6. Autocorrelation

6.1 Form and impact

The second implication of the adoption of a scalar covariance matrix,

(2.7) $$\mathrm{Cov}(\mathbf{u}) = \mathrm{E}(\mathbf{uu}') = \sigma^2 \underset{\text{nxn}}{\mathbf{I}}$$

is that disturbance terms of the different observations are not influencing each other, and so are uncorrelated. In the multiple regression model (2.5) an absence of **autocorrelation** was hence assumed. Autocorrelated disturbance terms indicate that the dependent variables tracks systematic influences which are not explained by the included regressors. The autocorrelation of the disturbance variable can be taken account of in three ways:

1. by including additional economic variables through which the autocorrelation of the disturbance variable can be eliminated,
2. by a generalized least-squares-estimation (variable transformation) which takes into account the autocorrelation of the disturbance variable,
3. by the use of corrected standard deviations of the estimated regression coefficients (autocorrelation-consistent standard errors)

We assume in this chapter that all economic variables, for which data are available in the period of investigation, have been included in the regression model.

What form has the variance-covariance-matrix with autocorrelated disturbance variables? If we now solely assume a violation of the assumption of non-correlation of the disturbance term, then their variance-covariance-matrix is given by

(6.1)
$$\mathrm{Cov}(\mathbf{u}) = \mathrm{E}(\mathbf{uu}') = \begin{bmatrix} \sigma^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma^2 \end{bmatrix}$$

The constant variances σ^2 on the main diagonal reflect in this context the assumption of homoscedasticity.

The off-diagonal elements

$$\sigma_{ts} = \mathrm{Cov}\,(u_t u_s) = \mathrm{E}\,(u_t \cdot u_s),\ \mathrm{T},\ s = 1,2,\ldots,n;\ t \neq s,$$

are the **autocovariances** between the disturbance terms of the t-th and the s-th observation. Because $\sigma_{ts}=\sigma_{st}$ applies, a total of n(n-1)/2 autocovariances can be distinguished. As in the case of heteroscedasticity an estimation of the multiple regression model on the basis of the covariance matrix (6.1) can only be carried out when certain assumptions about the disturbance process are made.

An autocorrelation of disturbance variables has the same consequences as heteroscedasticity with respect to the quality of the OLS estimation. The OLS estimator of the vector of the regression coefficients remains in fact unbiased, but is, however, no longer efficient. This means that the standard errors of the OLS-estimated regression coefficients are biased so that the significance tests and the confidence intervals loses their validity.

In what follows, we first introduce the concept of a **coefficient of autocorrelation** (Sect. 6.2). Then we discuss two tests for autocorrelation (Sect. 6.3). Here we elaborate at the same time how disturbance processes in the event of autocorrelation of disturbance variable u can be modelled. While the **Durbin-Watson test** is aimed at an autocorrelation of 1st order, the **Breusch-Godfrey-test** can also uncover autocorrelation of higher orders. In section 6.4 we then present **various approaches** to the **estimation of multiple regression model with autocorrelated** disturbance variables. Finally, in section 6.5 the use of **autocorrelation-consistent standard errors** by means of a variable transformation as an alternative to generalized least-Squares estimation is discussed.

6.2 Autocorrelation coefficient

In the presence of autocorrelation, the disturbance terms relating to different observations are stochastically dependent. This dependence is measured with the **autocorrelation coefficient**. To understand the concept of the autocorrelation coefficient, we resort to **Pearson‘s correlation coefficient**, r_{xy} , between two attributes X and Y that is known from descriptive statistics. It is defined by the ratio of the covariance s_{xy} and the product of the two standard deviations s_x and s_y:

$$r_{xy} = \frac{s_{xy}}{s_x \cdot s_y}$$

We now assume that for each of the two attributes X and Y n values in the form of two time series are available. The data then consist of the ordered observations $(x_1, y_1), (x_2, y_2), \ldots , (x_n, y_n)$, from which Pearson’s correlation coefficient can be calculated:

(6.2)
$$r_{xy} = \frac{\frac{1}{n}\sum_{t=1}^{n}(x_t - \bar{x})(y_t - \bar{y})}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(x_t - \bar{x})^2}\sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t - \bar{y})^2}} = \frac{\sum_{t=1}^{n}(x_i - \bar{x})(y_t - \bar{y})}{\sqrt{\sum_{t=1}^{n}(x_t - \bar{x})^2}\sqrt{\sum_{t=1}^{n}(y_t - \bar{y})^2}}$$

In econometrics it is investigated whether the **residuals** $\hat{u}_t$ of a regression are correlated "with themselves" or are **autocorrelated**. The **autocorrelation** is measured by, for example, the relationship between successive values, values with a time lag of two periods, three periods, etc.. One speaks in this connection of an autocorrelation of the first order, second order, third order, etc.

