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# Lowest bending mode of ${ }^{13} \mathrm{C}$-substituted $\mathrm{C}_{3}$ and an experimentally derived structure 

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#### Abstract

The $v_{2}$ lowest bending mode of linear $\mathrm{C}_{3}$ and of all its ${ }^{13} \mathrm{C}$-substituted isotopologues was recorded using a terahertz-supersonic jet spectrometer in combination with a laser ablation source. Sixtyfive ro-vibrational transitions between 1.8 and 1.9 THz have been assigned to linear ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C}$, ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C},{ }^{12} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C},{ }^{13} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C},{ }^{13} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$, and ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{13} \mathrm{C}$. For each isotopologue, molecular parameters were obtained and the $\mathrm{C}-\mathrm{C}$-bond length was derived experimentally. All results are in excellent agreement with recent ab initio calculations [B. Schröder and P. Sebald, J. Chem. Phys. 144, 044307 (2016)]. The new measurements explain why the interstellar search for singly substituted ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ has failed so far. A spectral line list with recommended transition frequencies based on global data fits is given to foster future interstellar detections. Published by AIP Publishing. [http://dx.doi.org/10.1063/1.4971854]


## I. INTRODUCTION

Linear $\mathrm{C}_{3}$ is formed from carbonaceous materials in high temperature processes and in carbon rich plasmas. ${ }^{1}$ Tricarbon has been found in flames, electrical discharges, and in laser ablation processes, as well as in astrophysical environments, such as cometary tails, ${ }^{2}$ envelopes of aging carbonstars, ${ }^{3}$ the diffuse interstellar medium, ${ }^{4}$ and towards star forming regions. ${ }^{5-9}$ First recognized by the turn of the twentieth century as unknown cometary emission features near 4050 $\AA$, the unambiguous assignment to linear $C_{3}$ lasted another five decades until Douglas ${ }^{10}$ in 1951 and Gausset et al. ${ }^{11}$ in 1965 performed spectroscopic laboratory investigations on a methane-discharge and flash photolysis of diazomethane. These laboratory spectra revealed the non-rigid nature of the $\mathrm{C}_{3}$ radical which possesses a large amplitude $v_{2}$ bending motion at an unusually low vibrational frequency of roughly 63 $\mathrm{cm}^{-1}$. Because of its relevance to astrophysics and its unusual molecular properties, $\mathrm{C}_{3}$ has been studied extensively over the last couple of decades, both by experiment and theory. To begin with, high resolution ro-vibrational spectra of the $v_{3}$ asymmetric stretching mode and associated $v_{2}$ hot bands were reported by Matsumura et al. ${ }^{12}$ and Kawaguchi et al. ${ }^{13}$ which confirmed the large amplitude motion of the low bending mode. Schmuttenmaer et al. ${ }^{14}$ used a fixed frequency far infrared laser mixed with tunable microwave radiation to measure seven ro-vibrational absorption lines of the $v_{2}$ bending mode of $\mathrm{C}_{3}$ between 58 and $69.5 \mathrm{~cm}^{-1}$, data which have later on been

[^0]extended by Gendriesch et al. ${ }^{15,16}$ in 2003. For review articles on $\mathrm{C}_{3}$ and other small carbon clusters, see Van Orden and Saykally ${ }^{17}$ and Weltner and Van Zee. ${ }^{18}$ Recently rotationally resolved infrared hot band spectra at $3 \mu \mathrm{~m}$ have been published by Krieg et al. ${ }^{19}$ using an optical parametric oscillator (OPO) to provide high power tunable IR-radiation. MoazzenAhmadi and McKellar ${ }^{20}$ performed infrared measurements using ${ }^{13} \mathrm{C}$-enriched precursors and derived molecular parameters for the isotopologues ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ and ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{13} \mathrm{C}$. Later, Krieg et al. ${ }^{19}$ measured the first spectra of ${ }^{12} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$ at $3 \mu \mathrm{~m}$. Optical spectra of ${ }^{13} \mathrm{C}$-isotopologues were reported by Haddad et al., ${ }^{21}$ although with less frequency accuracy compared to the infrared studies. So far there have been no spectra published of the ${ }^{13} \mathrm{C}$-substituted $\mathrm{C}_{3}$ at the $v_{2}$ lowest bending mode except Krieg et al. ${ }^{19}$ who reported a weak absorption band, centered at $60.747 \mathrm{~cm}^{-1}$ which was incorrectly assigned to singly substituted ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$. Our present data give evidence that the $60.747 \mathrm{~cm}^{-1}$ absorption belongs to the $v_{2}$ band of ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$. Based on the faulty assignment, our search for interstellar ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ using the Herschel space telescope failed, which finally led to the present systematic study on the low bending mode of $\mathrm{C}_{3}$ and its ${ }^{13} \mathrm{C}$-isotopologues.

This new study makes use of a substantially improved supersonic jet spectrometer and high quality samples of well defined isotopic mixing ratios. In the course of our present study, high level $a b$ initio data became available by Schröder and Sebald ${ }^{22}$ which were communicated to us prior to publication and which helped assign our laboratory data to the six isotopologues of $\mathrm{C}_{3}$. From a least squares fit analysis, we derived molecular parameters for each of the six isotopologues including all rotationally resolved experimental data available from the literature. Finally we derived an


FIG. 1. Experimental setup of the supersonic jet spectrometer. The absorption signal detected by a hot electron bolometer is enhanced by multi-passing the THz beam through the supersonic jet.
equilibrium $\mathrm{C}-\mathrm{C}$ bond distance from experimental data and compared results with $a b$ initio values by Schröder and Sebald. ${ }^{22}$ We give recommended frequency positions of all six isotopologues for rotational quantum numbers up to $J=12$ emphasizing their possible future detection in astrophysical environments.

## II. EXPERIMENT

$\mathrm{C}_{3}$ was produced by the laser ablation of a sintered carbon target using the fourth harmonic of a Q-switched Nd: YAG laser at a 30 Hz repetition rate. The sample rod rotated and translated simultaneously to constantly provide a pristine carbon surface during the measurements. The ablated carbon was seeded in a

TABLE I. Isotopic mole fractions and sample densities of the sintered carbon rods.

