Thresholds for employment and unemployment: A spatial analysis of German regional labour markets, 1992–2000*

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Abstract. This paper applies Verdoorn's and Okun's law to derive efficient estimates of the employment and unemployment threshold in the Unified Germany. The analysis is built on a disaggregated dataset of regional labour markets, where spatial dependencies are taken into account. Especially, a spatial SUR model is proposed utilising the eigenfunction decomposition approach suggested by Griffith (1996, 2000). The thresholds turn out to be unstable over time. However, minimum output growth sufficient for a rise in employment is below the level needed for a drop in the unemployment rate. If spatial effects are ignored, the thresholds seem to be markedly overrated.

JEL classification: C21, C23, E24, E32

Key words: Employment and unemployment thresholds, regional labour markets, spatial filtering techniques, spatial SUR analysis

1 Introduction

Changes in production and employment are closely related over the course of the business cycle. However, as exemplified by the laws of Verdoorn (1949, 1993) and

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Okun (1962, 1970), thresholds seem to be present in the relationship. Due to capacity reserves of the firms, output growth must exceed certain levels for the creation of new jobs or a fall in the unemployment rate. In order to assess the future development of employment and unemployment, these thresholds have to be taken into account. They serve as important guidelines for policymakers.

In contrast to previous studies, we present joint estimates for both the employment and unemployment threshold. Due to demographic patterns and labour market institutions, the two thresholds can differ, implying that minimum output growth needed for a rise in employment may not be sufficient for a simultaneous drop in the unemployment rate. For the economy as a whole a distinct reaction of employment and unemployment to changes in output can be mainly attributed to changes in the participation rate (see Blanchard 2003, p. 185).[1] The analysis builds to a large extent on regional information. In particular, a sample of 180 German regional labour markets is employed (Eckey 2001). Since the cross-sections are separated by the flows of job commuters, they correspond to travel-to-work areas. Labour mobility is high within a market, but low between each other. As the sectoral decomposition of economic activities varies across the regions, the thresholds are based on a heterogeneous experience, leading to more reliable estimates.

The contribution to the literature is twofold. First, to the best of our knowledge, no previous paper has investigated a similar broad regional dataset based on travel-to-work areas for the unified German economy before. As a panel data set is used, better insight on the stability of the laws of Verdoorn and Okun can be provided. By means of spatial methods, fluctuations in employment and unemployment in response to changes in output growth can be split into spatial and non-spatial effects. Second, the methods applied are of new type. They involve a mixture of pooled and spatial econometric techniques. Dependencies across the regions may result from common or idiosyncratic (region specific) shocks. In particular, the eigenfunction decomposition approach suggested by Griffith (1996, 2000) is used to identify spatial and non-spatial components in the regression analysis. As the spatial pattern may vary over time, inference is conducted on the base of a spatial seemingly unrelated regressions (spatial SUR) model. Due to this setting, efficient estimates of the thresholds are obtained.

Our results indicate strong spatial dependencies in the development of employment and unemployment across German regional labour markets. Moreover, both the Verdoorn and Okun coefficient turn out to be unstable. Time-varying parameters can be detected even in sub-periods, thereby questioning TAR or threshold models as in Harris and Silverstone (2001) or Mayes and Virén (2004). However, minimum output growth sufficient for a rise in employment is well below the level needed for a simultaneous drop in the unemployment rate in all years of the sample period. If spatial effects are not controlled for, the thresholds seem to be markedly overrated.

[1] On the regional level, changes in net migration and commuting are also important, because labour mobility is generally much higher across regions than across national borders (see Armstrong and Taylor, 2000, pp. 7, pp. 29 and pp. 166). In the EU and Germany, however, the migration response to wage and unemployment differentials turns out be relatively low (Büttner 1999; Armstrong and Taylor 2000).

The rest of this article is organised as follows. In the next two sections (2 and 3), we review the laws of Verdoorn (1949, 1993) and Okun (1962, 1970), as they mark the cornerstones of the analysis. Both identify the threshold in terms of a lower bound of output growth. In Verdoorn's law, the growth rate is sufficient for an increase in employment, while in Okun's law the focus is on a fall in unemployment. Then, econometric methods are discussed in Sect. 4. The dataset is described in Sect. 5. Empirical results are presented in Sect. 6 and 7, and the final section concludes.

2 Verdoorn's law and employment threshold

The law of Verdoorn (1949, 1993) states that faster output growth (y) will induce gains in labour productivity growth (p). Formally, the relationship:

$$p_t = \beta_0 + \beta_1 y_t,\ \beta_1 > 0 \tag{1}$$

predicts increasing returns to scale if the Verdoorn coefficient β_1 turns out to be larger than zero. A positive, but declining slope parameter is reported in most empirical studies of EU countries (see, e.g., Harris and Lau 1998; Pons-Novell and Viladecans-Marsal 1999; León-Ledesma 2000; Fingleton 2001a; Walwei 2002). Increasing returns may be explained by a variety of endogeneous growth models (Aghion and Howitt 1998).

A serious issue with Verdoorn's law lies in the ignorance of the role of capital, which can be substituted for labour. Because of the omitted variable problem, estimation of the parameters β_0 and β_1 from the relationship (1) seem to be biased. Suppose output is produced by a Cobb-Douglas technology,

$$y_t = \tau + \eta l_t + \lambda k_t, \tag{2}$$

where l, k and τ are the growth rates of labour, capital and technology, respectively. As employment growth is the difference between output and productivity growth, the relationship:

$$p_t = \tau/\eta + [(\eta - 1)/\eta]\, y_t + (\lambda/\eta)\, k_t \tag{3}$$

is implied. In general, the bias is proportional to the coefficient from a regression of capital growth on output growth (Greene 2003, p. 148). However, the link (1) between productivity and output growth can be justified if capital growth equals output growth. This is in line with the stylised fact of a more or less constant capital-output ratio (see, e.g., Jones 1998, p. 12; Maußner and Klump 1996, p. 7). Then, the parameters β_0 and β_1 are specified as $\beta_0 = \tau/\eta$ and $\beta_1 = (\eta + \lambda - 1)/\eta$, for which unbiased estimates can be obtained from equation (1). Note that the returns to scale parameter cannot be revealed from the Verdoorn coefficient β_1 without knowledge of the production elasticities. The constant β_0 corresponds with the rate of technological progress, divided by the production elasticity of labour.

