absolute error rate (i.e., $AER$ as the absolute value of the error rate; cf. Booth & Siegler, 2006). $ER$s and $AER$s were then averaged across target numbers. Positive mean $ER$s indicate overestimations and negative mean $ER$s indicate underestimations. However, when averaged across numbers or participants, these deviations might cancel each other out. Therefore, mean $AER$s were calculated that reflect the overall accuracy of the estimations irrespectively of over- or underestimations. The variability of estimations was assessed as standard deviations ($SD$) that were first computed separately per child and number and then averaged across numbers for each child.

Three stepwise hierarchical regression analyses were computed to investigate whether the familiar number range and the number of reference points would explain variance of the mean $ER$s, mean $AER$s, and the mean $SD$s. As there was a significant correlation between age in months across all age groups and familiarity with numbers, $r(120) = 0.66$, $p < 0.001$, age was entered in a first step in each of the regression analyses and familiar number range and number of reference points in a second step. All models were significant (see Table 1) with age as a significant
TABLE 1
Stepwise Regression Analyses to Predict Mean Error Rate (Mean ER), Mean Absolute Error Rate (Mean AER), and Mean Standard Deviation (Mean SD)

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Model</th>
<th>corrR²</th>
<th>F</th>
<th>p</th>
<th>Predictors</th>
<th>β</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean ER</td>
<td>1</td>
<td>0.34</td>
<td>61.02</td>
<td>&lt; 0.001</td>
<td>Age</td>
<td>−0.58</td>
<td>−7.89</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.46</td>
<td>49.64</td>
<td>&lt; 0.001</td>
<td>Age</td>
<td>−0.28</td>
<td>−3.13</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Familiar number range</td>
<td>−0.46</td>
<td>−5.06</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Reference points</td>
<td>−0.10</td>
<td>−1.42</td>
<td>0.156</td>
</tr>
<tr>
<td>mean AER</td>
<td>1</td>
<td>0.55</td>
<td>143.65</td>
<td>&lt; 0.001</td>
<td>Age</td>
<td>−0.74</td>
<td>−12.11</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.71</td>
<td>141.79</td>
<td>&lt; 0.001</td>
<td>Age</td>
<td>−0.40</td>
<td>−6.04</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Familiar number range</td>
<td>−0.52</td>
<td>−7.85</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Reference points</td>
<td>−0.02</td>
<td>−0.35</td>
<td>0.730</td>
</tr>
<tr>
<td>mean SD</td>
<td>1</td>
<td>0.45</td>
<td>96.94</td>
<td>&lt; 0.001</td>
<td>Age</td>
<td>−0.67</td>
<td>−9.85</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.56</td>
<td>74.55</td>
<td>&lt; 0.001</td>
<td>Age</td>
<td>−0.38</td>
<td>−4.69</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Familiar number range</td>
<td>−0.44</td>
<td>−5.39</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Reference points</td>
<td>−0.07</td>
<td>−1.18</td>
<td>0.240</td>
</tr>
</tbody>
</table>

Note. Model 1: df: 1, 118; Model 2: df: 2, 119.

predictor and familiar number range as further predictor explaining additional variance of the data. Older children and, beyond that, children who were familiar with a larger number range made more accurate and less variable estimations, while there was no significant effect of the number of reference points.

Comparing the Performance of Children Who Were Familiar and Being Not Familiar with Numbers up to 100 in the Bounded and Unbounded Condition

The following analyses served to test the second main hypothesis that in particular those children who were familiar with the number of the upper reference point would profit from it in the bounded condition. Three subordinate aspects of this main hypothesis were pursued, that is, whether children who were familiar with numbers up to 100 made in the bounded condition (a) more accurate and less variable estimations, (b) more systematic estimations that could be described by a mathematical function, and (c) more accurate estimations across the whole number range. In comparison to the previous analyses, in which the effect of familiarity with numbers as a gradual variable was tested, familiarity was now coded dichotomously. Using the data from the familiarity test, we classified children in being either familiar with numbers up to 100 (i.e., those who successfully solved the highest decade change to 100) or not.

