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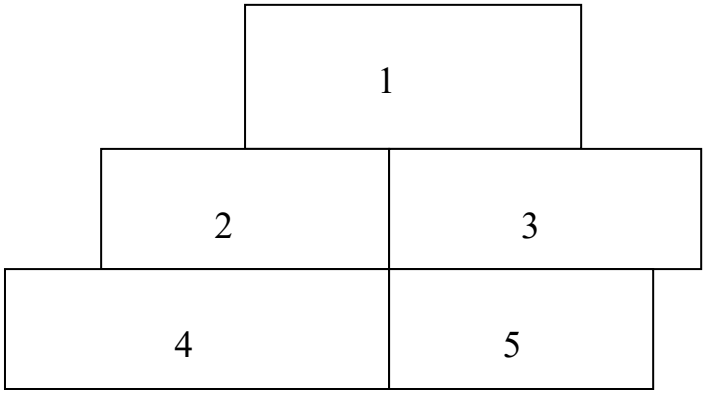
Spatial Econometrics
Assignments

General information on a system of regions and data

Five regions are arranged in the following form:



Distances between regional centres (in km):

Region	1	2	3	4	5
1		8	10	20	22
2			16	12	14
3				20	12
4					10

Data on geo-referenced variables:

Region	u	v
1	6	3
2	8	3
3	8	2
4	11	1
5	12	1

u: Unemployment rate
v: Vacancy rate

Beveridge curve:
Relation between the unemployment rate (u) and vacancy rate (v)

Assignment 1 (Connectivity)

The arrangement of five regions is given along with the distances between regional centres (see "General information on a system of regions and data").

a) Specify the contiguity matrix **W*** and the corresponding standardized weight matrix **W** for the given irregular arrangement!

b) Which spatial weight matrix **W*** is implied by the gravity model of spatial interaction?

c) Determine the spatial weight matrix **W*** on the basis of the negative exponential function with ß calculated at $\gamma = 0.5$!

d) Compare the standardized weight matrices arsing from the distance measures used in parts b) and c)!

Assignment 2 (Spatial Lag)

The data on the vacancy rate of five regions

Region	1	2	3	4	5
Vacancy rate (v)	3	3	2	1	1

are given along with the unstandardized and standardized contiguity matrices:

$$\mathbf{W}^* = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \text{ and } \mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}$$

a) Determine the first-order spatial lag of the vacancy rate on the basis of the

- unstandardized contiguity matrix **W***,
- standardized contiguity matrix **W**!

How can the spatial lags be interpreted?

b) What is the second-order spatial lag of the vacancy rate based on the standardized contiguity matrix?

Assignment 3 (Moran coefficient I)

Given are the data on the unemployment and vacancy rate of five regions

Region	1	2	3	4	5
Unemploy-ment rate (u)	6	8	8	11	12
Vacancy rate (v)	3	3	2	1	1

and the unstandardized and standardized contiguity matrices:

$$\mathbf{W}^* = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \text{ and } \mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}$$

a) Compute Moran's I for the unemployment rate on the basis of the

- unstandardized contiguity matrix **W***,
- standardized contiuity matrix **W**

with the sum formula and interpret it!

b) Compute Moran's I for the vacancy rate on the basis of the

- unstandardized contiguity matrix **W***,
- standardized contiuity matrix **W**

with the matrix formula and interpret it!

Assignment 4 (Moran coefficient II)

Given are the data on the vacancy rate of five regions:

Region	1	2	3	4	5
Vacancy rate (v)	3	3	2	1	1

a) Compute Moran's I for the vacancy rate on the basis of the unstandardized distance-based weights matrix **W***,

$$\mathbf{W}^{*} = \begin{bmatrix} 0 & 0.0156 & 0.0100 & 0.0025 & 0.0021 \\ 0.0156 & 0 & 0.0039 & 0.0069 & 0.0051 \\ 0.0100 & 0.0039 & 0 & 0.0025 & 0.0069 \\ 0.0025 & 0.0069 & 0.0025 & 0 & 0.0100 \\ 0.0021 & 0.0051 & 0.0069 & 0.0100 & 0 \end{bmatrix},$$

with the matrix formula!

b) Compute Moran's I for the vacancy rate on the basis of the standardized distance-based weights matrix **W**,

$$\mathbf{W} = \begin{bmatrix} 0 & 0.5166 & 0.3311 & 0.0828 & 0.0695 \\ 0.4952 & 0 & 0.1238 & 0.2190 & 0.1619 \\ 0.4292 & 0.1674 & 0 & 0.1073 & 0.2961 \\ 0.1142 & 0.3151 & 0.1142 & 0 & 0.4566 \\ 0.0871 & 0.2116 & 0.2863 & 0.4149 & 0 \end{bmatrix}$$

with the matrix formula!

