7. Models with qualitative variables

7.1 Models with qualitative regressors

Qualitative regressors in econometric models:

- different preferences between different groups of economic agents
- Capturing spatial effects (country or regional effects),
- Capturing temporal effects (e.g. seasonal effects)

Example:

- Differences in consumption and savings behavior in different social classes of the population can be captured by a qualitative regressor „social class“.
- For certain products different purchasing behavior of woman and men is to be expected, which can be considered in the form of a qualitative regressor „gender". □

When modelling qualitative variables it should be noted that in the standard regression analysis models only metrically scaled variables are in actual fact allowed. The only exception to this are **dummy variables** (0,1 variables), which in the simplest case measure constant additions and deductions. They can however also be combined with other explanatory variables. **Qualitative variables** must therefore always be included in the econometric single equation model (2.1) in the form of **one or more dummy variables**.

In the folowing we treat **three variants** of an inclusion of a **qualitative variable** in a multiple regression model:

A) Additive binary regressor
B) Multiplicative binary regressor (interaction)
C) Additive polytomous regressor

A) Additive binary regressor

We want to show the additive inclusion of a binary, i.e. a two-valued, regressor in a multiple regression model using a simple **demand function**, in which the demand q_t for a certain type of good is modelled in dependence of total expenditure x_t:

(7.1) $$q_t = \beta_1 + \beta_2 x_t + u_t$$

The disturbance variable u_t should satisfy the standard assumptions of the econometric one equation model.

Suppose now a type of good whose demand depends on gender. Examples include the demand for cosmetics or technical products which generally varies greatly between men and women.

How can then the influence of the qualitative variable „gender“ be taken account of in the analysis of demand? We define to this end a dummy variable g_t, which assumes the value 1 if the statistical unit t is female and the value 0 if the unit t is a male:

$$g_t = \begin{cases} 1, & \text{falls Frau} \\ 0, & \text{falls Mann}. \end{cases}$$

The demand function (7.1), with an additive extension with the qualitative regressor gender, is then

(7.2) $$q_t = \beta_1 + \beta_2 x_t + \beta_3 g_t + u_t$$

With eight statistical units arranged in the order 1x male, 1x female, 5x male, 1x female based on the attribute „gender“, the observation matrix **X** would have the following form:

$$\mathbf{X} = \begin{bmatrix} 1 & x_1 & 0 \\ 1 & x_2 & 1 \\ 1 & x_3 & 0 \\ 1 & x_4 & 0 \\ 1 & x_5 & 0 \\ 1 & x_6 & 0 \\ 1 & x_7 & 0 \\ 1 & x_8 & 1 \end{bmatrix}$$

As can be seen, a very specific structure or design of the observation matrix **X** hereby appears, so that with the inclusion of qualitative variables it is also sometimes called a design matrix.

