Introduction to Earthquake Engineering

Response Analysis

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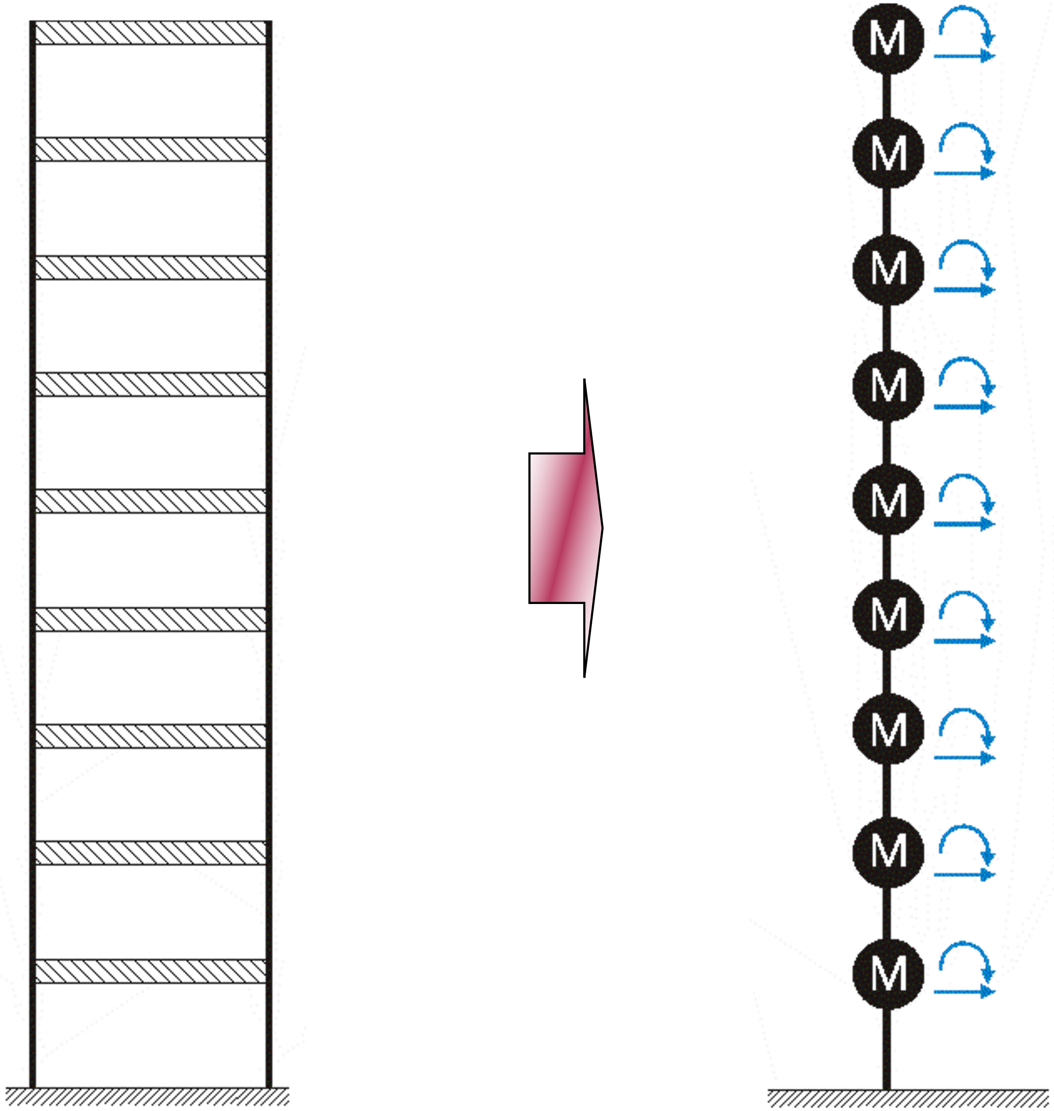
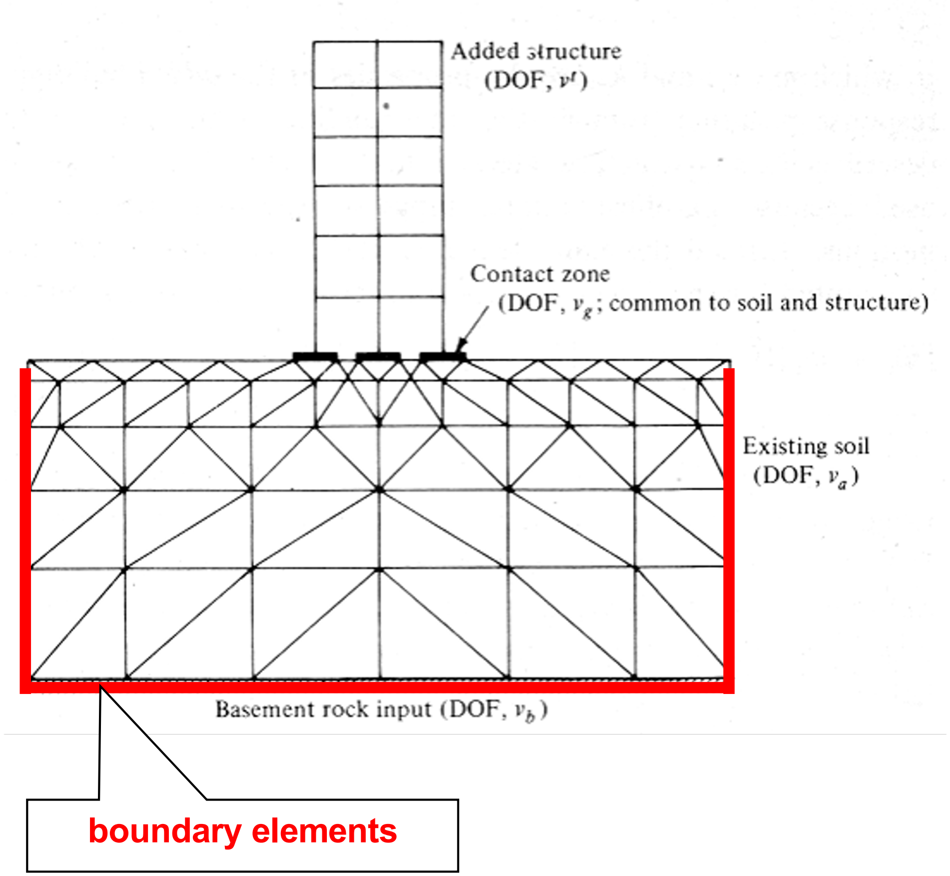
Modeling of buildings



