

# Quantum gate in the decoherence-free subspace of trapped ion qubits

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We propose a geometric phase gate in a decoherence-free subspace with trapped ions. The quantum information is encoded in the Zeeman sublevels of the ground state and two physical qubits to make up one logical qubit with ultra long coherence time. Single- and two-qubit operations together with the transport and splitting of linear ion crystals allow for a robust and decoherence-free scalable quantum processor. For the ease of the phase gate realization we employ one Raman laser field on four ions simultaneously, i.e. no tight focus for addressing. The decoherence-free subspace is left neither during gate operations nor during the transport of quantum information.

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## I. INTRODUCTION

Trapped ions are among the most promising physical systems for implementing quantum information due to their long coherence time as compared with the times required for quantum logic operations [1]. A robust quantum memory is a crucial part of the realization of an ion trap based quantum computer [2]. One may distinguish different possibilities for encoding a qubit in a trapped ion, either one uses a long lived metastable state and drives coherent transitions on the corresponding optical transition [3], which sets challenging requirements on the laser source and ultimately limits the coherence time to the lifetime of the metastable state. Alternatively, a qubit can be encoded in sublevels of the electronic ground state. This may be either hyperfine ground state levels [4] or Zeeman ground states [5] which are coherently manipulated by means of stimulated Raman transitions. For conventional single-ion qubits encoded in the Zeeman sublevels as in  $^{40}\text{Ca}^+$  are less favorable, as compared with hyperfine qubits in magnetically insensitive clock states [6, 7], as their energy splitting depends linearly on the magnetic field. Already small magnetic field fluctuations of about 0.2 T limit the coherence to 250  $\mu\text{s}$  [8].

We follow in our proposal an elegant alternative [9, 10] to boost the robustness of such qubits by using a decoherence-free subspace (DFS) [11–15]. We employ odd Bell states as the computational basis of *logical qubits*  $|0\rangle \equiv |\uparrow_1\downarrow_2\rangle$  and  $|1\rangle \equiv |\downarrow_1\uparrow_2\rangle$  with the overhead of having two physical spin qubits. Ground states  $|\uparrow\rangle$  and  $|\downarrow\rangle$  do not perform any bit flip errors. Magnetic field or laser phase fluctuation would lead to errors for a single ion, but in the chosen Bell states such fluctuations are identical for both ions in the logical qubit. This assures that such states can maintain coherent of up to 20 s and single qubit operations in DFS, have been demonstrated [16, 17]. A universal set of single and two qubit operations between logical qubits has been proposed [14] and recently performed with a fidelity of 89% [18], and it would be desirable to reach a fidelity of better than 99% also for DFS gates.

In this paper we show how these two-qubit gate opera-

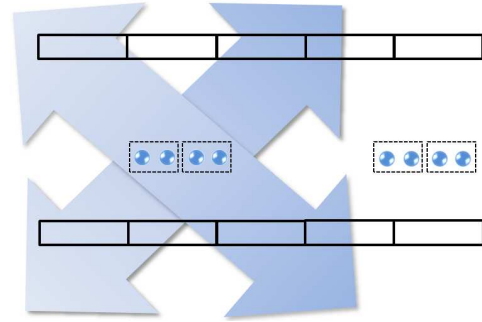


FIG. 1: (color online). Sketch of a segmented ion trapping device holding logic qubits in the DFS (dashed boxes). The qubits are moved to specific zones, merged or separated by applying time-dependent trap control voltages. A pair of logical qubits collectively interact with the laser fields for implementing a two qubit gate. Quantum coherence in the DFS is preserved during the gate operation and the transports.

tions could be improved by a novel scheme, which do not require individual ion addressing. The single ion addressing was identified as one of the major difficulties and one of the main source of loss in fidelity in the experiment. Our scheme is based on *homogeneous* illumination of the four ions. Additionally, our proposed gate operates in a Raman type of laser excitation between ground state DFS logic qubits, an additional promising a fidelity improvement as compared with the metastable, thus 1.2 s long lived basis states in Ref. [18]. We carefully investigated all contributions to the spin-dependent light force [19–23], and optimized the scheme. Furthermore, we generalize the scheme for a scalable approach of quantum computing, Fig.1. With gate times of about 25  $\mu\text{s}$ , more than 5 orders of magnitude faster the decoherence time, the required overhead to handle two ions for one qubit appears to be relatively small. Even more important, such favorable separation of time scales would pave the way to realize quantum error correction [24]. It further allows for transporting ion crystals in segmented micro ion traps [25–27] for the creation of cluster states [28] and operations with a large number of ions for ultimate