Concerning the first aspect that only children who were familiar with numbers up to 100 would make more accurate and less variable estimations in the bounded condition, a MANCOVA was calculated with the number of reference points and familiarity with 100 (yes vs. no) as independent variables, age as covariate, and mean ERs, mean AERs, and mean SDs as dependent variables. Familiarity with 100 yielded a main effect, $F(3, 113) = 17.75, p < 0.001, \eta^2 = 0.32$, that emerged for mean ERs, $F(1, 115) = 30.90, \eta^2 = 0.21$, mean AERs, $F(1, 115) = 52.60, \eta^2 = 0.31$, and mean SDs, $F(1, 115) = 14.15, \eta^2 = 0.11$, all $ps < 0.001$. Furthermore, age was a significant covariate, $F(3, 113) = 17.41, p < 0.001, \eta^2 = 0.32$, with effects on all three dependent
variables: mean $ERs, F(1, 115) = 7.72, \eta^2 = 0.06$, mean $AERs, F(1, 115) = 36.71, \eta^2 = 0.24$, and mean $SDs, F(1, 115) = 27.18, \eta^2 = 0.19$, all $ps < 0.01$. However, neither a main effect of the number of reference points was revealed, $F(3, 113) = 2.23, p = 0.088, \eta^2 = 0.06$—which is in line with the regression results—and the expected interaction between the number of reference points and familiarity, $F(3, 113) = 0.99, p = 0.400, \eta^2 = 0.03$. Thus, children who were familiar with numbers up to 100 made in general more accurate and less variable estimations compared to children who were familiar only with smaller numbers—this effect was independent of the presence of a reference point at the upper end of the number line.

With regard to the second aspect of whether in particular those children who were familiar with numbers up to 100 would make systematic estimations in the bounded condition, the number of children with at least one significant model fit (i.e., linear or logarithmic model; see Appendix C) was inspected. A Chi-square test revealed that 59 of the 62 children (i.e., 95%) who were familiar with numbers up to 100 yielded at least one significant model fit, whereas among the 58 children who were not familiar with numbers up to 100, only 17 (i.e., 29%) yielded a significant model fit, $\chi^2(1, n = 120) = 55.96, p < 0.001$. The same effect was revealed if only first graders were considered who belonged to the same age group but included a similar number of children who were familiar with numbers up to 100 (i.e., $n = 16$) or not (i.e., $n = 24$). All first graders who were familiar with numbers up to 100 yielded at least one significant model fit (i.e., 100%), whereas among those first graders who were not familiar with numbers up to 100 only 54% yielded a significant model fit, $\chi^2(1, n = 40) = 10.12, p = 0.001$ (exact test). Thus, being familiar with the whole number range resulted in general in more systematic number line estimations.

The third aspect addressed potential performance differences between children who were familiar or not familiar with numbers up to 100 in regard to the single target numbers to describe their poorer accuracy in more detail. An ANCOVA with repeated measures was conducted with familiarity with numbers up to 100 (yes vs. no) and target number as independent variables, age as covariate, and mean $ERs$ as dependent variable indicating over- and underestimations. The reference point condition was not considered as it yielded no effects in the previous analyses. There were main effects of age, $F(1, 117) = 8.46, p = 0.004, \eta^2 = 0.07$, familiarity, $F(1, 117) = 30.55, p < 0.001, \eta^2 = 0.21$, and target number, $F(2.82, 330.44) = 23.81, p < 0.001, \eta^2 = 0.17$ (Greenhouse-Geisser, $\varepsilon = 0.35$), an interaction of age and target number, $F(2.82, 330.44) = 11.65, p < 0.001, \eta^2 = 0.09$, as well as an interaction between familiarity with 100 and target number, $F(2.82, 330.44) = 14.59, p < 0.001, \eta^2 = 0.11$. As shown in Figure 2, the slopes of the estimations suggest that children who were unfamiliar with 100 discriminate poorer between target numbers by means of their estimations. Post-hoc tests revealed that children who were unfamiliar with 100 significantly overestimated the position of the numbers 17, 26, 35, 44, and 53, and underestimated the position of two largest numbers 84 and 92 compared to children who were familiar with the whole number range (all $ps < 0.05$, Bonferroni corrected), which suggests that those children used the given reference points for their estimations much less compared to children who were familiar with numbers to 100.