Assignment 5 (Moran's I test under normality)

Given are the data on the vacancy rate of five regions

Region	1	2	3	4	5
vacancy rate (v)	3	3	2	1	1

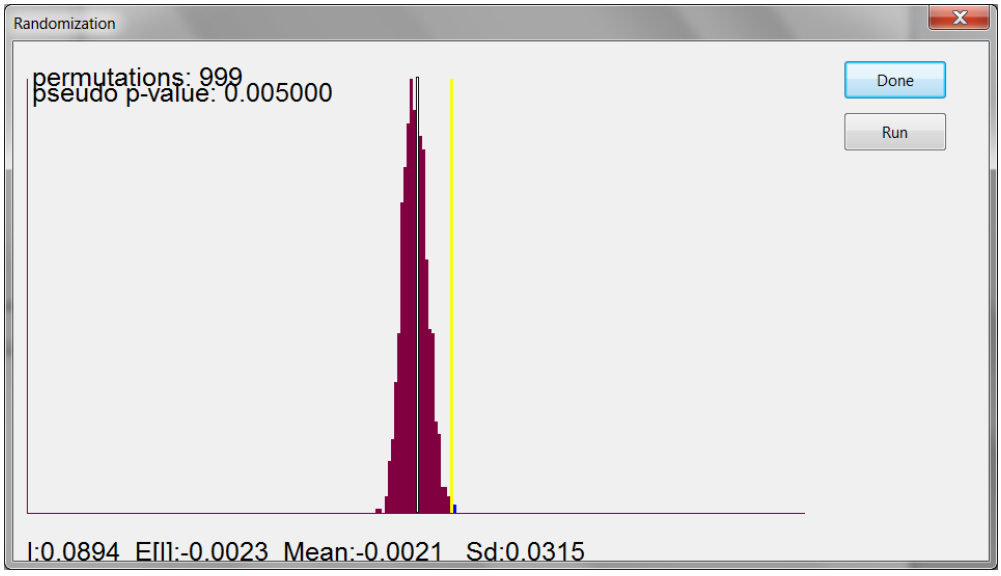
and the standardized contiguity matrix:

$$\mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}$$

Is the Moran coefficient under normality of the vacancy rate significant at the 5% level (right-sided test)?

Assignment 6 (Moran's I permutation test)

Using GeoDa, the Moran's I test under randomization for the share of employees in manufacture of electrical and optical instruments across German districts (n=439) yields the following results:



a) Explain the idea of the permutation test!

b) Interprete the statistics I, Mean and Sd and the parameter E[I]!

c) Test the null hypothesis of spatial randomness at a significance level of 5%! How can the pseudo p-value be interpreted?

Assignment 7 (Moran scatterplot)

Given are the data on the vacancy rate of five regions,

Region	1	2	3	4	5
Vacancy rate (v)	3	3	2	1	1

and the standardized spatial weight matrix

$$\mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}.$$

a) Determine the regression line of Lv on v!

b) Draw the Moran scatterplot for the vacancy rate!

c) Interpret the Moran scatterplot with respect to local characteristics!

Assignment 8 (Regression outliers)

Identity outliers in the regression of Lv on v (see Assignment 7) on the basis of the

- 2s rule for the residuals,
- hat matrix and leverage points,
- Cook's distance!

Assignment 9 (Geary's c)

Compute Geary's c for the vacancy rate

Region	1	2	3	4	5
Vacancy rate (v)	3	3	2	1	1

on the basis of the standardized contiguity matrix

$$\mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}$$

and interpret it!