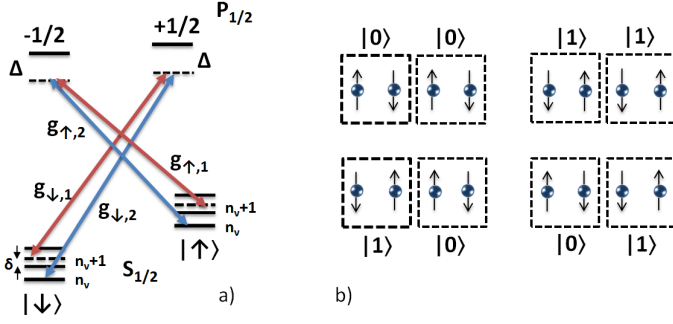


FIG. 2: (color online) a) Level scheme of a  $^{40}\text{Ca}^+$ . The qubit is encoded in the Zeeman sublevels  $|m_J = -1/2\rangle = |\downarrow\rangle$  and  $|m_J = 1/2\rangle = |\uparrow\rangle$  of the  $S_{1/2}$  ground state.  $S_{1/2} \rightarrow P_{1/2}$  transition is driven by a pair of laser beams. Each of the lasers is circularly polarized with balanced  $\sigma^+$  and  $\sigma^-$  components.  $g_{s_i,a}$  is the single beam Rabi frequency of the coupling between the ground states  $s_i = \downarrow, \uparrow$  and the excited state of the  $i$ th ion. b) Logical qubits basis states are indicated by the dashed boxes. When the spins of middle two ions are aligned in opposite directions the light force driving the gate cancels. If and only if the middle two spins are aligned in the same directions the force pushes the ions in different direction.

scalability of quantum information processing.

The present paper is organized as follows: Sec. II describes the optimized scheme of the DFS gate between two logical qubits. In Sec. III we show that the gate is suitable for the scalable gate operations of many logical qubits and specifically for the creation of cluster states without leaving the DFS. In Sec. IV we analyze several error sources relevant to an experimental implementation of our method. Some of the errors stem from a single gate operation, some of them occur when transporting logical qubits to achieve scalability. Finally, in Sec. V we give a summary of the results.

## II. STATE-DEPENDENT FORCE

We consider a linear crystal of four ions confined in a linear Paul trap with trap frequency  $\omega_z$ . The qubit is encoded in the Zeeman ground states levels  $|\uparrow\rangle = |m_J = 1/2\rangle$  and  $|\downarrow\rangle = |m_J = -1/2\rangle$  of the  $S_{1/2}$  ground state of the ion [5]. The linear ion crystal simultaneously interacts with two non-copropagating laser beams with frequency difference  $\omega_p + \delta$ , where  $\omega_p$  is the  $p$ th vibrational frequency of the ion crystal and  $\delta$  is the detuning from the vibrational frequency ( $\omega_p \gg |\delta|$ ). In contrast to the center-of-mass mode, the higher energy vibrational modes are less sensitive to the heating due to fluctuating ambient electric field because it requires short-wavelength components of the field to heat it [30]. The laser is detuned from the  $S_{1/2} \rightarrow P_{1/2}$  transition with large detuning  $\Delta$  and couples only the vibrational levels for each of the spin states according to Fig. 2a. The interaction Hamiltonian for a string of four ions simulta-

neously interacting with a single laser pulse in the Lamb-Dicke limit and rotating wave approximation is given by [31]

$$\hat{H}_I(t) = \sum_{\mathbf{s}_i} \left( F_{\mathbf{s}_i}^{(p)} z_p e^{-i\delta t} \hat{a}^\dagger + F_{\mathbf{s}_i}^{(p)*} z_p e^{i\delta t} \hat{a} \right) |\mathbf{s}_i\rangle \langle \mathbf{s}_i|. \quad (1)$$

