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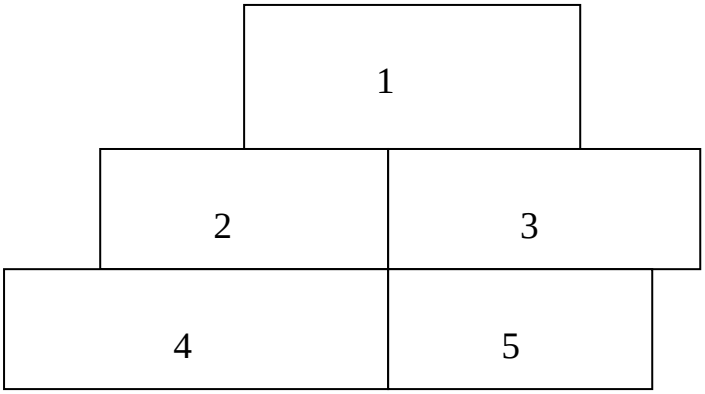
Spatial Econometrics

Assignments with Solutions

Part III – Assignments 15 - 17

General information on a system of regions and data

Five regions are arranged in the following form:



Distances between regional centres (in km):

Region	1	2	3	4	5
1		8	10	20	22
2			16	12	14
3				20	12
4					10

Data on geo-referenced variables:

Region	u	v
1	6	3
2	8	3
3	8	2
4	11	1
5	12	1

u: Unemployment rate
v: Vacancy rate

Beveridge curve:
Relation between the unemployment rate (u) and vacancy rate (v)

Assignment 15 (LM error test)

Apply the LM error test (α=0.10) with the standardized weight matrix

$$\mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}$$

to the residuals of the OLS estimated Beveridge curve:

Region	1	2	3	4	5
OLS residuals	-0.75	1.25	-1	-0.25	0.75

Linear regression model with spatially autocorrelated errors:

$$\mathbf{y} = \mathbf{X} \cdot \boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad \text{with} \quad \boldsymbol{\varepsilon} = \lambda \mathbf{W} \boldsymbol{\varepsilon} + \boldsymbol{\nu}$$

Null hypothesis H_0: $\lambda = 0$
Alternative hypothesis H_1: $\lambda \neq 0$

Test statistic: $LM_e = \dfrac{(\mathbf{e}'\mathbf{W}\mathbf{e}/s^2)^2}{T}$

with $s^2 = \mathbf{e}'\mathbf{e}/n$ and $T = tr[(\mathbf{W} + \mathbf{W}')\mathbf{W}]$

Vector of residuals: $\mathbf{e} = \begin{bmatrix} -0.75 \\ 1.25 \\ -1 \\ -0.25 \\ 0.75 \end{bmatrix}$

Numerator:

$$s^2 = \mathbf{e}'\mathbf{e}/n = \frac{1}{5} \cdot \begin{bmatrix} -0.75 & 1.25 & -1 & -0.25 & 0.75 \end{bmatrix} \begin{bmatrix} -0.75 \\ 1.25 \\ -1 \\ -0.25 \\ 0.75 \end{bmatrix} = \frac{1}{5} \cdot 3.75 = 0.75$$

$$\mathbf{e}'\mathbf{W}\mathbf{e} = \begin{bmatrix} -0.75 & 1.25 & -1 & -0.25 & 0.75 \end{bmatrix} \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} -0.75 \\ 1.25 \\ -1 \\ -0.25 \\ 0.75 \end{bmatrix}$$

$$\begin{bmatrix} -0.75 & 1.25 & -1 & -0.25 & 0.75 \end{bmatrix} \begin{bmatrix} 0.083 \\ -0.833 \\ 0.417 \\ 0.792 \\ -0.458 \end{bmatrix} = -2.0625$$