Verdoorn's law was originally considered for the industrial sector. Since the service sector has become more important, the hypothesis should be examined using total output data. Due to the high correlation of output and productivity growth, however, spurious regressions can easily occur. For example, if employment growth is constant, a perfect correlation between productivity and output growth would appear which is not informative at all. This problem is avoided in a specification between employment and output growth,

$$l_t = \alpha_0 + \alpha_1 y_t, \quad \alpha_0 = -\beta_0, \quad \alpha_1 = 1 - \beta_1, \tag{4}$$

that has been already favoured by Kaldor (1975). A high correlation between productivity and output growth does not imply the same for employment and output growth. If the former variables are perfectly correlated, for example, the latter variables will not correlate at all. Thus, spurious regressions arising from different trends driving productivity and output growth are avoided. Moreover, the best linear predictor of employment growth is given by a regression with the right-hand-side of equation (4) as its systematic part.

A Verdoorn coefficient of 0.5 in (1) implies a marginal employment intensity α_1 of the same size in (4). This means that a one-percent growth in output would stimulate employment by half a percent on average. The weaker than proportional reaction is due to efficiency gains, which can be realised more easily in periods of higher output growth. They may be traced to inter alia manpower reserves, increases in working hours and higher labour intensities.

The threshold of employment (y_E) indicates output growth for which employment is constant ($l_t = 0$). In terms of the model parameters the threshold level y_E reads as:

$$y_E = -\alpha_0 / \alpha_1 . \tag{5}$$

According to the parameter $-\alpha_0$, it is positively related to the rate of technological progress (τ) and negatively related to the production elasticity of labour (η). Also, a higher marginal employment intensity (α_1) reduces the threshold. Provided that output growth is above this bound, employment will be stimulated. If output growth falls below the threshold, losses in employment are expected on average. In this case, output growth is not sufficient to compensate for the rise in productivity due to technological progress and employment shrinks. According to (4) and (5), the evolution of employment,

$$l_t = \alpha_1 (y_t - y_E), \tag{6}$$

depends on the deviation of actual output growth from the threshold level. Each percentage point of output growth above (below) the threshold comes along with positive (negative) employment reaction that is determined by the marginal employment intensity.

3 Okun's law and unemployment threshold

An increase of employment is often seen to go hand in hand with a simultaneous decrease of unemployment. However, demographic factors and institutional factors can weaken the relationship. For example, if population growth is accompanied by a proportional rise in the labour force but a less increase of employment, unemployment will accelerate. Also, structural developments like a rising female labour force participation rate have to be taken into account. Generally, more favourable institutional settings on the labour market can attract people from outside the labour force. Thus, a strong relationship between changes in employment and unemployment cannot be expected. In particular, the minimum output growth rate needed for a rise in employment may not be sufficient for a drop in unemployment. The threshold for the latter is estimated by means of Okun's law.

According to Okun (1962, 1970), a negative relationship between unemployment and output fluctuations exists. Due to rigidities like menu costs, i.e. the costs of changing prices, firms tend to adjust output to aggregate demand in the short run. A rise in demand will stimulate production and employment, thereby lowering the unemployment rate. In particular, unemployment u will fall below its natural rate u^*,

$$(u_t - u^*) = \delta_1(y_t - y^*), \quad \delta_1 < 0, \tag{7}$$

if actual output growth is above its long run trend or potential y^*, that is driven by total factor productivity. Extended by a Phillips curve, the equation renders the aggregate supply curve of the economy. In this setting, u^* can be interpreted as the unemployment rate that is consistent with a fixed inflation rate. Also, the sacrifice ratio – the cumulative output loss arising from a permanent decrease in inflation – can be assessed (see Cechetti and Rich 1999). The lower the Okun coefficient δ_1 in absolute value, the lower the responsiveness of unemployment to growth and the higher the income loss resulting from a policy of disinflation.

Due to labour market conditions like the unemployment benefit system, unemployment is presumably of higher persistence than employment. Hence, δ_1 is expected to be lower than the employment intensity (α_1) in absolute terms. Like the latter, δ_1 seems to be unstable and has increased in recent times (Moosa 1997; Lee 2000; Freeman 2001). The stronger response of employment to output fluctuations may be caused by the productivity slowdown, stronger international competition, less legal protections of the employed and lower turnover costs, which encourage firms to reduce labour hoarding in periods of economic downturns. Assuming that potential output growth is roughly constant at least over sufficiently long intervals of time, the law can be rewritten,

$$(u_t - u^*) = \delta_0 + \delta_1 y_t, \quad \delta_1 < 0, \tag{8}$$

where the trend growth rate can be obtained from the intercept term. As a shortcoming, the gap specifications (7) and (8) of Okun's law are not directly suited for estimation. They involve unobservable variables, and there is no consensus on the proper procedure on how to identify them. In fact, a variety of filter techniques and trend decomposition methods are available, but they can lead to different conclusions. Therefore, the first difference (FD) specification of Okun's law,

$$\Delta u_t = \gamma_0 + \gamma_1 y_t, \quad \gamma_1 < 0, \tag{9}$$

might be more suitable for empirical reasons (see Okun 1970; Prachowny 1993). In contrast to (8), the FD specification relates the change in the actual unemployment rate to actual output growth.

If actual output growth is on the threshold level (y_U), unemployment is equal to its natural rate, implying that its change is equal to zero. Plugging this condition into (8) or (9) shows that the threshold can be derived in terms of the model parameters. Specifically, the threshold for a drop in unemployment:

$$y_{U,GAP} = -\delta_0/\delta_1, \tag{10a}$$

$$y_{U,FD} = -\gamma_0/\gamma_1, \tag{10b}$$

is supposed to decline, if the Okun coefficient rises in absolute value. In addition, a slower trend growth can contribute to a reduction. Unemployment dynamics depend on the threshold according to:

$$(u_t - u^*) = \delta_1(y_t - y_{U,GAP}), \tag{11a}$$

$$\Delta u_t = \gamma_1(y_t - y_{U,FD}), \tag{11b}$$

that is, unemployment will remain at its previous level if actual growth is just as high as the threshold for unemployment. For a drop of the unemployment rate output growth must exceed this level.