Assignment 10 (Getis-Ord G)

The arrangement of five regions is given along with the distances between regional centres (see "General information on a system of regions and data").

a) What value does Getis-Ord G statistic take for the vacancy rate

Region	1	2	3	4	5
Vacancy rate (v)	3	3	2	1	1

with a critical distance of $\bar{d}$?

b) Interpret the value of the G statistic for the vacancy rate!

Assignment 11 (Local Moran coefficients)

Given are the data on the vacancy rate of five regions,

Region	1	2	3	4	5
Vacancy rate (v)	3	3	2	1	1

and the standardized spatial weight matrix

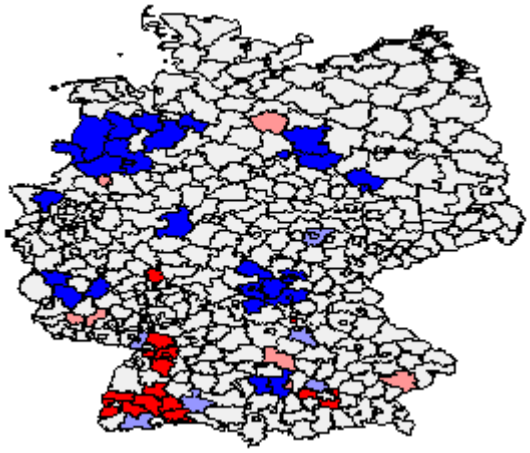
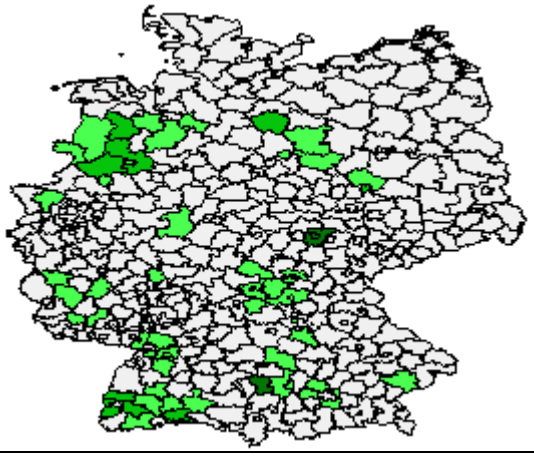
$$\mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}.$$

a) Compute the local Moran coefficients and interpret them!

b) Show the relationship between the local Moran cofficients and global Moran's I!

Assignment 12 (LISA Cluster and Significance Map)

GeoDa has created the following LISA Cluster and Significance Map for the share of employees of manufacture of electrical and optical instruments with the default settings:

LISA Cluster Map	LISA Significance Map
	

a) Interpret the LISA Cluster Map!

b) What additional information does the LISA Significance Map provide?

Assignment 13 (Local Getis-Ord G_i and G_i^*)

The vacancy rate (v) is unequally distributed across space:

Region	1	2	3	4	5
Vacancy rate (v)	3	3	2	1	1

The distance-based contiguity matrix with a critical distance of d=14.4 reads

$$\mathbf{W}(d=14.4)=\begin{bmatrix}1&1&1&0&0\\1&1&0&1&1\\1&0&1&0&1\\0&1&0&1&1\\0&1&1&1&1\end{bmatrix}.$$

a) Compute the G_i statistics and interpret them!

b) Compute the G_i^* statistics and interpret them!

Assignment 14 (OLS regression)

Data on the unemployment rate (u) and the vacancy rate (v) are given for 5 regions.

Region	1	2	3	4	5
Unemployment rate (u)	6	8	8	11	12
Vacancy rate (v)	3	3	2	1	1

a) Estimate the Beveridge curve by ordinary least squares (OLS)!

b) Compute the residual variance and the standard error of regression!

c) Are the regression coefficients significant (α=0.05)?

d) Compute the coefficient of determination and interpret it!