Here  $\hat{a}$  and  $\hat{a}^\dagger$  are the creation and annihilation operators of phonons in the  $p$ th vibrational mode,  $z_p = \sqrt{\hbar/2M\omega_p}$  is the spread of the ground state wavepacket for the respective vibrational mode,  $M$  is the ion mass and  $\mathbf{s}_i = \{s_1, s_2, s_3, s_4\}$  runs over all spin configurations of the four ions. The absolute static ac Stark shift of the energies of the qubit states is given by  $\chi_{s_i} = -(|g_{s_i,1}|^2 + |g_{s_i,2}|^2)/2\Delta$ , where  $g_{s_i,a}$  ( $a = 1, 2$ ) is the Rabi frequency pertaining to single beam, Fig. 2a. This shift is generally different for the qubit states  $|\uparrow\rangle$  and  $|\downarrow\rangle$ , thereby induces additional phase in the qubit evolution. The spatiotemporally varying differential shift, which gives rise to the spin-dependent force is  $\Omega_{s_i} = g_{s_i,1}^* g_{s_i,2} / \Delta$ . For the E-mode ( $\omega_3 \approx \sqrt{5.81}\omega_z$ ) the first and fourth ions oscillate out of phase and with equal amplitudes with the second and third ions [29]. Therefore, the magnitude of the laser-ion coupling is the same for all four ions, but opposite in sign with respect to the middle two ions. The force on the collective spin states due to differential Stark shift for the E-mode is given by

$$F_{s_1, s_2, s_3, s_4}^{(3)} = \frac{\hbar \Delta k}{2} (\Omega_{s_1} e^{i\zeta_1} - \Omega_{s_2} e^{i\zeta_2} - \Omega_{s_3} e^{i\zeta_3} + \Omega_{s_4} e^{i\zeta_4}), \quad (2)$$

where  $\Delta k$  is the laser wave vector difference along the trap axis. The position-dependent phase is equal to  $\zeta_i = \Delta k z_i^0 - \Delta\phi$ , where  $z_i^0 = l u_i$  with  $u_i$  is the dimensionless equilibrium position of the  $i$ th ion and  $l^3 = Z^2 e^2 / 4\pi\epsilon_0 M \omega_z^2$  is the length scale parameter.  $\Delta\phi$  is the phase difference between the driving fields. The unitary operator for the Hamiltonian (1) is given by

$$\hat{U}_0(t) = \prod_{\mathbf{s}_i} \hat{D}(\alpha_{\mathbf{s}_i}) e^{i\Phi_{\mathbf{s}_i}}, \quad (3)$$

where

$$\hat{D}(\alpha_{\mathbf{s}_i}) = \exp[(\alpha_{\mathbf{s}_i} \hat{a}^\dagger - \alpha_{\mathbf{s}_i}^* \hat{a})] |\mathbf{s}_i\rangle \langle \mathbf{s}_i| \quad (4)$$

is the state-dependent displacement operator with

$$\alpha_{\mathbf{s}_i} = (e^{-i\delta t} - 1) F_{\mathbf{s}_i}^{(3)} z_3 / \hbar \delta. \quad (5)$$

The state-dependent geometric phase

$$\Phi_{\mathbf{s}_i} = (\delta t - \sin \delta t) \left| F_{\mathbf{s}_i}^{(3)} z_3 \right|^2 / |\hbar \delta|^2 \quad (6)$$

appears due to non-commutativity of the interaction Hamiltonian at different times. We project the unitary operator (3) onto the DFS under consideration:  $\{|\uparrow\downarrow\uparrow\downarrow\rangle, |\downarrow\uparrow\uparrow\downarrow\rangle, |\uparrow\downarrow\downarrow\uparrow\rangle, |\downarrow\uparrow\downarrow\uparrow\rangle\}$ . These states are immune to collective dephasing caused by magnetic field fluctuations. We adjust the trap potential such that