In calculating a first-order autocorrelation coefficient one forms from an available time series of the residuals $\hat{u}_1, \hat{u}_2, ..., \hat{u}_n$ two series that are shifted from each other by one period:

$$\hat{u}_2, \hat{u}_3, ..., \hat{u}_n \quad \text{und} \quad \hat{u}_1, \hat{u}_2, ..., \hat{u}_{n-1}$$

Hence, from the n time series values n-1 ordered pairs are formed:

$$(\hat{u}_2, \hat{u}_1), (\hat{u}_3, \hat{u}_2), ..., (\hat{u}_n, \hat{u}_{n-1})$$

The **empirical first-order autocorrelation coefficient** then reads

(6.3a)
$$\hat{\rho}_1 = \frac{\sum_{t=2}^{n} \hat{u}_t \cdot \hat{u}_{t-1}}{\sum_{t=1}^{n} \hat{u}_t^2}$$

Its conceptual comparability with Pearson's correlation coefficient becomes apparent when for both residual series the mean of the complete residual series, $\bar{\hat{u}} = 0$, and in the denominator the product of the standard deviations of the complete residual series are used, which is equal to the residual variance.

Example:

In a regression of the share prices of a stock A (Y) on the stock index (X) the following residuals arose:

t	1	2	3	4	5	6	7	8
$\hat{u}_t$	0,6	0,5	-0,4	-0,7	-0,3	0,3	0,4	-0,4

How large is the first-order autocorrelation coefficient?

Work table:

t	$\hat{u}_t$	$\hat{u}_{t-1}$	$\hat{u}_t \cdot \hat{u}_{t-1}$	$\hat{u}_t^2$
1	0,6	-	-	0,36
2	0,5	0,6	0,30	0,25
3	-0,4	0,5	-0,20	0,16
4	-0,7	-0,4	0,28	0,49
5	-0,3	-0,7	0,21	0,09
6	0,3	-0,3	-0,09	0,09
7	0,4	0,3	0,12	0,16
8	-0,4	0,4	-0,16	0,16
Σ	0		0,46	1,76

$$\hat{\rho}_1 = \frac{\sum_{t=2}^{8}(\hat{u}_t \cdot \hat{u}_{t-1})}{\sum_{t=1}^{8}\hat{u}_t^2} = \frac{0,46}{1,76} = 0,261$$

To determine the **empirical second-order autocorrelation coefficient**, n-2 ordered pairs

$$(\hat{u}_3, \hat{u}_1), (\hat{u}_4, \hat{u}_2), ..., (\hat{u}_n, \hat{u}_{n-2})$$

are compared with each other:

(6.4)
$$\hat{\rho}_2 = \frac{\sum_{t=3}^{n} \hat{u}_t \cdot \hat{u}_{t-2}}{\sum_{t=1}^{n} \hat{u}_t^2}$$

In general, the **empirical autocorrelation coefficient of the j-th order** is given by

(6.5)
$$\hat{\rho}_j = \frac{\sum_{t=j+1}^{n} \hat{u}_t \cdot \hat{u}_{t-j}}{\sum_{t=1}^{n} \hat{u}_t^2}$$

It indicates the strength of the dependency between the residuals that lie j periods apart.

Example:

Which values do the autocorrelation coefficients of the second and the third order relating to the residuals from the regression of share prices (Y) on the stock index (X) take.

