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Übungen zur Vorlesung
Quantenmechanik für Nanostrukturwissenschaftler und Lehrer

Exercise 3

Task 1

If possible, normalize the following wavefunctions:

a) $\psi(x) = A \cdot \sin(kx)$

b) $\psi(x) = A \cdot e^{-x^2/\alpha}$ (**AID:** $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$)

c) $\psi(x) = A \cdot (1 + x^2)^{-1/2}$

Task 2

The solutions for a particle in an infinitely high potential well

$$V(x) = \begin{cases} 0 & , 0 \leq x \leq L \\ \infty & , \text{sonst} \end{cases}$$

is given by $u_n(x) = A \cdot \sin\left(\frac{n\pi x}{L}\right)$ with $E_n = \frac{\hbar^2\pi^2 n^2}{2mL^2}$.

a) Show, that the solutions can be normalized by calculating A. Show that the normalized $u_n(x)$ fulfill the following orthonormality condition $\int_0^L u_n^*(x) u_m(x) = \delta_{nm}$.

b) Calculate the time evolution $\Psi(x,t)$ for the wavefunction $\Psi(x,0) = \frac{1}{\sqrt{L}}(\sin(\frac{\pi x}{L}) + \sin(\frac{2\pi x}{L}))$.

Task 3

The quantum system is given by the potential $V(x)$. The ground state ψ_0 with the energy E_0 and the first two excited states ψ_1 and ψ_2 with the eigenenergy E_1 and E_2 are known. The system is now brought into the state

$$\Psi(x, t = 0) = \frac{1}{2}\psi_0 + \frac{1}{2}\psi_1 + \frac{1}{\sqrt{2}}\psi_2.$$

a) Show that $\Psi(x, t = 0)$ is normalized correctly. **AID:** ψ_0, ψ_1 and ψ_2 are already normalized.

b) At the time $t = 10$ s the energy is measured. Which results for the measurments are possible and how possible are those?