4 Spatially filtering and spatial SUR models

As the thresholds for employment and unemployment are estimated with regional data, dependencies between the cross-sections have to be taken into account. They may stem from common or idiosyncratic (region specific) shocks, which may generate spillovers among the cross-sections. Eventually, variables are spatially autocorrelated over the entities, and the particular pattern can bias the results (Anselin 1988, p. 58). Therefore, appropriate filters have to be employed in order to separate the spatial and non-spatial components of the series that enter the regression model.

Basically, two approaches are available to identify spatial effects in the data (see Getis and Griffith 2002, for a recent survey). Getis and Ord (1992) have proposed a spatial distance statistic. It requires that all variables are positive and can be measured relative to natural origins. These conditions are not met in this study, as growth rates and changes of variables are involved. Thus, the eigenfunction decomposition approach suggested by Griffith (1996, 2000) is preferred. Here, filtering relies on a decomposition of Moran's I (MI) statistic:

$$MI = \frac{\mathbf{x}'\mathbf{W}\mathbf{x}}{\mathbf{x}'\mathbf{x}}, \tag{12}$$

which is a quantity of the global spatial autocorrelation structure for a given variable. In particular, $\mathbf{x}$ holds the N observations of the variable under consideration, measured in deviations from the mean. $\mathbf{W}$ is an $N \times N$ matrix of spatial weights, where the elements of each row sum up to 1, and N the number of regions (see Anselin 1988, p. 16). The $\mathbf{W}$ matrix stores the information on the geographic map patterns, and is derived from a binary contiguity matrix. The elements of the latter are equal to 1 for neighbourhood regions and 0 otherwise. Moran's I can be expressed as a weighted sum of the eigenvalues of the matrix:

$$\mathbf{C} = (\mathbf{I}_N - \mathbf{1}\mathbf{1}'/N)\mathbf{W}(\mathbf{I}_N - \mathbf{1}\mathbf{1}'/N), \tag{13}$$

where $\mathbf{I}_N$ is the N-dimensional identity matrix and $\mathbf{1}$ a vector of ones (Tiefelsdorf and Boots 1995; Griffith 1996).

The eigenvectors of the $\mathbf{C}$ matrix are utilised to separate spatial from non-spatial components. Generally, spatial dependencies are represented by the system of eigenvectors, which identify near distinct geographic map patterns. The non-spatial part of a variable is given by the OLS residuals of a regression of that variable on the significant eigenvectors (Griffith 1996, 2000). Since the eigenvectors are near-orthogonal and near-uncorrelated,[2] stepwise regression can easily applied for selection. Based on this approach, the cross-section regression model:

$$\mathbf{y} = \beta_0 + \beta_1 \cdot \mathbf{x}^* + \sum_S \gamma_j \cdot \boldsymbol{\omega}_j + \mathbf{v}, \tag{14}$$

can be estimated via OLS. Here, $\mathbf{x}^*$ is the non-spatial component of the regressor and $\mathbf{v}$ the disturbance vector; the set S is formed by the relevant eigenvectors $\boldsymbol{\omega}_j$ of the $\mathbf{C}$ matrix. The eigenvectors must represent substantial spatial autocorrelations in order to be considered as relevant. In particular, the ratio $MI/MI_{\max}$, where $MI_{\max}$ denotes the largest Moran coefficient of any eigenvector of the $\mathbf{C}$ matrix, must exceed a bound of 0.25 for the selection of candidate vectors (Griffith 2003, p. 107). Because the linear combination of eigenvectors accounts for spatial dependencies, the errors of the regression are whitened.

Note that with panel data, the decomposition is required for each period t, as the spatial patterns may vary over time. Here, dependencies are expected to exist as well in time. For example, shocks arising in the regional labour markets do not diminish immediately. Instead of estimating unrelated equations in the style of (14) for different periods of time, a spatial SUR analysis is preferred. Contrary to the familiar SUR model, the system:

[2] Due to the standardisation of the weighting matrix the eigenvectors are only near-orthogonal and near-uncorrelated (Griffith 1996). In the application, nearly all correlations between the spatial components lie in a narrow range about zero.

$$
\begin{aligned}
y_{1i} &= \beta_0 + \beta_{11} \cdot x^*_{1i} + \sum_{S_1} \gamma_{1j} \cdot \omega_{ji} + v_{1i} \\
y_{2i} &= \beta_0 + \beta_{21} \cdot x^*_{2i} + \sum_{S_2} \gamma_{2j} \cdot \omega_{ji} + v_{2i} \\
&\vdots \\
y_{Ti} &= \beta_0 + \beta_{T1} \cdot x^*_{Ti} + \sum_{S_T} \gamma_{Tj} \cdot \omega_{ji} + v_{Ti}
\end{aligned}
\tag{15}
$$

is formed by the time periods for all the N spatial units ($i = 1, 2, \ldots, N$); see Anselin (1988, p. 137). The sets $S_1, S_2, \ldots, S_T$ of eigenvectors ω_j may include of different elements. The regression parameters β_{t1}, $t = 1, 2, \ldots, T$ are allowed to vary over time. The intercepts are mainly determined by technical progress.

