Tasks with Solutions for Introductory Econometrics

6. Autocorrelation

6.1 Which problems are associated with autocorrelation of the error terms of an econometric model?

Solution:

The OLS estimators are no longer efficient, i.e. the usual least squares estimate loses accuracy. It can be improved with modified estimation methods.

The standard errors (= square roots of the variances) of the regression coefficients are estimated unbiased with OLS. Thus, the t-tests and the confidence intervals for the regression coefficients are no longer valid.

6.2 What are the characteristics of a Markov process?

Solution:

A Markov process is a first-order autoregressive process (AR (1) process). In a Markov process, the current disturbance depends on the disturbance of the previous period,

$$u_t = \phi \cdot u_{t-1} + v_t, \quad |\phi| < 1,$$

with standard noise (innovation) assuming white noise.

ϕ is the autoregressive parameter, which must be absolutely smaller than one, because otherwise shocks from the past would be persistent. A shock that occurs in the period t would then increase continuously over time.

6.3 The residuals of the demand function for natural gas in relation to the gas price (Energy Model Ia) are as follows:

t	1	2	3	4	5	6	7
$\hat{u}_t$	-0,062	0,069	-0,067	-0,051	-0,119	0,038	0,191

Determine the 1st, 2nd and 3rd order autocorrelation coefficients!

Solution:

Work table:

t	$\hat{u}_t$	$\hat{u}_{t-1}$	$\hat{u}_{t-2}$	$\hat{u}_{t-3}$	$\hat{u}_t \cdot \hat{u}_{t-1}$	$\hat{u}_t \cdot \hat{u}_{t-2}$	$\hat{u}_t \cdot \hat{u}_{t-3}$	$\hat{u}_t^2$
1	-0,062	-	-	-	-	-	-	0,0038
2	0,069	-0,062	-	-	-0,0043	-	-	0,0048
3	-0,067	0,069	-0,062	-	-0,0046	0,0042	-	0,0045
4	-0,051	-0,067	0,069	-0,062	0,0034	-0,0035	0,0032	0,0026
5	-0,119	-0,051	-0,067	0,069	0,0061	0,0080	-0,0082	0,0142
6	0,038	-0,119	-0,051	-0,067	-0,0045	-0,0019	-0,0025	0,0014
7	0,191	0,038	-0,119	-0,051	0,0073	-0,0227	-0,0097	0,0365
Σ					0,0034	-0,0159	-0,0172	0,0678

1st order autocorrelation coefficients:

$$\hat{\rho}_1 = \frac{\sum_{t=2}^{7} \hat{u}_t \cdot \hat{u}_{t-1}}{\sum_{t=1}^{7} \hat{u}_t^2} = \frac{0,0034}{0,0678} = 0,050$$

2nd order autocorrelation coefficients:

$$\hat{\rho}_2 = \frac{\sum_{t=3}^{7} \hat{u}_t \cdot \hat{u}_{t-2}}{\sum_{t=1}^{7} \hat{u}_t^2} = \frac{-0,0159}{0,0678} = -0,235$$

3rd order autocorrelation coefficients:

$$\hat{\rho}_3 = \frac{\sum_{t=4}^{7} \hat{u}_t \cdot \hat{u}_{t-3}}{\sum_{t=1}^{7} \hat{u}_t^2} = \frac{-0,0172}{0,0678} = -0,254$$

6.4 Test the disturbance term (residuals see Aufg. 6.3) of the demand function for natural gas as a function of the gas price (Energy Model Ia) with the Durbin-Watson test for autocorrelation (α=0.05)! (with extrapolation)

Solution:

Step 1: **Formulation of the null hypothesis**: $H_0 : \phi = 0 \quad (\hat{=} \rho_1 = 0)$

Step2: **Determination of the significance level α:** $\alpha = 0{,}05$

Step 3: **Calculation of the test statistic:**

Working table:

t	$\hat{u}_t$	$\hat{u}_{t-1}$	$\hat{u}_t - \hat{u}_{t-1}$	$(\hat{u}_t - \hat{u}_{t-1})^2$	$\hat{u}_t^2$
1	-0,062	-	-	-	0,0038
2	0,069	-0,062	0,131	0,0172	0,0048
3	-0,067	0,069	-0,136	0,0185	0,0045
4	-0,051	-0,067	0,016	0,0003	0,0026
5	-0,119	-0,051	-0,068	0,0046	0,0142
6	0,038	-0,119	0,157	0,0246	0,0014
7	0,191	0,038	0,153	0,0234	0,0365
Σ				0,0886	0,0678

Durbin Watson statistic:

$$DW = \frac{\sum_{t=2}^{7}(\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{7}\hat{u}_t^2} = \frac{0{,}0886}{0{,}0678} = 1{,}307$$

Step 4: **Determination of the critical value:**

Since no table values are available for n = 7, k = 2 incl. dummy variable), we determine d_u and d_o by extrapolation:

Difference of the d_u-values for n=15 und 16 (1,08 und 1,10): 0,02

→ lower limit for n=7: $d_u = 1{,}08 - 8 \cdot 0{,}02 = 1{,}08 - 0{,}16 = 0{,}92$

Difference of the d_o-values for n=15 und 16 (1,36 und 1,37): 0,01

→ upper limit for n=7: $d_o = 1{,}36 - 8 \cdot 0{,}01 = 1{,}36 - 0{,}08 = 1{,}28$

Step 5: **Test decision:**

$DW = 1{,}307 \in [d_o = 1{,}28;\ 4 - d_o = 4 - 1{,}28 = 2{,}72]$

$\Rightarrow$ no autocorrelation

6.5 Determine regression coefficients β_1 and β_2 2 of the demand function for natural gas as a function of the gas price (energy model Ia) using the Cochran-Orcutt method!

Note: Calculate the transformed variables with three decimal places!

Solution:

Step 1

Execution of OLS regression (original variables):

$$\underbrace{\hat{GASV}_t}_{\hat{y}_t} = 5{,}809 - 0{,}368 \cdot \underbrace{GASP_t}_{x_t}$$ → see task. 1

OLS-Residuen (s. Aufg. 2):

t	1	2	3	4	5	6	7
$\hat{u}_t$	-0,062	0,069	-0,067	-0,051	-0,119	0,038	0,191

Step 2

Implementation of auxiliary regression

$$\hat{u}_t = \phi \cdot \hat{u}_{t-1} + v_t$$

OLS estimator for the Autoregressive Parameter ϕ:

$$\hat{\phi} \approx \frac{\sum_{t=2}^{7} \hat{u}_t \cdot \hat{u}_{t-1}}{\sum_{t=1}^{7} \hat{u}_t^2} = \frac{0{,}0034}{0{,}0678} = 0{,}050$$ (Numerator and denominator see task 6.3)

Step 3

Transformation of the variables

Dependent variable: $y_t^* = y_t - \hat{\phi} \cdot y_{t-1} = y_t - 0{,}050 \cdot y_{t-1}$

t=2: $y_2^* = y_2 - \hat{\phi} \cdot y_1 = 0{,}8 - 0{,}050 \cdot 1{,}0 = 0{,}750$

t=3: $y_3^* = y_3 - \hat{\phi} \cdot y_2 = 0{,}7 - 0{,}050 \cdot 0{,}8 = 0{,}660$

t=4: $y_4^* = y_4 - \hat{\phi} \cdot y_3 = 0{,}9 - 0{,}050 \cdot 0{,}7 = 0{,}865$

t=5: $y_5^* = y_5 - \hat{\phi} \cdot y_4 = 1{,}2 - 0{,}050 \cdot 0{,}9 = 1{,}155$

t=6: $y_6^* = y_6 - \hat{\phi} \cdot y_5 = 1{,}1 - 0{,}050 \cdot 1{,}2 = 1{,}040$

t=7: $y_7^* = y_7 - \hat{\phi} \cdot y_6 = 1{,}4 - 0{,}050 \cdot 1{,}1 = 1{,}345$

Independent variable: $x_t^* = x_t - \hat{\phi} \cdot x_{t-1} = x_t - 0{,}050 \cdot x_{t-1}$

t=2: $x_2^* = x_2 - \hat{\phi} \cdot x_1 = 13{,}8 - 0{,}050 \cdot 11{,}9 = 13{,}155$

t=3: $x_3^* = x_3 - \hat{\phi} \cdot x_2 = 13{,}7 - 0{,}050 \cdot 13{,}8 = 13{,}010$

t=4: $x_4^* = x_4 - \hat{\phi} \cdot x_3 = 13{,}2 - 0{,}050 \cdot 13{,}7 = 12{,}515$

t=5: $x_5^* = x_5 - \hat{\phi} \cdot x_4 = 12{,}2 - 0{,}050 \cdot 13{,}2 = 11{,}540$

t=6: $x_6^* = x_6 - \hat{\phi} \cdot x_5 = 12{,}9 - 0{,}050 \cdot 12{,}2 = 12{,}290$

t=7: $x_7^* = x_7 - \hat{\phi} \cdot x_6 = 12{,}5 - 0{,}050 \cdot 12{,}9 = 11{,}855$

Step 4:

Perform GLS regression (transformed variables):

$$y_t^* = \beta_1 \cdot (1-\phi) + \beta_2 \cdot x_t^* + u_t^*$$

Working table:

t	x_t^*	y_t^*	$(x_t^*)^2$	$x_t^* y_t^*$
2	13,155	0,750	173,054	9,8663
3	13,010	0,660	169,260	8,5866
4	12,515	0,865	156,625	10,8255
5	11,540	1.155	133,172	13,3287
6	12,290	1,040	151,044	12,7816
7	11,855	1,345	140,541	15,9450
Σ	74,365	5,815	923,696	71,337

