Introduction to Earthquake Engineering

Seismology

Prof. Dr.-Ing. Uwe E. Dorka

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Plate tectonics
Plate tectonics

- Normal fault
- Reverse fault
- Left lateral fault
- Right lateral fault
Tectonics and seismicity
Tectonics and seismicity

Epicentres for all earthquakes of 7 or larger that occurred between 1900 and 1980

From Wakabayashi (1)

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Definitions

Epicenter
Intensity $I_0$

Epicentral distance $\Delta$

Building
Intensity $I$

Ground surface

Sediment layer

Hypocenter
Magnitude $M$

Hypocentral depth $h$

Fault

Hypocentral distance $s$

From Müller, Keintzel (2)
Seismic waves

Body waves
P-wave

S-wave

From Grundbau-Taschenbuch (10)

Surface waves
Love wave

Rayleigh wave

From Grundbau-Taschenbuch (10)

\[
V_P = \sqrt{\frac{E}{\rho} \cdot \frac{1 - \nu}{(1 + \nu) \cdot (1 - 2 \cdot \nu)}} \quad \text{Velocity of P-Wave}
\]

\[
V_S = \sqrt{\frac{G}{\rho} = \sqrt{\frac{E}{\rho} \cdot \frac{1}{2 \cdot (1 + \nu)}}} \quad \text{Velocity of S-Wave}
\]

\[
E \quad \text{- Young's modulus}
\]

\[
G \quad \text{- shear modulus}
\]

\[
\rho \quad \text{- mass density}
\]

\[
\nu \quad \text{- Poisson ratio}
\]

i.e.: $V_P = 260$ to $690$ m/s and $V_S = 150$ to $400$ m/s

In comparison with the atmospheric speed of sound: $V = 360$ m/s $\sim 0.36$ km/s
Determination of Hypocentre

Arrival of P-wave

Arrival of S-wave

Arrival of L-wave

From Wakabayashi (1)

\[ \Delta = \frac{T_{SP}}{\left( \frac{1}{V_s} - \frac{1}{V_p} \right)} \]
Determination of Hypocentre

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Wave propagation in an elastic solid

Waves in elastic solids are described by the following partial differential equation, which is called the „wave-equation“:

\[ \ddot{y} - c^2 \cdot \ddots = 0 \]

It is satisfied in general by the following solution:

\[ y = f_1(x - c \cdot t) + f_2(x + c \cdot t) \]

The functions \( f_1 \) and \( f_2 \) define the form of the wave, which propagates with velocity \( c \).

Choosing:

\[ f_1(x - c \cdot t) = \dot{y} \cdot \sin \frac{2 \cdot \pi}{\lambda} \cdot (x - c \cdot t) \]

and \( f_2 = 0 \), the figure to the right shows the propagation of its wave form in x-direction versus time \( t \).
Wave propagation in layered bodies

Reflection and refraction on layers

Refraction in layers towards the surface

\[ \sin \theta_n = \frac{c_n}{c_1} \cdot \sin \theta_1 \]

From Wakabayashi (1)

\[ \sin \theta_1 = \frac{\sin \theta_2}{c_1} \cdot c_2 \]

From Wakabayashi (1)
Reflection on a free or ground surface

At the surface (x=0), the shear stress is zero, thus

$$G \cdot \frac{\partial y}{\partial x} = 0$$

Applying this boundary condition to the general solution of the wave equation, we obtain:

$$\frac{\partial f_1}{\partial t} = \frac{\partial f_2}{\partial t}$$

which gives \( f_1 = f_2 = f \), and therefore

$$y = f(x - c \cdot t) + f(x + c \cdot t)$$

Thus, we find for the displacement \( y_g \) on the surface:

$$y_g = 2 \cdot f(t)$$

With this solution, the wave in an arbitrary point in the ground is given by:

$$y(t, x) = \frac{1}{2} \left[ y_g \left( t - \frac{x}{c} \right) + y_g \left( t + \frac{x}{c} \right) \right]$$
Reflection and transmission between two layers