Work table:

t	$\hat{u}_t$	$\hat{u}_{t-2}$	$\hat{u}_{t-3}$	$\hat{u}_t \cdot \hat{u}_{t-2}$	$\hat{u}_t \cdot \hat{u}_{t-3}$	$\hat{u}_t^2$
1	0,6	-	-	-	-	0,36
2	0,5	-	-	-	-	0,25
3	-0,4	0,6	-	-0,24	-	0,16
4	-0,7	0,5	0,6	-0,35	-0,42	0,49
5	-0,3	-0,4	0,5	0,12	-0,15	0,09
6	0,3	-0,7	-0,4	-0,21	-0,12	0,09
7	0,4	-0,3	-0,7	-0,12	-0,28	0,16
8	-0,4	0,3	-0,3	-0,12	0,12	0,16
Σ	0			-0,92	-0,85	1,76

$$\hat{\rho}_2 = \frac{\sum_{t=3}^{n} \hat{u}_t \cdot \hat{u}_{t-2}}{\sum_{t=1}^{n} \hat{u}_t^2} = \frac{-0,92}{1,76} = -0,523 \quad \text{and} \quad \hat{\rho}_3 = \frac{\sum_{t=4}^{n} \hat{u}_t \cdot \hat{u}_{t-3}}{\sum_{t=1}^{n} \hat{u}_t^2} = \frac{-0,85}{1,76} = -0,483$$

6.3 Testing for Autocorrelation

6.3.1 Durbin-Watson test

In the presence of autocorrelation, the disturbance terms of the different observations are stochastically dependent. But when in a regression is the assumption of an uncorrelated disturbance term being violated? The empirical autocorrelation coefficient $\hat{\rho}_1$ will differ from 0 due to the sampling error, even if the disturbance term is uncorrelated in the population. A test of this assumption is called **autocorrelation test**.
It can often be expected that an autocorrelation of the disturbance term will show up immediately in a dependency on the neighbouring residuals. The **Durbin-Watson test** is an autocorrelation test, which tests the residuals for first-order autocorrelation. In the null hypothesis it is therefore assumed that the theoretical autocorrelation coefficient ρ_1 is equal to 0:

$$H_0: \rho_1 = 0.$$

Against this the alternative hypothesis of an autocorrelated disturbance term is contrasted:

$$H_1: \rho_1 \neq 0.$$

From the test result of the Durbin-Watson test it is possible in the event of a rejection of the null hypothesis to also discover whether a positive or a negative autocorrelation is present.

A stochastic process, which generates an autocorrelation of the first order of the disturbance term, is a **first-order autoregressive process (Markov process):**

(6.6) $$u_t = \phi \cdot u_{t-1} + v_t \,, \quad |\phi| < 1 \,, \quad t = 2,...,n$$

In this process, the current disturbance term u_t is dependent on the disturbance term of the previous period u_{t-1}. The strength of the dependence is depicted by the autoregressive parameter ϕ . v is a White-Noise disturbance variable that superimposes this relationship and is also called innovation:

(6.7) $$E(v_t) = 0 \quad \text{für } t = 2,...,n$$

(6.8) $$Cov(v_t, v_s) = E(v_t \cdot v_s) = \begin{cases} \sigma_v^2 & \text{für } t = s \\ 0 & \text{sonst} \end{cases}$$

The restriction $-1 < \phi < 1$ prevents past disturbance variables from obtaining a dominant influence on the current development (= nonstationarity). It is also furthermore necessary because the autoregressive parameter can be interpreted as a coefficient of (auto)correlation, which is limited to this value range.

The autoregressive parameter ϕ can be estimated when one replaces the disturbance terms u_t in (6.6) by the OLS residuals of the underlying regression model:

(6.9) $$\hat{u}_t = \phi \cdot \hat{u}_{t-1} + v_t \,, \quad |\phi| < 1$$

A regression of $\hat{u}_t$ on $\hat{u}_{t-1}$ then leads to the OLS-estimator

(6.10) $$\hat{\phi} = \frac{\frac{1}{n-1}\sum_{t=2}^{n} \hat{u}_t \hat{u}_{t-1} - \bar{\hat{u}}_0 \bar{\hat{u}}_1}{\frac{1}{n-1}\sum_{t=2}^{n} \hat{u}_{t-1}^2 - \bar{\hat{u}}_1^2}$$

with $$\bar{\hat{u}}_0 = \frac{1}{n-1}\sum_{t=2}^{n} \hat{u}_t \quad \text{and} \quad \bar{\hat{u}}_1 = \frac{1}{n-1}\sum_{t=2}^{n} \hat{u}_{t-1}$$