GLS-estimator for β_1 and β_2 with the Cochran-Orcutt-Method:

$$\hat{\beta}_{2,GLS}^{*CO} = \hat{\beta}_{2,GLS}^{CO} = \frac{(n-1) \cdot \sum x_{1t}^* y_t^* - \sum x_{1t}^* \sum y_t^*}{(n-1) \cdot \sum (x_{1t}^*)^2 - (\sum x_{1t}^*)^2}$$

$$= \frac{6 \cdot 71{,}3337 - 74{,}365 \cdot 5{,}815}{6 \cdot 923{,}696 - 74{,}365^2} = \frac{-4{,}4303}{12{,}0228} = -0{,}368$$

$$\hat{\beta}_{1,GLS}^{*CO} = \frac{\sum y_t^*}{n-1} - \hat{\beta}_{2,GLS}^{CO} \cdot \frac{\sum x_{1t}^*}{n-1}$$

$$= \frac{5{,}815}{6} - (-0{,}368) \cdot \frac{74{,}365}{6} = 0{,}969 + \underbrace{0{,}368 \cdot 12{,}394}_{=4{,}561} = 5{,}530$$

$$\hat{\beta}_{1,GLS}^{CO} = \hat{\beta}_{1,GLS}^{*CO} / (1-\hat{\phi}) = 5{,}530 / (1 - 0{,}050) = 5{,}821$$

GLS-estimated demand for natural gas (Cochran-Orcutt-Verfahren):

$$\hat{GASV}_t = 5{,}821 - 0{,}368 \cdot GASP_t$$

6.6 The OLS estimation of energy model III (n = 16, k = 4) reports an Durbin-Watson statistic of 1,461. The result of the Breusch-Godfrey test is given:

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.829749	Probability	0.464089
Obs*R-squared	2.277282	Probability	0.320254

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 01/25/07 Time: 15:21
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.082397	5.704853	0.365022	0.7227
GASPR	-1.018045	3.637415	-0.279881	0.7853
FERNWPR	-0.029593	4.465805	-0.006627	0.9948
VEINKR	-0.000772	0.002214	-0.348600	0.7346
RESID(-1)	0.314813	0.313126	1.005389	0.3384
RESID(-2)	-0.340237	0.367477	-0.925873	0.3763

R-squared	0.142330	Mean dependent var	-5.90E-16
Adjusted R-squared	-0.286505	S.D. dependent var	0.633593
S.E. of regression	0.718648	Akaike info criterion	2.457107
Sum squared resid	5.164552	Schwarz criterion	2.746828
Log likelihood	-13.65685	F-statistic	0.331899
Durbin-Watson stat	2.257740	Prob(F-statistic)	0.882416

a) What is the empirical autocorrelation coefficient of the 1st order of the residuals?

Solution:

Autocorrelation coefficient 1st order

$DW \approx 2 \cdot (1 - \hat{\phi}) = 2 - 2 \cdot \hat{\phi}$

$2 \cdot \hat{\phi} \approx 2 - DW$

$\hat{\phi} \approx 1 - 0{,}5 \cdot DW = 1 - 0{,}5 \cdot 1{,}461$

$= 1 - 0{,}731 = 0{,}269 \; (\approx \hat{\rho}_1)$

b) Use the Durbin-Watson test to check the disturbance for autocorrelation ($\alpha = 0.05$) and interpret the test result!

Solution:

Durbin-Watson test:

DW statistic = 1,461 (see specification in the task)

Nullhypothese: H_0: $\rho_1 = 0$

Critical value (α=0,05; n=16; k=4):

$d_u = 0{,}86$; $d_o = 1{,}73$

test decision:

$DW = 1{,}461 \in [d_u = 0{,}86 ;\ d_o = 1{,}73]$

$\Rightarrow$ There is no statement about first-order autocorrelation possible because DW is in the lower uncertainty region.

c) What is checked with the given results of the Breusch-Godfrey test? Enter the test decision using the critical value and the p-value ($\alpha = 0.05$)!

Solution:

Breusch-Godfrey test:

Check of the disturbance term for 2nd-order autocorrelation, since the auxiliary regression contains the residuals lagged by one and two periods.

Null hypothese H_0: $\alpha_1 = \alpha_2 = 0$

Test statistic:

$BG = n \cdot R^2 = 16 \cdot 0{,}14233 = 2{,}277$ (R^2 see auxiliary regression)

Critical value: (α=0,05, with two degrees of freedom):

$\chi^2_{2;0,95} = 5{,}991$

Test decision:

$BG = 2{,}277 < \chi^2_{2;0,95} = 5{,}991 \Rightarrow H_0$ can not be rejected

Alternative:

$p = 0{,}320 > \alpha = 0{,}05 \Rightarrow H_0$ can not be rejected

d) The first five residuals are: 0.5015, 0.6295, 0.7923, 0.7100, 0.0579. Indicate the first five values of the regressors RESID (-1) and RESID (-2) used in the Breusch-Godfrey test!

Solution:

RESID(-1): 0 0,5015 0,6295 0,7923 0,7100

RESID(-2): 0 0 0,5015 0,6295 0,7923