Denominator:

$$\mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}, \quad \mathbf{W}' = \begin{bmatrix} 0 & 1/3 & 1/3 & 0 & 0 \\ 0.5 & 0 & 1/3 & 0.5 & 0 \\ 0.5 & 1/3 & 0 & 0 & 0.5 \\ 0 & 1/3 & 0 & 0 & 0.5 \\ 0 & 0 & 1/3 & 0.5 & 0 \end{bmatrix}$$

$$\mathbf{W} + \mathbf{W}' = \begin{bmatrix} 0 & 5/6 & 5/6 & 0 & 0 \\ 5/6 & 0 & 2/3 & 5/6 & 0 \\ 5/6 & 2/3 & 0 & 0 & 5/6 \\ 0 & 5/6 & 0 & 0 & 1 \\ 0 & 0 & 5/6 & 1 & 0 \end{bmatrix}$$

$$(\mathbf{W} + \mathbf{W}')\mathbf{W} = \begin{bmatrix} 0.5556 & 0.2778 & 0.2778 & 0.2778 & 0.2778 \\ 0.2222 & 1.0556 & 0.4167 & 0 & 0.6389 \\ 0.2222 & 0.4167 & 1.0556 & 0.6389 & 0 \\ 0.2778 & 0 & 0.7778 & 0.7778 & 0 \\ 0.2778 & 0.7778 & 0 & 0 & 0.7778 \end{bmatrix}$$

$$\mathrm{T} = \mathrm{tr}[(\mathbf{W} + \mathbf{W}')\mathbf{W}] = 4.2222$$

Test statistic:

$$LM_e = \frac{(\mathbf{e}'\mathbf{W}\mathbf{e}/s^2)^2}{T} = \frac{(-2.0625/0.75)^2}{4.2222} = \frac{7.5625}{4.2222} = 1.7911$$

Critical value (α=0.10): $\chi^2(1;0.90) = 2.706$

Testing decision:
($LM_e = 1.7911$) < ($\chi^2(1;0.90) = 2.706$) => Accept H_0

Assignment 16 (Spatial cross-regressive model)

In 5 regions data on the unemployment rate (u) and the vacancy rate (v) are observed:

Region	1	2	3	4	5
Unemployment rate (u)	6	8	8	11	12
Vacancy rate (v)	3	3	2	1	1

The row-standardized contiguity matrix reads

$$\mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}.$$

a) Give the contents of the observation matrix [**X** **X***] that underlies the cross-regressive model of the Beveridge curve!

Spatial Lag of the vacancy rate:

$$\mathbf{W}\cdot\mathbf{v} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5\cdot 3 + 0.5\cdot 2 \\ (1/3)\cdot 3 + (1/3)\cdot 2 + (1/3)\cdot 1 \\ (1/3)\cdot 3 + (1/3)\cdot 3 + (1/3)\cdot 1 \\ 0.5\cdot 3 + 0.5\cdot 1 \\ 0.5\cdot 2 + 0.5\cdot 1 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 2 \\ 7/3 \\ 2 \\ 1.5 \end{bmatrix}$$

Observation matrix [**X** **X***]:

$$[\mathbf{X}\ \mathbf{X}^*] = \begin{bmatrix} 1 & 3 & 2.5 \\ 1 & 3 & 2 \\ 1 & 2 & 7/3 \\ 1 & 1 & 2 \\ 1 & 1 & 1.5 \end{bmatrix}$$

b) Estimate the cross-regressive model of the Beveridge curve using

$$([\mathbf{X}\ \ \mathbf{X}^*]'[\mathbf{X}\ \ \mathbf{X}^*])^{-1} = \begin{bmatrix} 8.44426 & 0.6557 & -4.6230 \\ 0.6557 & 0.4344 & -0.7377 \\ -4.6230 & -0.7377 & 2.9508 \end{bmatrix}$$

by OLS!

Dependent variable Y: unemployment rate (u)
Independent variable X: vacancy rate (v)

Vector of the endogenous variable **y**:

$$\mathbf{y} = \begin{bmatrix} 6 \\ 8 \\ 8 \\ 11 \\ 12 \end{bmatrix}$$

Observation matrix [**X X***]:

$$[\mathbf{X}\ \ \mathbf{X}^*] = \begin{bmatrix} 1 & 3 & 2.5 \\ 1 & 3 & 2 \\ 1 & 2 & 7/3 \\ 1 & 1 & 2 \\ 1 & 1 & 1.5 \end{bmatrix}$$