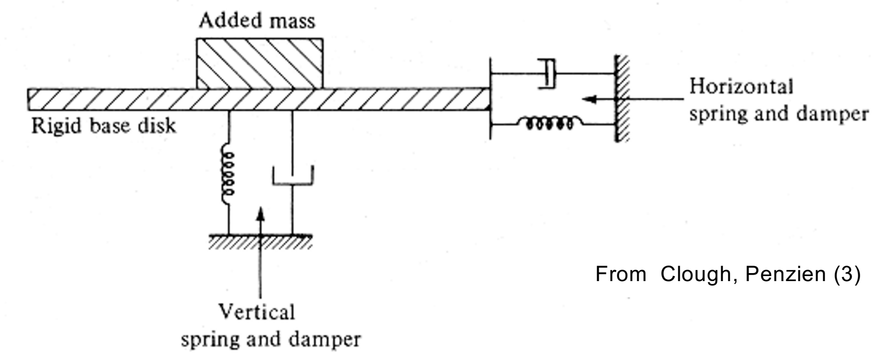
masses lumped in floors
frames with plastic hinges

Soil-structure interaction

structure with soil in FE and BE



simplified model

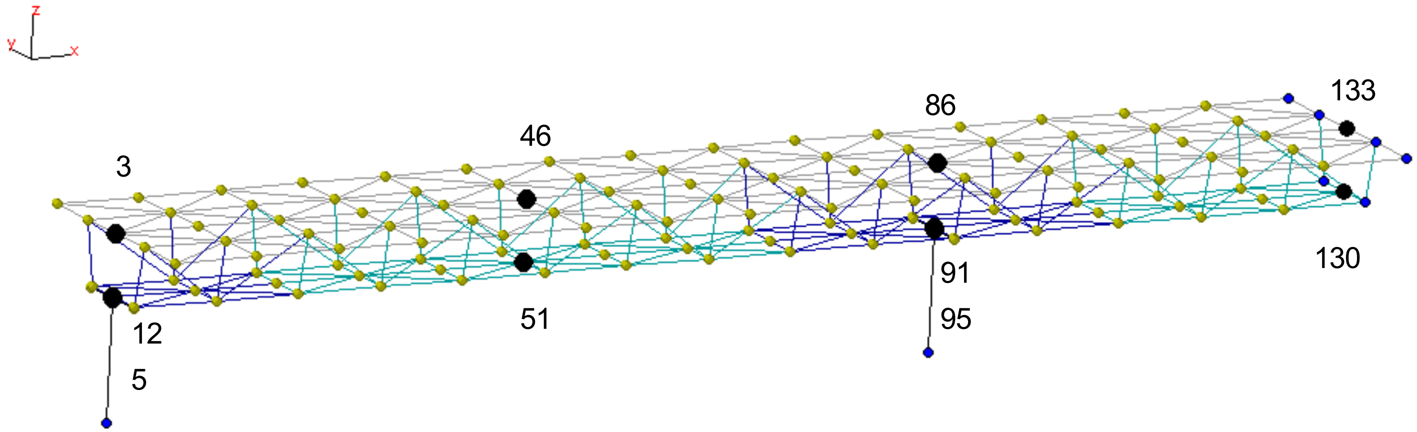


From Clough, Penzien (3)

Modeling of bridges

Model with 20 dynamic DOFs

superstructure, bearings and columns with FE ⇒ multiple base input



master node	DOF	
3	1	x
	2	y
5	3	x
	4	y
12	5	x
	6	y
46	7	y
	8	z
51	9	y
	10	z
86	11	x
	12	y
91	13	x
	14	y
95	15	x
	16	y
130	17	y
	18	z
133	19	y
	20	z

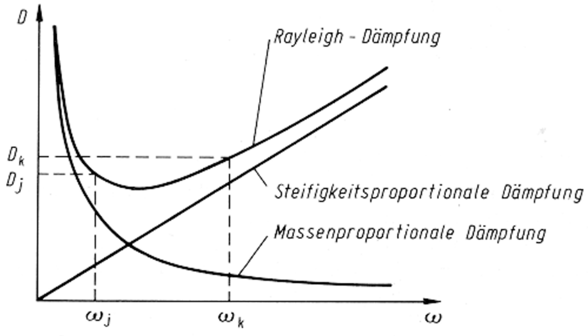
Rayleigh-damping:

$$\underline{C} = \alpha \underline{M} + \beta \underline{K} \qquad \alpha = \frac{2 \cdot \omega_1 \cdot \omega_2}{\omega_2^2 - \omega_1^2}(\omega_2 \cdot D_1 - \omega_1 \cdot D_2) \qquad \beta = \frac{2 \cdot (\omega_2 \cdot D_2 - \omega_1 \cdot D_1)}{\omega_2^2 - \omega_1^2}$$

