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Übungen zur Vorlesung
Quantenmechanik für Nanostrukturwissenschaftler und Lehrer

Exercise 4

Task 1

$\Psi(x) = Nxe^{-\frac{m\omega}{2\hbar}x^2}$ is a eigenstate of an unknown potential $V(x)$ with $V(0) = 0$.
a) Determine the normalization factor N.
b) Sketch $\Psi(x)$.
c) Determine and sketch the probability density$P(x)$.
d) What is the associated Schrödinger equation?
e) Determine $V(x)$ from the Schrödinger equation and the condition $V(0) = 0$.
f) Which state is $\Psi(x)$ in this potential?
g) Determine $\Psi(x, t)$.

Task 2

Given is the potential step of the form $V(x) = \begin{cases} 0 & , x \leq 0 \\ V_0 & , x > 0 \end{cases}$.
a) Draw this potential.
b) What is the associated Schrödinger equation?
c) What are the continuity conditions for Ψ and Ψ'?
d) Determine the a result $\Psi(x)$ of the time independent Schrödinger equation with $E = \frac{1}{3}V_0$.
e) Give a meaningful definition of the penetration depth into the the classically forbidden area, and determine the value for the above given result.

Task 3

Consider an electron in a one dimensional, infinitely high potential well of the width 1 mm. Which value of n belongs to a state with the energy 0,01 eV?

Task 4

Besides the photoeffect, there exist also other possibilities to release an electron from a metal. For example there would be the heating of the system which can be described classically, as well as the applying of an electric field, which is a purely quantum mechanical effect. We will consider the second effect. We assume, that the electron is in a big box with a constant potential barrier of the height W. By applying the electric field, the potential for the electrons change from $W \to W - eE_{el}x$.

a) Sketch the potential before and after applying the electric field. For this, assume that the right potential wall is at $x = 0$.

b) Calculate the transmission coefficient for electrons with the energy $E = 0$ within WKB approximation.

c) Think about which physical application this effect has.