When Does the Introduction of a New Currency Improve Welfare?

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Abstract: In recent years, cryptocurrencies such as Bitcoin have emerged, in upcoming years, corporate currencies such as Libra (Diem) and central bank digital currencies will emerge even in low-inflation developed economies. Using the dual currency search model of Kiyotaki and Wright (1993), we show how the introduction of a supplement to traditional money affects average utility. The room for a welfare improvement depends on differences in returns and costs, but, in particular, on the fraction of cash traders who will be replaced by digital money traders.

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1 Introduction

The process of digitalization accelerates the emergence of new currencies. Cryptocurrencies such as Bitcoin, corporate currencies such as Libra (Diem) and central bank digital currencies may serve as examples. These currencies do not substitute an ill-functioning (high inflation) traditional currency, but they are new competitors on the markets for payment systems, they may be used as new medium of exchange. From a government point of view, the question of an optimal degree of regulation arises (see Lotz, 2004). The answer very much depends on the sign of the net welfare effect of the new currency: does it improve average utility in the economy or not.

To address this topic, we develop a dual-currency search model, which builds on Kiyotaki and Wright (1993). In contrast to, for instance, Lagos and Wright (2005), this framework allows for a convincing modelling of a partially accepted currency. The economics of dual currency regimes is the topic of a wide body of theoretical and empirical literature (see, e.g., Aiyagari et al., 1996; Curtis and Waller, 2000; Craig and Waller, 2000; Chang, 2006; Rietz, 2019). Surprisingly, this literature says very little about the welfare effects. This paper aims to fill this gap.

2 The Setup

Our basic setup is an economy with a $[0,1]$ continuum of infinitely lived agents, agents discount the future at rate r. There is also a $[0,1]$ continuum of non-storable and indivisible consumption goods. Only the fraction x of the consumption goods enters the utility function of an agent, $U(x) > 0$, consumption of other commodities is not beneficial, $U(1-x) = 0$. Agents are divided into commodity traders (sellers) and money traders (buyers), money traders in turn are divided into cash traders and digital money traders. In each period, the economy consists of a fraction μ_S of agents each endowed with a commodity, μ_C with one unit of cash and μ_D with one unit of the digital currency, where $\mu_S + \mu_C + \mu_D = 1$. These fractions are exogenously given (policy) parameters, in particular, they are not the result of an optimization-based decision of the agents. To model trade, we borrow from Matsuyama et al. (1993) and assume that agent i is endowed with good i (part of $1-x$) but prefers good $i+1$ (part of x). There is thus no pure barter.

All agents are looking for a trading partner. Meetings are pairwise and occur according to a Poisson process with the arrival rate β. Let V_j, $j = S, C, D$, be the value functions of a seller, a cash trader and a digital money trader, then the expected returns to search are given by the Bellman equations:

$$rV_S = \mu_C \max_{\pi_C(i)} [\pi_C(i)(V_C - V_S)] + \mu_D \max_{\pi_D(i)} [\pi_D(i)(V_D - V_S)] \tag{1}$$

$$rV_C = \gamma_C + \mu_S \Pi_C (U - \eta_C + V_S - V_C) \tag{2}$$

$$rV_D = \gamma_D + \mu_S \Pi_D (U - \eta_D + V_S - V_C) \tag{3}$$

The derivation makes use of the normalization $\beta x = 1$ (see Trejos and Wright, 1995). The right side of (1) denotes the expected return for a seller. This is the probability of meeting a cash trader times the return of switching from a seller to a cash trader plus the probability of meeting a digital trader times the return of switching from a seller to a digital trader. If the return of switching is positive (negative), seller i always accepts (rejects) the currencies and sets the optimal response $\pi_C(i)$ respective $\pi_D(i)$ to unity (zero). If the seller is indifferent, he flips a coin with $0 < \pi_C(i), \pi_D(i) < 1$, a currency is partially accepted.

For a cash trader, the expected return from trading is equal to the probability of meeting a seller times the overall acceptance of cash Π_C times the utility of consuming the good minus the costs of using cash η_C plus the gain of switching from cash holder to seller. If no trading takes place, the cash holder receives a permanent monetary benefit γ_C. In the case of storage costs, $\gamma_C < 0$ holds. For a digital money trader, the line of argument is very much the same, see Eq. (3). Here, η_D are the costs of using the digital currency, the Bitcoin transaction fees may serve as an example. If the digital money pays a dividend (note the introduction of an interest-bearing CBDC) or the holding of digital money has some intrinsic value, we have $\gamma_D > 0$. Again, storage costs are captured by $\gamma_D < 0$. A trade between a cash and a digital money trader does not make both agents better off. Since we rule out additional payments in terms of some goods (see Aiyagari et al, 1996), each money trader continues with his own money.