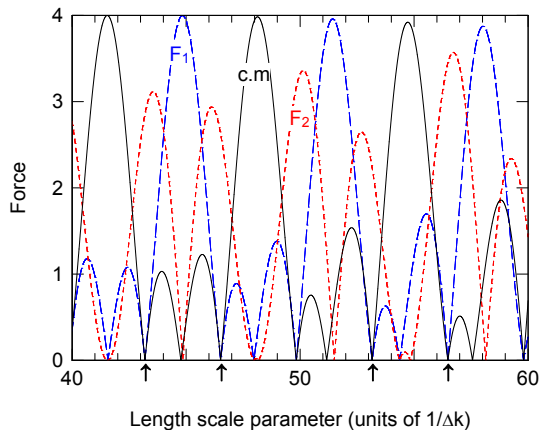


FIG. 3: (color online). Absolute values of the spin-dependent forces  $F_1 = F_{\uparrow\downarrow\uparrow}^{(3)} = F_{\downarrow\uparrow\downarrow}^{(3)}$  (dashed) and  $F_2 = F_{\downarrow\uparrow\downarrow}^{(3)} = F_{\uparrow\downarrow\uparrow}^{(3)}$  (dotted), Eq. (2) as a function of the length scale parameter  $l$  for a gate mediated by the E-mode. Here we assume that  $\Omega_{\uparrow} = -\Omega_{\downarrow}$ . The forces  $F_1$  vanish for  $\Delta kl = 2\pi n/(u_4 - u_2)$ , with  $n$  integer. At the same points the forces  $F_{\uparrow\downarrow\uparrow}^{(1)}$  and  $F_{\downarrow\uparrow\downarrow}^{(1)}$  (solid) for the center of mass mode (in brief c.m) are zero, as indicated by the arrows.

$$\begin{aligned}\Delta k(z_4^0 - z_1^0) &= \Delta k(z_2^0 - z_1^0) + 2n\pi, \\ \Delta k(z_4^0 - z_2^0) &= \Delta k(z_3^0 - z_1^0) = 2n\pi,\end{aligned}\quad (7)$$

where  $n$  is integer. This optimizes the motional coupling to the E-mode as can be seen from Eq. (2). In Fig. 3 we plot the spin-dependent forces for the E-mode as a function of the length scale parameter  $l$ . The spin-dependent forces  $F_{\uparrow\downarrow\uparrow}^{(3)}$  and  $F_{\downarrow\uparrow\downarrow}^{(3)}$  vary as a function of  $\Delta kl$  and vanish for  $\Delta kl = 2\pi n/(u_4 - u_2)$ , while the forces  $F_{\downarrow\uparrow\downarrow}^{(3)}$  and  $F_{\uparrow\downarrow\uparrow}^{(3)}$  displace the motional state in opposite directions. At the same point two of the spin-dependent forces for the center-of-mass mode are zero, hence the off-resonant excitations of this mode do not take place. During the time evolution the motional state moves along a circular path in phase space and returns to the origin after time  $T_g = 2\pi/\delta$ , while the spin states acquire geometric phases  $\Phi_{s_i} = 2\pi |F_{s_i}^{(3)} z_3|^2 / |\hbar\delta|^2$ . In this case the force displaces the ions if the spins of the middle two ions are aligned in the same direction and cancel if the spins are opposite, Fig. 2b. After time  $T_g$  the states for which the force is not canceled acquire geometric phases  $\Phi_{\downarrow\uparrow\downarrow} = \Phi_{\uparrow\downarrow\uparrow}$ . By proper choice of the Rabi frequencies  $\Omega_{\uparrow}$  and  $\Omega_{\downarrow}$  one can adjust the geometric phases to be  $\pi/2$  and the action of the gate onto the DFS states is given by

$$\begin{aligned}|\uparrow\downarrow\uparrow\downarrow\rangle &\rightarrow |\uparrow\downarrow\uparrow\downarrow\rangle, \\ |\downarrow\uparrow\uparrow\downarrow\rangle &\rightarrow i|\downarrow\uparrow\uparrow\downarrow\rangle, \\ |\uparrow\downarrow\downarrow\uparrow\rangle &\rightarrow i|\uparrow\downarrow\downarrow\uparrow\rangle, \\ |\downarrow\uparrow\downarrow\uparrow\rangle &\rightarrow |\downarrow\uparrow\downarrow\uparrow\rangle.\end{aligned}\quad (8)$$

The DFS gate (8) is a controlled-phase gate between two logical decoherence-free qubits. Hence, the unitary evo-