**DISCUSSION**

The present study investigated whether the accuracy and variability of children’s number line estimations were affected by their age, their familiarity with numbers, and the absence or presence of
a reference point at the upper end of the number line (i.e., bounded versus unbounded condition). Furthermore, it was examined whether in particular those children who were familiar with the number 100, provided as upper reference point, would profit from it in the bounded condition in terms of more accurate and less variable estimations.

In line with the first hypothesis, estimations were more accurate and less variable in older children and, beyond age, in children who were familiar with a larger number range (see also Simms et al., 2013). Contrary to our predictions, the presence of the upper reference point (i.e., 100) in the bounded condition affected neither the accuracy nor the variability of children’s estimations. In addition, the hypothesis that only those children who were familiar with the number of the upper reference point would profit from it, could not be confirmed. Instead, children who were familiar with numbers up to 100 made in general more accurate, less variable and more systematic estimations as indicated by significant model fits (i.e., linear or logarithmic). Finally, children who were unfamiliar with 100 made less accurate estimations not only in the larger number range in terms of underestimations but also in the smaller number range in terms of considerable overestimations, compared to children who were familiar with numbers up to 100.

The Role of Familiarity with Numbers in Children’s Number Line Estimations

Obviously, familiarity with numbers in a given number range is a pivotal factor affecting both the accuracy and variability of children’s number line estimations in that number range (cf. LeFevre et al., 2013). The present study confirms and extends findings of Newman and Berger (1984), who reported a positive relationship between children’s familiarity with numbers in a small number range—assessed by their ability to count forward and backward in multiple steps up to 30—and

![Figure 2](image-url) Median estimations of children across age groups being familiar (black dots) or not familiar (white dots) with numbers up to 100.
their estimation accuracy on a number line ranging up to 23. We showed that such a relationship exists also for a larger number range (i.e., to 100) including numbers with which younger children were not familiar, and by using a familiarity test that differed from the approach of Newman and Berger. Furthermore, we provided evidence that the relationship between familiarity with numbers and number line estimation accuracy also applies to estimation tasks, in which—in contrast to Newman and Berger’s study—no feedback on the accuracy was given. Finally, we showed that not only the accuracy of the estimations was affected by children’s familiarity with numbers but also their variability. Children who were familiar with larger numbers provided more systematic and thus less variable number line estimations.

However, it also became clear that familiarity with numbers cannot fully explain the variance of children’s performance in number line tasks (see Table 1). Other potential predictors are conceivable, such as the grasp of the place value system or more general spatial skills (LeFevre et al., 2013). Table 1 also shows that the two accuracy measures (i.e., mean ER and mean AER) and the variability measure (i.e., mean SD) were explained differently well by our predictors. This might first suggest that accuracy and variability are different measures that do not necessarily have to overlap. Estimations might for instance show a relatively low variability but a poor accuracy if they are systematically biased. Similarly, mean ER and mean AER might differ—but for mathematical reasons: In mean ER, over- and underestimations might level each other out, yielding relatively low mean ER even if single estimations over- or underestimate the actual positions, while in mean AER, estimation errors are accumulated. We therefore used both measures as they indicate different aspects of accuracy.

The Lack of an Effect of Reference Point Condition

Contrary to our expectations, but in line with the findings of Link and colleagues (2014) that children only from third grade on perform differently in bounded and unbounded number line tasks, the reference point condition revealed neither a main effect nor an interaction with familiarity. Thus, children who were familiar with numbers up to 100 provided better estimations than children who were familiar with smaller numbers—not only in the bounded condition but also in the unbounded condition. This finding suggests that familiarity with larger numbers might go along with a more precise mental representation of numbers (e.g., Dehaene et al., 2008) and/or more effective estimation strategies that lead to an advantage also in unbounded number line tasks. These two aspects, by which children who were familiar with numbers up to 100 might differ from children who were not, will be discussed in the following.