Adjustment of employment and unemployment to changes in output may be accomplished with some delays due to institutional settings like employment protection and imperfect information. Persistency of employment and unemployment, however, is partly accounted for in the spatial SUR model (15), as the sets $S_1, S_2, \ldots, S_T$ will usually include common and different eigenvectors. By explaining parts of the spatial components of the dependent variable in different periods, spatially mediated delays of adjustment can be captured by common eigenvectors over time. Additional persistence can be picked up by the lagged non-spatial parts $y^*_{t-1,i}$ of the dependent variable in the spatial SUR model:

$$
y_{ti} = \beta_0 + \beta_{t1} \cdot y^*_{t-1,i} + \beta_{t2} \cdot x^*_{ti} + \sum_{S_t} \gamma_{tj} \cdot \omega_{ji} + v_{ti}, \quad t = 2, 3, \ldots, T. \tag{16}
$$

The sets of eigenvectors $S_1, S_2, \ldots, S_T$ can be determined by stepwise regression. In the first step, we test all candidate eigenvectors successively for significance at each period. Eigenvectors with significance equal or lower than 0.05 will enter the model. At the end of the pass, eigenvectors with a p-value above 0.05 will be removed. If the residuals of the t-th year pass the Moran test, the set S_t is determined. Otherwise, in a second step, eigenvectors which are significant at the 0.10-level are additionally included. This selection procedure is also feasible for spatially filtering of geo-referenced variables.

If the choice of spatial components for the sets S_t, $t = 1, 2, \ldots, T$, proves to be successful, the disturbances v_{ti} will be contemporaneous uncorrelated:

$$
\mathrm{Cov}(v_{ti}, v_{tl}) = \mathrm{E}(v_{ti} \cdot v_{tl}) = 0. \tag{17}
$$

Dependencies over time can be modelled by defining the $N \times N$ covariance matrix $\mathbf{\Sigma}$,

$$
\Sigma = \mathrm{E}(\mathbf{v}_t \cdot \mathbf{v}'_s) = \sigma_{ts} \cdot \mathbf{I}_N, \tag{18}
$$

in which σ_{ts} denotes the covariance between the periods t and s, $t \neq s$, and $\boldsymbol{\nu}_t$ a $N \times 1$ disturbance vector in period t. With this the time-dependent autocorrelation structure of the spatial SUR system (16) or (17) is given by the $NT \times NT$ covariance matrix:

$$\Sigma^* = \Sigma \otimes \mathbf{I}_N . \tag{19}$$

Provided that $T < N$ the covariance matrix $\boldsymbol{\Sigma}^*$ can be estimated directly from the data. An initial estimator can be obtained from the OLS residuals. By means of an estimate for the coefficient vector we determine an improved estimator of the temporal covariance matrix of the errors. Then, both estimators are simultaneously updated. In each iteration, estimators of both the coefficient vector and the covariance matrix are improved. Thus, spatial and temporal effects are determined with regard to one another.

As dependencies over time are taken into account, parameters are estimated more efficiently. In previous studies space-time models have been employed where spatial effects were captured by first order autoregressive lags (see, for instance, Beck and Katz 2001; Elhorst 2001; Beck and Gleditsch 2003). Spatial SUR modelling with spatial filtering allows for varying patterns of spatial dependencies over time. Probably due to high computational demands, applications of the spatial SUR model are rarely found. Exceptions are the works of Florax (1992) and Fingleton (2001b). The advantage of Griffith's approach consists in allowing setting up the model straightforwardly with tools usually available in econometric programs.

5 Regional labour markets and data

Threshold estimates are based on a sample of regional labour markets. As Eckey, Horn and Klemmer (1990) have pointed out, regions defined on behavioural settings are generally preferable over administrative units, as the latter may distort the economic conditions. Regional labour markets are defined on the basis of job commuters and correspond to travel-to-work areas.

Starting from 440 administrative districts (*Kreise*), Eckey (2001) constructed 180 regional labour markets of which 133 are located in the western and 47 in the eastern part of Germany. As the three overlapping regions consisting of both eastern and western districts are mainly located in the western part, they are counted as West German regions. On average 53 percent of the employees bounded to the social security system in local authority areas are commuters who travel to their workplaces across administrative boundaries. The average share of commuters decreases to 21 percent when labour market regions are used as regional units. While data on GDP and employment are available for administrative districts, figures on the unemployed refer to labour market agencies (*Dienststellenbezirke*). Both classifications do not match with the borders of regional labour markets. Hence the data must be aggregated. On average a regional labour market consists of 2.4 districts and 4.8 agencies.

The analysis is based on annual data. Nominal GDP and employed persons are obtained from the *Volkswirtschaftliche Gesamtrechnung der Länder* published by the Statistical state office of Baden-Württemberg. GDP is deflated by the regional GDP price indices, which are available at the state-wide level. Growth rates of real GDP and employment are calculated in the continuous compounding form. Data on unemployment is taken from the *Amtliche Nachrichten der Bundesanstalt für Arbeit* edited by the Federal Employment Services.

The sample runs from 1992 to 2000 and covers the recent experience of the German unification. As instabilities of the Verdoorn and Okun coefficients are reported in most studies, the thresholds should be based on a reasonable short time series dimension. Because GDP in 1993 is not available for districts, it has been interpolated referring to GDP on the state level.

6 Spatial autocorrelation patterns

As a first step of the analysis, Moran's I is computed as an overall measure of spatial autocorrelation for the variables considered. For the Okun relation, the FD specification is preferred, as it is based on observable series. Spatial dependencies are striking for all variables in most years. However, the Moran coefficient fluctuates over time (Fig. 1). The measure is significant at least at the 5 percent level, except of GDP growth in 1997 and 1999. The significance holds for both randomisation and normal approximation.

As spatial autocorrelation varies over time, it will not be removed by uniformly applying Griffith's filtering method to the variables over the entire sample period. This view is supported by the distinct map patterns of the variables in individual years. Although some common spatial components of the variables are at work over time, other eigenvectors of the **C** matrix change their significance from year to year. Hence, a varying spatial pattern has to be taken into account.