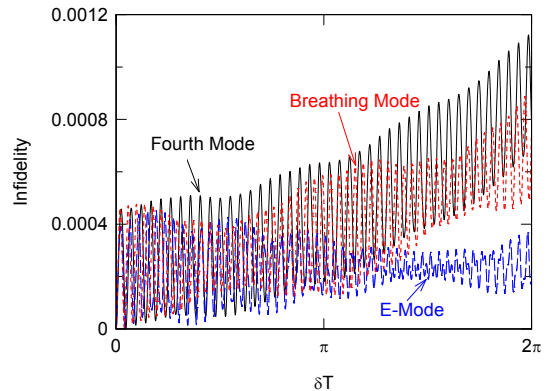


FIG. 4: (color online). The part of the infidelity  $1 - F$  due to off-resonant excitations of the parasitic vibrational modes as a function of  $\delta t$ . The string of four ions is simultaneously addressed with laser fields with frequency close to the breathing mode (dotted), E-mode (dashed), and fourth mode (solid). The Rabi frequencies and the axial trap frequencies are listed in Table I.

lution transforms any superposition of the states belonging to the DFS into another superposition of those states. Note that the gate operation does not require an ion localization well within the Lamb Dicke regime but no ground state cooling of the gate mode, similar to the geometric phase two-qubit gate [19].

Additionally to the geometric phase the qubit states acquire extra phase due to ac Stark shift  $\chi_{s_i}$  so that we have  $|\downarrow\rangle \rightarrow e^{-i\varphi_{\downarrow}}|\downarrow\rangle$  and  $|\uparrow\rangle \rightarrow e^{-i\varphi_{\uparrow}}|\uparrow\rangle$ , where  $\varphi_{s_i} = \int_0^t \chi_{s_i} dt$ . The phase is proportional to the laser intensity thereby it is making the gate implementation very sensitive to laser-intensity fluctuations. However, as long as the four ion chain is addressed uniformly and the quantum information is encoded in the DFS the ac Stark shift causes only global phase in Eq. (8). The effect of slightly inhomogeneous illumination of the ions - accounting for the experimental reality - and the implication of the gate fidelity will be discussed in Section IV

### III. SCALABILITY AND CREATION OF A LINEAR CLUSTER STATE

In general, two approaches for scalable quantum computing with ion string are viable. The one aims for a long ion crystal, where all ions share common modes of vibration which allow to drive gate operations between them. The seminal paper by Cirac and Zoller [32] proposed axial modes and more recently it has been proposed to use the radial modes of a large ion crystal [33]. The fundamentally different approach aims to shuttle ions in segmented traps [11, 34], such that only two logical qubits are in the processor region during the quantum gate operation.

If the second approach is joined with gate operations in DFS and logical qubits, a maximum of four ions is

kept in the central processor unit and pairs of ions are shuttled together using the control segments of the trap (1). A large separation of time scales for a gate with about 100 ms transport operations are viable in the ms range [35], as well as obtaining error syndromes if one aims for quantum error correction, where the readout of fluorescence is necessary. In order to implement a multi qubit controlled phase gate we can decompose it in a sequence of single qubit gates and geometric phase gates between the logical qubits [36]. The single logical qubits (pair of ions) can be trapped in the storage zone and then can be transported to the processing zone where the gate operations are performed, Fig. 1. For instance, a three bit decoherence-free geometric gate is decomposed as  $G_3 = T_{2,0}G_{3,0}T_{2,0}$ . Here  $T_{2,0}$  is the Toffoli gate between the first two control logical qubits and the auxiliary logical qubit which is prepared initially in the state  $|0\rangle$ . The second operation  $G_{3,0}$  is the geometric phase gate between the target logical qubit and the auxiliary qubit. The final Toffoli gate is applied to restore the auxiliary qubit in the state  $|0\rangle$ . The Toffoli gate also can be decomposed as a succession of Hadamard gates and geometric phase gates between the logical qubits. Therefore any of the gate operations is realized with two ions (single qubit operations) and with four ions (two qubit operations) where the proposed method can be applied. The generalization to  $n$ -bit geometric phase gate is straightforward:  $G_n = T_{n-1,0}G_{n,0}T_{n-1,0}$ .