Our focus will be on symmetric equilibria with $\pi_C(i) = \Pi_C$ and $\pi_D(i) = \Pi_D$, that is, the optimal response is equal to the overall acceptance. Moreover, welfare is defined by average utility, which can be written as

$$W = \mu_S V_S + \mu_C V_C + \mu_D V_D \tag{4}$$

3 Three Scenarios

The welfare effect of a new currency very much depends on the acceptance of both the traditional and the new currency. We thus distinguish between three scenarios, first, both currencies are partially accepted, second, cash is fully and the digital currency is partially accepted, and third, both currencies are fully accepted.

3.1 Both Currencies Partially Accepted

Partial acceptance of cash and digital money requires $V_C = V_S$ and $V_C = V_D$, respectively, see Eq. (1). From (1) also follows $V_S = 0$, so that $V_C = V_D = 0$ and $W = 0$. Using (2) and (3), we compute the equilibrium acceptance rates as

$$\Pi_C = -\frac{\gamma_C}{\mu_S(U - \eta_C)} \quad \textit{and} \quad \Pi_D = -\frac{\gamma_D}{\mu_S(U - \eta_D)} \tag{5}$$

An equilibrium requires storage costs, $\gamma_C, \gamma_D < 0$. If there are no such costs, the value of being a money trader would be positive, and each seller would have an incentive to switch from a seller to a money trader, each seller would always accept both cash and digital money. But this contradicts the assumption of a partial equilibrium. Similarly, if the storage costs exceed the expected value from trading, each seller avoids a negative realization of utility by always rejecting a currency.

In the initial equilibrium, the traditional currency (cash) is only partially accepted. There is thus some room for a welfare improvement by making trade easier via the introduction of a new currency. But if the new currency is only partially accepted too, there will be no improvement in terms of welfare, welfare remains at zero.

Proposition 1: *Suppose that in the initial equilibrium, the traditional currency is partially accepted. (i) The introduction of a new partially accepted currency is neutral with respect to welfare. (ii) A steady state equilibrium requires storage costs for both currencies.*

3.2 Cash Fully Accepted, Digital Money Partially Accepted

Partial acceptance of cash is more the exception than the rule. From our point of view, full acceptance of cash is still the more realistic scenario. In our model, full acceptance of cash, $\pi_C(i) = \Pi_C = 1$, requires $V_C > V_S$. As before, partial acceptance of the digital money, $\pi_D(i) = \Pi_D < 1$, requires $V_D = V_S$. By combining Eqs. (1) and (2), it is straightforward to show that $V_C - V_S$ is positive, if

$$\rho_C \equiv \gamma_C + \mu_S(U - \eta_C) > 0 \tag{6}$$

holds. Here, ρ_C is the expected per period return of cash. If the sum of the expected net utility from buying and consuming a good plus the monetary benefit (or minus the storage costs) is positive, cash will be universally accepted. For the acceptance rate of the digital money, we obtain $\Pi_D = \frac{\mu_C \nu_C \rho_C - \gamma_D}{\mu_S(U - \eta_D)}$ with $\nu_C \equiv (r + \mu_S + \mu_C)^{-1}$.

Let us turn to welfare. For the initial equilibrium, we assume a single currency regime, just the fully accepted cash circulates. Setting $\mu_D = 0$, welfare (4) can be derived as

$$rW^{SR} = \mu_C^{SR} \rho_C^{SR} \tag{7}$$

where the superscript SR denotes the single currency regime. Average utility is given by the share of cash traders times the expected per period return of cash. Notice that the welfare of the sellers "vanishes". Any change in a parameter which, for instance, increases the welfare of the sellers, implies an equal sized decrease in the welfare of the money traders. The comparative static of (7) is straightforward, most interesting is the welfare-maximizing share of cash traders, which is given by $\mu_C^{*SR} = \frac{1}{2} + \frac{\gamma_C}{2(U-\eta_C)}$. For $\gamma_C = 0$, the optimal share is one half. A monetary benefit, $\gamma_C > 0$, (storage costs, $\gamma_C < 0$) shifts the maximum to the right (left).