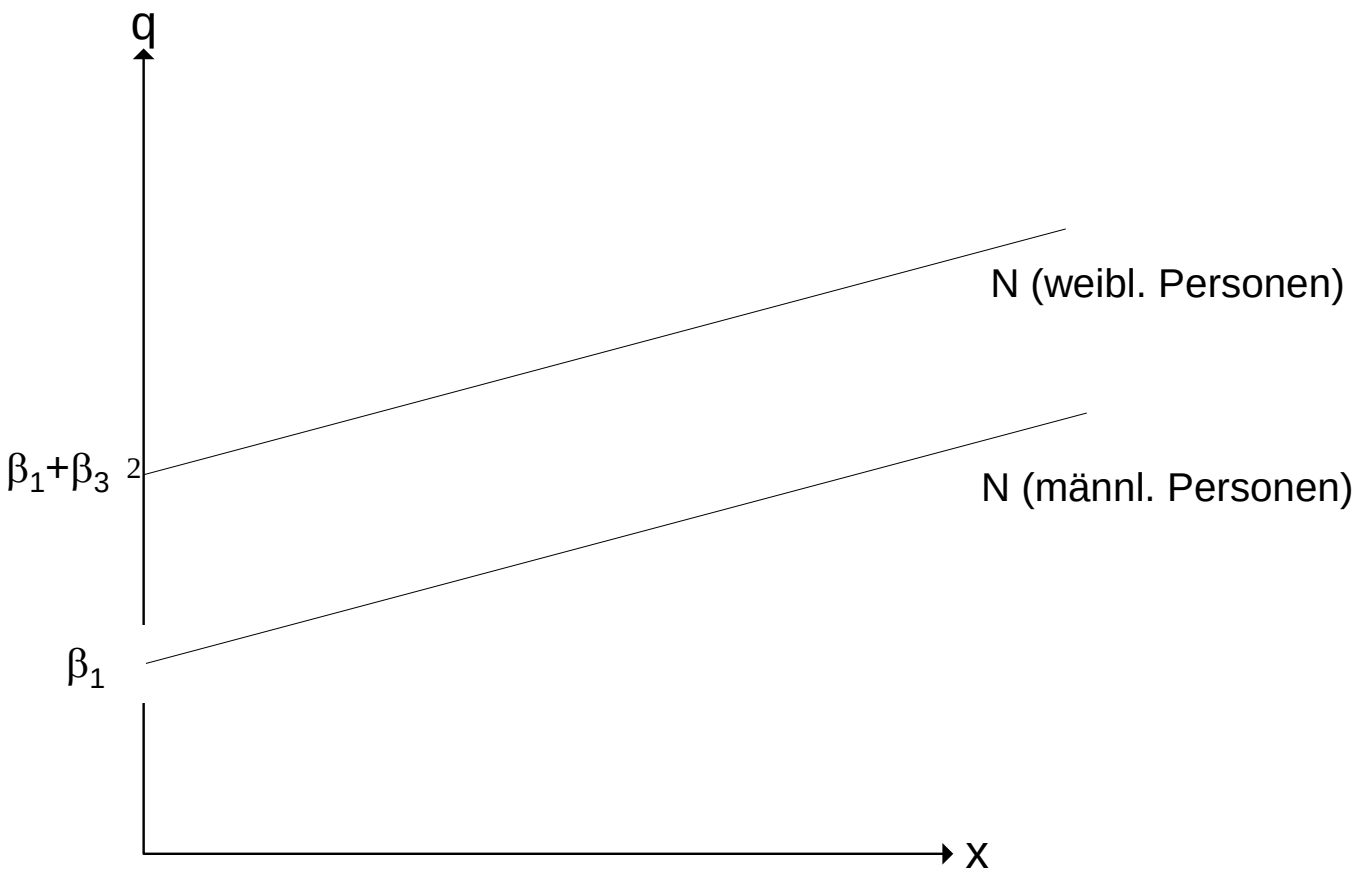
One achieves an interpretation of the influence of the qualitative regressor gender when one considers the demand function (7.2) separately for male and female persons:

for male persons: $q_t = \beta_1 + \beta_2 x_t + u_t$

for female persons: $q_t = (\beta_1 + \beta_3) + \beta_2 x_t + u_t$

The inclusion of the qualitative variable gender causes a parallel intercept shift of the demand function (N), which is shown in figure 7.1 for the case $ß_2 > 0$. The dummy variable measures in this case the larger demand of women compared to men for the considered type of good at a given level of total expenditure.

Fig. 7.1: Parallel shift of the demand function



Econometric estimation of a regression model with qualitative regressors (with validity of the standard assumptions):
ordinary least squares method (OLS method)

Advantage compared to a **separate estimation:**
Efficiency gain due to the greater number of degrees of freedom

Significance test of the OLS estimator $\hat{\beta}_3$: t-test

Significance of the estimator $\hat{\beta}_3$ means in the case of the demand function (7.2) that a difference in demand behaviour between women and men relating to the considered type of goods is statistically secure.

Example:

We consider the demand for a technical diy article. A sample of eight buyers in a hardware store (diy store) has generated the following data:

Buyer	1	2	3	4	5	6	7	8
Purchased amount (y)	9	1	4	8	14	4	12	2
Total expenditure (x_2)	48	30	23	56	72	32	42	43
Gender (x_3)	male	female	male	male	male	male	male	female

To econometrically estimate the demand function of the technical diy product (y) in dependence on total expenditure (x_2) and gender (x_3) we code the qualitative regressor "gender" as follows:

$$x_{3t} = \begin{cases} 1, & \text{falls Frau} \\ 0, & \text{falls Mann}. \end{cases}$$

With the coding of the regressor “gender" as a dummy variable we get the observation matrix (design matrix)

$$\mathbf{X} = \begin{bmatrix} 1 & 48 & 0 \\ 1 & 30 & 1 \\ 1 & 23 & 0 \\ 1 & 56 & 0 \\ 1 & 72 & 0 \\ 1 & 32 & 0 \\ 1 & 42 & 0 \\ 1 & 43 & 1 \end{bmatrix}$$