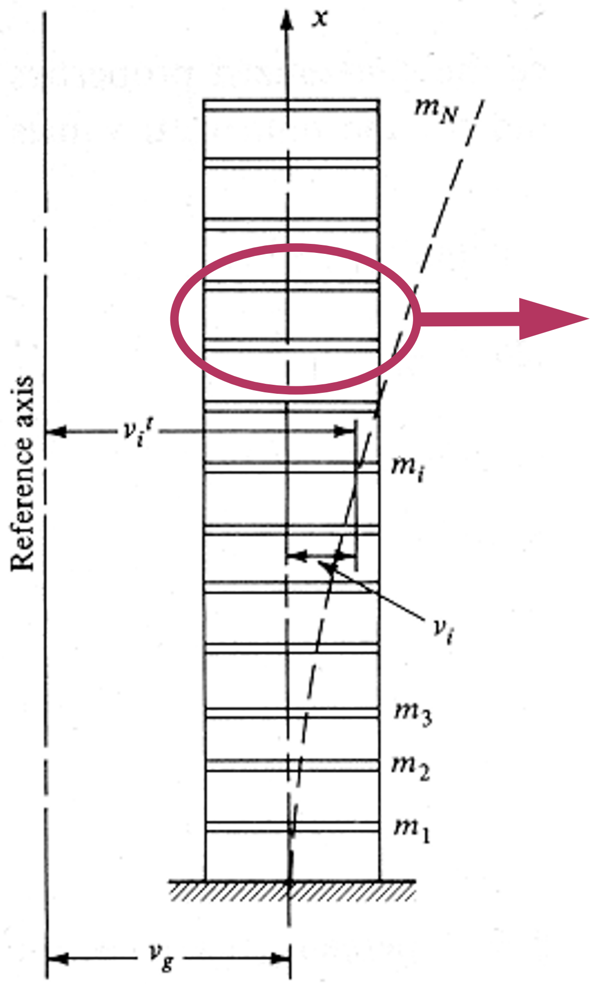
with $D_1 = D_2 = 5\%$ and $\omega_1 = 100.06$, $\omega_2 = 429.44$ gives:

$\alpha = 8.11515890$

$\beta = 0.00018886$

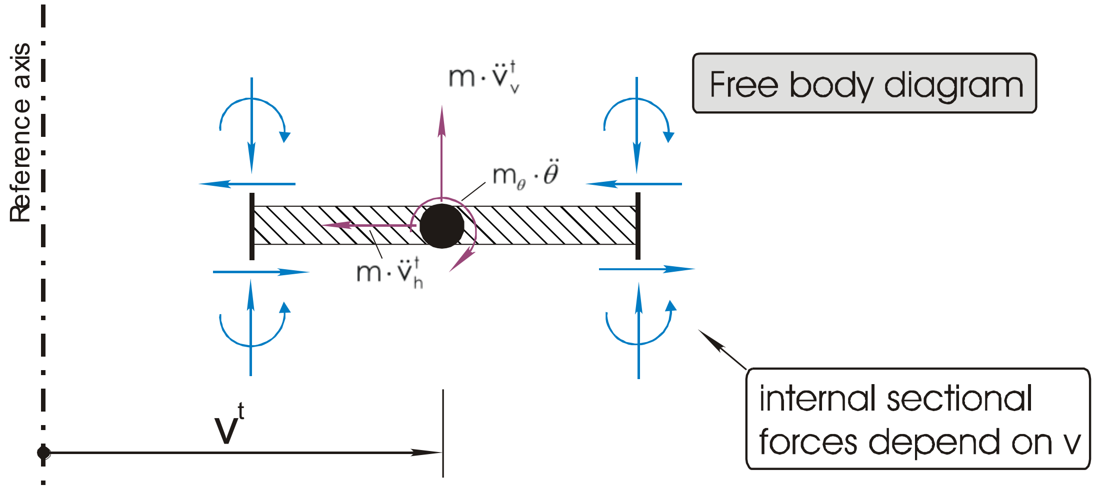


Earthquake loading



From Clough, Penzien (3)

lumped MDOF-sytem with rigid base translation (horizontal case)



$$\mathbf{m} \cdot \ddot{\mathbf{v}}^{\mathbf{t}} + \mathbf{c} \cdot \dot{\mathbf{v}} + \mathbf{k} \cdot \mathbf{v} = \mathbf{0}$$

$$\mathbf{v}^{\mathbf{t}} = \mathbf{v} + \{\mathbf{j}\} \cdot \mathbf{v}_{\mathbf{g}}$$