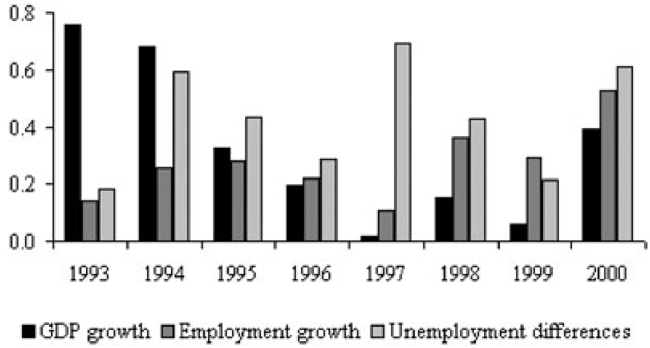

Fig. 1. Moran's I of geo-referenced variables

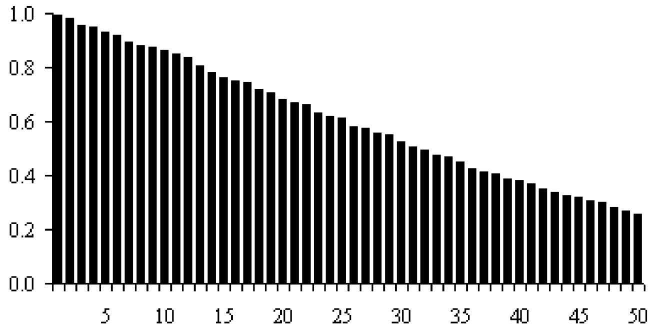

Fig. 2. Moran's *I* of candidate eigenvectors

The spatial SUR models (15) and (16) suppose that the explanatory variables are independent from the error term.[3] Thus, GDP growth and the lagged endogenous variable are pre-filtered to remove spatial effects. Employment growth and the change of the unemployment rate are explained by the spatially filtered lagged labour market variable, spatially filtered GDP growth rate and the map patterns of the endogeneous variables extracted from the **C** matrix. The spatial autocorrelations of the candidate eigenvectors are shown in Fig. 2. $MI_{\max}$ takes a value of 0.988. According to the $MI/MI_{\max}$ criterion the number of relevant eigenvectors is restricted to 50.

The map patterns of the eigenvectors corresponding to the eight largest eigenvalues of the **C** matrix are portrayed in Fig. 3. Only a very limited number of clusters exists, which tends to be larger, as positive spatial autocorrelation declines. While the first seven eigenvectors are associated with marked spatial autocorrelation ($MI/MI_{\max} > 0.9$), the eighth eigenvector depicts still strong spatial dependencies. The clusters appear to lie on trend surfaces with positive or negative gradients depending upon the chosen starting point. Obviously, a smooth transition from high-valued to low-valued clusters and vice versa takes place in the case of strong spatial autocorrelation. With a lower degree of positive spatial autocorrelation the map pattern becomes more speckled.

The eigenvectors of the matrix **C** represent purely spatial components, which may be accessible to a geographical interpretation (see Griffith 2003, p. 93). The map patterns of the eigenvectors corresponding with the highest eigenvalues are marked by high and low values at opposite sides. While $\boldsymbol{\omega}_1$ exhibits a north-south trend, $\boldsymbol{\omega}_2$ portrays a west-east trend. Both eigenvectors $\boldsymbol{\omega}_3$ and $\boldsymbol{\omega}_4$ seem to render two trend surfaces. An east-west trend appears in the northern and a west east trend in the southern part of the economy. The eigenvector $\boldsymbol{\omega}_4$ depicts a north-east trend as well as and a trend subsiding from an elevation at the northern border with France in all domestic directions. The eigenvectors $\boldsymbol{\omega}_6$ and $\boldsymbol{\omega}_8$ portray three trend surfaces with a central depression.

[3] The issue of exogeneity of output in Verdoorn's hypothesis is addressed by Fingleton (2000) in a spatial lag model. Although the Hausman test provides some evidence for endogeneity of output, Fingleton favours output to be treated as exogenous.