Now assume that a new partially accepted currency is introduced, so that $\gamma_D > 0$. Computing the new expected returns to search and inserting into Eq. (4) yields

$$rW^{DP} = (1 + \mu_D \nu_C^{DP}) \mu_C^{DP} \rho_C^{DP} \tag{8}$$

Here, the superscript DP stands for the <u>d</u>ual currency regime with <u>p</u>artial acceptance of the new currency. In order to compare (8) with (7), we need a hypothesis on the replacement of sellers and cash traders by the digital money traders. This is done by

$$\mu_S^{DP} = \mu_S^{SR} - \lambda \mu_D \tag{9}$$

$$\mu_C^{DP} = \mu_C^{SR} - (1 - \lambda) \mu_D \tag{10}$$

Let us focus on the polar cases, $\lambda = 0$ and $\lambda = 1$. For $\lambda = 0$, the digital money trader do not replace any seller, the economy's endowment with goods remains the same, $\mu_S^{DP} = \mu_S^{SR}$. Instead, the digital money trader replace one-to-one cash trader, $\mu_C^{DP} = \mu_C^{SR} - \mu_D$. In this case, the new currency does not change the endowment of the economy with money, but the money supply is now made up by two fiat currencies. For $\lambda = 1$, the digital money trader replace only sellers, the proportion of sellers declines one-to-one with the proportion of digital money traders, $\mu_S^{DP} = \mu_S^{SR} - \mu_D$. Since the proportion of cash traders remains constant, $\mu_C^{DP} = \mu_C^{SR}$, the new currency implies an increase in the economys money supply. Again, we have to emphasize that a switch for example from cash to digital money is not the result of an optimization process but just an assumption on the endowment of the economy.

Suppose the digital traders replace only cash traders, $\lambda = 0$. Then we can show that $W^{DP} > W^{SR}$ requires $r + \mu_S^{DP} < 0$. But this condition is never fulfilled. Therefore, in the case of $\lambda = 0$, the introduction of a new partially accepted currency unambiguously lowers welfare. The reason is quite simple. The cash traders, who switch their status to a digital money trader, switch from a currency with full acceptance to a currency with partial acceptance. The aggregate money supply does not change, but the probability of consuming (finding a seller) goes down. To say it a little bit sloppy, the "quality" of the economy's money as medium of exchange decreases.

Turn to the case, where the digital money traders replace sellers but no cash traders, $\lambda = 1$. Now we get $W^{DP} > W^{SR}$ if

$$\gamma_C > (r + \mu_C^{SR})(U - \eta_C) \tag{11}$$

If cash has some storage costs, $\gamma_C < 0$, the condition is not fulfilled, the new currency again lowers welfare. Interestingly, a monetary benefit of the traditional currency turns out to be a necessary condition for a positive welfare effect of the new currency. If γ_C exceeds a threshold, we will observe a positive welfare effect.

There are countervailing forces at work. On the one hand, the sellers who switch status to digital traders, improve the probability of consuming in the initial period from zero to the acceptance rate Π_D, trade becomes easier. On the other hand, the cash traders face a loss. Because of the lower number of sellers at the market place, the probability of consuming goes down. The loss is increasing in the net utility gain from consumption, $U - \eta_C$, the share of cash traders, μ_C^{SR}, and the discount rate r, but decreasing in the monetary benefit γ_C. If (11) is fulfilled, the loss of the cash traders is lower than the gain of the remaining agents. Proposition 2 summarizes:

Proposition 2: *Suppose that in the initial equilibrium, cash is fully accepted, the new currency is partially accepted. (i) If the digital traders replace only cash traders, the new currency lowers welfare. (ii) If the digital traders replace only sellers, a positive welfare effect requires a monetary benefit of cash which exceeds a positive threshold.*

3.3 Both Currencies Fully Accepted

Our third scenario assumes that both the traditional and the new currency are universally accepted, $\Pi_C = \Pi_D = 1$. This requires $V_C > V_S$ and $V_D > V_S$. Rearranging the Bellman equations (1) to (3) shows that these constraints are fulfilled if and only if the parameter constellation

$$-(1 - \mu_C^{DF}\nu_C^{DF})\rho_C^{DF} < \mu_D^{DF} - \mu_C^{DF} < (1 - \mu_D\nu_D^{DF})\rho_D^{DF} \tag{12}$$