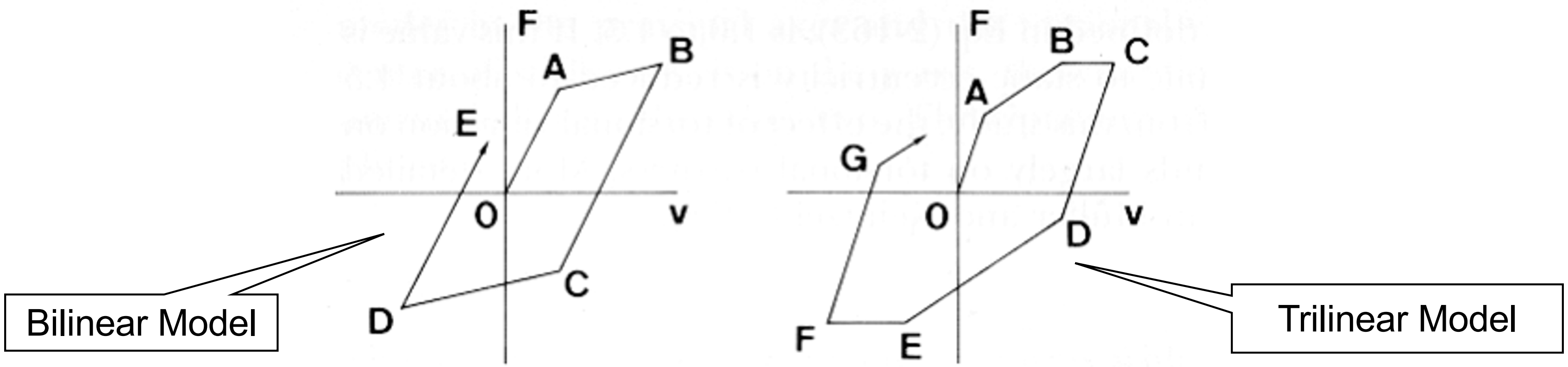
where $\{j\}$ – direction cosine, for buildings typically 1

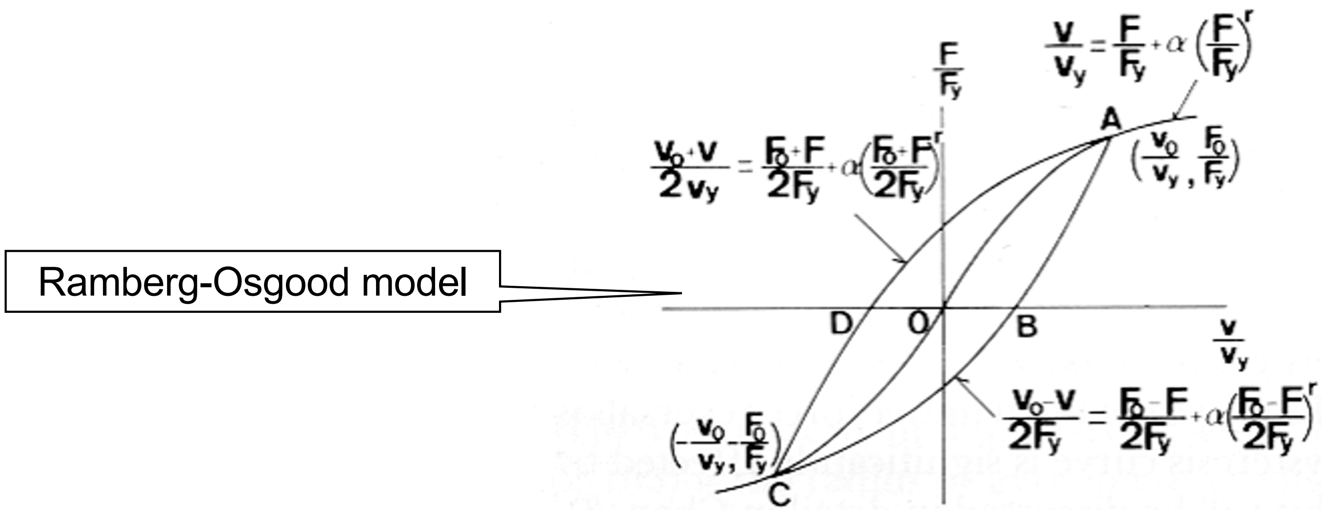
$$\mathbf{m} \cdot \ddot{\mathbf{v}} + \mathbf{c} \cdot \dot{\mathbf{v}} + \mathbf{k} \cdot \mathbf{v} = \mathbf{p}_{\mathbf{eff}}(\mathbf{t})$$

$$\mathbf{p}_{\mathbf{eff}}(\mathbf{t}) = -\mathbf{m} \cdot \{\mathbf{j}\} \cdot \ddot{\mathbf{v}}_{\mathbf{g}}(\mathbf{t})$$