Assignment 15 (LM error test)

Apply the LM error test (α=0.10) with the standardized weight matrix

$$\mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}$$

to the residuals of the OLS estimated Beveridge curve:

Region	1	2	3	4	5
OLS residuals	-0.75	1.25	-1	-0.25	0.75

Assignment 16 (Spatial cross-regressive model)

In 5 regions data on the unemployment rate (u) and the vacancy rate (v) are observed:

Region	1	2	3	4	5
Unemployment rate (u)	6	8	8	11	12
Vacancy rate (v)	3	3	2	1	1

The row-standardized contiguity matrix reads

$$\mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}.$$

a) Give the contents of the observation matrix [**X** **X***] that underlies the cross-regressive model of the Beveridge curve!

b) Estimate the cross-regressive model of the Beveridge curve using

$$([\mathbf{X}\ \mathbf{X}^*]'[\mathbf{X}\ \mathbf{X}^*])^{-1} = \begin{bmatrix} 8.44426 & 0.6557 & -4.6230 \\ 0.6557 & 0.4344 & -0.7377 \\ -4.6230 & -0.7377 & 2.9508 \end{bmatrix}$$

by OLS!

c) Given $\hat{\sigma} = 0.3787$ test the significance of the estimated regression coefficient $\hat{\gamma}_2$ (α=0.05)!

d) Apply the F test on omitted spatially lagged exogenous variables using the residual vector

$$\mathbf{e} = \begin{bmatrix} -0.1639 \\ 0.2377 \\ -0.1475 \\ 0.3361 \\ -0.2623 \end{bmatrix}$$

(α=0.05)!

Assignment 17 (Spatial lag model – IV Estimation)

In 5 regions data on the unemployment rate (u) and the vacancy rate (v) are observed:

Region	1	2	3	4	5
Unemployment rate (u)	6	8	8	11	12
Vacancy rate (v)	3	3	2	1	1

The row-standardized contiguity matrix reads

$$\mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}.$$

a) Use the vector of spatially lagged values of the unemployment rate in the previous period,

$$\mathrm{L}\mathbf{y}_{t-1} = \mathbf{W}\mathbf{y}_{t-1} = \begin{bmatrix} 5.9 \\ 6.9 \\ 8.5 \\ 11.8 \\ 10.1 \end{bmatrix},$$

as an instrument for L**y** = **Wy** and specify the matrix of exploratory variables **Z** and the matrix of instruments **Q** for IV estimation of the spatial lag model of the Beveridge curve!

b) Compute projection matrix $\mathbf{P}_Q$ and give the predicted values of a regression of the variables in **Z** on the instruments in **Q**!

c) What is the IV estimator for the parameter vector θ of the spatial lag model? Interpret the IV estimator for θ!

d) Compute the standard error of regression of IV estimation!

e) Is the IV estimator for the autoregressive parameter ρ significant (α=0,05)?

Assignment 18 (Maximum Likelihood I)

A simple random sample ($x_1, x_2, \ldots, x_n$) is drawn from a Poission distributed population with an unkown parameter $\lambda>0$:

$$f(x|\lambda) = \frac{\lambda^x}{x!} e^{-\lambda}, \; x=0,1,2,\ldots$$

Derive the maximum likelihood (ML) estimator for λ and interpret it!

Assignment 19 (Maximum Likelihood II)

A simple random sample ($x_1, x_2, \ldots, x_n$) is drawn from an exponentially distributed population with an unknown parameter $\lambda>0$:

$$f(x \mid \lambda) = \lambda \cdot e^{-\lambda x}, \quad x \geq 0$$

Derive the maximum likelihood (ML) estimator for λ and interpret it!

Assignment 20 (Spatial lag model – ML Estimation I)

In 5 regions data on the unemployment rate (u) and the vacancy rate (v) are observed:

Region	1	2	3	4	5
Unemployment rate (u)	6	8	8	11	12
Vacancy rate (v)	3	3	2	1	1

The row-standardized contiguity matrix reads

$$\mathbf{W} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}.$$

a) Estimate the OLS estimators of a regression of the spatial lag of the unemploymnt rate (SL(u)) on the vacancy rate (v) using the inverse of the product matrix **X'X** (s. assigment 14 a)) :

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 1.2 & -0.5 \\ -0.5 & 0.25 \end{bmatrix}!$$

b) Compute the vector of residuals, $\mathbf{e}_L$, of the regression of SL(u) on v!

c) Determine the ML estimate for the autoregressive parameter ρ by grid search with the aid of the R file LCLAGPLOT using the residual vector (s. assignment 14 b)

$$\mathbf{e}_O = \begin{bmatrix} -0.75 \\ 1.25 \\ -1 \\ -0.25 \\ 0.75 \end{bmatrix}!$$

d) Determine the ML estimator $\hat{\boldsymbol{\beta}}_{ML}$ of the vector of regression coefficients $\boldsymbol{\beta}$ using the OLS estimator (s. assignment 14 a):

$$\hat{\boldsymbol{\beta}}_O = \begin{bmatrix} 13.5 \\ -2.25 \end{bmatrix}!$$

e) Compute the standard error of regression of ML estimation!