First, the requirements of the decade change test, by which familiarity with numbers was examined in the present study, tapped the ability to flexibly retrieve successors of given numbers and to infer the next decade change. Only children with a reliable mental concept of how numbers are ordered spatially and how they are constructed regarding how the decades and units could successfully solve the decade change test. This profound number knowledge might be based on a more advanced mental representation of numbers that affects children’s estimation performance in particular in number line tasks, which require a grasp of the ordinal structure of numbers, too. This assumption is supported by the finding that children who were familiar with numbers up to 100 also provided more systematic estimations that could be described by mathematical models. In contrast, children unfamiliar with numbers up to 100 made significant overestimations in the small number range—even for the number 17 that should fall into the familiar number range.
of most kindergartners. Thus, a limited familiarity with numbers affects not only estimations of unfamiliar numbers but is also reflected by less accurate estimations in the familiar number range. Furthermore, the estimation pattern of children who were unfamiliar with numbers up to 100 revealed that they discriminated less between target numbers than children who were familiar with 100.

Second, being familiar with a larger number range might also be associated with more advanced estimation strategies, such as using the upper reference point for calibration, extrapolating given units in terms of a “dead reckoning strategy” (Cohen & Sarnecka, 2014), or constructing internal reference points (cf. Ashcraft & Moore, 2012). It is important to note at this point that estimation strategies were not assessed directly in our study by asking participants—nor was it in other studies (for an exception, see Verschaffel et al., 2013)—but could only be inferred from the patterns and quality of the estimations. The assumed relationship between familiarity with numbers and the application of estimation strategies still deserves further research.

A differential effect of familiarity with numbers up to 100 in the bounded number line task was mainly expected among the group of first graders that included a comparable number of children who were familiar and unfamiliar with numbers up to 100. Among kindergartners, there were only a few children who were familiar with numbers up to 100. Moreover, the performance of kindergartners was quite unsystematic, which is surprising given the fact that young children in other studies involving bounded number line estimations up to 100 appeared to perform more systematically (e.g., Booth & Siegler, 2006; Slusser et al., 2012). However, the performance in these studies was often analyzed on group level rather than inspecting individual data, which might generate an apparently more systematic estimation pattern. This discrepancy occurred also in our study where the estimations of kindergartners as a group yielded a significant model fit, while on an individual level this was true only for 25% of kindergartners (see Appendix C). Moreover, our sample had not yet received formal instruction on numbers and counting in their kindergartens, therefore revealing a quite variable performance. Finally, numbers were presented in our study only in a symbolic format, as numeral and number word. Young children might have difficulties to infer the corresponding magnitude from symbols compared to cases in which the magnitude is additionally represented nonsymbolically (e.g., as number of dots; Ebersbach et al., 2008). However, a remaining question is whether children who provide estimations that do not systematically increase with target number possess an explicit mental representation of large numbers at all (cf. Moeller & Nuerk, 2011).

The estimations of second graders, in contrast, were quite linear in both conditions and were also unaffected by the additional, upper reference point. In second grade, children usually become highly acquainted with numbers up to 100—as also reflected by our measure of familiarity. They might have assumed that the upper end of the number line in the unbounded condition corresponded to 100—either because it is a prototypical large number for them or because they extrapolated the section between 1 and 10 to come to a rough estimation of 100 (cf. White & Szucs, 2012). This might have resulted in relatively accurate and systematic estimations even in the unbounded condition. In this respect, one might think of using more challenging tasks in future studies, like include less common reference points (e.g., 13 instead of 10) or reference points that are located only in the middle of the number line (e.g., 36 and 37) but not at both ends. Furthermore, one might use unbounded number lines whose number range is less easy to infer, for instance by stretching the number line out far beyond the tested number range (cf. Link et al., 2014).
Conclusions and Future Research