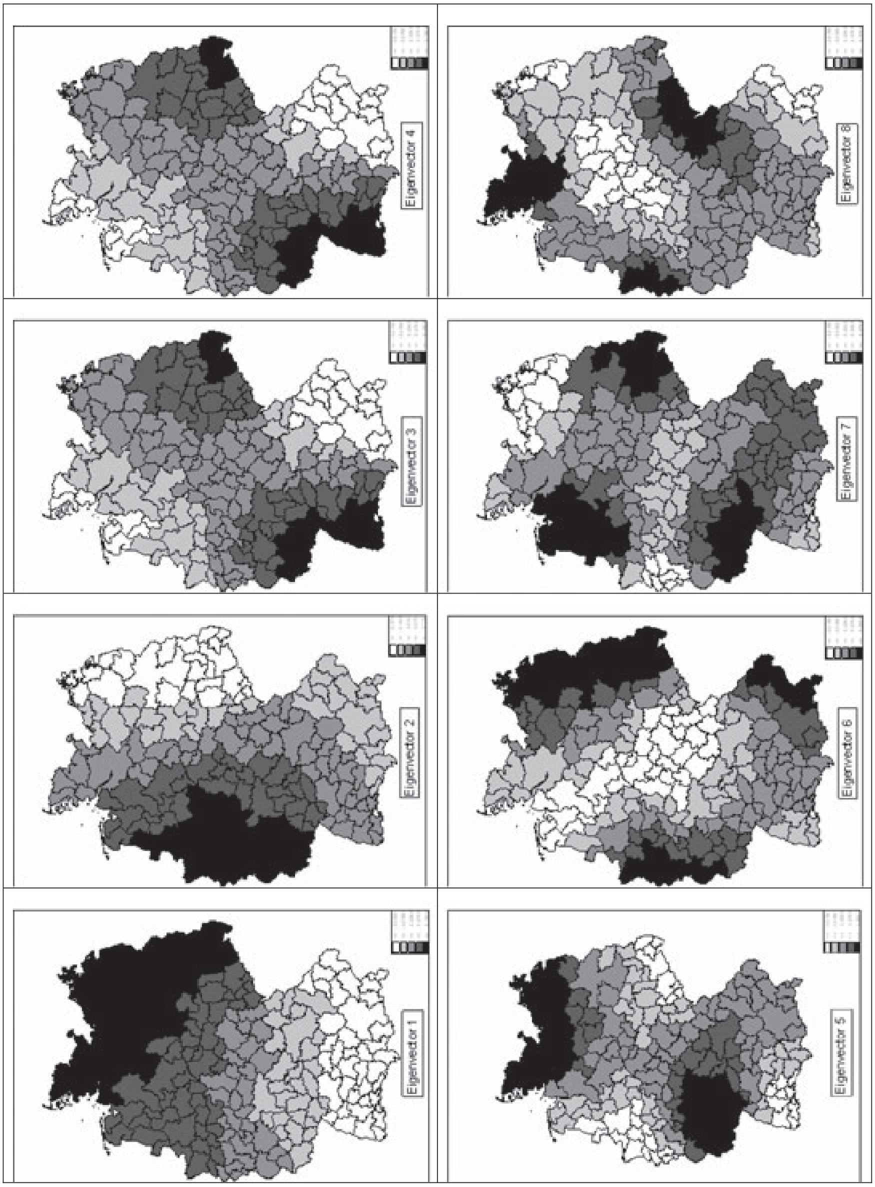

Fig. 3. Map patterns of eigenvectors

7 Empirical analysis of the employment threshold

As indicated by Moran's *I*, all spatial correlations of the residuals from the spatial SUR model are near to zero (see Table 1). In none of the years the

Table 1. Verdoorn's law: estimation and testing results[a]

	Non-spatial GLS estimation			Spatial SUR estimation		
	Coefficient		z-statistic	Coefficient		z-statistic
Constant	–0.0062		–10.027	–0.0023		–4.467
Output growth 93	–0.0604		–2.692	0.1769		3.183
Output growth 94	0.2850		15.540	0.3044		6.241
Output growth 95	0.2875		8.507	0.1643		3.336
Output growth 96	0.2440		4.650	0.3842		6.826
Output growth 97	0.1391		4.095	0.0675		1.464
Output growth 98	0.2367		7.283	0.2096		7.400
Output growth 99	0.0947		2.406	0.1833		5.494
Output growth 00	0.2848		5.806	0.2319		4.487
D98	0.0127		11.178	0.0122		15.871
D99	0.0154		12.124	0.0127		15.031
D00	0.0101		6.161	0.0130		15.154
	R^2	P-DW	Wald	R^2	P-DW	Wald
	0.344	1.612	178.384	0.520	1.837	25.888

Year	Non-spatial GLS	Spatial SUR estimation		
	Moran's I residuals (z-value)	Moran's I residuals (z-value)	Significant eigenvectors	Moran's I spatial components
1993	0.022 (0.685)	–0.044 (–0.464)	1, 2, 4	0.967
1994	0.071 (1.606)	–0.005 (0.369)	1, 2, 8	0.975
1995	0.169 (3.716)	0.042 (1.656)	1, 2, 4, 12, 18, 29, 46	0.805
1996	0.287 (6.219)	–0.022 (0.516)	2, 3, 6, 7, 11, 16, 17, 18, 22	0.858
1997	0.146 (3.192)	–0.056 (–0.726)	1, 2, 3	0.972
1998	0.347 (7.484)	–0.059 (0.329)	1, 2, 3, 4, 6, 8, 10, 12, 24, 25, 27, 29, 31, 32 38	0.848
1999	0.318 (6.826)	0.018 (1.923)	1, 2, 3, 5, 8, 17, 23, 26, 27, 28, 38, 39, 41, 44, 47, 49	0.757
2000	0.394 (8.543)	0.006 (1.486)	1, 2, 3, 4, 7, 8, 14, 15, 29, 33, 43, 50	0.893

[a] Non-spatial GLS estimation is based on the SUR model (3.4) without applying the spatial filtering approach. D98, D99, D00: Dummies for 1998, 1999 and 2000, R^2: coefficient of determination, P-DW: Panel Durbin-Watson statistic, lower critical value: 1.813 (interpolated value using tables from Bhargava et al. 1982). Wald test on equal coefficients of output growth $\chi^2_{7;0.95} = 14.1$.

z-value for Moran's I exceeds the quantile $z_{0.975}$ of the normal distribution.[4] The non-spatial GLS residuals are highly spatially autocorrelated. Although the panel Durbin-Watson statistic (Bhargava et al. 1982) is close to 2, the test result is indeterminate. Revisions of the employment statistics led to a structural break in the late 1990s. Since April 1999 the minor employed are liable to the

[4] For some problems with Moran's I in space-time models see Hooper and Hewings (1981).

social security system. Their registration has brought a noticeably upward revision of the total number of employees. This is captured by means of time dummies.

The Wald coefficient test clearly rejects the null of an equal response of employment to output growth over the entire sample period. The coefficients differ also significantly from one another in years with GDP growth above average, 1994–1995 and 1997–2000, ($\chi_5^2 = 14.516$, $p = 0.013$) and in years with output growth below average, 1993 and 1996 ($\chi_1^2 = 6.946$, $p = 0.008$). Although employment intensities vary from year to year, all coefficients in the spatial SUR model are significant with the correct sign. The average output elasticity of employment of 0.2 implies a rather weak reaction of employment to GDP growth. Factors like labour hoarding and adjustments of labour intensity have played a prominent role in the 1990s. For the entire sample period, the spatial SUR model implies a threshold of 1.4 percent for employment.[5] This level is in line with previous findings of Logeay (2001) and Walwei (2002), who report estimates in the range of 1–1.5 percent, on the base of time-series methods. Even though no clear pattern can be identified from the small sample, it is conspicuous that the thresholds of 0.8 percent and 1.0 percent at the peaks in 1994 and 2000 are both below average. The value of 3.2 percent implied by non-spatial GLS estimation overrates the threshold to a large extent.

In principle, inertia in the development of employment could be attributed to both spatial and non-spatial components. If lagged spatially adjusted employment growth is included as an additional regressor, the employment threshold turns out to be negative, and Verdoorn's law would be rejected. This finding, however, is an indication that dependencies are over-specified.[6] In fact, dependencies over time might itself result from the spatial structure. In this case, the inclusion of both variables would lead to serious multicollinearities. Hence the implied threshold is based on the assumption that lagged employment adjustment can be primarily traced back to the spatial components. Time-varying employment intensities arise as well in the model with a lagged non-spatial employment component. Moreover, the inferences regarding temporal and spatial autocorrelation turn out to be robust. Spatial components are essentially unaffected by modelling inertia of employment adjustment.

