$\underline{Y} = \underline{X}\underline{\beta} + \underline{\varepsilon}$

$\hat{\underline{\beta}} = (\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Y}$
$= (\underline{X}'\underline{X})^{-1}\underline{X}'(\underline{X}\underline{\beta} + \underline{\varepsilon})$
$= (\underline{X}'\underline{X})^{-1}\underline{X}'\underline{X}\underline{\beta} + (\underline{X}'\underline{X})^{-1}\underline{X}'\underline{\varepsilon}$
$= \underline{\beta} + (\underline{X}'\underline{X})^{-1}\underline{X}'\underline{\varepsilon}$

$\hat{\underline{\beta}} - \underline{\beta} = (\underline{X}'\underline{X})^{-1}\underline{X}'\underline{\varepsilon}$
sampling error

$Cov(\hat{\underline{\beta}}) = E\{[\hat{\underline{\beta}} - \underbrace{E(\hat{\underline{\beta}})}_{=\underline{\beta}}][\hat{\underline{\beta}} - \underbrace{E(\hat{\underline{\beta}})}_{=\underline{\beta}}]'\}$
$= E\{[(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{\varepsilon}][(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{\varepsilon}]'\}$
$= E[(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{\varepsilon}\,\underline{\varepsilon}'\underline{X}(\underline{X}'\underline{X})^{-1}]$
$= (\underline{X}'\underline{X})^{-1}\underline{X}'\underbrace{E(\underline{\varepsilon}\,\underline{\varepsilon}')}_{=\sigma^2\underline{I}}\underline{X}(\underline{X}'\underline{X})^{-1}$
$= \sigma^2(\underline{X}'\underline{X})^{-1}\underbrace{\underline{X}'\underline{X}(\underline{X}'\underline{X})^{-1}}_{=\underline{I}} = \sigma^2(\underline{X}'\underline{X})^{-1}$

- **Hypothesis tests**

-Test of significance of regression coefficients

$E(\hat{\underline{\beta}}) = E[\underline{\beta} + (\underline{X}'\underline{X})^{-1}\underline{X}'\underline{\varepsilon}]$
$= \underline{\beta} + E(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{\varepsilon}$
$= \underline{\beta} + (\underline{X}'\underline{X})^{-1}\underline{X}'\underbrace{E(\underline{\varepsilon})}_{=0}$
$E(\hat{\underline{\beta}}) = \underline{\beta}$
unbiasedness

Null hypothesis H_0: $\beta_j = 0$

Distribution of the OLS estimator $\hat{\boldsymbol{\beta}}$ under H_0 for normally distributed errors:

$$\hat{\boldsymbol{\beta}} \sim N(\boldsymbol{\beta}, \sigma^2(\mathbf{X}'\mathbf{X})^{-1}) \text{ because of } E(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta} \text{ and } Cov(\hat{\boldsymbol{\beta}}) = \sigma^2 \cdot (\mathbf{X}'\mathbf{X})^{-1}$$

variances of $\hat{\beta}_j$ on the main diagonal

Test statistic:

$$(4.16) \quad t_j = \frac{\hat{\beta}_j}{\hat{\sigma}\sqrt{xx^{jj}}}$$

superscript

superscript (= hochgestellt, Hochstellung, Hochzahl)
t_j subscript (= tiefgestellt, tiefgestelltes Zeichen)

xx^{jj}: *jth* main diagonal element of the inverse $(\mathbf{X}'\mathbf{X})^{-1}$

t_j follows a t distribution with n-k degrees of freedom.

Significance level: α

Critical value (two-sided test): t(n-k;1-α/2)

Testing decision: $| t_j | > t(n-k;1-\alpha/2)$ => Reject H_0

or

$p < \alpha$ => Reject H_0

p-value: Probability of obtaining an (absolute) higher t statistic than | tj |

(Exceeding probability)