Taken together, we showed that children’s familiarity with numbers is a crucial predictor of their performance in bounded and unbounded number line tasks. Familiarity, as we have conceived and operationalized it, denotes a flexibly accessible and robust number knowledge that includes a basic understanding of the decade system. Nevertheless, familiarity with numbers is not sufficient to predict whether children would use the upper reference point in a bounded version of the number line task. One might think of other factors, such as place value knowledge (LeFevre et al., 2013), that might support this use. Furthermore, our cross-sectional data suggest that kindergartners, at least in our sample, still have problems with providing systematic estimations if the tasks involve symbolic numerical stimuli (even if they were also stated verbally). Second graders, in contrast, appear to have well-developed strategies to make systematic estimations even in the unbounded number line task.

Future research should examine longitudinally whether there is a causal relationship between familiarity with numbers and estimation skills in the number line task as well as in other estimation tasks (cf. LeFevre et al., 2013; Muldoon et al., 2013). Assessing children’s estimation strategies either more directly by means of verbal protocols (cf. Verschaffel et al., 2013) or indirectly by means of eye-movement measures (cf. Schneider et al., 2008) could be suited to uncover the relationship between children’s familiarity with numbers and their estimation strategies. If this would be the case, interventions could be developed to enhance children’s familiarity with numbers, such as asking them to count forwards or backwards in steps of two or three (cf. Newman & Berger, 1984). These interventions could be combined with training aiming at enhancing the linearity of estimations, as for instance by linear number games (e.g., Siegler & Ramani, 2008; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). However, it is also conceivable that children with a larger familiar number range possess at the same time more efficient proportional estimation strategies, such as to internally generate and use additional reference points (cf. White & Szucs, 2012). A longitudinal design might also help to unravel causal relationships. To conclude, even if the performance in number line tasks does not directly reflect the mental representation of numbers, the paradigm can be used to uncover how number skills and the use of strategies affect numerical estimations.

FUNDING

The preparation of this article was supported by a grant of the Deutsche Forschungsgemeinschaft (DFG: EB462/1-1) to the first author and a GOA grant 2012/010 from the Research Fund KU Leuven, Belgium.

REFERENCES


Verschaffel, L., Peeters, D., Degrande, T., Vanbussel, M., Ebersbach, M., & Luwel, K. (2013, August). The mental number line, the external number line, and elementary school mathematics. In K. Reiss (Chair), *Numeracy and arithmetic competence*. Invited symposium at the Biennial Conference of the European Association for Research on Learning and Instruction, München, Germany.


APPENDIX A

TABLE A1
Children’s Familiarity with Numbers per Age Group

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Mean Familiar Number Range (SD)</th>
<th>Children Who Were Familiar with Numbers up to 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kindergartners</td>
<td>36 (30)</td>
<td>6 (15%)</td>
</tr>
<tr>
<td>First graders</td>
<td>59 (38)</td>
<td>16 (40%)</td>
</tr>
<tr>
<td>Second graders</td>
<td>100 (0)</td>
<td>40 (100%)</td>
</tr>
</tbody>
</table>

Note. Age groups differed significantly concerning their familiarity with numbers, $F(2, 117) = 54.19, p < 0.001$, $\eta^2 = 0.48$. Repeated contrasts revealed significant differences between all successive age groups, $p_s < 0.001$. The same effect of age group was revealed if children who were familiar with numbers up to 100 were considered, $\chi^2 (N = 120) = 61.34, p < 0.001$.

APPENDIX B

TABLE B1
Age, Familiarity with Numbers, and Proportion of Girls and Boys in the Bounded and Unbounded Condition