Description of non-linear behaviour

Hysteresis models (1D) – non-degrading models





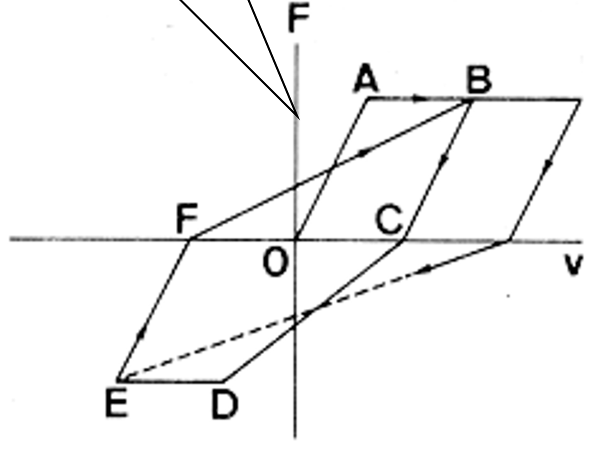
From Wakabayashi (1)

Description of non-linear behaviour

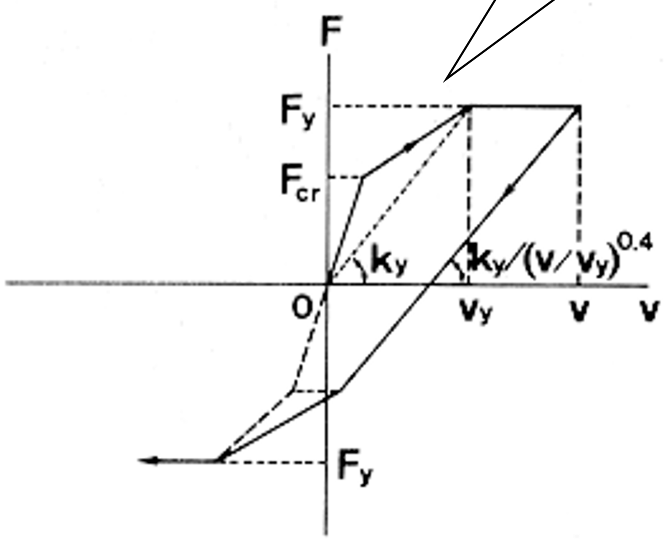
Hysteresis models (1D) – degrading models

Piecewise linear model
(Clough and Johnston)

Trilinear Takeda model
for rc-members



(a)



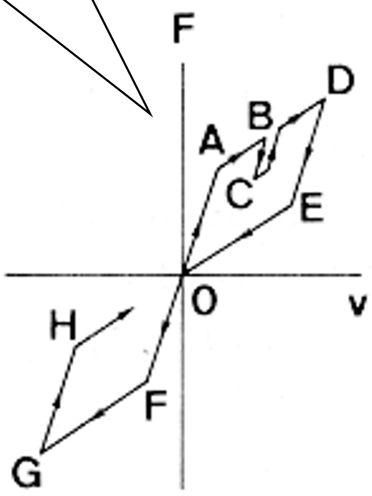
(b)

From Wakabayashi (1)

Description of non-linear behaviour

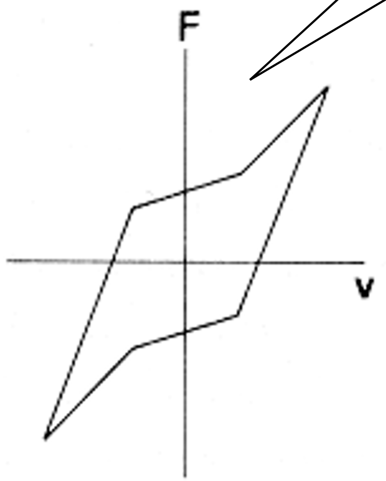
Hysteresis models (1D) – slip-type models

Double bilinear model by Tanabashi and Kaneta



(a)

Slip-type model



(b)

From Wakabayashi (1)

Description of non-linear behaviour

Example for slip-type hysteresis model
(friction connection with multiple stops)