| Sample | $x_{12} \mathrm{C}$ | $x_{13} \mathrm{C}$ | $\rho\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ |
| :--- | :---: | :---: | :---: |
| A | 1 | 0 | 0.94 |
| B | $2 / 3$ | $1 / 3$ | 1.07 |
| C | $1 / 3$ | $2 / 3$ | 1.20 |
| D | 0 | 1 | 1.35 |

flow of a helium gas pulse at 2 bar stagnation pressure which pre-expanded into a reaction channel of 3 mm length and $10 \times 1 \mathrm{~mm}^{2}$ cross section. The reaction channel allowed the formation of small and medium sized carbon clusters by re-condensing at moderate temperatures before the gas flow expanded adiabatically into a vacuum chamber at typically $10^{-3} \mathrm{mbar}$ background pressure in the form of a supersonic jet. The adiabatic expansion reduced the rotational temperature of the carbon clusters to a few tens Kelvin. A schematic sketch of the experiment is shown in Figure 1. Radiation in the range of 9 to 14 GHz was produced by a tunable synthesizer, and the output signal was amplified and frequency multiplied by a factor of 144 in a cascaded multiplier chain (Virginia Diodes) to generate radiation between 1.8 and 1.9 THz . The terahertz-beam intersected the supersonic jet perpendicularly 20 mm downstream the slit-nozzle exit in a multi-pass optics (arrangement: folded Heriott type). The transmitted THz-radiation was focused onto a liquid-He cooled InSb hot-electron bolometer (QMC instruments). The data were recorded during a 100 $\mu \mathrm{s}$ time frame allowing for the background subtraction of the signal shortly before the pulsed absorption signal appeared. A low-noise amplifier and band-pass filter (SR560, Scientific Instruments) was used to reduce the low frequency noise of the signal before storing the time-dependent signal on a computer. An absorption spectrum was obtained when tuning the THzfrequency in steps of 0.25 MHz over the entire spectral range, by taking the averaged $100 \mu$ s time-signals of 30 laser shots

TABLE II. Molecular parameters of the ground and lowest bending states of the $\mathrm{C}_{3}$ isotopologues (in MHz ).

| Parameter | ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C}^{\text {a }}$ | ${ }^{12} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}^{\text {b }}$ | ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}^{\mathrm{b}}$ | ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}^{\mathrm{c}}$ | ${ }^{13} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ | ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{13} \mathrm{C}^{\mathrm{d}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left(0,0^{0}, 0\right)$ |  |  |  |  |  |  |
| $B^{\prime \prime} \times 10^{-3}$ | 12.908 678(4) | $12.900325(96)$ | 12.403170 (11) | $12.395038(86)$ | $11.910794(31)$ | 11.903 524(32) |
| $D^{\prime \prime}$ | 0.046 68(24) | 0.042 57(128) | 0.040 97(66) | $0.04374{ }^{\text {e }}$ | $0.04210^{\text {e }}$ | $0.03839(51)$ |
| $H^{\prime \prime} \times 10^{6}$ | 6.95 (39) |  | 3.42(57) |  |  | 4.05(46) |
| $L^{\prime \prime} \times 10^{9}$ | -0.95(16) |  |  |  |  |  |
| $\left(0,1^{1}, 0\right)$ |  |  |  |  |  |  |
| $v \times 10^{-3}$ | 1901.181 630(9) | 1830.243173 (69) | 1892.864 561(18) | $1821.165337(63)$ | 1884.687 261(57) | $1812.201110(22)$ |
| $B^{\prime} \times 10^{-3}$ | 13.263 270(5) | $13.245166(74)$ | 12.743 487(12) | 12.725 848(65) | $12.237327(36)$ | $12.220861(22)$ |
| $D^{\prime}$ | 0.070 62(22) | 0.08171 (128) | 0.061 60(116) | $0.06911^{\text {e }}$ | 0.075 26(15) | 0.060 28(44) |
| $H^{\prime} \times 10^{6}$ | 6.55(60) |  |  | 4.51(19) |  | 8.63(47) |
| $L^{\prime} \times 10^{9}$ | 1.33(46) |  |  |  |  |  |
| $q$ | 170.938(6) | 176.495(47) | 158.614(9) | 164.148(43) | 146.893(26) | 152.381(22) |
| $q_{J}$ | $-0.03119(32)$ |  | -0.030 70(126) | -0.021 78(7) |  | -0.022 90(62) |
| $q_{J J} \times 10^{6}$ | 11.5(11) |  |  |  |  |  |
| $q_{J J J} \times 10^{9}$ | -5.48(87) |  |  |  |  |  |

${ }^{\text {a }}$ Global fit including high resolution ${ }^{12} \mathrm{C}_{3}$ data. ${ }^{13,14,16,19}$
${ }^{\mathrm{b}}$ Global fit including data from Krieg et al. ${ }^{19}$ and Haddad et al. ${ }^{21}$
${ }^{\mathrm{c}}$ Global fit including optical data set. ${ }^{21}$
${ }^{\mathrm{d}}$ Global fit including data from Moazzen-Ahmadi and McKellar. ${ }^{20}$
${ }^{e}$ Fixed estimated D values. ${ }^{30}$
at each frequency position. The accuracy of line center positions was typically 50 kHz with line widths (FWHM) around 4 MHz .

For measurements on $\mathrm{C}_{3}$ isotopologues, ${ }^{13} \mathrm{C}$-enriched target samples were produced in stoichiometric mixtures. Sample pellets of 10 mm in diameter and of 10 mm length were sintered from ${ }^{12} \mathrm{C}$ - and ${ }^{13} \mathrm{C}$-amorphous powders (Sigma Aldrich, $99 \%$ grade) at the Tycho Sinterlab, Rostock, using a HP D5 unit (FCT Systeme GmbH). To achieve homogeneous samples of reproducible high quality, the field assisted sintering technique (FAST) was used, applying a force of 5 kN on the sample at $1700^{\circ} \mathrm{C}$ final temperature and a previous heating rate of $100 \mathrm{~K} / \mathrm{min}$ (for technical details, see Garay ${ }^{23}$ ). The sample density depends linearly on the ${ }^{13} \mathrm{C}$ mole fraction, which has been used as a sample quality parameter (see Table I).

## III. MEASUREMENTS AND DATA REDUCTION

In the present study, sixty-five ro-vibrational transitions of the $v_{2}$ fundamental bending mode, $\left(0,1^{1}, 0\right)-\left(0,0^{0}, 0\right)$, were measured in the frequency region between 1.8 and 1.9 THz