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Bounded</th>
<th>Unbounded</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kindergartners</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age (in years)</td>
<td>5.95 (0.41)</td>
<td>5.80 (0.37)</td>
<td>0.21</td>
</tr>
<tr>
<td>Familiar number range</td>
<td>34 (30)</td>
<td>39 (30)</td>
<td>0.64</td>
</tr>
<tr>
<td>Gender (girls/boys)</td>
<td>10/10</td>
<td>10/10</td>
<td>1.00</td>
</tr>
<tr>
<td>First graders</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age (in years)</td>
<td>6.63 (0.50)</td>
<td>6.37 (0.50)</td>
<td>0.06</td>
</tr>
<tr>
<td>Familiar number range</td>
<td>67 (37)</td>
<td>52 (38)</td>
<td>0.23</td>
</tr>
<tr>
<td>Gender (girls/boys)</td>
<td>9/11</td>
<td>10/10</td>
<td>0.75</td>
</tr>
<tr>
<td>Second graders</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age (in years)</td>
<td>7.96 (0.43)</td>
<td>8.11 (0.38)</td>
<td>0.24</td>
</tr>
<tr>
<td>Familiar number range</td>
<td>100 (0)</td>
<td>100 (0)</td>
<td>1.00</td>
</tr>
<tr>
<td>Gender (girls/boys)</td>
<td>10/10</td>
<td>10/10</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note. Standard deviations in parentheses. Statistical comparisons were based on t-tests and Chi-square tests.

APPENDIX C

Preliminary Analyses Testing the Systematics of the Estimations

To examine whether children provided systematic estimations that increased for larger numbers, two common models (i.e., a linear and a logarithmic model) were fitted to the median estimations of each age group in each reference point condition. Both models yielded in each age group and condition a significant fit ($p_s < 0.05$) and thus suggest systematically increasing estimations on group level. To check whether this holds true for individual children, a linear and a logarithmic model were fitted to the estimations of each child even as the results have to be regarded...
cautiously given the small number of trials per child. At least one of the two models yielded a significant fit in the unbounded reference point condition in five kindergartners (i.e., 25%), 13 first graders (i.e., 65%), and 20 second graders (i.e., 100%). In the bounded reference point conditions, the models yielded a significant fit in four kindergartners (i.e., 20%), 14 first graders (i.e., 70%), and 20 second graders (i.e., 100%). It can thus be assumed that at least from Grade 1 on, children made estimations that systematically increased with larger target magnitudes.

Next, we analyzed the functional relationship between the estimations and the target magnitudes per age group and condition, as this relationship might indicate differences in the estimation strategies. In addition to a linear and a logarithmic model, a one-cycle power model was fitted to the data to account for a potential proportional estimation strategy (cf. Barth & Paladino, 2011). This model was not included earlier as it does not allow for testing the significance of the model fit. Akaike Information Criteria (AIC\textsubscript{corr}), adjusted for small sample sizes, were computed to compare the three model fits, with smaller AIC\textsubscript{corr} indicating a relative better fit. Please note that the nonlinear regression, by which the one-cycle power model was estimated, allows no significance testing. Accordingly, the inspection of AIC\textsubscript{corr} does not imply a significant model fit but serves only to compare the relative model fit.

As shown in Table C1, the median estimations of kindergartners were best fitted by a linear model in the unbounded condition and by a logarithmic model in the bounded condition. Estimations of first graders were fitted best by a logarithmic model in both conditions while those of second graders were best described by a linear model in both conditions.