A part of the incident wave passes through the boundary surface into the upper layer, while the rest is reflected at the interface:

\[
y_1 = f_1\left(t - \frac{x}{c_1}\right) + g_1\left(t + \frac{x}{c_1}\right)
\]

\[
y_2 = f_2\left(t - \frac{x}{c_2}\right)
\]

where \( c_1 = \sqrt{\frac{G_1}{\rho_1}} \) and \( c_2 = \sqrt{\frac{G_2}{\rho_2}} \)

The compatibility conditions at the boundary between both layers are:

\[
V_1\big|_{x=0} = V_2\big|_{x=0}
\]

\[
G_1 \cdot \frac{\partial V_1}{\partial x}\big|_{x=0} = G_2 \cdot \frac{\partial V_2}{\partial x}\big|_{x=0}
\]

From Wakabayashi (1)
Reflection and transmission between two layers

\[
f_2\left(t - \frac{x}{c_2}\right) = \frac{2}{1 + \alpha} \cdot f_1\left(t - \frac{x}{c_1}\right)
\]

\[
g_1\left(t + \frac{x}{c_1}\right) = \frac{1 - \alpha}{1 + \alpha} \cdot f_1\left(t + \frac{x}{c_1}\right)
\]

where \(\alpha\), \(\beta\) and \(\gamma\) are the wave-propagation impedances:

\[
\alpha = \frac{\rho_2 \cdot c_2}{\rho_1 \cdot c_1}
\]

\[
\beta = \frac{1 - \alpha}{1 + \alpha}
\]

\[
\gamma = \frac{2}{1 + \alpha}
\]

From Wakabayashi (1)
Wave propagation in a surface layer on bedrock

The solutions for the free surface and between two layers must be combined. The solution for the surface layer is:

\[ v_2(t, x_2) = \frac{1}{2} \left[ v_g \left( t - \frac{x_2}{c_2} \right) + v_g \left( t + \frac{x_2}{c_2} \right) \right] \]

The solution for the bedrock is:

\[ v_1(t, x_1) = f_1 \left( t - \frac{x_1}{c_1} \right) + g_1 \left( t + \frac{x_1}{c_1} \right) \]

The compatibility conditions at the boundary between both layers are

\[ v_2(t, -H) = v_1(t, 0) \]

\[ G_2 \cdot \frac{\partial}{\partial x_2} \cdot v_2(t, -H) = G_1 \cdot \frac{\partial}{\partial x_1} \cdot v_1(t, 0) \]

These three equations lead to the expression on the right:

\[ f_1(t) = \frac{1}{4} \cdot \left[ (1 + \alpha) \cdot v_g \left( t + \frac{H}{c_2} \right) + (1 - \alpha) \cdot v_g \left( t - \frac{H}{c_2} \right) \right] \]

From Wakabayashi (1)
Wave propagation in a surface layer on bedrock

Assuming that the waveform $v_g$ at the surface is $A_0 e^{i\omega t}$ and the incident wave $f_1(t)$ is $a e^{i\omega t}$, $a$ is given by

$$a = \frac{A_g}{4} \cdot \left[ (1 + \alpha) \cdot e^{\frac{i\omega H}{c_2}} + (1 - \alpha) \cdot e^{\frac{-i\omega H}{c_2}} \right]$$

$$= \frac{A_g}{2} \cdot \left[ \cos \frac{\omega H}{c_2} + i \cdot \alpha \cdot \sin \frac{\omega H}{c_2} \right]$$

The amplitude ratio $A_g$ to $2a$ at the boundary between surface layer and bedrock is:

$$\left| \frac{A_g}{2a} \right| = \left( \cos^2 \frac{\omega H}{c_2} + \alpha^2 \cdot \sin^2 \frac{\omega H}{c_2} \right)^{\frac{1}{2}}$$

From Wakabayashi (1)
Amplification characteristics of multi-layers

From Wakabayashi (1)
Earthquake motions on ground surface

The seismometer

viscously damped SDOF oscillator

\[ v(t) = v_g(t) + v(t) \]