FIG. 2. Stick diagram of measured $v_{2}$ bending mode transitions, $\left(0,1^{1}, 0\right)$ $\left(0,0^{0}, 0\right)$, for $C_{3}$ and its isotopologues; in gray, spectra of ${ }^{13} \mathrm{C}$ enriched samples. The second plot from the top (in orange) shows the simulated spectra for all isotopologues with the parameters and temperatures determined from the present study. The lower six plots (in green) show the simulations of each assigned $\mathrm{C}_{3}$ isotopologue with respect to the fitted rotational temperature.
and assigned to six different $\mathrm{C}_{3}$ isotopologues. Table III lists line positions for the main isotopologue as well as those from the singly, doubly, and fully ${ }^{13} \mathrm{C}$-substituted $\mathrm{C}_{3}$ isotopologues. For transitions outside the covered frequency range, we give calculated values based on a least squares fit analysis using a standard Hamiltonian for a linear molecule from all available data from the literature. We used the PGOPHER program $^{24}$ to derive molecular parameters for all six $\mathrm{C}_{3}$ isotopologues in the ground and vibrational excited states as given in Table II. Ground state and excited bending state energies are given by

$$
\begin{equation*}
E_{0}(J) / h=B^{\prime \prime} f-D^{\prime \prime} f^{2}+H^{\prime \prime} f^{3}+L^{\prime \prime} f^{4} \tag{1}
\end{equation*}
$$

for the $\left(0,0^{0}, 0\right)$ ground state, and

$$
\begin{align*}
E_{ \pm}(J) / h= & v+B^{\prime} f_{\ell}-D^{\prime} f_{\ell}^{2}+H^{\prime} f_{\ell}^{3}+L^{\prime} f_{\ell}^{4} \\
& \pm \frac{1}{2}\left(q f+q_{J J} f^{2}+q_{J J} f^{3}+q_{J J J} f^{4}\right) \tag{2}
\end{align*}
$$

for the $\left(0,1^{1}, 0\right)$ exited state, with $f=J(J+1)$ and $f_{\ell}$ $=J(J+1)-\ell^{2}$ the squared angular momenta. $B^{\prime \prime}, D^{\prime \prime}, H^{\prime \prime}$, and $L^{\prime \prime}$ denote the rotational ground state constant and centrifugal distortion constants, $B^{\prime}, D^{\prime}, H^{\prime}$, and $L^{\prime}$ the corresponding excited state constants, $v$ the band center, $q$ the $\ell$-type doubling constant with $q_{J}, q_{J J}$, and $q_{J J J}$ the higher order correction terms. The sign convention of $\ell$-type doubling constants follows the definition given in Yamada ${ }^{25}$ (positive $q$ for e(lower) and $f(u p p e r)$ states). Each isotopologue was fitted separately including the new measured data and available data from the literature. A stick spectrum of measured $\mathrm{C}_{3}$ lines in the frequency region of $1.8-1.9 \mathrm{THz}$ and simulated spectra based on best fit parameters are shown in Figure 2.


FIG. 3. Combination differences of the low $\mathrm{C}_{3}$-transitions. The lower state rotational constant $B^{\prime \prime}$ is derived from the $R(0)-P(2)$ combination difference, and the upper state $B^{\prime}$ is derived from the $R(2)-P(2)$ combination difference.


FIG. 4. Stick spectrum of measured ${ }^{12} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$ lines and calculated line intensities at $T=16 \mathrm{~K}$ featuring a $0: 1$ statistical weight alternation of odd/even $J$ transitions. Upper trace frames show recorded spectra from $33 \%{ }^{13} \mathrm{C}$-enriched samples (blue) and fitted Gaussian line shapes. A weaker signal (orange) is recorded when samples of $67 \%{ }^{13} \mathrm{C}$ were used.

## A. ${ }^{12} C^{12} C^{12} C$

In order to improve the accuracy of the main isotopologue data, we re-measured the transitions $P(2), P(4), Q(2)$, $Q(4), Q(8), Q(10)$, and $R(0)$, which in combination with our previous measurement of the $R(2)$ line ${ }^{15,16}$ allow for a separate determination of upper and lower state constants, $B^{\prime}$, $q^{\prime}$, and $B^{\prime \prime}$ via combination differences (see Figure 3). The remeasured $P(2)$ line was found 2 MHz higher in frequency than the value given by Schmuttenmaer et al., ${ }^{14}$ a finding which may
explain the deviations of astrophysically detected $P(J)$ transitions from line positions given in the Cologne Database for Molecular Spectroscopy (CDMS). ${ }^{26}$ From the improved data set, an accurate line list for ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C}$ bending mode transitions was derived from a weighted global fit analysis of all available data. As a result, improved rotational constants and the band center position were obtained which are in excellent agreement with recent $a b$ initio results published by Schröder and Sebald ${ }^{22}$ and which reproduce the astrophysical data within the error limits. The obtained molecular parameters


FIG. 5. The lower frame shows the measured experimental transitions (black upwards) in the $v_{2}$ band of ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ in the frequency range $1.8-1.9 \mathrm{THz}$ and the simulation (green downwards) at a rotational temperature of 13 K as stick spectrum. The upper frames show measured line profiles (dots) with a mole fraction of $1 / 3{ }^{13} \mathrm{C}$ in the ablation target (sample B) and their Gaussian fits (blue solid line). The isotopologue assignment, ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$, is confirmed by the $Q(3)$-transition, which was much weaker using a $2 / 3$ ${ }^{13} \mathrm{C}$ (sample C), i.e., at a higher ${ }^{13} \mathrm{C}$ mole fraction as indicated in orange.

TABLE III. Transition frequencies of the $\left(0,1^{1}, 0\right)-\left(0,0^{0}, 0\right)$ bending mode (in MHz ) of ${ }^{12} \mathrm{C}_{3}$ and for ${ }^{13} \mathrm{C}$ substituted $\mathrm{C}_{3}$ isotopologues. Experimental uncertainties are given in parentheses with $1 \sigma$ deviation.