Because $\bar{\hat{u}}_0 \approx \bar{\hat{u}}_1 \approx 0$ and $$\sum_{t=2}^{n} \hat{u}_{t-1}^2 = \sum_{t=1}^{n-1} \hat{u}_t^2 \approx \sum_{t=1}^{n} \hat{u}_t^2$$

one obtains the OLS-estimator $\hat{\phi}$ which thus approximately corresponds to the definition of the empirical autocorrelation coefficient $\hat{\rho}_1$ in (6.3a):

(6.3b) $$\hat{\phi} \approx \frac{\sum_{t=2}^{n} \hat{u}_t \hat{u}_{t-1}}{\sum_{t=1}^{n} \hat{u}_t^2}$$

The null hypothesis of the absence of first-order autocorrelation (H_0: $\rho_1 = 0$) can therefore be stated equivalently as

(6.11) H_0: $\phi = 0$

Provided that the null hypothesis is true, it follows from the approach (6.6) that $u_t = v_t$, so that the disturbance term u meets the standard assumptions of the multiple regression model.

The null hypothesis (6.11) can be checked with the **Durbin-Watson test**. The test statistic, known as the Durbin-Watson statistic DW, is based on the OLS residuals and is given by

(6.12a)
$$DW = \frac{\sum_{t=2}^{n} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{n} \hat{u}_t^2}$$

Because the numerator and the denominator each have squared summands, the Durbin-Watson statistic is always non-negative (DW≥0).

Positive autocorrelation: Current residuals $\hat{u}_t$ and residuals of the previous period, $\hat{u}_{t-1}$, have in the majority of instances the same sign (both + or both -)

Perfect positive autocorrelation: The residuals $\hat{u}_t$ and $\hat{u}_{t-1}$ match
→ numerator in (6.12a) is equal to 0 → DW = 0

The numerator in (6.12a) can be further transformed, what in particular is recommended with regard to the interpretation of the test statistic. By solving the binomial one gets firstly

$$\sum_{t=2}^{n}\left(\hat{u}_t-\hat{u}_{t-1}\right)^2=\sum_{t=2}^{n}\left(\hat{u}_t^2-2\hat{u}_t\hat{u}_{t-1}+\hat{u}_{t-1}^2\right)=\sum_{t=2}^{n}\hat{u}_t^2+\sum_{t=2}^{n}\hat{u}_{t-1}^2-2\sum_{t=2}^{n}\hat{u}_t\hat{u}_{t-1}$$

If the number of observations n is sufficiently large, the approximation

$$\sum_{t=2}^{n}\hat{u}_t^2\approx\sum_{t=2}^{n}\hat{u}_{t-1}^2\approx\sum_{t=1}^{n}\hat{u}_t^2 \quad .$$

applies, so that

$$\sum_{t=2}^{n}\left(\hat{u}_t-\hat{u}_{t-1}\right)^2\approx 2\sum_{t=1}^{n}\hat{u}_t^2-2\sum_{t=2}^{n}\hat{u}_t\hat{u}_{t-1}$$

results. Substituting this relationship in (6.12a) one obtains

$$DW\approx\frac{2\sum_{t=1}^{n}\hat{u}_t^2-2\sum_{t=2}^{n}\hat{u}_t\hat{u}_{t-1}}{\sum_{t=1}^{n}\hat{u}_t^2}=2\left(1-\frac{\sum_{t=2}^{n}\hat{u}_t\hat{u}_{t-1}}{\sum_{t=1}^{n}\hat{u}_t^2}\right)$$

and, taking account of (6.3b), finally

(6.12b) $$DW \approx 2\left(1-\hat{\phi}\right),$$

Thus with this, the relationship between the Durbin-Watson statistic and the empirical first-order autocorrelation coefficient of the residuals, $\hat{\rho}_1$, is established. Because of $|\hat{\rho}_1| < 1$ the Durbin-Watson statistic always take values in the interval (0,4)

