Tasks for Introductory Econometrics

3. Significance tests and confidence intervals

3.1 What is the standard error of the regression (SER) from GASD to DIINC? (energy model Ib, see solutions of exercise 2.8) (with interpretation)?

3.2 The OLS estimators of the regression coefficients β_1 and β_2 of the natural gas demand function (energy model Ia) are (n = 7): $\hat{\beta}_1 = 5{,}809$ and $\hat{\beta}_2 = -0{,}368$. The sum of the squared residuals is 0.067, the sum of the gas prices is 91.2 and the sum of the squared gas prices is 1190.28.

Test the OLS estimators of the regression coefficients for significance (α = 0.01)!

3.3 Given is the OLS-estimated demand function (≙ energy model II)

$$GASD_t = 2{,}706 + 0{,}199 \cdot GASP_t + 0{,}074 \cdot DIINC_t$$

and the inverse of the product matrix **X'X**

$$(\mathbf{X'X})^{-1} = \begin{pmatrix} 255{,}7066 & -15{,}7459 & -4{,}1520 \\ -15{,}7459 & 0{,}997745 & 0{,}2262 \\ -4{,}1520 & 0{,}2262 & 0{,}099228 \end{pmatrix}.$$

The sum of the squared residuals is 0,0127.

Test the significance of the impact of gas price (GASP) and disposable income (DIINC) on the demand for natural gas (α = 0.05)!

3.4 Determine the 95% confidence intervals of the unknown regression coefficients β_1 and β_2 of the energy model Ia (for details see Exercise 3.2)!

3.5 Calculate for the energy model II (see exercise 2.5) the two residuals for the first and last year of the data interval (Stützbereich)! Which statement about the quality of the adjustment at the margines can be derived from them?

3.6 The OLS-estimated demand function for natural gas is given.

$$\widehat{GASD}_t = 5{,}809 - 0{,}368 GASP_t, t = 1,2, \dots 7.$$

a) Reflect the sources of dispersion for natural gas demand in the form of an ANOVA table!

b) Test the overall connection of the model using the sum of squared deviations found in the ANOVA table (α=0.05)!

c) Test the overall connection of the model using the coefficient of determination (α=0.05)!

3.7 Using the time series data on the investments (i), the nominal interest rate (i) and the inflation rate (π)

t	I_t	i_t	π_t
1	100	6	2
2	108	5,8	2,2
3	122	5	2,6
4	128	4,8	3,2
5	134	4	3,5

The investment function

$$I_t = \beta_1 + \beta_2 \cdot i_t + \beta_3 \cdot \pi_t + u_t$$

should be econometrically estimated and tested. The disturbance variable u_t fulfills the standard assumptions.

a) Estimate the investment function with the ordinary least squares method using the matrix calculus!

<u>Note:</u> The inverse $(\mathbf{X'X})^{-1}$ reads as follows:

$$(\mathbf{X'X})^{-1} = \begin{bmatrix} 423,482 & -49,786 & -62,361 \\ -49,786 & 5,918 & 7,217 \\ -62,361 & 7,217 & 9,411 \end{bmatrix}.$$

b) In macroeconomic theory it is assumed that investments i depend only on the real interest rate i-π and that the inflation rate have no further effects. Give the linear restriction behind this statement in scalar and matrix notation!

c) Test the linear restriction given in part b) with the t-test (α=0.05) using the unbiased estimator of the variance of the error term of 16.950! Interpret the test result!

d) Test the linear restriction given in part b with an t-test (α=0,05)!

3.8 The OLS estimate of demand for natural gas (GASD) as a function of natural gas price (GASP), district heating price (DHC) and disposable income (DIINC) (energy model III) is as follows:

Dependent Variable: GASD
Method: Least Squares
Sample: 1980 1995
Included observations: 16

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-8.670524	5.240572	-1.654500	0.1239
GASP	-4.510243	2.910106	-1.549855	0.1471
DHC	16.21776	4.035146	4.019125	0.0017
DIINC	0.006006	0.001900	3.160758	0.0082

R-squared	0.795878	Mean dependent var	12.65000
Adjusted R-squared	0.744847	S.D. dependent var	1.402379
S.E. of regression	0.708379	Akaike info criterion	2.360643
Sum squared resid	6.021608	Schwarz criterion	2.553790
Log likelihood	-14.88514	F-statistic	15.59609
Durbin-Watson stat	1.461440	Prob(F-statistic)	0.000193

a) What information on the overall adjustment of demand for natural gas can be found in the output table?

b) Show the relation between „S.E. of regression“ and „Sum squared resid“!

c) Determine 99% confidence intervals for the regression coefficients of the dependend variables (without a absolut term)! What conclusions can you make about the significance of the estimated regression coefficients?