Matrix product **X'X** and matrix product **X'y**:

$$[\mathbf{X}\ \ \mathbf{X}^*]'[\mathbf{X}\ \ \mathbf{X}^*] = \begin{bmatrix} 5 & 10 & 10.3333 \\ 10 & 24 & 21.6667 \\ 10.3333 & 21.6667 & 21.9444 \end{bmatrix}, \quad [\mathbf{X}\ \ \mathbf{X}^*]'\mathbf{y} = \begin{bmatrix} 45 \\ 81 \\ 89.6667 \end{bmatrix}$$

OLS estimator of **β**:

$$\begin{bmatrix} \hat{\boldsymbol{\beta}} \\ \hat{\boldsymbol{\gamma}} \end{bmatrix} = \left([\mathbf{X}\ \ \mathbf{X}^*]'[\mathbf{X}\ \ \mathbf{X}^*]\right)^{-1}[\mathbf{X}\ \ \mathbf{X}^*]'\mathbf{y}$$

$$= \begin{bmatrix} 8.44426 & 0.6557 & -4.6230 \\ 0.6557 & 0.4344 & -0.7377 \\ -4.6230 & -0.7377 & 2.9508 \end{bmatrix} \begin{bmatrix} 45 \\ 81 \\ 89.6667 \end{bmatrix} = \begin{bmatrix} \hat{\beta}_1 = 18.5082 \\ \hat{\beta}_2 = -1.4508 \\ \hat{\gamma}_2 = -3.1967 \end{bmatrix}$$

c) Given $\hat{\sigma} = 0.3787$ test the significance of the estimated regression coefficient $\hat{\gamma}_2$ (α=0.05)!

- for γ_2 (H_0: $\gamma_2 = 0$)

OLS estimator for γ_2: $\hat{\gamma}_2 = -3.1967$

Test statistic: $t_{\gamma 2} = \frac{\hat{\gamma}_2}{\hat{\sigma} \cdot \sqrt{x * x *^{11}}} = \frac{-3.1967}{0.3787 \cdot \sqrt{2.9508}} = -4.9140$

Critical value (α=0.05, two-sided test): t(2,0.975) = 4.303

Testing decision: (| $t_{\gamma 2}$ |=4.9140) > [t(2;0.975)=4.303] => Reject H_0

d) Apply the F test on omitted spatially lagged exogenous variables using the residual vector

$$\mathbf{e} = \begin{bmatrix} -0.1639 \\ 0.2377 \\ -0.1475 \\ 0.3361 \\ -0.2623 \end{bmatrix}$$

(α=0.05)!

Null hypothesis H_0: $\gamma_2 = 0$

Constrained residual sum of squares (regression of Y on X_1 (=constant) and X_2 = v): SSR_c = 3.75 (= SSR of standard regression model without spatial lag LX_2 = Lv, see assigment 12)

Unconstrained residual sum of squares (regression of Y on X_1 (constant), X_2 = v and LX_2 = Lv): SSR_u = 0.2869 (= SSR of the regression model with spatial lag LX_2 = Lv):

$$SSR_{\mathbf{U}} = \mathbf{e}'\mathbf{e} = \begin{bmatrix} -0.1639 & 0.2377 & -0.1475 & 0.3361 & -0.2623 \end{bmatrix} \begin{bmatrix} -0.1639 \\ 0.2377 \\ -0.1475 \\ 0.3361 \\ -0.2623 \end{bmatrix} = 0.2869$$

Test statistic

$$F = \frac{(SSR_c - SSR_u)}{SSR_u/(n-k-1)} = \frac{(3.75 - 0.2869)}{0.2869/(5-2-1)} = 24.1415$$

Critical value (α=0.05): F(1;2;0.95) = 18.5

Testing decision: (F=24.1415) < [F(1;2;0.95)=18.5] => Reject H_0

Assignment 17 (Spatial lag model – IV Estimation)

In 5 regions data on the unemployment rate (u) and the vacancy rate (v) are observed:

Region	1	2	3	4	5
Unemployment rate (u)	6	8	8	11	12
Vacancy rate (v)	3	3	2	1	1

The row-standardized contiguity matrix reads

$$\mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}.$$

a) Use the vector of spatially lagged values of the unemployment rate in the previous period,

$$\mathrm{L}\mathbf{y}_{t-1} = \mathbf{W}\mathbf{y}_{t-1} = \begin{bmatrix} 5.9 \\ 6.9 \\ 8.5 \\ 11.8 \\ 10.1 \end{bmatrix},$$

as an instrument for L**y** = **Wy** and specify the matrix of exploratory variables **Z** and the matrix of instruments **Q** for IV estimation of the spatial lag model of the Beveridge curve!

$$\underset{5x3}{\mathbf{Z}} = [\underset{5x2}{\mathbf{X}} \quad \underset{5x1}{\mathbf{Wy}}] = \begin{bmatrix} 1 & x_1 & \mathrm{L}y_1 \\ 1 & x_2 & \mathrm{L}y_2 \\ 1 & x_3 & \mathrm{L}y_3 \\ 1 & x_4 & \mathrm{L}y_4 \\ 1 & x_5 & \mathrm{L}y_5 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 8 \\ 1 & 3 & 8\ 1/3 \\ 1 & 2 & 8\ 2/3 \\ 1 & 1 & 10 \\ 1 & 1 & 9.5 \end{bmatrix}$$

$$\underset{5x3}{\mathbf{Q}} = [\underset{5x2}{\mathbf{X}} \quad \underset{5x1}{\mathbf{Wy}_{t-1}}] = \begin{bmatrix} 1 & x_1 & \mathrm{L}y_{t-1} \\ 1 & x_2 & \mathrm{L}y_{t-2} \\ 1 & x_3 & \mathrm{L}y_{t-3} \\ 1 & x_4 & \mathrm{L}y_{t-4} \\ 1 & x_5 & \mathrm{L}y_{t-5} \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5.9 \\ 1 & 3 & 6.9 \\ 1 & 2 & 8.5 \\ 1 & 1 & 11.8 \\ 1 & 1 & 10.1 \end{bmatrix}$$

b) Compute projection matrix $\mathbf{P}_Q$ and give the predicted values of a regression of the variables in **Z** on the instruments in **Q**!

• Projection matrix $\mathbf{P}_Q$:

$$\underset{5x5}{P_Q} = \underset{5x3}{\mathbf{Q}}\,\underset{3x3}{(\mathbf{Q}'\mathbf{Q})^{-1}}\,\underset{3x5}{\mathbf{Q}'}$$

$$\mathbf{Q}'\mathbf{Q} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & 3 & 2 & 1 & 1 \\ 5.9 & 6.9 & 8.5 & 11.8 & 10.1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5.9 \\ 1 & 3 & 6.9 \\ 1 & 2 & 8.5 \\ 1 & 1 & 11.8 \\ 1 & 1 & 10.1 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 43.2 \\ 10 & 24 & 77.3 \\ 43.2 & 77.3 & 395.92 \end{bmatrix}$$

$$(\mathbf{Q}'\mathbf{Q})^{-1} = \begin{bmatrix} 89.5352 & -15.7360 & -6.6971 \\ 15.7360 & 2.8779 & 1.1551 \\ -6.6971 & 1.1551 & 0.5077 \end{bmatrix}$$

$$\mathbf{P}_Q = \mathbf{Q}(\mathbf{Q}'\mathbf{Q})^{-1}\mathbf{Q}'$$

$$= \begin{bmatrix} 1 & 3 & 5.9 \\ 1 & 3 & 6.9 \\ 1 & 2 & 8.5 \\ 1 & 1 & 11.8 \\ 1 & 1 & 10.1 \end{bmatrix} \begin{bmatrix} 89.5352 & -15.7360 & -6.6971 \\ 15.7360 & 2.8779 & 1.1551 \\ -6.6971 & 1.1551 & 0.5077 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & 3 & 2 & 1 & 1 \\ 5.9 & 6.9 & 8.5 & 11.8 & 10.1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.8142 & -0.2871 & -0.2361 \\ -3.8830 & 0.8680 & 0.2716 \\ 1.1376 & -0.1617 & -0.0711 \\ -5.2270 & 0.7723 & 0.4494 \\ 6.1582 & -1.1914 & -0.4138 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & 3 & 2 & 1 & 1 \\ 5.9 & 6.9 & 8.5 & 11.8 & 10.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5598 & 0.3237 & 0.2331 & -0.2589 & 0.1424 \\ 0.3237 & 0.5953 & 0.1620 & 0.1904 & -0.2714 \\ 0.2331 & 0.1620 & 0.2100 & 0.1371 & 0.2579 \\ -0.2589 & 0.1904 & 0.1371 & 0.8477 & 0.0838 \\ 0.1424 & -0.2714 & 0.2579 & 0.0838 & 0.7873 \end{bmatrix}$$