| Isotopologue | $J$ | $P(J)^{\mathrm{a}}$ | о.-c. | $Q(J)^{\mathrm{a}}$ | о.-c. | $R(J)^{\mathrm{a}}$ | o.-c. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C}$ | 0 |  |  |  |  | 1914 273.912(13) | -0.010 |
|  | 2 | $1836823.502(25)$ | -0.028 | $1890558.188(11)$ | +0.020 | $1968595.391(200)^{\text {b }}$ | +0.299 |
|  | 4 | $1787890.534(14)$ | +0.003 | $1896706.838(9)$ | -0.008 | $2025051.586(899)^{\text {c }}$ | -0.043 |
|  | 6 | $1741122.646(6895)^{\text {c }}$ | -1.275 | $1906337.903(250)^{\text {b }}$ | -0.103 | $2083626.535(899)^{\text {c }}$ | -0.586 |
|  | 8 | 1696525.363 (801) |  | 1919410.791 (7) | +0.001 | 2144301.38 (158) |  |
|  | 10 | 1654 087.90(209) |  | 1935870.390 (158) | -0.068 | 2207049.97 (342) |  |
|  | 12 | 1613 805.25(435) |  | $1955648.963(200)^{\text {b }}$ | -0.301 | 2271843.88 (652) |  |
|  | 14 | 1575 661.17(795) |  | 1978 667.579(150) ${ }^{\text {b }}$ | +0.049 | 2338 649.4(115) |  |
|  | 16 | $1539634.2(134)$ |  | $2004835.934(3000)^{\text {b }}$ | +1.031 | 2407428.6 (193) |  |
| ${ }^{12} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$ | 0 |  |  |  |  | 1843311.778 (215) | +0.016 |
|  | 2 | $1765911.343(490)$ |  | 1819 596.013(89) | -0.014 | 1897470.722 (103) | -0.001 |
|  | 4 | 1716 881.67(136) |  | $1825647.312(49)$ | +0.007 | 1953 647.368(599) |  |
|  | 6 | 1669 898.28(277) |  | $1835125.454(65)$ | -0.008 | 2011 799.71(186) |  |
|  | 8 | $1624935.55(493)$ |  | $1847989.176(128)$ | +0.007 | 2071 870.74(401) |  |
|  | 10 | 1581952.80 (809) |  | 1864182.073 (210) |  | 2133788.40 (729) |  |
|  | 12 | 1540 894.3(125) |  | $1883632.789(558)$ |  | $2197465.6(120)$ |  |
| ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ | 0 |  |  |  |  | $1905449.403(31)$ |  |
|  | 1 |  |  | $1880960.405(18)$ | +0.012 | 1931298.897 (62) | -0.000 |
|  | 2 | $1831031.852(111)$ | -0.002 | $1882638.282(15)$ | -0.008 | 1957 668.258(142) |  |
|  | 3 | $1807272.923(38)$ | +0.000 | $1885152.952(16)$ | -0.021 | $1984556.117(451)$ |  |
|  | 4 | $1784038.758(42)$ | -0.000 | 1888501.875 (15) | +0.035 | 2011960.97 (110) |  |
|  | 5 | $1761329.936(195)$ |  | 1892681.327 (30) | -0.083 | 2039 881.18(225) |  |
|  | 6 | 1739 146.880(605) |  | $1897687.328(28)$ | +0.015 | 2068314.95 (409) |  |
|  | 7 | $1717489.85(140)$ |  | 1903514.278 (57) |  | 2097 260.31(685) |  |
|  | 8 | 1696358.93 (277) |  | $1910156.118(131)$ |  | $2126715.1(108)$ |  |
|  | 9 | 1675753.99 (493) |  | 1917 605.715(269) |  | 2156 677.1(162) |  |
|  | 10 | 1655 674.71(814) |  | 1925854.996 (513) |  | $2187143.6(235)$ |  |
|  | 11 | $1636120.5(127)$ |  | $1934894.914(921)$ |  | $2218111.8(330)$ |  |
|  | 12 | 1617 090.6(189) |  | 1944715.45 (158) |  | $2249578.8(452)$ |  |
| ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$ | 0 |  |  |  |  | 1833 727.029(135) | +0.040 |
|  | 1 |  |  | $1809265.353(47)$ | +0.013 | 1859510.580 (340) | -0.256 |
|  | 2 | 1759358.370 (435) |  | 1810916.271 (46) | -0.040 | $1885789.206(51)$ | +0.002 |
|  | 3 | 1735 566.630(777) |  | 1813 390.624(32) | +0.027 | 1912 559.993(182) |  |
|  | 4 | 1712 274.65(120) |  | $1816685.578(28)$ | -0.024 | 1939 820.765(435) |  |
|  | 5 | 1689 482.42(172) |  | $1820797.925(40)$ | +0.046 | 1967 568.758(775) |  |
|  | 6 | 1667 189.60(231) |  | $1825723.103(52)$ | -0.041 | 1995800.91 (120) |  |
|  | 7 | 1645 395.54(299) |  | $1831456.425(142)$ | +0.148 | $2024513.85(171)$ |  |
|  | 8 | 1624 099.24(375) |  | 1837 991.464(284) | +0.042 | 2053703.98 (231) |  |
|  | 9 | 1603 299.66(460) |  | 1845321.794 (80) | -0.049 | 2083 367.42(300) |  |
|  | 10 | 1582994.66 (552) |  | 1853440.173 (129) | +0.054 | 2113500.12 (379) |  |
|  | 11 | $1563183.09(653)$ |  | $1862338.136(137)$ | +0.047 | 2144097.83 (470) |  |
|  | 12 | 1543862.77 (763) |  | $1872006.862(164)$ | -0.038 | 2175156.16 (575) |  |
| ${ }^{13} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ | 0 |  |  |  |  | $1896777.619(91)$ |  |
|  | 1 |  |  | $1873249.945(28)$ | -0.042 | 1921 609.918(120) | +0.003 |
|  | 2 | $1825315.282(1000)$ | +0.908 | $1874849.468(62)$ | +0.021 | $1946944.145(378)$ |  |
|  | 3 | $1802507.865(137)$ | -0.009 | 1877 246.689(20) | +0.041 | 1972 776.911(601) |  |
|  | 4 | 1780208.360 (389) |  | $1880439.089(76)$ | -0.113 | 1999104.018 (874) |  |
|  | 5 | $1758414.455(596)$ |  | $1884423.867(34)$ | -0.059 | 2025920.48 (120) |  |
|  | 6 | $1737123.985(855)$ |  | 1889 196.899(92) | +0.058 | 2053220.51 (159) |  |
|  | 7 | $1716333.98(117)$ |  | $1894753.384(157)$ | -0.213 | $2080997.52(204)$ |  |
|  | 8 | $1696040.68(154)$ |  | $1901087.345(444)$ |  | 2109 244.14(258) |  |
|  | 9 | $1676239.52(199)$ |  | 1908 192.997(803) |  | 2137952.20 (323) |  |
|  | 10 | $1656925.15(252)$ |  | 1916062.96 (133) |  | 2167112.73 (403) |  |
|  | 11 | 1638 091.41(316) |  | 1924 689.29(206) |  | 2196715.96 (502) |  |
|  | 12 | 1619731.36 (396) |  | 1934063.21 (303) |  | $2226751.33(625)$ |  |

TABLE III. (Continued.)

| Isotopologue | $J$ | $P(J)^{\mathrm{a}}$ | о.-c. | $Q(J)^{\text {a }}$ | о.-c. | $R(J)^{\mathrm{a}}$ | о.-c. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{13} \mathrm{C}$ | 0 |  |  |  |  | 1824 269.584(67) | +0.031 |
|  | 1 |  |  | $1800767.425(28)$ | +0.050 | $1849040.217(34)$ | -0.002 |
|  | 2 | $1752849.792(174)$ |  | $1802341.000(40)$ | +0.051 | 1874 290.759(52) | -0.012 |
|  | 3 | $1730010.350(313)$ |  | $1804699.272(15)$ | -0.043 | $1900019.564(46)$ | +0.005 |
|  | 4 | $1707655.383(473)$ |  | $1807840.160(34)$ | +0.075 | $1926224.713(133)$ |  |
|  | 5 | $1685785.055(651)$ |  | $1811760.101(18)$ | +0.007 | $1952904.156(406)$ |  |
|  | 6 | 1664 399.291(878) |  | $1816455.333(58)$ | -0.077 | $1980055.627(928)$ |  |
|  | 7 | 1643497.78 (125) |  | $1821921.380(26)$ | +0.032 | 2007 676.70(180) |  |
|  | 8 | 1623 080.01(190) |  | $1828152.253(386)$ | -0.244 | 2035764.81 (313) |  |
|  | 9 | 1603145.23 (297) |  | $1835142.712(38)$ | -0.023 | 2064317.29 (506) |  |
|  | 10 | 1583 692.55(459) |  | 1842 885.099(155) | -0.161 | 2093 331.39(774) |  |
|  | 11 | 1564720.90 (690) |  | $1851372.802(245)$ | +0.181 | 2122 804.3(113) |  |
|  | 12 | 1546229.1 (100) |  | $1860597.147(402)$ | +0.400 | $2152733.3(160)$ |  |
|  | 13 | $1528215.8(141)$ |  | 1870 548.993(79) | +0.007 | 2183115.6 (220) |  |