Output proportional to relative displacement \( v(t) \)

\[ \ddot{v}_g(t) = \ddot{v}_{g0} \sin \omega t \] (Base motion input)

From Clough, Penzien (4)

free body equilibrium

\[ k \cdot v = F_k \]

\[ m \cdot \ddot{v} = F_m \]

\[ c \cdot \dot{v} = F_c \]

\[ \sum F = 0 \Rightarrow F_k + F_m + F_c = 0 \]

\[ m \cdot \ddot{v}(t) + c \cdot \dot{v}(t) + k \cdot v(t) = -m \cdot \ddot{v}_g(t) \]
The seismometer

2\textsuperscript{nd} order differential equation for a SDOF system under sinusoidal base excitation:

\[ \ddot{v}(t) + 2\xi\omega \cdot \dot{v}(t) + \omega^2 \cdot v(t) = -\ddot{v}_g \cdot \sin \omega t \]

\[ \ddot{v}_g \Rightarrow \text{ground acceleration amplitude} \]

with the following definitions:

\[ \omega^2 = \frac{K}{m} \Rightarrow \text{frequency} \quad \xi = \frac{1}{2} \sqrt{\frac{c^2}{K \cdot m}} \Rightarrow \text{damping ratio} \]

The general solution is:

\[ V_c + V_p \] Homogeneous plus particular solution
The seismometer

Homogeneous solution: free vibration response

\[ v_c = e^{-\xi \omega t} \cdot (A \cdot \sin \omega_D t + B \cdot \cos \omega_D t) \]

damped free vibration

damped frequency:

\[ \omega_D = \omega \sqrt{1 - \xi^2} \]

logarithmic decrement:

\[ \delta = \ln \frac{v_n}{v_{n+1}} = \frac{2\pi \xi}{\sqrt{1 - \xi^2}} \]
The seismometer

Particular solution: steady-state response (free vibration has damped out)

\[ v_p = G_1 \cdot \sin \bar{\omega} t + G_2 \cdot \cos \bar{\omega} t \]

\[ v(t) = -\ddot{v}_g \cdot \frac{1}{\omega} \cdot \frac{1}{(1 - \xi^2)^2 + (2\xi\beta)^2} \cdot \left[(1 - \xi^2) \sin \bar{\omega} t - 2\xi\beta \cos \bar{\omega} t\right] \]

\[ \beta = \frac{\bar{\omega}}{\omega} \]

---

sine amplitude:

\[ \rho_s = \frac{\left(1 - \xi^2\right)}{(1 - \xi^2)^2 + (2\xi\beta)^2} \cdot \ddot{v}_g \]

cosine amplitude:

\[ \rho_c = \frac{2\xi\beta}{(1 - \xi^2)^2 + (2\xi\beta)^2} \cdot \ddot{v}_g \]

amplitude:

\[ \rho = \frac{\ddot{v}_g \cdot \sqrt{(1 - \beta^2)^2 + (2\xi\beta)^2}}{\omega^2} \]

phase angle:

\[ \theta = \tan^{-1} \frac{2\xi\beta}{(1 - \beta^2)^2} \]
The seismometer

dynamic magnification:

\[ D = \frac{\rho}{\rho_s} \]

\[ p_s = \frac{\ddot{y}_g}{\omega^2} \quad - \text{static response} \]

\[ D = \frac{1}{\sqrt{(1 - \beta)^2 + (2 \xi \beta)^2}} \]

amplitude:

\[ \rho = \frac{\ddot{y}_g}{\omega^2} \cdot D \]

Accelerometer:

- high tuned SDOF
- damping ratio $\xi \sim 0.7$
The seismometer

harmonic base displacement:

\[ v_g(t) = -\omega^2 \cdot v_{g0} \cdot \sin \omega t \]

amplitude:

\[ \rho = \frac{\omega^2}{\omega^2} \cdot v_{g0} \cdot D = v_{g0} \left( \beta^2 \cdot D \right) \]

Displacement meter:

- low tuned SDOF
- damping ratio \( \xi \sim 0.5 \)
Data processing of measured acceleration