• Predicted values of a regression of the variables in **Z** on the instruments in **Q**

$$\underset{5x3}{\hat{\mathbf{Z}}} = \underset{5x3}{\mathbf{Q}}\,\underset{3x3}{(\mathbf{Q}'\mathbf{Q})^{-1}}\,\underset{3x5}{\mathbf{Q}'}\,\underset{5x3}{\mathbf{Z}} = \underset{5x5}{\mathbf{P}_Q}\cdot\underset{5x3}{\mathbf{Z}}$$

$$= \begin{bmatrix} 0.5598 & 0.3237 & 0.2331 & -0.2589 & 0.1424 \\ 0.3237 & 0.5953 & 0.1620 & 0.1904 & -0.2714 \\ 0.2331 & 0.1620 & 0.2100 & 0.1371 & 0.2579 \\ -0.2589 & 0.1904 & 0.1371 & 0.8477 & 0.0838 \\ 0.1424 & -0.2714 & 0.2579 & 0.0838 & 0.7873 \end{bmatrix}$$

$$\cdot \begin{bmatrix} 1 & 3 & 8 \\ 1 & 3 & 8\ 1/3 \\ 1 & 2 & 8\ 2/3 \\ 1 & 1 & 10 \\ 1 & 1 & 9.5 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 7.9590 \\ 1 & 3 & 8.2801 \\ 1 & 2 & 8.8550 \\ 1 & 1 & 9.9759 \\ 1 & 1 & 9.4299 \end{bmatrix}$$

c) What is the IV estimator for the parameter vector θ of the spatial lag model? Interpret the IV estimator for θ!

$$\underset{3x1}{\hat{\boldsymbol{\theta}}_{IV}} [= \underset{3x1}{\hat{\boldsymbol{\theta}}_{2SLS}}] = \underset{3x3}{(\hat{\mathbf{Z}}'\hat{\mathbf{Z}})^{-1}} \underset{3x5}{\hat{\mathbf{Z}}'} \underset{5x1}{\mathbf{y}}$$

$$\underset{3x3}{\hat{\mathbf{Z}}'\hat{\mathbf{Z}}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & 3 & 2 & 1 & 1 \\ 7.9590 & 8.2801 & 8.8550 & 9.5759 & 9.4299 \end{bmatrix}$$

$$\cdot \begin{bmatrix} 1 & 3 & 7.9590 \\ 1 & 3 & 8.2801 \\ 1 & 2 & 8.8550 \\ 1 & 1 & 9.9759 \\ 1 & 1 & 9.4299 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 44.5 \\ 10 & 24 & 85.8333 \\ 44.5 & 85.8333 & 398.7601 \end{bmatrix}$$

$$\underset{3x3}{(\hat{\mathbf{Z}}'\hat{\mathbf{Z}})^{-1}} = \begin{bmatrix} 542.2454 & -41.3580 & -51.6101 \\ -41.3580 & 3.3355 & 3.8974 \\ -51.6101 & 3.8974 & 4.9231 \end{bmatrix}$$

$$\underset{3x1}{\hat{\boldsymbol{\theta}}_{IV}} = \underset{3x3}{(\hat{\mathbf{Z}}'\hat{\mathbf{Z}})^{-1}} \underset{3x5}{\hat{\mathbf{Z}}'} \underset{5x1}{\mathbf{y}} = \begin{bmatrix} 542.2454 & -41.3580 & -51.6101 \\ -41.3580 & 3.3355 & 3.8974 \\ -51.6101 & 3.8974 & 4.9231 \end{bmatrix}$$

$$\cdot \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & 3 & 2 & 1 & 1 \\ 7.9590 & 8.2801 & 8.8550 & 9.5759 & 9.4299 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 8 \\ 8 \\ 11 \\ 12 \end{bmatrix} = \begin{bmatrix} \hat{\beta}_1 = 8.1133 \\ \hat{\beta}_2 = -1.8432 \\ \hat{\rho} = 0.5138 \end{bmatrix}$$

d) Compute the standard error of regression of IV estimation!

