

Übungen zur Vorlesung
Quantenmechanik für Nanostrukturwissenschaftler und Lehrer

Exercise 5

Task 1

Given is $D_{xy}(\theta, \varphi) = \frac{xy}{r^2}$.

- a) What is this function in spherical coordinates?
- b) Apply the operator

$$\hat{L}^2 = -\hbar^2 \left(\frac{\partial^2}{\partial \theta^2} + \cot(\theta) \frac{\partial}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right)$$

on the above function.

- c) Show that D_{xy} is an eigenfunction of the operator \hat{L}^2 with the eigenvalue $6\hbar^2$.
- d) How big is l ?

Task 2

Given is $F_{xyz}(\theta, \varphi) = \frac{xyz}{r^3}$.

- a) Show that $F_{xyz}(\theta, \varphi)$ is an eigenfunction of \hat{L}^2 (see above).
- b) Determine l .

Task 3

The states of the energy in a three dimensional potential well $L \times L \times L$ are $E_{n_x, n_y, n_z} = \frac{\hbar^2 \pi^2}{2mL^2} (n_x^2 + n_y^2 + n_z^2)$. What is the lowest energy with an amount of 24 fermions (e.g. electrons)? How big would it be for bosons?