The high Moran coefficients of the linear combinations of the eigenvectors highlight the common spatial characteristics across neighbouring regions in all sub-periods. According to the qualitative classification scheme introduced by Griffith (2003, p. 107), the MI scatterplot trend is pronounced in three years ($MI/MI_{\max} \geq 0.9$). In all other years the $MI/MI_{\max}$ values indicate a conspicuous *MI* scatterplot trend ($MI/MI_{\max} \geq 0.75$). Figure 4 shows that both west-east and

[5] The thresholds for the entire sample are always determined by averaging yearly thresholds provided that they are properly defined.

[6] The problem of non-significant positive intercepts in time-series specifications with lagged endogenous employment is known from country studies (see, e.g., Döpke 2001). In studying Verdoorn's law, however, the issue of lagged adjustment is usually not addressed (see, e.g., Fingleton 2001a; León-Ledesma 2000; Logeay 2001; Pons-Novell and Viladecans-Marsal 1999; Walwei 2002).

Fig. 4. Verdoorn's law: map patterns of spatial components

east-west trends of the spatial components dominate the map patterns with Verdoorn's law. The trend surfaces slope down from a north-south stripe of high values along the western border in three years (1996, 1997, partly 1999). In other three years (1993, 1998, 2000) the trend is emanating inward from mounds in the north-west and south-east direction of the country. In two years, 1994 and 1995, a clear east-west trend is revealed by the map patterns.

8 Empirical analysis of the unemployment threshold

Table 2 reports the results of the FD specification of Okun's law. As the non-spatial lagged difference of the unemployment rate turns out not to be significant, adjustment in unemployment seems to be essentially captured by the spatial components. In case of asymmetric adjustment of unemployment to changes in output (see, e.g., Harris and Silverstone 2001; Silvapulle et al. 2004; Mayes and Virén 2004), time-varying Okun coefficients are to be expected. The Okun coefficients, however, differ not only significant from each other over the whole sample period, but as well within sub-samples of above and below average growth years. The Wald test statistics are $\chi_5^2 = 25{,}666$ ($p = 0.000$) and $\chi_5^2 = 5{,}266$ ($p = 0.022$), respectively. This questions the assumption of constant coefficients within the regimes of TAR and threshold models (Harris and Silverstone 2001; Mayes and Virén 2004). Note that all but one coefficient are significant with the expected sign, but their magnitudes appear to be very low. Due to persistence effects, unemployment does not react very much in response to a change in the growth rate.

For the entire sample period the spatial SUR estimates imply an unemployment threshold of 2.8 percent. The findings on the threshold are not unequivocal. While Schalk and Untiedt (2000) establish a decline of the unemployment threshold to 2 percent since the 1970s, Walwei (2002) estimates a threshold of 4.6 percent for the 1990s. However, such a high value fails to be consistent with the author's estimates of 3.0 for the period 1980–2000 and 2.8 percent for the 1980s. Also, the non-spatial GLS threshold of 3.4 percent seems to be overrated, as spatial effects are neglected.

As before, the spatial effects can be splitted into common and diverging map patterns. From Moran's I it becomes obvious that the spatial effects capture the largest part of the cross-section dependencies in the change of the unemployment rate. On average, the spatial correlations of the residuals are reduced by a factor of 14 by spatial SUR estimation compared to non-spatial GLS estimation. In six out of eight sub-periods the standardized Moran's I of the spatial SUR residuals is well below the 5 percent critical value.

Figure 5 points out west-east and east-west trends, which dominate the map patterns of the spatial components. With Okun's law the Moran scatterplot trends are pronounced ($MI/MI_{\max} \geq 0.9$) in four years. For 1996 and 1998 the $MI/MI_{\max}$ ratios indicate strong spatial autocorrelation. It is conspicuous that the east-west trend inclinations run considerably smoother than the corresponding declining trend surfaces. In 1996 and 1998 a west-east trend in northern and mid-Germany is combined with trend surfaces declining from the Czech and Austrian/Swiss border, respectively, in the south. Several clusters of high values dispersed across the landscape arise in 1993 and 1999.

The difference between the unemployment and employment threshold is noticeable. The gap might be attributed to the growth of the working population,

Table 2. Okun's law: estimation and testing results[a]

	Non-spatial GLS estimation			Spatial SUR estimation		
	Coefficient		z-statistic	Coefficient		z-statistic
Constant	0.0017		8.515	0.0012		11.690
Output growth 93	–0.0945		–11.226	–0.0647		–4.168
Output growth 94	–0.0883		–10.156	–0.0792		–4.970
Output growth 95	–0.0203		–1.781	0.0024		0.160
Output growth 96	–0.0823		–4.122	–0.1259		–5.879
Output growth 97	0.0070		0.534	–0.0164		–1.304
Output growth 98	–0.0959		–7.963	–0.0602		–5.292
Output growth 99	–0.0525		–4.034	–0.0625		–6.374
Output growth 00	–0.0350		–2.660	–0.0316		–2.192
D93	0.0083		13.398	0.0130		25.970
D00	–0.0092		–19.507	–0.0063		–22.494
	R^2	P-DW	Wald	R^2	P-DW	Wald
	0.583	1.346	92.188	0.744	1.810	40.645

Year	Non-spatial GLS	Spatial SUR estimation		
	Moran's I residuals (z-value)	Moran's I residuals (z-value)	Significant eigenvectors	Moran's I spatial components
1993	0.161 (3.631)	–0.015 (0.850)	2, 4, 7, 12, 14, 16, 21, 22, 27, 33, 37	0.718
1994	0.155 (3.492)	–0.033 (0.793)	1, 2, 3, 5, 6, 7, 10, 12, 14, 16, 30	0.961
1995	0.342 (7.388)	0.020 (2.097)	1, 2, 3, 4, 7, 8, 10, 13, 20, 25, 31, 36	0.914
1996	0.297 (6.425)	–0.014 (0.849)	1, 2, 3, 9, 15, 19, 25, 29, 42	0.885
1997	0.695 (14.801)	0.028 (2.211)	1, 2, 3, 4, 6, 7, 18, 29, 32, 33, 34, 38, 39, 40, 41, 44	0.905
1998	0.345 (7.433)	–0.036 (0.367)	1, 2, 6, 9, 10, 23, 24, 25, 39	0.817
1999	0.227 (4.911)	–0.041 (0.386)	6, 8, 13, 14, 16, 17, 25, 26, 27, 28, 29, 31, 33	0.678
2000	0.438 (9.484)	0.003 (1.482)	1, 2, 3, 4, 5, 7, 17, 19, 22, 25, 28, 33	0.943