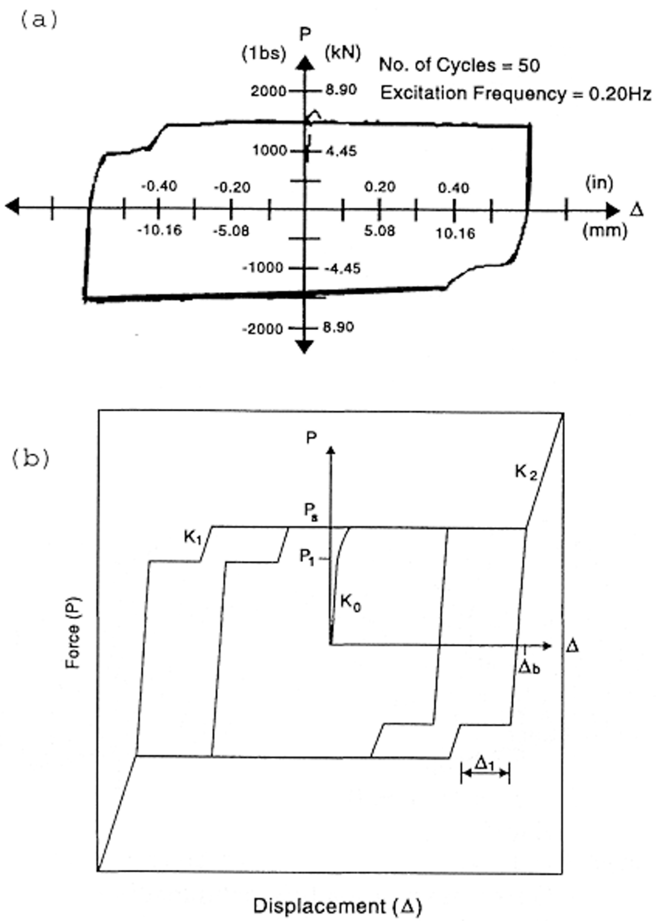
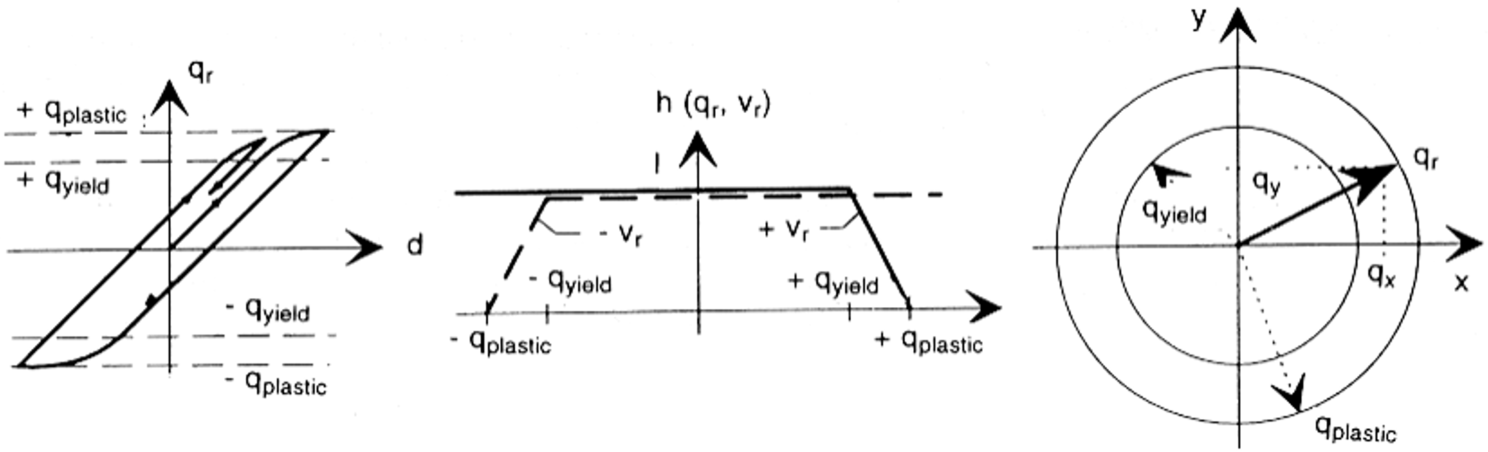