$\bar{a}$ Values without o.-c. values are calculated values from a best fit analysis based on measured frequencies.
${ }^{\mathrm{b}}$ Taken from Gendriesch et al. ${ }^{16}$
${ }^{\text {c }}$ Taken from Schmuttenmaer et al. ${ }^{14}$
are listed in Table II. Figure 2 shows a stick spectrum of ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C}$ with only even $J$ quantum numbers, featuring Bose-Einstein statistical weights of two identical carbon nuclei of $\operatorname{spin} I\left({ }^{12} \mathrm{C}\right)=0$.

## B. ${ }^{12} C^{13} C^{12} C$

Using ${ }^{13} \mathrm{C}$-enriched target rods, a new band at 1.830 THz appeared 71 GHz below the main isotopologue band center. Strongest line intensities were observed when one-third of the carbon sample was of amorphous ${ }^{13} \mathrm{C}$, i.e., presumably favoring the formation of singly ${ }^{13} \mathrm{C}$-substituted $\mathrm{C}_{3}$ species, see Figure 4 . The lack of odd-numbered $J$-transitions in the measured spectrum was in support of assigning two $R$-branch and four $Q$-branch transitions to centro-symmetric ${ }^{12} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$.

These new data together with IR-data reported by Krieg et al. ${ }^{19}$ and optical data from Haddad et al. ${ }^{21}$ were analyzed in a weighted least squares fit to derive molecular parameters as given in Table II. We found excellent agreement with $a b$ initio data of Schröder and Sebald; ${ }^{22}$ in particular the measured and calculated line center positions agree within 6 GHz . For transitions of $J>8$, line intensities became rather faint due to an unfavorable Boltzmann-population of rotational levels at temperatures below 20 K .

## C. ${ }^{12} C^{12} C^{13} C$

A second band at 1.893 THz was found only 8 GHz below the main isotopologue band when a $33 \%{ }^{13} \mathrm{C}$-enriched sample was used. We assigned one $R$-branch, two $P$-branch,


FIG. 6. Stick spectrum of measured ${ }^{13} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ lines and calculated line intensities at $T=13 \mathrm{~K}$ featuring a $3: 1$ statistical weight alternation of odd/even $J$ transitions. Upper trace frames show recorded spectra from $67 \%{ }^{13} \mathrm{C}$-enriched samples (blue) and fitted Gaussian line shapes. A weaker signal (orange) is recorded when samples of $33 \%{ }^{13} \mathrm{C}$ were used.
and six $Q$-branch transitions of the $v_{2}$-bending mode to ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ (see Figure 5). Even and odd numbered transitions were found of comparable intensities, according to an asymmetric molecule of point group $\mathrm{C}_{\infty v}$. All measured line positions were determined very precisely with uncertainties of less than 100 kHz . Accurate line positions are given in Table III, and molecular parameters derived from a weighted global fit analysis including IR-data published by Krieg et al. ${ }^{19}$ and optical data by Haddad et al. ${ }^{21}$ can be found in Table II.

## D. ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$

In order to identify the lowest rotation-vibrational transitions of the doubly ${ }^{13} \mathrm{C}$-substituted asymmetric ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$ species, a $67 \%{ }^{13} \mathrm{C}$-enriched target sample was used. The strongest lines of a band originating at 1.821 THz were assigned to three $R$ - and twelve $Q$-branch transitions of this molecule. For a global fit analysis, optical measurements by Haddad et al. were included. The lack of high $J$-transitions did not allow us to determine the centrifugal distortion constants of ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$. Instead we used ground and excited state centrifugal parameters of ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{13} \mathrm{C}$ and scaled these with the corresponding ratios of ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C}$ and ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$.

## E. ${ }^{13} C^{12} C^{13} C$

A set of ten lines around the band center at 1.885 THz showed maximum intensity when using a sintered target rod of $67 \%{ }^{13} \mathrm{C}$-enriched carbon. In addition to the standard Boltzmann thermal population, individual lines follow an intensity alternation with an odd numbered $J$-transition being three times stronger than the even numbered ones. These findings led to the assignments of these lines to ${ }^{13} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$. Here the line intensities are affected by Dirac-spin statistical weights of two identical nuclei of $\operatorname{spin} I\left({ }^{13} \mathrm{C}\right)=1 / 2$. Figure 6 shows the recorded ${ }^{13} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ spectrum at temperatures around


FIG. 7. Measured (filled symbols) and calculated (open symbols) Qtransitions of all $\mathrm{C}_{3}$ isotopologues are shown. The band center $v_{\mathrm{xyz}}^{\left(0,1^{1}, 0\right)}$ of the respective isotopologues are marked with a vertical double arrow. The Q-branches of isotopologues split up into two groups at 1.82 and 1.89 THz , which are defined by the centro-symmetric atom of each molecule. The deviation within the groups is less than one percent of the band center frequency. The symbol size represents the transition intensity at 30 K .

13(2) K. We scaled the ground state centrifugal distortion constants of ${ }^{13} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ with ground state centrifugal parameters of ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{13} \mathrm{C},{ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C}$, and ${ }^{12} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$ due to the lack of measured higher $J$-transitions.

## F. ${ }^{13} C{ }^{13} C{ }^{13} C$

In order to identify the bending mode of the fully ${ }^{13} \mathrm{C}$-substituted $\mathrm{C}_{3}$ isotopologue, we used a $67 \%{ }^{13} \mathrm{C}$-enriched sample. Four $R$-branch and thirteen $Q$-branch transitions of a 3:1 intensity alternation were measured in the accessible range of the radiation source. Molecular parameters were obtained from a least squares fit analysis including IR-data published by Moazzen-Ahmadi and McKellar. ${ }^{20}$

In total 65 transitions were assigned to the six isotopologues of $\mathrm{C}_{3}$. Accurate line center positions, rotational constants for ground and excited states as well as the $\ell$-type doubling parameters were derived. Higher centrifugal parameters were obtained when rotationally resolved IR-data of high $J$-transitions were introduced to the global fit.