[a] Non-spatial GSL estimation is based on the SUR model (3.4) without applying the spatial filtering approach. D93, D00: Dummies for 1993 and 2000, R^2: coefficient of determination, P-DW: Panel Durbin-Watson statistic, lower critical value: 1.802 (interpolated value using tables from Bhargava et al. 1982). Wald test on equal coefficients of output growth $\chi^2_{7;0.95} = 14.1$.

mainly due to a perceptible rise in the participation rate of women, which increased by 1.1 percentage points in the sample period (Statistisches Bundesamt 2002). Also, the unemployment benefit system could be relevant. In periods of economic upturns new jobs are partially filled with people from outside the labour force. Thus, employment is affected, while unemployment is eventually stable. In case of downturns, job losses will imply a simultaneous rise in unemployment to receive financial support.

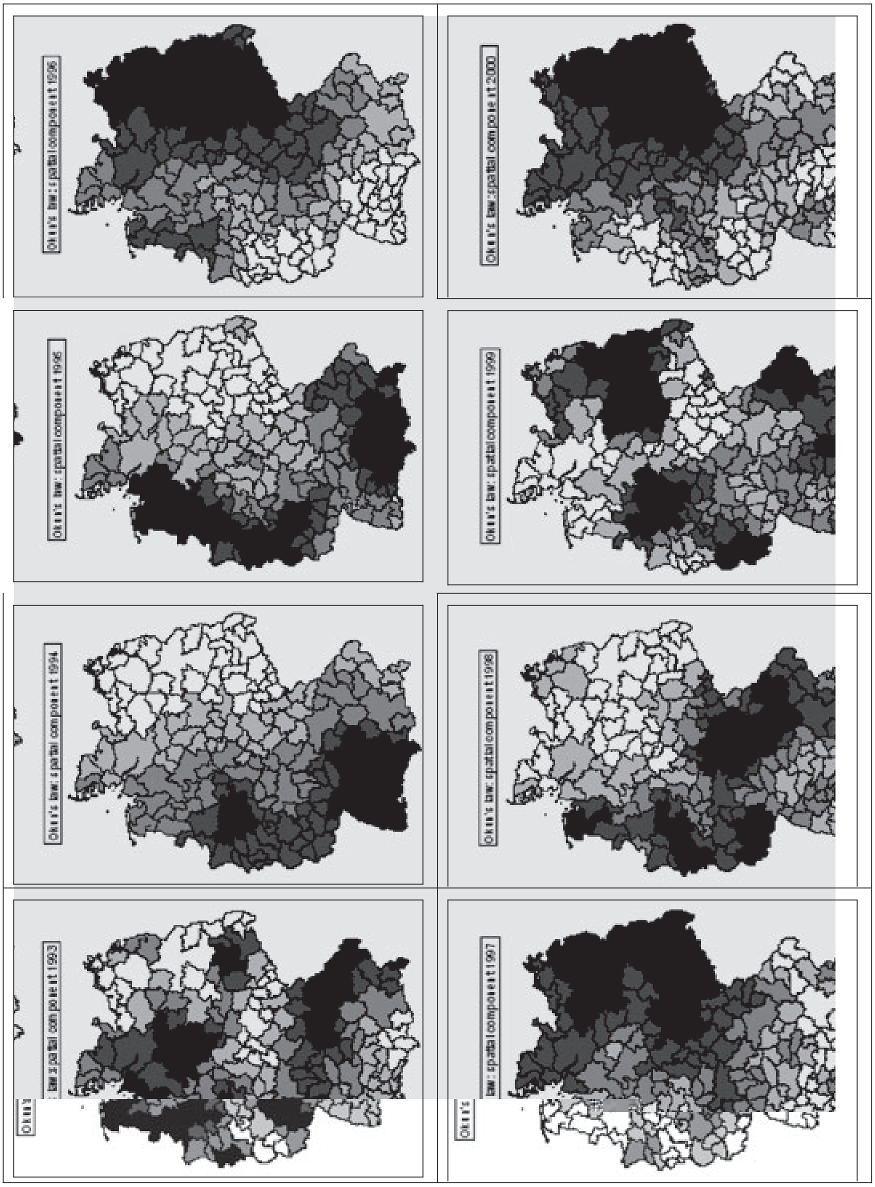

Fig. 5. Okun's law: map patterns of spatial components

9 Conclusions

In this article, we have investigated the laws of Verdoorn and Okun in the 1990s for the unified Germany from a spatial perspective. A spatial analysis of German regional labour market regions reveals that the relations between employment, unemployment and production may be distorted by strong spatial dependencies. In order to capture spatial and temporal autocorrelations, feasible spatial SUR techniques are proposed. It turns out that both the Verdoorn and Okun coefficients vary over time. Even in subperiods of high and low GDP growth, the employment and unemployment response to changes in output seems to be unstable. The latter finding casts serious doubts in the adequacy of TAR models to study labour market dynamics.

In most periods, employment and unemployment do not react very much in response to a change in output. Yet it turns out that minimum output growth sufficient for a rise in employment is well below the level that is needed for a drop in the unemployment rate. While the employment threshold amounts to 1.4 percent on average in the sample period 1993–2000, the threshold of unemployment takes an average value of 2.8 percent. The ordering between the thresholds might be related both to demographic changes and institutional settings on the labour market. But these numbers are only rough guidelines for the policymakers, as the estimates are hardly stable even in shorter periods of time.

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