The vector **y** of the dependent variable contains the quantities purchased:

$$\mathbf{y} = \begin{bmatrix} 9 \\ 1 \\ 4 \\ 8 \\ 14 \\ 4 \\ 12 \\ 2 \end{bmatrix}$$

Using the EViews program, we obtain the estimated demand function with various statistics and test results:

Dependent Variable: Y (Nachfrage q)

Method: Least Squares

Sample: 1 8 Included observations: 8

Variable	Coefficient	Std. Error	t-Statistic	Prob.
X_1 (C)	0.046758	2.889029	0.016185	0.9877
X_2 (x)	0.185786	0.059754	3.109164	0.0266
X_3 (g)	-5.327930	2.026657	-2.628926	0.0466

R-squared	0.818185	Mean dependent var	6.750000
Adjusted R-squared	0.745459	S.D. dependent var	4.743416
S.E. of regression	2.393153	Akaike info criterion	4.863097
Sum squared resid	28.63591	Schwarz criterion	4.892888
Log likelihood	-16.45239	F-statistic	11.25022
Durbin-Watson stat	2.903200	Prob(F-statistic)	0.014095

Interpretation:

The coefficient of determination of 0,818 indicates that the demand for the diy article(y) is at 81,8% explained by total expenditure (x_2) and gender (x_3). The corresponding F-value of 11,25 is significant at the 5% level. The empirical significance level (p-value) amounts to 0,014.

While the intercept of the demand function is not significant, both considered influences are statistically secured at a significance level of 5%:

Total expenditure (x_2): $| t = 3,109 | > t_{5;\ 0,975} = 2,571 \rightarrow$ reject H_0

or

$p = 0,0266 < \alpha = 0,05 \rightarrow$ reject H_0

Gender (x_3): $| t = -2,629 | > t_{5;\ 0,975} = 2,571 \rightarrow$ reject H_0

or

$p = 0,0466 < \alpha = 0,05 \rightarrow$ reject H_0

Reaction of demand to changes in the explanatory variables:

Increase in total expenditure by 1 unit
→ Increase in demand of 0,186 units

On average, the demand of women is 5,328 units less than that of men

Demand function of male buyers:

$$\hat{y}_{männl} = 0{,}047 + 0{,}186 \cdot x_2$$

Demand function of female buyers:

$$\hat{y}_{weibl} = 0{,}047 + 0{,}186 \cdot x_2 - 5{,}328 \cdot x_3$$

□

B) Multiplicative binary regressors (interactions)

Note that the demand function (7.2) implicitly contains the hypothesis of an equal marginal propensity to consume with respect to the considered good, because the regression coefficient ß$_1$ for both groups of people is regarded as equal. With the econometric model (7.2) only the relevance of a parallel shift of the demand function can be appraised.

The possibility of a **different marginal propensity to consume** can be captured by the econometric model

(7.3) $$q_t = \beta_1 + \beta_2 x_t + \beta_3 x_t g_t + u_t$$

The binary regressor "gender" (g) is in (7.3) taken account of in a multiplicative form, i.e. as interaction. The interaction term $x_t \cdot g_t$ does not apply to male consumers, since g is 0 in this case:

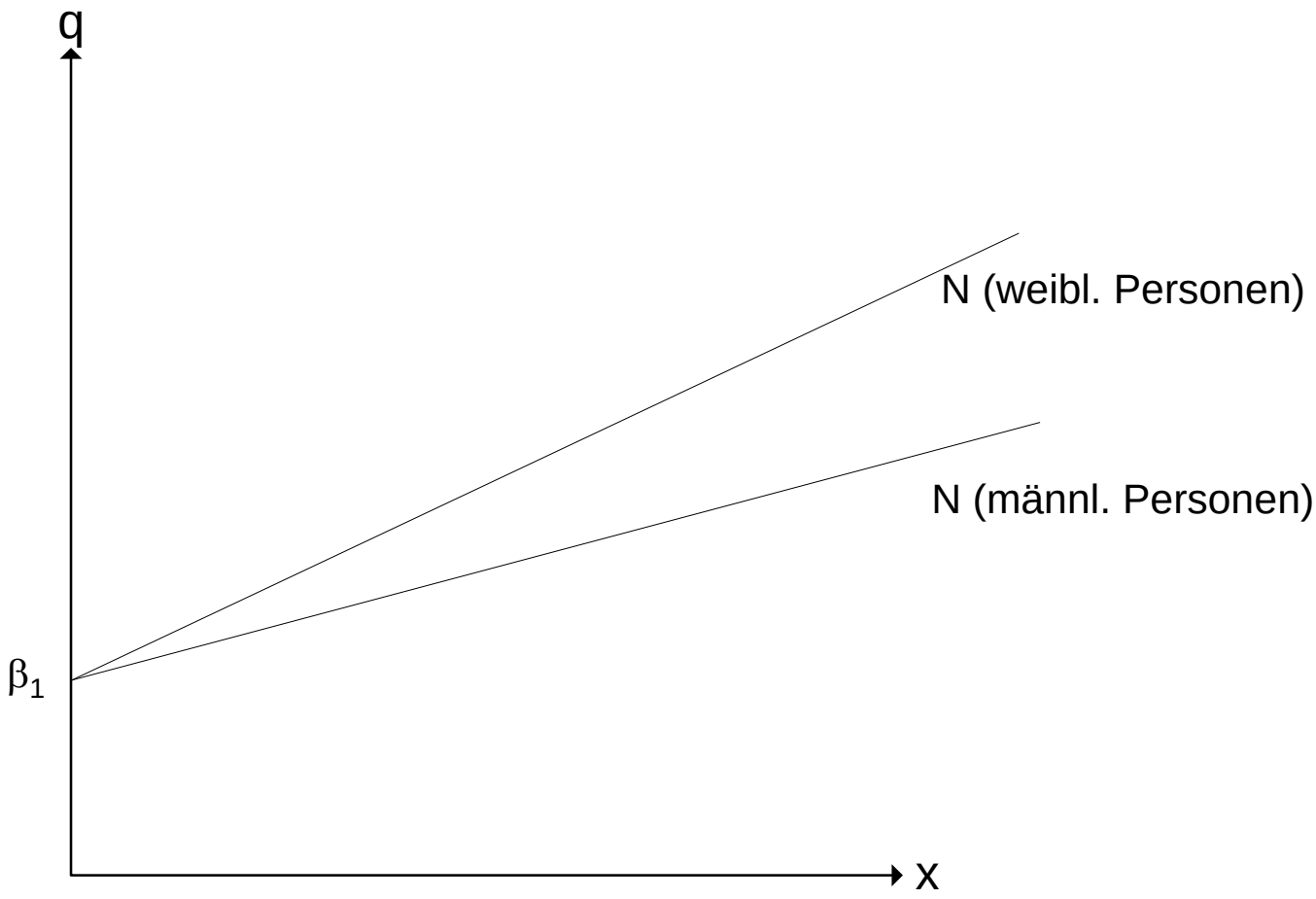
for male persons: $$q_t = \beta_1 + \beta_2 x_t + u_t$$

For female consumers the interaction term $x_t \cdot g_t$ corresponds at the same time to the respective total expenditures, since g is equal to 1. The regression coefficient ß$_2$ then indicates the change in the marginal propensity to consume of women compared to men:

for female persons: $$q_t = \beta_1 + (\beta_2 + \beta_3) x_t + u_t$$

Figure 7.2 depicts the situation $ß_2 > 0$, where the marginal propensity to consume the considered product is higher among women than among men.

Figure 7.2: Change in the marginal propensity to consume



Of course it is also possible to conceive both behavioral differences between the groups of persons simultaneously in an econometric model.

Example:

We now estimate the demand function for the technical diy product while deploying a multiplicative link between total expenditure (x) and gender (g).

For this we once again code the qualitative variable “gender" (g) as a dummy variable:

$$g_t = \begin{cases} 1, & \text{falls Frau} \\ 0, & \text{falls Mann}. \end{cases}$$

The vector of the dependent variable (demand q), **y**, remains unchanged compared to the model with an additive regressor. The observation matrix **X** now contains in the third column the values of the interaction term $x_t \cdot g_t$ (= Variable x_3):

$$\mathbf{X} = \begin{bmatrix} 1 & 48 & 0 \\ 1 & 30 & 30 \\ 1 & 23 & 0 \\ 1 & 56 & 0 \\ 1 & 72 & 0 \\ 1 & 32 & 0 \\ 1 & 42 & 0 \\ 1 & 43 & 43 \end{bmatrix}$$

The OLS estimation of the model (7.3) yields the following result:

Dependent Variable: Y (Nachfrage q)
Method: Least Squares
Sample: 1 8 Included observations: 8

Variable	Coefficient	Std. Error	t-Statistic	Prob.
X_1 (C)	-0.321410	2.779895	-0.115619	0.9125
X_2 (x)	0.193655	0.057966	3.340858	0.0205
X_3 (x*g)	-0.142923	0.052759	-2.708974	0.0423