The experimental observation shows that the band centers have to be predicted theoretically with an accuracy better than half percent to distinguish different isotopologues, see Figure 7. The $a b$ initio calculated band centers of Schröder and Sebald ${ }^{22}$ are systematically lower by $6 \mathrm{GHz}(0.3 \%)$ compared to experimental values. The calculated excited state rotational constants $B^{\left(0,1^{1}, 1\right)}$ and the $\ell$-type doubling constants $q$ agree almost perfectly well with the corresponding experimental values. In Table IV experimental vibrationrotation coupling constants $\alpha_{2}$ for the $\nu_{2}$ bending mode are given.

## IV. MOLECULAR STRUCTURE

The structure parameters, bond lengths $r_{1}, r_{2}$ and bending angle $\rho$, are not directly observable. In good approximation $\mathrm{C}_{3}$ is thought to be a linear molecule with two equivalent bond lengths $r_{1}=r_{2}=r$ and bending angle $\rho=0$. As a first approach, the bond length $r$ can be estimated by using the experimentally determined $B^{(0)}$-values of the isotopologues, resulting in the so-called " $r^{(0)}$-structure." Because of the unharmonicity of the potential even at the ground vibrational level, the $r^{(0)}$-distance between the outer and inner carbon atoms does not correspond to the equilibrium distance. In Sections IV B and IV C, the structural parameter $r$ is approximated by two methods. First, Watson's mass-dependent structure $r^{(m)}$ is calculated using experimental data only. Second, the vibrational correction $\alpha_{2}$ of the bending mode is used to derive an approximated value $r^{(0 e 0)}$ for the equilibrium structure $r^{(e)}$. It is expected that $r^{(m)}$ and $r^{(0 e 0)}$ are close to theoretical values as discussed in Section V.

## A. Effective vibrational ground state structure, $r^{(0)}$

The effective moment of inertia for the vibrational ground state, $I^{(0)}$, can be derived from the rotational ground state constant $B^{(0)}$,

$$
\begin{equation*}
I^{(0)}=\frac{\hbar}{4 \pi B^{(0)}} \tag{3}
\end{equation*}
$$

TABLE IV. Comparison of experimental and calculated molecular parameters (in MHz ) of all ${ }^{13} \mathrm{C}$ substituted $\mathrm{C}_{3}$ isotopologues.

| Parameter | This work | Previous expt. work | Theo. calc. ${ }^{\text {a }}$ |
| :--- | :---: | :---: | :---: |
| ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C}$ |  |  |  |
| $B^{\left(0,0^{0}, 0\right)} \times 10^{-3}$ | $12.908678(4)$ | $12.908248(57)^{\mathrm{b}}$ | 12.909963 |
| $v^{\left(0,1^{1}, 0\right)} \times 10^{-3}$ | $1901.181630(9)$ | $1901.181677(231)^{\mathrm{b}}$ | 1896.487 |
| $B^{\left(0,1^{1}, 0\right)} \times 10^{-3}$ | $13.263270(5)$ | $13.262944(48)^{\mathrm{b}}$ | 13.263718 |
| $q^{\left(0,1^{1}, 0\right)}$ | $170.938(6)$ | $170.672(36)^{\mathrm{b}}$ | 170.8 |
| $\alpha_{2}$ | $-354.592(6)$ | $-354.696(75)$ | -353.755 |
|  |  |  |  |
| ${ }^{12} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$ | $12.899350(809)^{\mathrm{b}}$ | 12.902168 |  |
| $B^{\left(0,0^{0}, 0\right)} \times 10^{-3}$ | $12.900325(96)$ |  | 1824.537 |
| $v^{\left(0,1^{1}, 0\right)} \times 10^{-3}$ | $1830.243173(69)$ | 13.246030 |  |
| $B^{\left(0,1^{1}, 0\right)} \times 10^{-3}$ | $13.245166(74)$ | 177.2 |  |
| $q^{\left(0,1^{1}, 0\right)}$ | $176.495(47)$ | -343.862 |  |

${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}^{\mathrm{b}}$

| $B^{\left(0,0^{0}, 0\right)} \times 10^{-3}$ | $12.403170(11)$ | $12.403643(507)^{\mathrm{b}}$ |
| :--- | :---: | :---: |
| $v^{\left(0,1^{1}, 0\right)} \times 10^{-3}$ | $1892.864561(18)$ | 12.404213 |
| $B^{\left(0,1^{1}, 0\right)} \times 10^{-3}$ | $12.743487(12)$ | 1888.093 |
| $q^{\left(0,1^{1}, 0\right)}$ | $158.614(9)$ | 12.743878 |
| $\alpha_{2}$ | $-340.317(16)$ | 158.6 |

${ }^{13} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$

| $B^{\left(0,0^{0}, 0\right)} \times 10^{-3}$ | $12.395038(86)$ | $12.4204(360)^{\mathrm{c}}$ | 12.396418 |
| :--- | :---: | :---: | :---: |
| $v^{\left(0,1^{1}, 0\right)} \times 10^{-3}$ | $1821.165337(63)$ | $1821.16984(114)^{\mathrm{b}}$ | 1815.543 |
| $B^{\left(0,1^{1}, 0\right)} \times 10^{-3}$ | $12.725848(65)$ | $12.729757(660)^{\mathrm{b}}$ | 12.726789 |
| $q^{\left(0,1^{1}, 0\right)}$ | $164.148(43)$ | $173.31(104)^{\mathrm{b}}$ | 164.3 |
| $\alpha_{2}$ | $-330.810(108)$ |  | -330.371 |


| ${ }^{13} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ |  |  |
| :--- | :---: | :---: |
| $B^{\left(0,0^{0}, 0\right)} \times 10^{-3}$ | $11.910794(31)$ | $11.9413(258)^{\mathrm{c}}$ |
| $v^{\left(0,1^{1}, 0\right)} \times 10^{-3}$ | $1884.687261(57)$ |  |
| $B^{\left(0,1^{1}, 0\right)} \times 10^{-3}$ | $12.237327(36)$ | 18.911654 |
| $q^{\left(0,1^{1}, 0\right)}$ | $146.893(26)$ | 12.237528 |
| $\alpha_{2}$ | $-326.533(48)$ | 147.2 |
|  |  | -325.874 |
| ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{13} \mathrm{C}$ |  |  |
| $B^{\left(0,0^{0}, 0\right)} \times 10^{-3}$ | $11.903524(32)$ | $11.902480(450)^{\mathrm{d}}$ |
| $v^{\left(0,1^{1}, 0\right)} \times 10^{-3}$ | $1812.201110(2)$ |  |
| $B^{\left(0,1^{1}, 0\right)} \times 10^{-3}$ | $12.220861(22)$ |  |
| $q^{\left(0,1^{1}, 0\right)}$ | $152.381(22)$ | 11.904759 |
| $\alpha_{2}$ | $-317.337(39)$ | 12.221639 |

