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On the Superneutrality of Money

Superneutralität des Geldes

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1. *The problem*

The determinants of the real interest rate have been a main topic in the literature on monetary theory since the days of Knut Wicksell and Irving Fisher, respectively. Is the real interest rate solely determined by non-monetary factors, such as the individual time preference rate and the population growth rate, or is it a monetary variable, which can be influenced by the monetary authority via the inflation rate? The literature discusses this problem as the "superneutrality of money". Money is said to be superneutral if – taking into account the dynamic processes of saving and capital accumulation – a change in the growth rate of the money supply leaves all steady state variables (i.e. real interest rate, real per capita income) unaffected.

The first monetary growth models (cf. Tobin, 1965, Sidrauski, 1967a) consistently denied the hypothesis of superneutrality. Tobin, for example, by employing a proportional savings function, came to the conclusion that an increase in the rate of monetary expansion increases capital accumulation and thus lowers the real interest rate (Tobin effect). His line of argument was that money and private shares of real assets are substitutes for the investor. Via higher inflation an increased money growth rate lowers the "interest" being paid on money, causing the investors to hold more real assets. As Dornbusch/Frenkel (1973) demonstrated, this result can be reversed, if increased inflation leads to a sufficiently large reduction of savings.

All these articles have been criticized, especially because of their *ad hoc* assumption on the savings behavior. It was claimed that, in general, a proportional savings function cannot be deduced from an individual utility maximization rationale. Sidrauski (1967b) was the first to employ such an optimization approach. He assumed the existence of an "immortal family", which maximizes the present value of per capita utility over all future periods. In this model, the real interest rate is invariably determined by the "Modified Golden Rule" and resistant to changes of the money growth rate.

In their overview article, Barro/Fischer (1976) hypothesize that superneutrality would not be given in case of a finite time horizon. Moreover, a second hypothesis contends that superneutrality can only occur if there are intergenerative transfers, or, to put it differently, if there is an operative bequest motive. Drazen (1981) proved the first hypothesis. Given a limited time horizon for the individual, variations in the growth of the money supply cause parallel effects for capital accumulation.

This article sets out to verify the second Barro/Fischer hypothesis. Using a monetary growth model with overlapping generations, it will be shown that the individuals counteract the Tobin effect of a higher inflation by altering the bequests to their children. A second goal is to emphasize the fact that – by changing the money growth rate – the government is in a position to decide whether money is superneutral or not.

The article is structured as follows. The next section lays out the model. Section 2.1 examines the individual optimization problem. Emphasis is placed on the microeconomic foundation of the cash-in-advance approach and on the bequest behavior. Following the description of the steady state features (Sec. 2.2), the superneutrality result will be discussed in great detail (Sec. 3). Section 4 summarizes the main results.

2. *The model*

In the literature, there are two ways of including the money functions as an accounting unit and especially as a medium of exchange into a growth theory framework: the cash-in-advance approach versus the money-in-the-utility-function approach. The first approach works by including the transaction and liquidity costs that are needed to finance consumption into an individual budget constraint. By utilizing money these costs can be lowered. In the case of the money-in-the-utility-function approach the argument is a little more subtle, or rather, less direct. The utility function of an individual is defined via the consumption and leisure time in each period of his life. In a barter economy, the search for bargaining partners, or the construction of transactional chains, requires time that could otherwise be spent on leisure activities. An economy with money reduces this time loss in favor of more leisure time. If money is inserted into the utility function, the direct utility of money reflects the increased utility caused by additional leisure time (cf. McCallum, 1983).

The vehement discussion on which approach would be more adequate (cf. Wallace, 1980, McCallum, 1983) was rendered superfluous following the publication of an article by Feenstra (1986). Both approaches are equivalent under certain preconditions. However, the proof of equivalence requires the exclusion of utility functions which are additively separable in respect to the "money" argument. In the following, the cash-in-advance approach is chosen because of its more obvious and plausible economic interpretation.

The analysis will be carried out within an overlapping generations model, in line with Barro (1974), who extended the Diamond model (1965) by adding the bequest motive.

3.1. The individual optimization problem

Each generation consists of identical individuals whose life-cycles are divided into the working and the retirement period. The utility u_t of an individual born at the beginning of period t is determined by the consumption in both periods and by the utility of its representative descendant (u_{t+1})[1]):

[1]) As opposed to this altruistic bequest motive à la *Barro,* a strategic bequest motive can be found in the literature. In these models, in which the bequest enters the utility function (cf. *Bernheim, Shleifer, Summers,* (1985), money is never superneutral.

$$u_t = u\left(c_t^1, c_t^2\right) + \frac{1}{1+\varrho} u_{t+1} \qquad \varrho > 0 \tag{1}$$

c_t^1 and c_t^2 denote per capita consumption during the working and the retirement period; ϱ is the intergenerational discount factor. The utility function u is assumed to be strictly quasi-concave in c_t^1 and c_t^2. Since u_{t+1} includes the utility of generation $t + 2$ (u_{t+2}), and one argument of u_{t+2} is u_{t+3} and so on, the individual faces an infinite planning horizon despite its finite life-time.

Because the individual budget constraints are influenced by the way the government implements monetary policy, the latter shall be modelled here. It shall be assumed, that the government (central bank) pursues a policy of constant growth of the money supply:

$$M_t = (1 + \theta)\, M_{t-1}. \tag{2}$$

M_t describes the nominal money supply in period t and θ is its growth rate. The money supply θM_{t-1} to be issued at the beginning of period t is being paid as a transfer to the N_t individuals of the young generation t. Each individual receives a real transfer g_t[2]: $g_t P_t N_t = \theta M_{t-1}$. In case of a diminishing money supply, g_t will become negative and can be viewed as a lump-sum tax. P_t stands for the price level in period t. m_t^1 and m_t^2 denote the real per capita money balances in the working period (retirement period).

The first and second period budget constraints are:

$$w_t + g_t = c_t^1 + \varphi\left(c_t^1, m_t^1\right) + m_t^1 + s_t \tag{3}$$

$$\begin{aligned}(1 + r_{t+1})s_t + (1 + r_{t+1})q_{t-1} + \frac{P_t}{P_{t+1}} m_t^1 \\ = c_t^2 + \varphi\left(c_t^2, m_t^2\right) + m_t^2 + (1+n)q_t\end{aligned} \tag{4}$$

During the working period an individual supplies a fixed amount of labor, which is rewarded by the wage w_t. The individual splits the wage and the transfer into consumption and savings. Saving takes place in the form of real money balances m_t^1 and in the form of private bonds s_t (real capital). Consumption can be split into consumption of goods c_t^1 and the "transaction costs" φ needed to finance c_t^1. The resources in the retirement period consist of bond savings bearing real interest r_{t+1}, real money balances from the previous period, and the bequest q_{t-1} received from generation $t - 1$[3]. We assume, for simplicity's sake, that bequests only take the form of private bonds. A bequest q_{t-1} provides the individual t with $(1 + r_{t+1})q_{t-1}$ of

[2]) In accordance with the literature, it will be assumed that the individuals will regard these transfers as independent of their money balances.

[3]) The chosen modelling of bequest behavior is in its fundamentals based on *Carmichael* (1982).