Tasks for Introductory Econometrics

5. Heteroscedasticity

5.1 What is meant by heteroskedasticity and what can cause this model defect?

5.2 Which inferential statistical consequences result for the OLS parameter estimators of a multiple regression model in the case of heteroscedasticity?

5.3 Explain the basic idea of the generalized least squares method (GLS method) to eliminate heteroscedasticity!

5.4 Test the disturbance term of the natural gas demand model as a function of the gas price (= energy model Ia, data see Exercise 2.3) with the Goldfeld-Quandt test for heteroscedasticity ($\alpha = 0.05$)!

5.5 Estimate the demand function for natural gas depending on the gas price (= energy model Ia) using the specification $\sigma_t^2 = \sigma^2 \cdot GASP_t^2$ of the error term variance with the weighted least squares method (WLS method)!
Note: Calculate the WLS estimators for β_1 and β_2 using the formulas for the OLS estimators with $y_t \to y_t^*, x_t \to x_{1t}^*$. Use the transformed variables $(x_{1t}^*)^2$ and $x_{1t}^* \cdot y_t^*$ with three decimal places!

5.6 The White test has produced the following result for the Energy Model III:

White Heteroskedasticity Test:

Obs*R-squared	5.696925	Probability	0.457982

Test Equation: Dependent Variable: RESID^2
Method: Least Squares
Sample: 1980 1995, Included observations: 16

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-29.74777	58.70411	-0.506741	0.6245
GASPR	-25.04399	28.17182	-0.888973	0.3972
GASPR^2	17.07002	15.58422	1.095340	0.3018
FERNWPR	-21.22605	89.98156	-0.235893	0.8188
FERNWPR^2	8.174083	44.31804	0.184441	0.8578
VEINKR	0.068311	0.062335	1.095866	0.3016
VEINKR^2	-2.23E-05	2.03E-05	-1.097094	0.3011

R-squared	0.356058	Mean dependent var	0.376351
Adjusted R-squared	-0.073237	S.D. dependent var	0.682307
S.E. of regression	0.706851	Akaike info criterion	2.443642
Sum squared resid	4.496743	Schwarz criterion	2.781649
Log likelihood	-12.54913	F-statistic	0.829402
Durbin-Watson stat	2.894434	Prob(F-statistic)	0.575525

a) Explain the basic idea of the White test with reference to the auxiliary regression carried out!

b) Enter the test decision of the White test shown here using the critical value and the p-value (α=0,10)! Interpret the result!

c) Enter the auxiliary regression of the full White test for the energy model III and interpret the corresponding abridged output:

White Heteroskedasticity Test:

Obs*R-squared	8.073769	Probability	0.526730

5.7 From the regression of natural gas demand on the the gas price (= energy model Ia, data see Assignm. 2.3), the following OLS residuals result:

t	1	2	3	4	5	6	7
$\hat{u}_t$	-0,062	0,069	-0,067	-0,051	-0,119	0,038	0,191

a) Compare the standard error of the regression coefficient of the gas price under the assumption of homoscedastic disturbances with White's heteroskedasticity-consistent estimator!

Note: Use the respective sum formulas to calculate the two standard errors!

b) Specify White's heteroskedasticity-consistent estimators of the standard errors of the two regression coefficients numerically using the variance-covariance matrix $\widehat{Cov}(\hat{\boldsymbol{\beta}})$!

5.8 Discuss the advantages and disadvantages of using heteroskeedasticity consistent (HC) standard errors instead of GLS estimation (generalized least squares estimation) of the regression model!

5.9 Why is the original White estimator usually used in a modified form in an HC estimation of the variance-covariance matrix $Cov(\hat{\boldsymbol{\beta}})$? Explain the three modifications of the White estimator available in the mathematical and statistical program R!

5.10 Some modifications of the White estimator make use of the hat matrix (projection matrix) $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$.

a) Demonstrate that the vector of the regression values $\hat{\mathbf{y}}$ can be calculated from the vector of the dependent variable **y** using the hat matrix **H**!

b) Show the relationship between the vector of the OLS residuals, $\hat{\mathbf{u}}$, and the vector of the disturbances, **u**, using the hat matrix **H**!