Measurement of the east-west acceleration component of the Montenegro earthquake, 1979

Base line correction

Filtering raw data

Numerical integration

Numerical integration
Fourier spectrum

Function $a(t)$ in the time domain

Fourier transformation

$$U(\omega) = \int_{-\infty}^{\infty} a(t) \cdot e^{-i\omega t} \cdot dt$$
Power spectrum

Correlation function \( \Phi(\tau) \) in the time domain

**Fourier Transformation**

\[
S(\omega) = \int_{-\infty}^{\infty} \Phi(\tau) \cdot e^{i\omega t} \cdot dt
\]
Generation of artificial accelerograms

Random process:

i. e. Kanai-Tajimi-Filter:

\[
H(\omega) = \frac{1 + 4 \xi_g^2 \left( \frac{\omega}{\omega_g} \right)^2}{1 - \left( \frac{\omega}{\omega_g} \right)^2} + 4 \xi_g^2 \left( \frac{\omega}{\omega_g} \right)^2
\]

Generated accelerogram; statistically equivalent to measured accelerograms

Intensity function

From Flesch (6)
Earthquake process simulation

Empirical $\text{Green}^{\prime}$s functions

From DGEB (9)
Earthquake process simulation

FEM simulation

two-dimensional model

From DGEB (9)
Earthquake process simulation

FEM-simulation

three-dimensional model

From DGEB (9)
Seismic moment and moment magnitude

Richter established an empirical relationship between the energy of the surface wave $E_s$ and the surface magnitude $M_s$:

$$\log E_s = 11.8 + 1.5M_s$$

with $E_s$ in erg, $1\text{ erg} = 10^{-4}\text{ kNm}$

$$M_0 = \mu \cdot d \cdot A$$

with:
- $d$ - average displacement of ruptured surface
- $A$ - area of ruptured surface
- $\mu$ - shear strength of rock

KANAMORI gives the following relationship between seismic moment $M_0$ and moment magnitude $M_w$:

$$M_w = \frac{2}{3} \log M_0 - 10.73 \text{ [dyn cm]}$$

with $1\text{ dyn} = 10^{-8}\text{ kN}$

$$\log L = -1.01 + 0.32M_w$$

The length of the fault rupture $L$ [km] is related to $M_w$:
Magnitude and intensities

One earthquake has only one magnitude but the intensity differs with the hypocentral distance $r$:

$$I = 8.16 + 1.46 \cdot M - 2.46 \cdot \ln r$$

The relationship between hypocentral depth $h$, Magnitude $M$ and the epicentral Intensity $I_0$ for European conditions is given by the following equation:

$$M = 0.5 \cdot I_0 + \log h + 0.35$$
## Intensity scales

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Not felt except under exceptionally favorable circumstances</td>
</tr>
<tr>
<td>II</td>
<td>Felt by persons at rest</td>
</tr>
<tr>
<td>III</td>
<td>Felt indoors; may not be recognized as an earthquake</td>
</tr>
<tr>
<td>IV</td>
<td>Windows, dishes, and doors disturbed; standing motor cars rock noticeably</td>
</tr>
<tr>
<td>V</td>
<td>Felt outdoors; sleepers wakened; doors swing</td>
</tr>
<tr>
<td>VI</td>
<td>Felt by all; walking unsteady; windows and dishes broken</td>
</tr>
<tr>
<td>VII</td>
<td>Difficult to stand; noticed by drivers; fall of plaster</td>
</tr>
<tr>
<td>VIII</td>
<td>Steering of motor cars affected; damage to ordinary masonry</td>
</tr>
<tr>
<td>IX</td>
<td>General panic; weak masonry destroyed, ordinary masonry heavily damaged</td>
</tr>
<tr>
<td>X</td>
<td>Most masonry and frame structures destroyed with foundations; rails bent slightly</td>
</tr>
<tr>
<td>XI</td>
<td>Rails bent greatly; underground pipes broken</td>
</tr>
<tr>
<td>XII</td>
<td>Damage total; objects thrown into the air</td>
</tr>
</tbody>
</table>
Intensity scales