holds. Here, $\rho_D^{DF} \equiv \gamma_D + \mu_S^{DF}(U - \eta_D)$ is the expected per period return of the digital currency, and $\nu_D^{DF} \equiv (r + \mu_S^{DF} + \mu_D)^{-1}$. The superscript DF denotes the dual currency regime with full acceptance of the new currency. The spread between the expected per period return of the digital currency and cash must not be too big, otherwise either the digital currency or cash is no longer fully accepted. Using the definitions of the per period returns, the spread is given by $\mu_D^{DF} - \mu_C^{DF} = \gamma_D - \gamma_C - \mu_S^{DF}(\eta_D - \eta_C)$. Welfare in the DF regime can be computed as

$$rW^{DF} = \mu_C^{DF}\rho_C^{DF} + \mu_D\rho_D^{DF} \tag{13}$$

The introduction of a universally accepted currency improves welfare, if $W^{DF} > W^{SR}$, or equivalently, if $\mu_D\rho_D^{DF} > \mu_C^{SR}\rho_C^{SR} - \mu_C^{DF}\rho_C^{DF} = \rho_C^{SR}(\mu_C^{SR} - \mu_C^{DF}) + \mu_C^{DF}(\rho_C^{SR} - \rho_C^{DF})$. If the welfare gain of the digital traders exceeds the welfare loss of the cash traders, then the new currency allows for a net welfare gain. The loss of the cash traders, in turn, is split into the decline of the number of cash traders (first summand) and the loss per cash trader (second summand). To sign the net effect on welfare, we again need a hypothesis on the replacement of sellers and cash traders by the digital money traders. We adapt (8) and (9) by assuming $\mu_S^{DF} = \mu_S^{SR} - \lambda\mu_D$ and $\mu_C^{DF} = \mu_C^{SR} - (1-\lambda)\mu_D$.

Let us again focus on the polar cases. Suppose that the digital traders replace only cash traders, the number of sellers remains constant, $\lambda = 0$. In this case, the condition for a positive net welfare effect boils down to

$$(W^{DF} > W^{SR})|_{\lambda=0} \quad \Leftrightarrow \quad \rho_D^{DF} > \rho_C^{DF} \tag{14}$$

The cash traders, who switch status to digital money traders, switch to a currency with the same liquidity value (acceptance rate), they gain the expected per period return ρ_D^{DF}, they lose the expected per period return ρ_C^{SR}, which corresponds for ρ_C^{DF} for $\lambda = 0$. If the former exceeds the latter, the economy yields a payoff.

Now suppose that the digital traders replace only sellers, the number of cash traders remains constant, $\lambda = 1$. Then we obtain

$$(W^{DF} > W^{SR})|_{\lambda=1} \quad \Leftrightarrow \quad \rho_D^{DF} > \mu_C^{DF}(U - \eta_C) \tag{15}$$

The sellers, who switch status to digital money traders, gain ρ_D^{DF}. The cash traders are affected indirectly. Because of the lower number of sellers the probability of trade and consumption declines.