R-squared	0.824481	Mean dependent var	6.750000
Adjusted R-squared	0.754273	S.D. dependent var	4.743416
S.E. of regression	2.351350	Akaike info criterion	4.827853
Sum squared resid	27.64423	Schwarz criterion	4.857643
Log likelihood	-16.31141	F-statistic	11.74348
Durbin-Watson stat	2.619748	Prob(F-statistic)	0.012907

Interpretation:

The coefficient of determination of 0,824 indicates that the demand for the technical diy product (y) is at 82,4% explained by total expenditure (x_2) and the interaction between total expenditure and gender (x_3). The corresponding F-value of 11,74 is significant at the 5% level. The empirical significance level (p-value) amounts to 0,013.

While the absolute term of the demand function is not significant, both considered influence factors are statistically secure on a significance level of 5%:

Total expenditure (x_2): $| t = 3{,}341 | > t_{5;\ 0{,}975} = 2{,}571 \rightarrow$ reject H_0
or
$p = 0{,}0205 < \alpha = 0{,}05 \rightarrow$ reject H_0

Interaction (x_3): $| t = -2{,}709 | > t_{5;\ 0{,}975} = 2{,}571 \rightarrow$ reject H_0
or
$p = 0{,}0423 < \alpha = 0{,}05 \rightarrow$ reject H_0

Reaction of demand to changes in the explanatory variables:

Increase in total expenditure by 1 unit
→ Increase in demand of 0,194 units

The marginal propensity to consume the technical diy product is 0,143 lower for women than for men

The demand function of male buyers (marginal propensity to consume: 0,194):

$$\hat{y}_{männl} = -0{,}321 + 0{,}194 \cdot x_2$$

The demand function of female buyers (marginal propensity to consume: 0,194 – 0,143 = 0,051):

$$\hat{y}_{weibl} = -0{,}321 + 0{,}051 \cdot x_2$$

□

C. Additive polytomous regressors

By means of a **qualitative variable** with **three attributes** we want to discuss in general the inclusion of qualitative regressors in econometric models. For this purpose, the demand for a goods type is considered in dependence on the **social class** of a household in addition to total expenditures. If one now for each of the three categories of social class (s) would introduce a dummy variable, then the demand function would be

$$q_t = \beta_1 + \beta_2 x_t + \beta_3 s_{1t} + \beta_4 s_{2t} + \beta_5 s_{3t} + u_t$$

with

$$s_{ht} = \begin{cases} 1, & \text{falls soziale Stellung } h \\ 0, & \text{sonst} \end{cases}, \quad h = 1,2,3$$

With this, e.g. with 8 households, the following design of the observation matrix **X** would appear:

$$X = \begin{bmatrix} 1 & x_1 & 0 & 1 & 0 \\ 1 & x_2 & 1 & 0 & 0 \\ 1 & x_3 & 1 & 0 & 0 \\ 1 & x_4 & 0 & 0 & 1 \\ 1 & x_5 & 0 & 0 & 1 \\ 1 & x_6 & 0 & 1 & 0 \\ 1 & x_7 & 1 & 0 & 0 \\ 1 & x_8 & 0 & 0 & 1 \end{bmatrix}$$

Household 1: middle social class
Household 1: lower social class
Household 1: lower social class
etc.

In this design, the sum of the last three columns yields exactly the first column:

Column 1 = Column 3 + Column 4 + column 5.

The columns of the observation matrix containing the variable values are thereby not linearly independent. The rank of the observation matrix **X** is one less than the number of its columns, from which follows that the product matrix **X'X** is not invertible. The OLS estimator $\hat{\boldsymbol{\beta}}$ would therefore with such a design not be determinable.

For this reason must, by the inclusion of a qualitative regressor in an econometric model, be proceeded as follows.

Qualitative regressor with **h categories**: Formation of **h-1 dummy variables**

In our example, one could e.g. for each of the categories 2 and 3 of the social class compose one dummy variable:

$$s_{2t} = \begin{cases} 1, & \text{falls mittlere soziale Stellung} \\ 0, & \text{sonst} \end{cases}$$

and

$$s_{3t} = \begin{cases} 1, & \text{falls obere soziale Stellung} \\ 0, & \text{sonst} \end{cases}$$

The qualitative regressor “social lass" is in the multiple regression model replaced by the two dummy variables s_2 and s_3, which are being labelled “middle social class" and “upper social class". If both dummy variables take on the value 0 for a buyer or a household, a low social class is present.