${ }^{\text {a }}$ Taken from Schröder and Sebald. ${ }^{22}$
${ }^{\mathrm{b}}$ Taken from Krieg et al. ${ }^{19}$ Note that the experimental values given for ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$ were originally assigned to the ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ isotopologue.
${ }^{\text {c }}$ Taken from Haddad et al. ${ }^{21}$
${ }^{\mathrm{d}}$ Taken from Moazzen-Ahmadi and McKellar. ${ }^{20}$

The effective structure parameter in the vibrational ground state, $r^{(0)}$, can be determined by

$$
\begin{align*}
I^{(0)}= & \sum_{i=1}^{3} m_{i}\left(z_{i}^{(0)}\right)^{2} \\
= & \frac{\left(r^{(0)}\right)^{2}}{M^{2}}\left[m_{1}\left(m_{2}+2 m_{3}\right)^{2}\right. \\
& \left.+m_{2}\left(m_{1}-m_{3}\right)^{2}+m_{3}\left(2 m_{1}+m_{2}\right)^{2}\right] \tag{4}
\end{align*}
$$

where $z_{i}^{(0)}$ and $m_{i}$ are the coordinate of the $i$ th atom with mass $m_{i}$ in the molecule-fixed center of the mass coordinate system, $r^{(0)}$ is the $\mathrm{C}-\mathrm{C}$ bond length, and $M=m_{1}+m_{2}+m_{3}$ is the total mass. The derived $r^{(0)}$ values are listed in Table V for each isotopologue.

## B. Watson's mass-dependent structure, $r^{(m)}$

Another approximation to the bond length $r$ can be made by using the approach by Watson. ${ }^{27}$ First, the so-called

TABLE V. Experimental structure parameters ${ }^{a}$ of the $C_{3}$ isotopologues.

| Parameter | Units | ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C}$ | ${ }^{12} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$ | ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ | ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$ | ${ }^{13} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$ | ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{13} \mathrm{C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B^{(0)}$ | GHz | 12.908 678(4) | $12.900325(96)$ | $12.403170(11)$ | 12.395 038(86) | 11.910 794(31) | 11.903 524(32) |
| $I^{(0)}$ | $\mathrm{u} \AA^{2}$ | 39.150330 (12) | 39.175 680(292) | 40.745 955(36) | 40.772 687(283) | 42.430337 (110) | 42.456 252(22) |
| $r^{(0)}$ | A | 1.277 209(0) | 1.277 623(10) | 1.277 261(1) | 1.277 661(9) | 1.277 308(3) | 1.277 698(3) |
| $I^{(s)}$ | $u \AA^{2}$ | 39.541 03(851) | 39.554 09(964) | 41.137 86(851) | 41.15473 (955) | 42.825 36(882) | 42.843 82(973) |
| $I^{(m)}$ | $u \AA^{2}$ | 39.931 72(1702) | 39.932 50(1928) | 41.529 77(1703) | $41.53677(1909)$ | 43.220 38(1764) | 43.231 38(1909) |
| $r^{(m)}$ | A | 1.289 89(27) | 1.289 90(31) | 1.289 49(26) | 1.289 58(30) | $1.28914(26)$ | 1.289 31(29) |
| $B^{(0 e 0)}$ | GHz | 12.554086 (8) | 12.555 484(155) | 12.062 853(20) | 12.064 228(138) | 11.584 261(57) | $11.586187(50)$ |
| $I^{(0 e 0)}$ | $u \AA^{2}$ | $40.256137(24)$ | 40.251 655(496) | 41.895 479(68) | 41.890 704(479) | 43.626349 (214) | 43.619 097(189) |
| $r^{(0 e 0)}$ | £ | $1.295121(1)$ | 1.295 049(16) | $1.295152(2)$ | $1.295060(15)$ | $1.295185(6)$ | $1.295077(6)$ |

${ }^{\mathrm{a}}$ For conversion of the units, the recommended values from CODATA ${ }^{31}$ were used.
" $r^{(s)}$-structure" has to be evaluated by applying the Kraitchman equation, e.g., see Chapter XIII in Gordy and Cook. ${ }^{28}$ It is well known that the Kraitchman coordinates fail in determining the position of atoms close to the center-of-mass. For the central carbon atom, the $z^{(s)}(2)$ coordinates are expected to be very small but the values obtained here are in fact noticeably large and positive for all isotopologues, i.e., about 0.16 $\AA$. Thus the $r^{(s)}$-structure could not be determined in the present study. Although the Kraitchman coordinates are of less significance than the coordinates of the atoms themselves; the substitution moment of inertia, $I^{(s)}$, calculated with the $\left[z^{(s)}(i)\right]^{2}$ for $i=1,2,3$ can be used to evaluate the $I^{(m)}$ defined by Watson, ${ }^{27}$ which is $I^{(m)}=2 I^{(s)}-I^{(0)}$. The values of $I^{(s)}$ and $I^{(m)}$ are listed in Table V. Because $I^{(m)}$ is a good approximation of the equilibrium moment of inertia $I^{(e)}$ at least for molecules built of heavy atoms, the structure parameter $r^{(m)}$ for the mass-dependent structure proposed by Watson ${ }^{27}$ can be derived by replacing $I^{(0)}$ and $r^{(0)}$ in Eq. (4) by $I^{(m)}$ and $r^{(m)}$. The obtained $r^{(m)}$ values are listed in Table V for each isotopologue. The variances of the $r^{(m)}$ values for isotopologues are larger than those of the $r^{(0)}$ values by one order of magnitude. This may be due to the model errors, which are enhanced in the process estimating the $I^{(s)}$. The $r^{(m)}$ values are systematically and noticeably larger than $r^{(0)}$ for all isotopologues.

## C. Vibrational corrected equilibrium structure, $r^{(0 e 0)}$

The effective rotational constant of a linear molecule for a given vibrational state $\left(v_{1}, v_{2}^{\ell}, v_{3}\right)$ can be denoted as $B^{\left(v_{1}, v_{2}^{\ell}, v_{3}\right)}$.