As an example we describe the method for the generation of a linear cluster state. Cluster states are highly entangled states, which are the fundamental resource of the one-way quantum computer [28]. Cluster states have been experimentally created with atoms in optical lattice [37] and with photons [38]. Multi-qubit cluster states have yet not been created with trapped ions. An ion-trap architecture for high-speed measurement-based quantum computer was proposed [39]. Ref. [40] proposed an efficient technique for the creation of four, five, and six qubit linear cluster states by collective bichromatic interaction, while in [41] creation of two-dimensional cluster state was suggested by using a spin-spin coupling induced by a magnetic-field gradient. Here, we propose the creation of four linear cluster state, without leaving the DFS. If the gate (8) is applied onto the decoherence-free initial state  $|\Psi_{in}\rangle = |B_{1,2}\rangle|B_{3,4}\rangle$ , which is a product of two Bell states  $|B_{i,j}\rangle = (|\uparrow_i\downarrow_j\rangle + |\downarrow_i\uparrow_j\rangle)/\sqrt{2}$ , then the geometric phase gate transforms the state to

$$|\Psi_c\rangle = (|\uparrow\uparrow\downarrow\downarrow\rangle + i|\uparrow\downarrow\downarrow\uparrow\rangle + i|\downarrow\uparrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle)/2. \quad (9)$$

The final state is highly entangled four qubit cluster state. Since the spin states are not mixed due to the unitary evolution (3), the initial state is transformed into the final cluster state without leaving the DFS. Combining the ion trap architecture and the transport of such states in a segmented trap, (see Fig. 1) large cluster states can be created by fusing several four linear cluster states with the DFS gate.

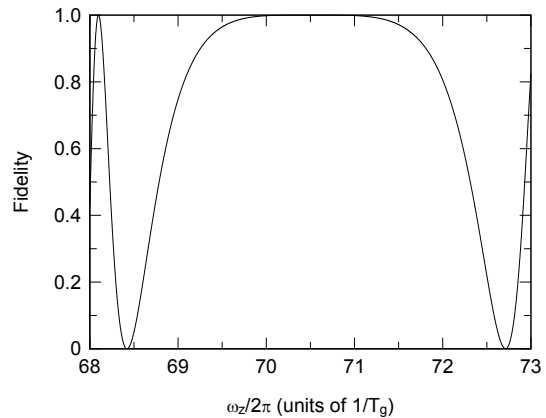


FIG. 5: The fidelity  $F$  Eq. (14) as a function of the axial trap frequency  $\omega_z$  for a gate Eq. (8) mediated by the E-mode.

#### IV. OPTIMIZING THE GATE ROBUSTNESS AND REMAINING ERRORS

During the gate evolution on the axial E-mode, also the other vibrational modes are off-resonantly excited there might be a non-vanishing coupling strength; however all radial motional degrees of freedom of the four ion crystal are excluded by the geometry of the laser excitation. We try to optimize the gate fidelity by analyzing and avoiding spurious ac light force driving on all other modes. For general coupling strengths and  $\delta_p = \omega_p - \omega_3 - \delta$ , the displacement and geometric phase associated with the  $p$ th vibrational mode is given by

$$\begin{aligned} \alpha_{\mathbf{s}_i}^{(p)}(t) &= \frac{F_{\mathbf{s}_i}^{(p)} z_p}{\hbar \delta_p} (e^{-i\delta_p t} - 1), \\ \Phi_{\mathbf{s}_i}^{(p)}(t) &= \left| \frac{F_{\mathbf{s}_i}^{(p)} z_p}{\hbar \delta_p} \right|^2 [\delta_p t - \sin(\delta_p t)]. \end{aligned} \quad (10)$$

As now the absolute force magnitude and the final time are fixed such that the geometric phase gate condition is fulfilled,  $T_g = 2\pi/\delta_3$ , one obtains parasitic displacement and geometric phases

$$\begin{aligned} \alpha_{\mathbf{s}_i}^{(p)}(T) &= \frac{F_{\mathbf{s}_i}^{(p)} z_p}{\hbar \delta_p} (e^{-i2\pi\delta_p/\delta_3} - 1), \\ \Phi_{\mathbf{s}_i}^{(p)}(T) &= \left| \frac{F_{\mathbf{s}_i}^{(p)} z_p}{\hbar \delta_p} \right|^2 [2\pi\delta_p/\delta_3 - \sin(2\pi\delta_p/\delta_3)]. \end{aligned} \quad (11)$$