With a correct dummy variable coding of "social class" the demand function is therefore

$$q_t = \beta_1 + \beta_2 x_t + \beta_3 s_{2t} + \beta_4 s_{3t} + u_t \qquad (7.4)$$

From this appears a design of the observation matrix **X**, which allows a determination of the OLS estimator $\hat{\boldsymbol{\beta}}$ for the unknown regression coefficients.

For the considered households the observation matrix takes the form

$$\mathbf{X} = \begin{bmatrix} 1 & x_1 & 1 & 0 \\ 1 & x_2 & 0 & 0 \\ 1 & x_3 & 0 & 0 \\ 1 & x_4 & 0 & 1 \\ 1 & x_5 & 0 & 1 \\ 1 & x_6 & 1 & 0 \\ 1 & x_7 & 0 & 0 \\ 1 & x_8 & 0 & 1 \end{bmatrix}$$

which now contains no linear dependence between the columns of the dummy variables and the column of the pseudo variable.

The interpretation of the coefficients follows from the separated presentation of the demand function of the three categories of social class:

for the lower social position: $q_t = \beta_1 + \beta_2 x_t + u_t$

for the middle social class: $q_t = (\beta_1 + \beta_3) + \beta_2 x_t + u_t$

for the upper social position: $q_t = (\beta_1 + \beta_4) + \beta_2 x_t + u_t$

The regression coefficients $ß_2$ and $ß_3$ measure the excess or deficit in demand of a household in the middle or upper social class against a household in the lowest category. The **lower social class** thus serves as **reference category**. Just as well could also the middle or upper social class have been used as reference category, which in the interpretion would have to be accordingly acknowledged.

Dummy variables are often included in econometric models with data covering less than one year to account for seasonal influences. In principle, the modelling is carried out in exactly the same manner as with the here considered qualitative variables. With quarterly data e.g. 3 and with monthly data 11 dummy variables have to be formed. The regression coefficients of the dummy variables indicate in this connection the seasonal fluctuations in the unit periods with respect to a reference quarter or month.

Example:

The demand function for the technical diy product shall be econometrically estimated in dependence on total expenditure (x) and "social class".

To this end, we replace the qualitative variable “social class" by the two dummy variables “middle social class" (s_2) and “upper social class" (s_3):

$$s_{2t} = \begin{cases} 1, & \text{falls mittlere soziale Stellung} \\ 0, & \text{sonst} \end{cases} \quad \text{and} \quad s_{3t} = \begin{cases} 1, & \text{falls obere soziale Stellung} \\ 0, & \text{sonst} \end{cases}$$

The vector of the dependent variable (demand q), **y**, remains unchanged with respect to the model with gender as the qualitative regressor. The observation matrix now has the following form:

$$\mathbf{X} = \begin{bmatrix} 1 & 48 & 1 & 0 \\ 1 & 30 & 0 & 0 \\ 1 & 23 & 0 & 0 \\ 1 & 56 & 0 & 1 \\ 1 & 72 & 0 & 1 \\ 1 & 32 & 1 & 0 \\ 1 & 42 & 0 & 0 \\ 1 & 43 & 0 & 1 \end{bmatrix}$$

OLS estimation of the demand model (7.4) gives the following results:

Dependent Variable: Y (Nachfrage q)
Method: Least Squares
Sample: 1 8 Included observations: 8

Variable	Coefficient	Std. Error	t-Statistic	Prob.
X1 (C)	-7.336207	3.277255	-2.238522	0.0888
X2 (x)	0.410617	0.092781	4.425640	0.0115
X3 (s_2)	-2.588475	2.422406	-1.068555	0.3455
X4 (s_3)	-8.068966	3.121037	-2.585348	0.0610

R-squared	0.839383	Mean dependent var	6.750000
Adjusted R-squared	0.718920	S.D. dependent var	4.743416
S.E. of regression	2.514815	Akaike info criterion	4.989129
Sum squared resid	25.29719	Schwarz criterion	5.028849
Log likelihood	-15.95651	F-statistic	6.967985
Durbin-Watson stat	2.187155	Prob(F-statistic)	0.045698

Interpretation:

The coefficient of determination of 0.839 indicates that the demand for the Diy product (y) is at 83.9% explained by total expenditure (x_2) and "social class" (dummy variables s_2 and s_3). The associated F-value of 6.968 is significant at the 5% level. The empirical significance level (p-value) amounts to 0.046.