TABLE VI. Bond length parameter of $\mathrm{C}_{3}$ in $\AA$.

| $\mathrm{C}-\mathrm{C}$ length | $r$ |
| :--- | :--- |
| $r^{(0)}$ | $1.27746(22)$ |
| $r^{(m)}$ | $1.28955(31)$ |
| $r^{(0 e 0)}$ | $1.29511(5)$ |
| $r_{\text {exp }}^{(e)}$ | $1.29471(5)^{\mathrm{a}}$ |
| $r_{\operatorname{mix}}^{(e)}$ | $1.29407(10)^{\mathrm{b}}$ |
| $r_{\text {theo }}^{(e)}$ | $1.29397(10)^{\mathrm{b}}$ |

[^1]The effective rotational constant for the ground vibrational state is thus denoted as $B^{\left(0,0^{0}, 0\right)}$ or simply as $B^{(0)}$ (as in Section IV A). In the first approximation, the equilibrium rotational constant $B^{(e)}$, i.e., the rotational constant for the $r^{(e)}$-structure, is related to the effective rotational constant of a vibrational state by

$$
\begin{equation*}
B^{\left(v_{1}, v_{2}^{\ell}, v_{3}\right)}=B^{(e)}-\sum_{i=1}^{3} \alpha_{i}\left(v_{i}+d_{i} / 2\right) \tag{5}
\end{equation*}
$$

where $\alpha_{i}$ and $d_{i}$ represent the vibration-rotation interaction constant and the degeneracy of the $i$ th vibrational mode. The equilibrium moment of inertia $I^{(e)}$ is readily obtained as the inverse of $B^{(e)}$, see Eq. (3). The equilibrium moment of inertia of $\mathrm{C}_{3}$ derived in this manner is expressed by a simple function of the mass $m_{i}$ and coordinate $z_{i}$ of each atom, see Eq. (4). The structure parameters $r^{(e)}$ for each isotopologue are thus determined from the corresponding values of $I^{(e)}$.

In the present study, experimental values $B^{(0)}$ and $\alpha_{2}$ were very accurately determined for all isotopologues. For the $v_{1}$ and $v_{3}$ symmetric and antisymmetric stretching mode of ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C}$, the values $\alpha_{1}=169.160(630) \mathrm{MHz}$ (derived from $B^{\left(1,0^{0}, 0\right)}=12739.681 \mathrm{MHz}^{29}$ ) and $\alpha_{3}=-153.023(208)$ MHz have almost the same value but with opposite signs. Thus, the effect of $\alpha_{1}$ and $\alpha_{3}$ in Eq. (5) nearly cancels out; the $a b$ initio calculation also supports this cancellation. ${ }^{22}$ Hence, taking only the $\alpha_{2}$ contribution into consideration, we obtained $B^{(0 e 0)}$ values for each isotopologue. The thus-derived $r^{(0 e 0)}$ values are listed in Table V. The value of $\alpha_{2}$ is smaller than $B^{(0)}$ by a factor of approximately $3 / 100$, which is larger than that of a semi-rigid molecule. We estimate the higher order contribution neglected in the approximate relation of Eq. (5) to be on the order of $(3 / 100)^{2} \approx 1 / 1000$. Thus, the relative uncertainty of $r^{(0 e 0)}$ may be in the order of $0.001 \AA$. They show very little deviation among the isotopologues.

## v. CONCLUSION

We observed the $v_{2}$ fundamental band of the $\mathrm{C}_{3}$ radical at high resolution for all isotopologues: ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C},{ }^{12} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C}$, ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C},{ }^{13} \mathrm{C}^{13} \mathrm{C}^{12} \mathrm{C},{ }^{13} \mathrm{C}^{12} \mathrm{C}^{13} \mathrm{C}$, and ${ }^{13} \mathrm{C}^{13} \mathrm{C}^{13} \mathrm{C}$. From the rotational constants of the ground state and the $v_{2}=1$ state, we determined experimentally the bond length of the $\mathrm{C}_{3}$ radical
to high precision. The C-C nuclear distances were determined using various models, see Table VI. In this table, the $r^{(0)}, r^{(m)}$, and $r^{(0 e 0)}$ values are the averages over those obtained for the isotopologues as listed in Table V. Their estimated uncertainties reflect the variance of the parameters among these isotopologues. For example, since the differences between the $r^{(0)}$ values of the isotopologues are very small, its calculated standard deviation of the mean value is underestimated. It is important to note that the $r^{(0)}$ value in Table VI is indeed smaller than the $r^{(m)}$ value and that the relation $r^{(0 e 0)}>r^{(m)}>r^{(0)}$ holds. This might be explained as an effect of a bending vibration with a large amplitude, which is not negligible even in the vibrational ground state through the zero-point vibration. Furthermore the probability of finding a carbon atom perpendicular to the molecular axis is expected to be distributed in a fairly large area. Thus, the effective rotational constant in the ground state should be significantly affected by these offaxis contributions. As mentioned in Section IV, the $r^{(0 e 0)}$ value should be a good approximation of the $r^{(e)}$ value in the present case. For ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C}$, where experimental values of $\alpha_{1}, \alpha_{2}$, and $\alpha_{3}$ are available, the derived $r_{\text {exp }}^{(e)}=1.29471(5) \AA$ value using Eq. (3) is slightly smaller than the $r^{(0 e 0)}=1.29511(5) \AA$ value. Thus, we expect the experimentally determined $r^{(0 e 0)}$ from all isotopologues to slightly overestimate the true bond length. This is also confirmed by the theoretically determined value of $r_{\text {theo }}^{(e)}=1.29397(10) \AA$ by a recent $a b$ initio calculation by Schröder and Sebald. ${ }^{22}$ The agreement between the two values is excellent, if we take into account the uncertainty of $0.001 \AA$ in $r^{(0 e 0)}$ mentioned at the end of Section IV C.

## SUPPLEMENTARY MATERIAL

See supplementary material for a complete list of fitted molecular parameters and line lists for all $\mathrm{C}_{3}$ isotopologues.

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${ }^{30}$ In case of floppy molecules, the Pekeris equation, $D_{e}=4 B_{e}^{3} / \omega_{e}{ }^{2}$, is not applicable to derive centrifugal distortion parameters. Therefore, we used scaled centrifugal parameters calculated from isotopic ratios, e.g., $D^{12-12-12} / D^{12-12-13}=D^{13-13-12} / D^{13-13-13}$.
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[^1]:    ${ }^{\mathrm{a}}$ Derived from rotational constants of ${ }^{12} \mathrm{C}^{12} \mathrm{C}^{12} \mathrm{C}$ only: $\mathrm{B}^{\left(0,0^{0}, 0\right)}, \mathrm{B}^{\left(0,1^{1}, 0\right)}, \mathrm{B}^{\left(0,0^{0}, 1\right)}$
    (see the supplementary material), and $\mathrm{B}^{\left(1,0^{0}, 0\right)}\left(\right.$ see Zhang et al. ${ }^{29}$ ).
    ${ }^{\text {b }}$ Values taken from Schröder and Sebald: ${ }^{22} r_{\text {theo }}^{(e)}\left(\right.$ ab initio calculation) and $r_{\text {mix }}^{(e)}$ (mixed experimental/theoretical approach).