Estimated error variance:

$$\hat{\sigma}^2 = \frac{1}{n}(\mathbf{y} - \mathbf{Z}\boldsymbol{\theta}_{IV})'(\mathbf{y} - \mathbf{Z}\boldsymbol{\theta}_{IV}) = \frac{1}{n}\mathbf{e}_{IV}'\mathbf{e}_{IV}$$

Vector of regression values:

$$\hat{\mathbf{y}} = \mathbf{Z} \cdot \hat{\boldsymbol{\theta}}_{IV} = \begin{bmatrix} 1 & 3 & 8 \\ 1 & 3 & 8\ 1/3 \\ 1 & 2 & 8\ 2/3 \\ 1 & 1 & 10 \\ 1 & 1 & 9.5 \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 = 8.1133 \\ \hat{\beta}_2 = -1.8432 \\ \hat{\rho} = 0.5138 \end{bmatrix} = \begin{bmatrix} 6.6943 \\ 6.8656 \\ 8.8801 \\ 11.4084 \\ 11.1515 \end{bmatrix}$$

Vector of IV residuals:

$$\mathbf{e}_{IV} = \mathbf{y} - \hat{\mathbf{y}} = \begin{bmatrix} 6 \\ 8 \\ 8 \\ 11 \\ 12 \end{bmatrix} - \begin{bmatrix} 6.6943 \\ 6.8656 \\ 8.8801 \\ 11.4084 \\ 11.1515 \end{bmatrix} = \begin{bmatrix} -0.6943 \\ 1.1344 \\ -0.8801 \\ -0.4084 \\ 0.8485 \end{bmatrix}$$

Estimated error variance:

$$\hat{\sigma}^2 = \frac{1}{n}\mathbf{e}_{IV}'\mathbf{e}_{IV} = \frac{1}{5}\begin{bmatrix} -0.6943 & 1.1344 & -0.8801 & -0.4084 & 0.8485 \end{bmatrix}$$
$$\cdot \begin{bmatrix} -0.6943 \\ 1.1344 \\ -0.8801 \\ -0.4084 \\ 0.8485 \end{bmatrix} = \frac{1}{5} \cdot 3.4303 = 0.6861$$

Standard error of regression (SER):

$$\text{SER} = \hat{\sigma} = \sqrt{0.6861} = 0.8283$$

e) Is the IV estimator for the autoregressive parameter ρ significant (α=0,05)?

Estimated covariance matrix of the regression coefficients:

$$\hat{\text{Cov}}(\hat{\theta}_{IV}) = \hat{\sigma}^2 \cdot (\hat{\mathbf{Z}}'\hat{\mathbf{Z}})^{-1} = 0.6861 \cdot \begin{bmatrix} 542.2454 & -41.3580 & -51.6101 \\ -41.3580 & 3.3355 & 3.8974 \\ -51.6101 & 3.8974 & 4.9231 \end{bmatrix}$$

3rd diagonal element of $(\hat{\mathbf{Z}}'\hat{\mathbf{Z}})^{-1}$: $zpz^{33} = 4.9231$

• **Significance test for ρ** (two-sided test):

H_0: $\rho = 0$, H_1: $\rho \neq 0$

IV estimator for ρ (see part c): $\hat{\rho} = 0.5138$

Standard error of regression (see part d): $\text{SER} = \hat{\sigma} = \sqrt{0.6861} = 0.8283$

Test statistic:

$$z_3 = \frac{\hat{\rho}}{\hat{\sigma} \cdot \sqrt{zpz^{33}}} = \frac{0.5138}{0.8283 \cdot \sqrt{4.9231}} = 0.2796$$

Critical value (α=0.05, two-sided test):

$z_{0.975} = 1.96$

Testing decision:

$|z_\rho| = 0.0375 < z_{0.975} = 1.96 \Rightarrow$ Accept H_0