The estimated regression coefficient $\hat{\beta}_2$ of total expenditure (x_2) has the expected positive sign and is statistically secured at a significance level of 5%:

Total expenditure (x_2): $| t = 4.426 | > t_{5;\ 0.975} = 2.571$ → reject H_0

or

$p = 0{,}012 < \alpha = 0.05$ → reject H_0

Increase in total expenditure by 1 unit
→ Increase in demand of 0.411 units

The two negative regression coefficients $\hat{\beta}_3$ and $\hat{\beta}_4$ relating to the dummy variables “middle social class" and "upper social class" indicate that the technical diy product tend to be more in demand by households in the lower social class. Since in addition $|\hat{\beta}_3| < |\hat{\beta}_4|$ applies, it can also be inferred that demand tends to retreat with a higher position in the social hierarchy.

Accordingly, the demand for the diy product falls with 2.588 units in households of the middle class compared to bottom class households. Demand of upper class households decreases by 5.481 (= 8.069 – 2.588) units compared with the households of the middle class and relative to the households of the lower class by 8.069 units.

The **group specific demand functions** are:

for the lower social class: $\hat{y}_{s1} = -7{,}336 + 0{,}411 \cdot x_t + u_t$

for the average social class: $\hat{y}_{s2} = -9{,}924 + 0{,}411 \cdot x_t + u_t$

for the upper social class: $\hat{y}_{s3} = -17{,}994 + 0{,}411 \cdot x_t + u_t$

However, since $\hat{\beta}_3$ is not significant at any of the usual levels, the reduction in the demand of the households of the middle position in comparison with the house-holds of the lower position is not statiscally secured. E.g. for α= 0.10 applies:

Middle social class (s_2): $| t = -1.069 | < t_{4;\,0,95} = 2.132 \rightarrow H_0$ accept

or

$p = 0.346 > \alpha = 0.10 \rightarrow H_0$ accept

The regression coefficient $\hat{\beta}_4$ of the dummy variable "upper social class" is in fact not significant at the 5% but at the 10% level ($t_{4;\,0,975} = 2.776$):

Upper social position (s_3): $| t = -2.585 | > t_{4;\,0,95} = 2.132 \rightarrow H_0$ reject

or

$p = 0.061 < \alpha = 0.10 \rightarrow H_0$ reject

The influence of the upper social class (s_3) on demand is therefore also referred to as weakly significant. □

7.2 Structural break test

In an economy from time to time events occur that permanently change its structure in certain ways. After a war, an oil price shock, basic innovations or a fundamental regulatory policy decision can the general economic framework have experienced a new orientation, which will not be without lasting influence on the structural parameters of an econometric model. If this is the case, the econometrician speaks of a **structural break.** Before the structural break in the period n_1+1 had the econometric model

(7.5) $$y_t = \sum_{j=1}^{k} \beta_j^1 \cdot x_{jt} + u_t, \quad t = 1,2,...,n_1$$

validity; after that the model has the form

(7.6) $$y_t = \sum_{j=1}^{k} \beta_j^2 \cdot x_{jt} + u_t, \quad t = n_1 + 1, n_1 + 2,...,n$$

For the entire period of observation, the econometric model can, with the use of two dummy variables d1 and d2 which take account of the structural break, be formulated as follows:

(7.7) $$y_t = \sum_{j=1}^{k} \beta_j^1 \cdot x_{jt} \cdot d_{1t} + \sum_{j=1}^{k} \beta_j^2 \cdot x_{jt} \cdot d_{2t} + u_t, \quad t = 1,2,...,n$$

with

$$d_{1t} = \begin{cases} 1 & \text{für } t=1,2,...,n_1 \\ 0 & \text{für } t=n_1+1, n_1+2, ...,n \end{cases}$$

and

$$d_{2t} = \begin{cases} 0 & \text{für } t=1,2,...,n_1 \\ 1 & \text{für } t=n_1+1, n_1+2, ..., n. \end{cases}$$

In this way, the fact is taken into account that with a structural break all regression coefficients can have changed.

The econometrician now has to judge whether a particular event actually has led to a structural break or not. It is thus to be **tested,** whether the economic framework conditions have changed to such an extent that it is no longer allowed to presume a **constancy of the structural parameters of the econometric model** for the entire observation period. A test that makes it possible to reach a statement on this point is the **Chow test**.