DW-Statistics and limiting cases of autocorrelation

No autocorrelation ($\rho_1 = 0$) : $\hat{\rho}_1 \approx \hat{\phi} \approx 0$ → DW ≈ 2

Perfect positive autocorrelation ($\rho_1 = 1$): $\hat{\rho}_1 \approx \hat{\phi} \approx 1$ → DW ≈ 0

Perfect negative autocorrelation ($\rho_1 = -1$): $\hat{\rho}_1 \approx \hat{\phi} \approx -1$ → DW ≈ 4

Value range of the DW statistic: $0 \leq DW \leq 4$

Specifying the acceptance and rejection regions

To be able to specify the acceptance and rejection regions of the DW-statistic, we need to know its distribution under the null hypothesis H_0: $\rho_1 = 0$,

Premises:
- Independent, identically normal distributed disturbance terms u_t
- Explanatory variables X_k are fixed quantities

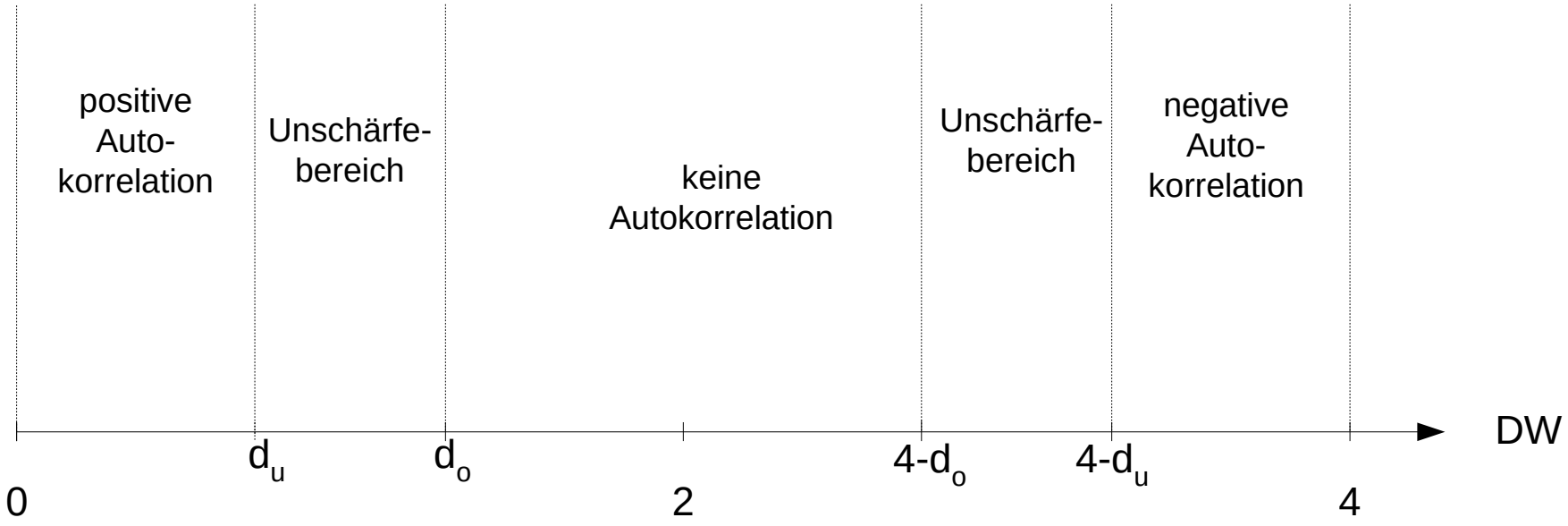
Under these premises the distribution of the DW-statistic depends on
- the number of observations n (sample size),
- the number of exogenous variables k,
- the values of the exogenous variables

In order to be able to deploy a universal distribution in the autocorrelation test, one takes into account a certain degree of fuzziness. In this way can the critical values be specified independently of the concrete variable values.

In the tabulation of the critical values **uncertainty regions** arise with respect to deciding on positive or negative autocorrelation and non-autocorrealtion. The uncertainty regions are defined by the lower (d_u) and upper (d_O) critical value:

- with positive autocorrelation: $d_u \leq \leq$ DW d_O
- with negative autocorrelation: $4\text{-}d_O \leq$ DW $\leq 4 - d_u$

Figure: Regions of the DW statistic



The critical values d_u and d_O are tabulated in dependence on the number of observations n and the number of exogenous variables k (incl. the pseudo variable) for error probabilities $\alpha = 0{,}05$ and $\alpha = 0{,}01$).

