• Ordinary least squares (OLS) estimation

An important task of regression analysis is to estimate the unknown vector of regression coefficients, **β**, in order to assess the influence of the regressors X_1, X_2, …, X_k on the dependent variable Y. Under the standard assumptions, ordinary least squares (OLS) estimation yields best linear unbiased estimators (blue property).

$(\underline{X}'\underline{X})^{-1} = \frac{1}{|\underline{X}'\underline{X}|}(\underline{X}'\underline{X})^{adj}$

adjoint matrix (= transpose of the matrix of adjoints)

Least squares criterion:

$\underset{2\times 2}{\underline{A}} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \underline{A}^{adj} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, if $\underline{A}$ symmetric: $\underline{A}^{adj} = \begin{bmatrix} d & -b \\ -b & a \end{bmatrix}$

$|\underline{A}| = ad - bc$ $\qquad |\underline{A}| = ad - b^2$

= prime

(4.2a) $$Q(\boldsymbol{\beta}) = \sum_{i=1}^{n} \varepsilon_i^2 = \boldsymbol{\varepsilon}'\boldsymbol{\varepsilon} = (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}).$$

$\underline{\beta}'\underline{X}'\underline{y} = (\underline{\beta}'\underline{X}'\underline{y})' = y'X\beta$

Q has to be minimized with respect to **β** for which we use the equivalent expression:

$-\underline{y}'\underline{X}\underline{\beta} - (X\beta)'y = -\underline{y}'\underline{X}\underline{\beta} - \underline{\beta}'\underline{X}'\underline{y}$

(4.2b) $$Q(\boldsymbol{\beta}) = \mathbf{y}'\mathbf{y} - 2\mathbf{y}'\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta}$$

Rule 1 (scalar product)

$f(x) = a \cdot x \rightarrow \frac{df(x)}{dx} = \frac{dax}{dx} = a$

First order condition for a minimum of Q:

$\underline{a}'\underline{x} \rightarrow \frac{d\underline{a}'\underline{x}}{d\underline{x}} = (\underline{a}')' = \underline{a}$

$$\frac{dQ(\boldsymbol{\beta})}{d\boldsymbol{\beta}} = -2\mathbf{X}'\mathbf{y} + 2(\mathbf{X}'\mathbf{X})\boldsymbol{\beta} = \mathbf{o}$$

$\left[\frac{d\, 2\underline{y}'\underline{X}\underline{\beta}}{d\underline{\beta}} = (2\underline{y}'\underline{X})' = 2\underline{X}'\underline{y} \right.$ $\left[\begin{matrix} \text{Rule:} \\ (\underline{A}\cdot\underline{B})' \\ = \underline{B}'\underline{A}' \end{matrix} \right.$

OLS estimator of **β**;

Rule 2: (quadratic form)

$f(x) = a x^2 [= x \cdot a \cdot x] \rightarrow \frac{df(x)}{dx} = \frac{dax^2}{dx} = 2ax$

(4.3) $$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

$\left[\frac{d\underline{\beta}'\underline{X}'\underline{X}\underline{\beta}}{d\underline{\beta}} = 2(\underline{X}'\underline{X})\underline{\beta} \right]$ $\underset{\text{symmetric}}{\underline{x}'\underline{A}\underline{x}} \rightarrow \frac{d\underline{x}'\underline{A}\underline{x}}{d\underline{x}} = (2\underline{x}'\underline{A})' = 2\underline{A}\underline{x}$