With the **Chow test** three regressions are to be be carried out. On the one hand, the regression equations (7.5) and (7.6) are estimated for the sub-period before and after the structural break. From this one obtains the two sums of squares of the residuals in the two sub-periods:

before the structural break: $Q_1 = \sum_{t=1}^{n_1} \hat{u}_t^2$ mit $n_1 - k$ Freiheitsgraden

after the structural break: $Q_2 = \sum_{t=n_1+1}^{n} \hat{u}_t^2$ mit $n_2 - k$ Freiheitsgraden

In addition, the regression equation

(7.8) $$y_t = \sum_{j=1}^{k} \beta_j \cdot x_{jt} + u_t, \quad t = 1,2,...,n$$

is being estimated for the entire observation period, leading to the following sum of squares of the residuals:

$$Q = \sum_{t=1}^{n} \hat{u}_t^2 \quad \text{mit n-k Freiheitsgraden}$$

• with stability of the structural parameter:

Model adaptation does not improve decisively when one considers separately the two sub-periods before and after a suspected structural break.

$\Rightarrow$ The sum of squares Q of the residuals will then not be substantially greater than the two sums of squares Q_1 and Q_2 taken together

• with instability of the structural parameter:

The residual sum of squares Q in contrast noticeably exceeds the two sums of squares Q_1 and Q_2

The null hypothesis of the Chow test

$$H_0 : \beta_j^1 = \beta_j^2 \quad \text{für alle } j = 1,2,...,k$$

is appraised using the deviation sums of squares Q, $Q_{1,}$ and Q_2.

The test statistic of the **Chow test**,

(7.9) $$F = \frac{[Q - (Q_1 + Q_2)] \ / \ k}{(Q_1 + Q_2) \ / \ (n - 2k)}$$

is F-distributed with

$$(n - k) - [(n_1 - k) + (n_2 - k)] = k \qquad \text{(for nominator term)}$$

and

$$(n_1 - k) + (n_2 - k) = n - 2k \qquad \text{(for denominator term)}$$

degrees of freedom.

Test decision:

$$F > F_{k;n-2k;1-\alpha} \rightarrow H_0 \qquad \rightarrow H_0 \text{ reject}$$

The difference between the residual sum of squares for the entire observation period and the two sub-periods before and after the presumed structural break can then no longer be regarded as coincidental.

Example:

For 1975, the German Bundesbank for the first time published a target relating to the Central bank money supply. The central bank money suply and later the money supply M3 replaced since then the free liquidity reserves and the interest rate level as indicators and intermediate targets of monetary policy. The question arises whether the re-orientation in monetary policy has shown up in effects on the demand for money in terms of a structural break, which here shall be examined in an exemplary manner with the Chow-test.

The starting point in this connection is the OLS estimated money demand function for the entire observation period from 1970 to 1989:

$$\widehat{\ln m_t} = \underset{}{-4{,}741} + \underset{(-10{,}900)}{1{,}424} \cdot \ln y_t - \underset{(24{,}903)}{0{,}077} \cdot \ln r_t \underset{(-4{,}076)}{},$$

$$R^2 = 0{,}978, \quad DW = 1{,}192,$$

for which the residual sum of squares Q takes the value of 0,014855.

- before a possible structural break (time period between 1970 and 1974):

$$\widehat{\ln m_t} = \underset{(-2,497)}{-2,267} + \underset{(8,552)}{1,090} \cdot \ln y_t - \underset{(-4,327)}{0,098} \cdot \ln r_t ,$$

$$R^2 = 0,974, \quad DW = 2,932$$

Residual sum of squares : $Q_1 = 0,000321$

- after a possible structural break (time period between 1975 and 1989):

$$\widehat{\ln m_t} = \underset{(-8,385)}{-5,637} + \underset{(17,079)}{1,544} \cdot \ln y_t - \underset{(-3,504)}{0,080} \cdot \ln r_t ,$$

$$R^2 = 0,961, \quad DW = 1,323$$

Residual sum of squares : $Q_2 = 0,010708$

Test statistic of the Chow-test (k = 3 and n = 20):

$$F = \frac{[Q - (Q_1 + Q_2)]/K}{(Q_1 + Q_2)/(n - 2k)} = \frac{[0{,}014855 - (0{,}00032 + 0{,}10708)]/3}{(0{,}000321 + 0{,}010708)/(20 - 2 \cdot 3)}$$
$$= \frac{0{,}001275}{0{,}000788} = 1{,}631$$

Critical value (α= 0,05):

$$F_{3;14;0,95} = 3{,}34$$

Test decision:

$$(F = 1{,}631) < (F_{3;14;0,95} = 3{,}34) \quad \rightarrow H_0 \text{ accept}$$

Thus, with the change in monetary policy strategy, a concurrent structural break pertaining to the money demand behavior of the audience can not be detected. □