Example:

The Durbin-Watson test is to be illustrated using the Keynesian consumption function (2.27). With this it can be assessed the extent to which the assumption of an absence of autocorrelation of the disturbance variable of the consumption function is justified. More specifically, the disturbance variable u will be tested with the Durbin-Watson-test for first-order autocorrelation. As the test statistic of this test will the Durbin-Watson-Statistic DW be numerically be specified using the OLS residuals $\hat{u}_t$.

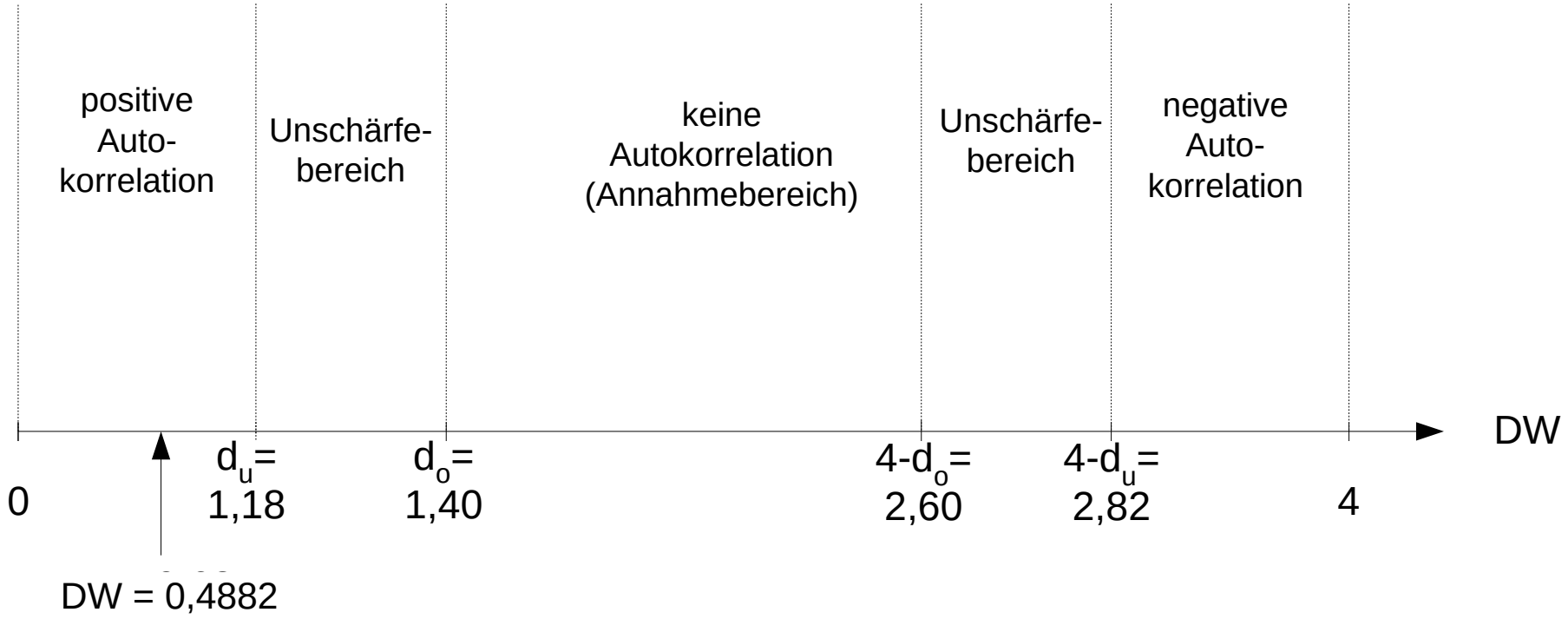
The deviation sums of squares required for this purpose are calculated in a work table:

t	$\hat{u}_t$	$\hat{u}_{t-1}$	$\hat{u}_t^2$	$(\hat{u}_t - \hat{u}_{t-1})^2$
1	-11,059452	-	122,311474	-
2	-6,484428	-11,059452	42,0478065	20,9308428
3	-4,945561	-6,484428	24,4585746	2,36811134
4	-0,278686	-4,945561	0,07766578	21,7797251
5	0,613325	-0,278686	0,37616731	0,79568291
6	6,707777	0,613325	44,9942736	37,1423488
7	19,312656	6,707777	372,978689	158,882977
8	13,751808	19,312656	189,112223	30,9230327
9	6,450680	13,751808	41,611266	53,3064774
10	1,105008	6,450680	1,22104334	28,5762006
11	-2,554586	1,105008	6,52590708	13,3926268
12	-6,378473	-2,554586	40,684914	14,6221133
13	-4,055685	-6,378473	16,4485841	5,39534084
14	-7,263435	-4,055685	52,7574822	10,2896549
15	-10,669292	-7,263435	113,833796	11,599866
16	-1,168320	-10,669292	1,36497139	90,2684746
17	-2,267566	-1,168320	5,14185511	1,20834177
18	4,388743	-2,267566	19,261066	44,3064495
19	4,795486	4,388743	22,9966831	0,16543954
			1118,20444	545,953706

The Durbin-Watson statistic thus takes the value

$$DW = \frac{\sum_{t=2}^{19} \left(\hat{u}_t - \hat{u}_{t-1}\right)^2}{\sum_{t=1}^{19} \hat{u}_t^2} = \frac{545{,}953706}{1118{,}20444} = 0{,}4882$$

The critical values of the Durbin-Watson test for a significance level of 5% with a sample size of n = 19 and k = 2 regressors (incl. the pseudo variables) are shown in the following diagram:



In our example, the Durbin-Watson-Statistic DW falls in the left region,

$$DW = 0{,}4882 \in [0; d_u = 1{,}18] \text{ ,}$$

which means that from the test result a presence of an autocorrelation in the OLS residuals of the Keynesian consumption function can be inferred. The Durbin-Watson test indicates that private consumption, despite the high level of explanation, can not solely be attributed to disposable income.

In the OLS residuals a systematic variation remains, which could be caused by the neglect of influences such as wealth, the interest rate, sentiment indicators (→ GfK) or time delayed consumption (→ Habit-Persistence-Hypothesis). But the autocorrelation of the disturbance term could also possibly be traced back to an inadequacy of the income variable used in the Keynesian consumption function. Clarification on this would be delivered by e.g. econometric tests of the permanent income hypothesis or the life cycle hypothesis. □

6.3.2 Breusch-Godfrey Test

A test for autocorrelation of not only the first but of any order is, for example, the **Breusch-Godfrey test**. The test is also applicable for stochastic regressors. The starting point is the multiple regression model

(2.1b) $$y_t = \mathbf{x}_t' \cdot \boldsymbol{\beta} + u_t$$

which can be extended to m lagged endogenous variable y_{t-1}, y_{t-2}, ... , y_{tm} :

(6.13) $$y_t = \mathbf{y}_{\_t}' \cdot \boldsymbol{\gamma} + \mathbf{x}_t' \cdot \boldsymbol{\beta} + u_t$$

with $$\mathbf{y}_{\_t}' = (y_{t-1} \;\; y_{t-2} \cdots y_{t-m})$$ and $$\boldsymbol{\gamma}' = (\gamma_1 \;\; \gamma_2 \cdots \gamma_m)$$

The extended regression model (6.13) contains in addition to the k non-stochastic explanatory terms (incl. the pseudo variables) x_1, x_2, ... , ... , x_k , m stochastisc regressors in the form of the lagged endogenous variables. For the disturbance variable u in (2.1b) or (6.13) an autoregressive process of the pth order is allowed

(6.14) $$u_t = \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \cdots + \alpha_p u_{t-p} + v_t$$

where v follows a white noise process. The disturbance process (6.14), on which the Breusch-Godfrey test is based, is an **autoregressive process of the pth order**, which is a generalization of the Markov process underlying the Durbin-Watson statistic (= first-order autoregressive process) (6.6).