Assignment 21 (Spatial lag model – ML estimation II)

ML estimation of the spatial lag model for the Beveridge curve brought out the following results (see "General information on a system of regions and data"):

REGRESSION
SUMMARY OF OUTPUT: SPATIAL LAG MODEL - MAXIMUM LIKELIHOOD ESTIMATION
Data set: **region5uv**
Spatial Weight: **region5_ex.GAL**
Dependent Variable: **U** Number of Observations: 5
Mean dependent var: 9 Number of Variables: 3
S.D. dependent var: 2.19089 Degrees of Freedom: 2
Lag coeff. (Rho): ?

R-squared: 0.849709 Log likelihood: -6.32298
Sq. Correlation: - Akaike info criterion: 18.646
Sigma-square: 0.721396 Bayes criterion: 17.4743
S.E of regression: 0.84935

Variable	Coefficient	Std.Error	?-value	Probability
W_U	?	0.3879349	0.5273139	0.5979756
CONSTANT	11.35549	3.991297	2.845064	0.0044404
V	-2.088054	0.4804627	-4.345923	0.0000139

a) Specify the regression equation of the spatial lag model and interpret it!

b) Explain the structure of the log-likelihood function of the spatial lag model!

c) Is the autoregressive parameter of the spatial lag model significant (α=0.05)?

d) Specify the variance-covariance matrix of the errors in equation (5.5b)/(5.6b)!

Hint: $(\mathbf{I}-\rho\mathbf{W}')^{-1}=[(\mathbf{I}-\rho\mathbf{W})^{-1}]'$

Assignment 22 (Spatial error model – ML estimation)

ML estimation of the spatial error model for the Beveridge curve brought out the following results (see "General information on a system of regions and data"):

```
REGRESSION
SUMMARY OF OUTPUT: SPATIAL ERROR MODEL - MAXIMUM LIKELIHOOD ESTIMATION
Data set: region5uv
Spatial Weight: region5_ex.GAL
Dependent Variable:U            Number of Observations: 5
Mean dependent var: 9.000000    Number of Variables: 2
S.D. dependent var: 2.190890    Degree of Freedom: 3
Lag coeff. (Lambda): -0.774416

R-squared:    0.937623          R-squared (BUSE): -
Sq. Correlation: -              Log likelihood: -4.803067
Sigma-square: 0.299409          Akaike info criterion:13.6061
S.E of regression: 0.547183     Bayes criterion: 12.825011
```

Variable	Coefficient	Std.Error	?-value	Probability
CONSTANT	13.95179	0.4773741	29.22611	0.0000000
V	-2.461901	0.2252333	-10.93045	0.0000000
LAMBDA	-0.7744158	0.2980455	?	?

a) Specify the spatial error process and interpret it!

b) Give the Jacobian term of the log-likelihood function and interpret its role in estimation!

c) Is the autoregressive parameter of the spatial error model significant (α=0.01)?

d) Specify the variance-covariance matrix of the errors!

Hint: $(\mathbf{I}-\lambda\mathbf{W}')^{-1}=[(\mathbf{I}-\lambda\mathbf{W})^{-1}]'$

Assignment 23 (Model comparison)

Estimation of the Beveridge curve (see "General information on a system of regions and data") has brought out the following summary statistics with four different regression models:

	Standard regression model	Spatial cross-regressive model	Spatial lag model	Spatial error model
R^2	0.844	0.988	0.850	0.938
Adj. R^2	0.792	0.976	-	-
AIC	16.761	5.899	18.646	13.606
BIC	15.970	4.727	17.474	12.825

a) Explain the summary statistics for model selection!

b) Which regression model would you select for modelling the Beveridge curve? Substantiate your selection!