The same analyses were conducted comparing the AIC\textsubscript{corr} on individual level using the estimations of each child largely confirming the analyses on group level. Among kindergartners, there was neither a dominant model in the unbounded condition ($p = 0.224$) nor in the bounded condition ($p = 0.224$, Fisher’s exact test)—with the limitation that only a few kindergartners yielded a significant model fit at all. For the majority of first graders, a logarithmic model yielded most frequently the best fit in both conditions, $\chi^2 (2, n = 20) = 15.70, p = 0.001$ and $\chi^2 (2, n = 20) = 7.90, p = 0.016$, while a linear model was the model fitting best most frequently in second graders in both conditions, $\chi^2 (2, n = 20) = 28.90, p < 0.001$ (no Chi-square statistic for the unbounded condition as the linear model yielded in all cases the best fit).
<table>
<thead>
<tr>
<th>Age Group</th>
<th>Model</th>
<th>F</th>
<th>p</th>
<th>$R^2$</th>
<th>$a$</th>
<th>$b$</th>
<th>AIC corr</th>
<th>Best Fitting Model on Individual Level</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kindergarten</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unbounded</td>
<td>linear</td>
<td>13.64</td>
<td>0.008</td>
<td>0.66</td>
<td>46.58</td>
<td>0.24</td>
<td><strong>17.40</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>logarithmic</td>
<td>8.93</td>
<td>0.020</td>
<td>0.56</td>
<td>20.19</td>
<td>10.21</td>
<td>18.41</td>
<td></td>
</tr>
<tr>
<td></td>
<td>one-cycle</td>
<td>0.63</td>
<td></td>
<td>122.71</td>
<td>0.21</td>
<td>17.69</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Bounded</td>
<td>linear</td>
<td>10.86</td>
<td>0.013</td>
<td>0.61</td>
<td>46.83</td>
<td>0.24</td>
<td>18.35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>logarithmic</td>
<td>15.64</td>
<td>0.006</td>
<td>0.69</td>
<td>13.97</td>
<td>11.89</td>
<td><strong>17.42</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>one-cycle</td>
<td>0.63</td>
<td></td>
<td>124.28</td>
<td>0.22</td>
<td>18.16</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>First grade</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unbounded</td>
<td>linear</td>
<td>36.86</td>
<td>0.001</td>
<td>0.84</td>
<td>32.86</td>
<td>0.50</td>
<td>19.29</td>
<td></td>
</tr>
<tr>
<td></td>
<td>logarithmic</td>
<td>52.46</td>
<td>&lt; 0.001</td>
<td>0.89</td>
<td>−32.29</td>
<td>23.93</td>
<td><strong>17.73</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>one-cycle</td>
<td>0.86</td>
<td></td>
<td>130.03</td>
<td>0.47</td>
<td>18.53</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Bounded</td>
<td>linear</td>
<td>50.04</td>
<td>&lt; 0.001</td>
<td>0.88</td>
<td>29.82</td>
<td>0.49</td>
<td>18.18</td>
<td></td>
</tr>
<tr>
<td></td>
<td>logarithmic</td>
<td>66.51</td>
<td>&lt; 0.001</td>
<td>0.91</td>
<td>−31.88</td>
<td>22.80</td>
<td><strong>16.23</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>one-cycle</td>
<td>0.88</td>
<td></td>
<td>118.09</td>
<td>0.43</td>
<td>17.23</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>Second grade</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unbounded</td>
<td>linear</td>
<td>2019.23</td>
<td>&lt; 0.001</td>
<td>10.0</td>
<td>3.97</td>
<td>0.90</td>
<td><strong>8.17</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>logarithmic</td>
<td>100.75</td>
<td>&lt; 0.001</td>
<td>00.94</td>
<td>−102.27</td>
<td>40.11</td>
<td>19.64</td>
<td></td>
</tr>
<tr>
<td></td>
<td>one-cycle</td>
<td>0.99</td>
<td></td>
<td>95.75</td>
<td>0.83</td>
<td>10.21</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Bounded</td>
<td>linear</td>
<td>1309.27</td>
<td>&lt; 0.001</td>
<td>10.0</td>
<td>2.74</td>
<td>0.91</td>
<td><strong>10.01</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>logarithmic</td>
<td>97.25</td>
<td>&lt; 0.001</td>
<td>00.93</td>
<td>−105.52</td>
<td>40.87</td>
<td>19.93</td>
<td></td>
</tr>
<tr>
<td></td>
<td>one-cycle</td>
<td>0.99</td>
<td></td>
<td>97.38</td>
<td>0.84</td>
<td>12.94</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Note.** Linear model: $y = a + bx$, logarithmic model: $y = a + b \times \ln(x)$; one-cycle power model: $y = a(x^b)/(x^b + (a-x)^b)$. $df$s: 1, 7. No significance testing available for nonlinear regression used to estimate the one-cycle power model. Smallest AIC corr indicating best fit are bold. Best fitting model on individual level: Frequency of the models with smallest AIC corr based on individual data (not implying a significant fit).