An appropriate measure of the gate fidelity is given by [42]:

$$F = \frac{1}{N^2} \sum_{i,j} \langle \tilde{\mathbf{s}}_i | \hat{U}_0^\dagger \hat{U} | \tilde{\mathbf{s}}_i \rangle \langle \tilde{\mathbf{s}}_j | \hat{U}^\dagger \hat{U}_0 | \tilde{\mathbf{s}}_j \rangle, \quad (12)$$

where  $N = 4$  is the dimension of the DFS and  $i, j$  each run over all spin configurations belonging to DFS.  $\hat{U}_0$  is the desired unitary transform given by Eq. 8 and  $\hat{U}$  is the

Mode	Breathing	E-mode	Fourth
$\omega_z/2\pi$ (MHz)	2.86	2.82	2.68
$\Omega/2\pi$ (kHz)	119.64	130.62	130.47
Infidelity	$8.1 \times 10^{-4}$	$1.8 \times 10^{-4}$	$7.7 \times 10^{-4}$
$\Delta\omega_z/2\pi$ (kHz)	53	65	92

TABLE I: The values of the axial trap frequency  $\omega_z$  and the Rabi frequency  $\Omega$  required for implementation of the DFS gate for three different vibrational modes. The infidelity due to the off-resonant excitation is also presented. The frequency plateaus, where the gate fidelity is better than 99% are listed in order to elucidate the sensitivity on this parameter.

actual one. Neither  $\hat{U}_0$  nor  $\hat{U}$  is mixing the spin states. The action of  $\hat{U}$  on any basis state is simply given by

$$|\tilde{\mathbf{s}}_i\rangle = |\mathbf{s}_i\rangle|vac_{1,2,3,4}\rangle \rightarrow e^{i\sum_p \Phi_{\mathbf{s}_i}^{(p)}} |\mathbf{s}_i\rangle \left| \alpha_{\mathbf{s}_i}^{(1)} \dots \alpha_{\mathbf{s}_i}^{(4)} \right\rangle. \quad (13)$$

where  $|vac_{1,2,3,4}\rangle$  indicates the ground state of all axial modes. The fidelity can then be evaluated under consideration of the matrix element  $\langle 0|\alpha\rangle = e^{-|\alpha|^2/2}$ . Hence, we obtain for the fidelity at time  $t$

$$F = \frac{1}{16} \left| \sum_i \prod_p e^{-\frac{1}{2}|\alpha_{\mathbf{s}_i}^{(p)}|^2 + i\Phi_{\mathbf{s}_i}^{(p)}} \right|^2. \quad (14)$$

As an example of qubit we consider the  $^{40}\text{Ca}^+$  ion with qubit states encoded in the Zeeman sublevels of  $S_{1/2}$  state. The two-photon Raman transition is driven by a laser field with a wave length of  $\lambda \approx 397$  nm and a wave vector difference along the trap axis  $\Delta k \approx 2\sqrt{2}\pi/\lambda$ . In order to cancel the spin forces  $F_{\uparrow\downarrow\uparrow}$  and  $F_{\downarrow\uparrow\downarrow}$ , Eq. (7) we choose distance parameter  $n = 15$ . The Rabi frequencies are  $\Omega_{\uparrow} = -\Omega_{\downarrow}$  with gate time  $T_g = 25 \mu\text{s}$  and a laser detuning  $\delta = 2\pi \times 40$  kHz. Table I lists the values of the axial trap frequency  $\omega_z$  and the Rabi frequency  $\Omega$  needed for the synthesis of the DFS gate for the three different vibrational modes. We compare the minimum gate infidelity  $1 - F$  mediated by different vibrational modes due to the off-resonant excitations, (see, Fig. 4). Even for one cycle in phase space  $\delta T_g = 2\pi$  the infidelity for the E-mode is smallest since the off-resonant excitation of the center-of-mass mode for spin states  $|\uparrow\downarrow\uparrow\rangle$  and  $|\downarrow\uparrow\downarrow\rangle$  vanishes.