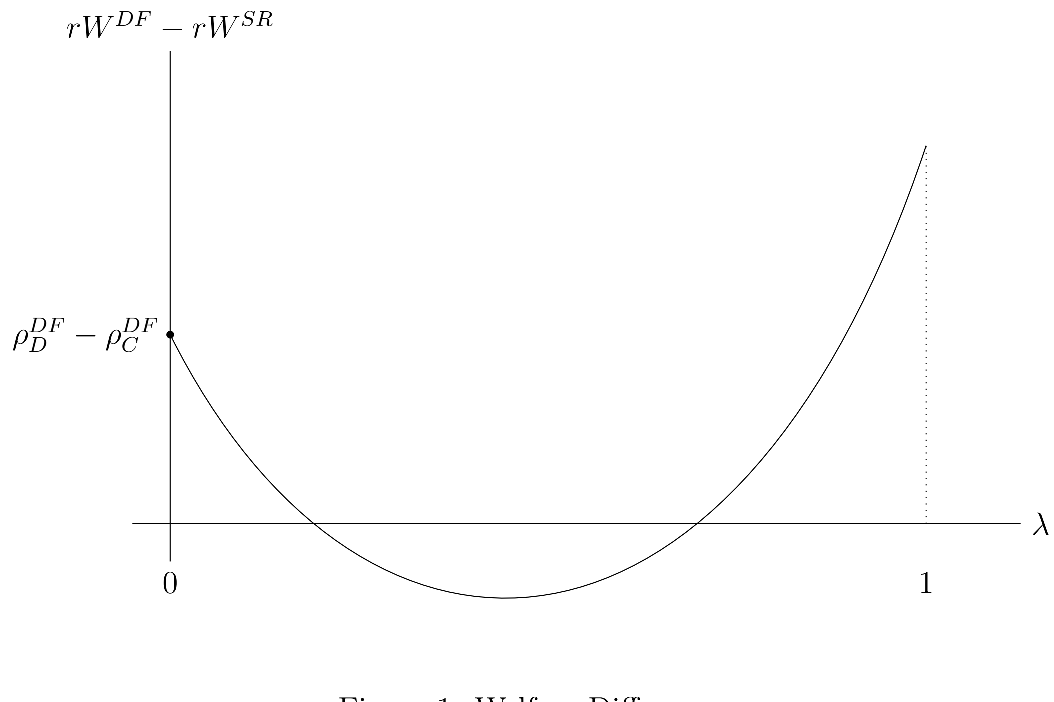

Figure 1: Welfare Difference

Net welfare is not linear, but a quadratic function in λ. Therefore, even if for both $\lambda = 0$ and $\lambda = 1$ the net welfare effect is positive, it is not guaranteed that the net effect is positive for all values of λ between zero and one. Figure 1 illustrates. Depending on the parameter constellation, we may observe a range for λ, where the net effect turns out to be negative. This is the result of three different forces, each depending on λ. First, the higher λ, the lower the per period return of a digital money trader, ρ_D^{DF}, the number of sellers and thus the number of trades declines in λ. Second, the higher λ, the lower is the decline in the number of cash traders and thus the lower is the loss of the cash traders due to a lower number of cash traders. Third, the loss per cash trader is increasing in λ, since the probability of a trade declines. The net effect may change the sign.

Proposition 3: *Suppose that in the initial equilibrium, cash is fully accepted, the new currency is fully accepted, too. (i) The existence of an equilibrium requires that the spread $\rho_D^{DF} - \rho_C^{DF}$ must not be too big. (ii) If the digital money traders replace only cash traders, a positive spread ensures a net welfare gain. (iii) If the digital money traders replace only sellers, the condition $\rho_D^{DF} > \mu_C^{DF}(U - \eta_C)$ ensures a net welfare gain. (iv) The sign of the net welfare effect is sensitive to the choice of λ, the parameter determining the replacement of sellers and cash traders by digital money traders.*

4 Conclusion

The main task of the work was to investigate how a second (digital) currency affects welfare. As proved, it mainly depends on the properties of the second currency, given by the monetary benefit, the transaction fee and the acceptance. Furthermore, it is also of importance whether the digital money traders replace commodity or cash traders. We can distinguish between the three mentioned cases: first, if digital money is also partially accepted next to cash, welfare remains zero. In this case digital money does not affect welfare. Second, if digital money is only partially accepted compared to a fully accepted cash, welfare only increases if the digital money traders replace commodity traders. Third, if both payment systems are fully accepted, it mainly depends on the replacement parameter. In particular, for the polar cases a welfare improvement is possible here.

Literature

Aiyagari, Rao S., Neil Wallace and Randall Wright (1996): "Coexistence of money and interest-bearing securities", *Journal of Monetary Economics* 37(3): 397-419.

Chang, Sander S. (2006): "Inflation and dollarization in a dual-currency search-theoretic model", *Economics Letters* 92(3): 353-359.

Craig, Ben R. and Christopher J. Waller (2000): "Dual-currency economies as multiple-payment systems", *Federal Reserve Bank of Cleveland Economic Review* 36(1): 2-13.

Curtis, Elisabeth S. and Christopher J. Waller (2000): "A search-theoretic model of legal and illegal currency", *Journal of Monetary Economics* 45(1): 155-184.

Kiyotaki, Nobuhiro and Randall Wright (1993): "A search-theoretic approach to monetary economics", *The American Economic Review* 1993: 63-77.

Lagos, Ricardo and Randall Wright (2005): "A unified framework for monetary theory and policy analysis", *Journal of Political Economy* 113(3): 463-484.

Lotz, Sébastien (2004): "Introducing a new currency: Government policy and prices", *European Economic Review* 48(5): 959-982.

Matsuyama, Kiminori, Nobuhiro Kiyotaki and Akihiko Matsui (1993): "Toward a theory of international currency", *The Review of Economic Studies* 60(2): 283-307.

Rietz, Justin (2019): "Secondary currency acceptance: Experimental evidence with a dual currency search model", *Journal of Economic Behavior and Organization* 166: 403-431.

Trejos, Alberto and Randall Wright (1995): "Search, bargaining, money, and prices", *Journal of Political Economy* 103(1): 118-141.