In the null hypothesis, the absence of autocorrelation is assumed which means that the autoregressive parameters α_j, j = 1,2, ... , p, are equal to 0:

(6.15) $H_0: \alpha_1 = \alpha_2 = ... = \alpha_p = 0$

Then $u_t = v_t$ applies, so that the disturbance variable in (2.1b) and (6.14) satisfies the property of a pure random process (white noise).

The Breusch-Godfrey test for autocorrelation is based on the general Lagrange multiplier principle which only renders necessary an estimation of the restricted model. From this follows that a simple test statistic can be derived. Accordingly, subsequent to the estimation of the model (2.1b) and (6.13), the **auxiliary regression**

(6.16) $$\hat{u}_t = \alpha_0 + \mathbf{z}_t'\boldsymbol{\delta} + \alpha_1\hat{u}_{t-1} + ... + \alpha_p\hat{u}_{t-p} + w_t$$

is run with w as disturbance term.

Variable vector $\mathbf{z}'_t$:

- with regression model (2.1b): $\mathbf{z}'_t = \mathbf{x}'_t$ (= $\mathbf{x}'_t$ without peudo variable x_{1t}= 1)
- with regression model (6.13): $\mathbf{z}'_t = \left(\mathbf{y}'_{t} \quad \mathbf{x}'_t\right)$

Rule of thumb for choosing the lags p:

with annual data: 1 to 4
with quarterly data: 4 or 8
with monthly data: 12 or 24

The **test statistic of the Breusch-Godfrey-Test**

(6.17) $$BG(p) = n \cdot R^2$$

is, with a normal distribution of the disturbance term u, under the null hypothesis (6.15) asymptotic χ^2-distributed with p degrees of freedom. R^2 is herein the coefficient of determination of the auxiliary regression (6.16).

Test decision:

$BG(p) > \chi^2_{p\ 1-\alpha}$ → H_0 to be rejected

Accepting the null hypothesis: acceptance of no p-order autocorrelation
Rejecting the null hypothesis: acceptance of p-order autocorrelation

Example:

The null hypothesis of a lack of autocorrelation of the disturbance term in the Keynesian consumption function (2.27) shall be investigated with the Breusch-Godfrey test. In contrast to monthly and quarterly data is by annual data mainly first-order autocorrelation of relevance, because the time distance for higher order autocorrelations here rapidly becomes big.

[Note: The Breusch-Godfrey test for first-order autocorrelation is carried out in detail with the help of the least-squares-estimate of the auxiliary regression (6.16) and the subsequent calculation of the test statistic (6.17). Supplementary are the test results for a higher order autocorrelation displayed, determined with the function bgtest of the R-Package lmtest.]

In the investigation of the first-order autocorrelation of the disturbance term we test the null hypothesis

$$H_0: \alpha_1 = 0$$

contrasting it with the alternative hypothesis

$$H_1: \alpha_1 \neq 0$$

Since the Keynesian consumption function has disposable income Y^v as the only explanatory variable, the variable vector $\mathbf{z}'_t$ in the auxiliary regression (6.16) includes solely the regressor Y^v . OLS estimation of the auxiliary regression (6.16), e.g. with R, yields

$$\hat{u}_t = -1{,}9913 + 0{,}0016 \cdot Y_t^V + 0{,}7075 \cdot \hat{u}_{t-1}$$

t-value → $(-0{,}155)\ (0{,}170)\qquad (3{,}912)$.

Note: The lagged residual series $\hat{u}_{t-1}$ is in the first period set equal to 0, so that the auxiliary regression (6.16) is always carried out with the complete n observations.

The t-value estimated autoregressive coefficient $\hat{\alpha}$ of 3,912 indicates significance at the 1% level (critical value: $t_{16;\,0{,}995} = 2{,}921$). With a measure of determination of the auxiliary regression of 0,4888 and a sample size of 19 one obtains the value for the test statistic BG of the Breusch-Godfrey test

$$BG(1) = n \cdot R^2 = 19 \cdot 0{,}4888 = 9{,}287$$

Critical value ($\alpha = 0{,}01$; $p = 1$): $\chi^2_{1;\,0{,}99} = 6{,}63$

Test decision:

BG (1) = 9,287> $\chi^2_{1;\,0{,}99} = 6{,}63$ (P = 0,002))
→ Acceptance of a first-order autocorrelation