In Fig. 5 we show the fidelity (14) as a function of the axial trap frequency  $\omega_z$  for fixed gate time  $T_g$ . The Rabi frequency is chosen such that the condition  $\Phi_{\downarrow\uparrow\downarrow} = \Phi_{\uparrow\downarrow\uparrow} = \pi/2$  is fulfilled. The frequency plateaus, where the minimum fidelity is better than 99% for the gate implementation mediated by the different vibrational modes are listed in Table I.

Additionally, the proposed gate scheme shows a remarkable robustness against laser intensity fluctuations. As both laser beams are derived from the same laser source, we assume that the intensity fluctuations are common. The corresponding Rabi frequencies fluctuate

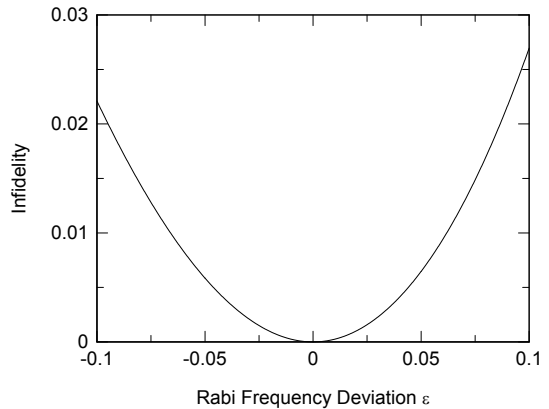


FIG. 6: Infidelity as a function of the deviation  $\epsilon$  of the Rabi frequencies defined as  $\Omega_{\{\mathbf{s}_i\}}(t) = \Omega_{\{\mathbf{s}_i\}}(1 + \epsilon)$  for the implementation of the decoherence-free gate (8) mediated by the E-mode.

therefore simultaneously for all ions. As a result, we observe that the fidelity only decrease quadratically with the fluctuations, see Fig. 6.

The ac Stark shift from the Raman beams can be compensated by a proper choice of polarizations of both beam or by employing an additional compensation beam [43]. The precision of this compensation is only limited by the spin coherence time, in a typical experiment one might reach an accuracy  $\delta_{ac}/(2\pi)$  well below 1 kHz. Such shifts scramble the spin qubit phases as they translate laser intensity fluctuations into effective magnetic field fluctuations. However, the logical qubits are inherently robust against spin phase fluctuations. Errors might only occur when the when an uncompensated ac Stark shift is accompanied by an inhomogeneous illumination of two ions which comprise a logical qubit. As this error of higher order, with a fraction of the above number, we do not take it into further consideration. In a fusing process of three logical qubits for the scalable scheme discussed above, when in the first step two of the qubits are exposed to the gate laser field while the third qubit is not illuminated, even an uncompensated ac Stark shift would not lead to errors as long as the logical qubits are illuminated equally.

Finally, the coherence of qubits in DFS is limited by the fluctuations of the gradient of the magnetic field over the distance of the two ion crystal, a few  $\mu\text{m}$ . When transporting such logical qubits in the trap one may expose them to varying magnetic gradients, however the phase evolution of the logical qubits is deterministic [15] and can be easily corrected for.

The motional decoherence of the vibrational motion of the trapped ions is the most serious limiting factor in ion trap quantum information processing. In order to achieve high fidelity the gate time  $T_g$  required for the implementation of DFS gate (8) must be much shorter than the heating time  $\tau$ . The measured heating time of the center of mass mode for  $^{40}\text{Ca}^+$  ion in a segmented micro-ion traps [5] was found to be 3.3 ms, a typical

value for this devices. The heating time for higher energy modes is expected to be much larger. Hence the gate time is more than two orders of magnitude faster than the heating time such that the fidelity of the gate is not affected.

## V. CONCLUSION

In conclusion we proposed a simple and robust technique for a decoherence-free controlled phase gate between two logical qubits. We studied in detail the fidelity of the gate implementation taking into account various error sources such as off-resonant transitions, laser fluctuations, and the deviation of the right choice of the axial trap frequency. We have compared the error sources for

a gate mediated by three different modes, and we have shown that the gate mediated by the Egyptian mode minimizes the off-resonant transitions. Our scheme includes the creation of a linear cluster state within a decoherence free subspace manifold of four qubits - a starting point for decoherence-free ion trap one way quantum computing.

## VI. ACKNOWLEDGMENTS

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