MM = Modified-Mercalli-Scale
MSK = Medvedev-Sponheuer-Karnik Scale
JMA = Japanese Meteorological Agency

<table>
<thead>
<tr>
<th>MM</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
<th>XI</th>
<th>XII</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSK</td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>IV</td>
<td>V</td>
<td>VI</td>
<td>VII</td>
<td>VIII</td>
<td>IX</td>
<td>X</td>
<td>XI</td>
<td>XII</td>
</tr>
<tr>
<td>JMA</td>
<td>0</td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>IV</td>
<td>V</td>
<td>VI</td>
<td>VII</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

0.5 1 2 5 10 20 50 100 200 500 1000  

maximum acceleration (cm/s²)

From Wakabayashi (1)
European earthquake intensities

Maximum observed intensities in Europe

From Wakabayashi (1)
Earthquake as statistical process

**Generalized Gumbel distribution:**

\[
\lambda_0(M) = \frac{1}{1 \text{ Jahr}} \left( \frac{f_1 - f_2}{\alpha_0} \right)^{\gamma/c} \text{ für } M < M_{\text{max}} = m_0 + \alpha_0 f_1/f_2
\]

\[
f_1 = \Gamma(1 + \tau) \quad f_2 = \sqrt{\Gamma(1 + 2\tau)} - \Gamma^2 \Gamma \text{ Gammafunktion}
\]

\[
\sigma_0 = \sigma \cdot \left( \frac{A \cdot T}{A_0 \cdot T_0} \right)^{\tau} \quad m_0 = m + (\sigma - \alpha_0) \cdot f_1/f_2 \quad \tau_0 = \tau > 0 \quad M_{\text{max0}} = M_{\text{max}}
\]

**Gutenberg-Richter Statistik:**

\[
\log_{10} \left[ \lambda_0(M) \right] = a_0 - b_0 M \quad b_0 = \frac{\pi}{\log_{10}(e)} \quad a_0 = b_0 m_0 - E \log_{10}(e) - 1.0 \quad E = 0.577 \ldots
\]

\[
\sigma_0 = \sigma \quad b_0 = b \quad m_0 = m + \frac{1}{b} \log_{10} \left( \frac{A_0 \cdot T_0}{A \cdot T} \right) \quad a_0 = a + \log_{10} \left( \frac{A_0 \cdot T_0}{A \cdot T} \right)
\]

From DGBE (11)
Earthquake as statistical process

From DGEB (11)
Earthquake as statistical process

Bild 3. Probabilistische seismische Gefährdungsbestimmung nach der Cornell-Methode

Fig. 3. Probabilistic seismic hazard assessment according to the Cornell method
Earthquake as statistical process

Probabilistic seismic hazard in Germany for the 1000-year return period

Probabilistic seismic hazard in Germany for the 10000-year return period
Paleo-Seismology

Earthquakes and gradual motions in active faults in the Lower Rhine embankment

From DGEB (7)
Paleo-Seismology

Relative Height Changes along the Erft Fault System 1933–1952

Geodetic data from Quitzow & Vahlensieck (1955)

Average Slip Rate 0.9 mm per Year

from DGEB (7)
Paleo-Seismology

Erft fault at the 66-m floor of an open-pit mine near Cologne

From DGEB (7)
Paleo-Seismology

Ancient dislocation of layers at the Erft fault:

Evidence for a single earthquake with more than 6.0 $M_w$?

From DGEB (7)
Paleo-Seismology

Three types of typical motion pattern in tectonic zones

From DGEB (7)
Paleo-Seismology

Gradual motions in active faults in California

From DGEB (7)
Paleo-Seismology

Magnitude Frequency Lower Rhine Embayment

Lower Rhine Embayment
\( \log(N/y) = 2.69 - 0.86M_w \)

10km fault segment
\( \log(N/y) = 1.10 - 0.86M_w \)

Observation Period
- 1700–1994
- 1900–1994
- 1980–1994

Estimated from tectonic balance
- 0.02/y
- 0.002/y

Bree event

From DGEB (7)
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