Table 4.2 Hysteretic Model with Multilevel Bearing Stops

Define $\Delta_2 = \Delta_1 + (P_s - P_1)/K_1$
If $|P| < P_1$ then
  Elastic loading or unloading
  $\dot{P} = K_0\dot{\Delta};\ \dot{\gamma} = 0$
Elseif $P\dot{\Delta} \leq 0$ and $|\Delta| \leq \Delta_b$ then
  Unloading within hysteresis block
  If $\Delta_1 < \gamma \mathrm{sgn}(\dot{\Delta}) < \Delta_2$ then
    Inner bearing unloading
    $\dot{P} = K_1\dot{\Delta};\ \dot{\gamma} = \dot{\Delta}$
  Else
    Elastic unloading
    $\dot{P} = K_0\dot{\Delta};\ \dot{\gamma} = 0$
  Endif
Elseif $|\Delta| < \Delta_b$ then
  Loading within hysteresis block
  If $\gamma \mathrm{sgn}(\dot{\Delta}) < \Delta_1$ then
    Slippage
    $\dot{P} = 0;\ \dot{\gamma} = \dot{\Delta}$
  Elseif $\gamma \mathrm{sgn}(\dot{\Delta}) < \Delta_2$ then
    Inner bearing
    $\dot{P} = K_1\dot{\Delta};\ \dot{\gamma} = \dot{\Delta}$
  Elseif $|P| < P_s$ then
    Elastic loading
    $\dot{P} = K_0\dot{\Delta};\ \dot{\gamma} = 0$
  Else
    Slippage
    $\dot{P} = 0;\ \dot{\gamma} = 0$
  Endif
Else
  Outer bearing loading or unloading
  $\dot{P} = K_2\dot{\Delta};\ \dot{\gamma} = 0$
Endif

Description of non-linear behaviour

2D hysteresis model (2D Bouc-When)



From Pradlwarter, Schuëller, Dorka (7)

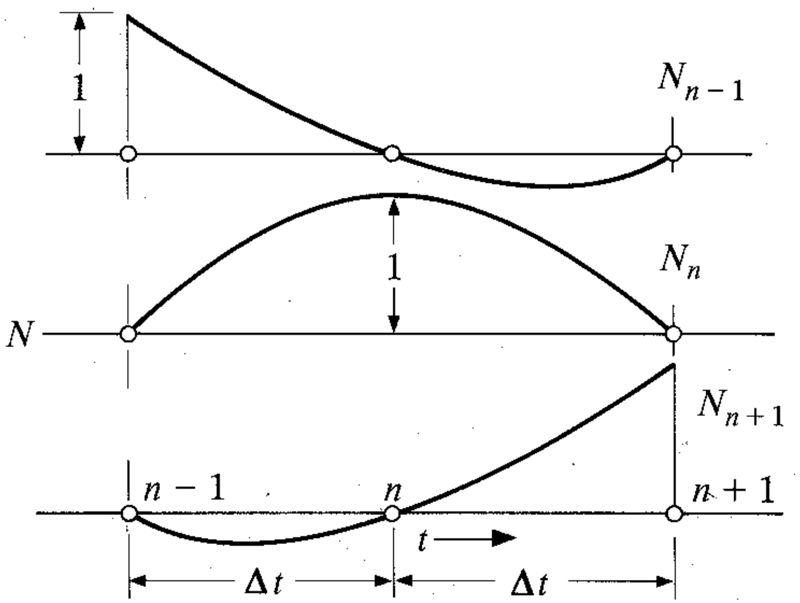
Time history analysis

Time discretization:

Dynamic equilibrium: $$M\frac{d^2}{dt^2}x + C\frac{d}{dt}x + Kx + f_r = p(t)$$

with f_r : vector of non-linear restoring forces

Shape functions for discretizing x in time:



$$N_{n-1} = \frac{\xi \cdot (1+\xi)}{2};$$
$$N_{n-1} = (1-\xi)\cdot(1+\xi);$$
$$N_{n+1} = -\frac{\xi \cdot (1-\xi)}{2}$$

with: $\xi = \frac{t}{\Delta t}$

Weighted residual formulation

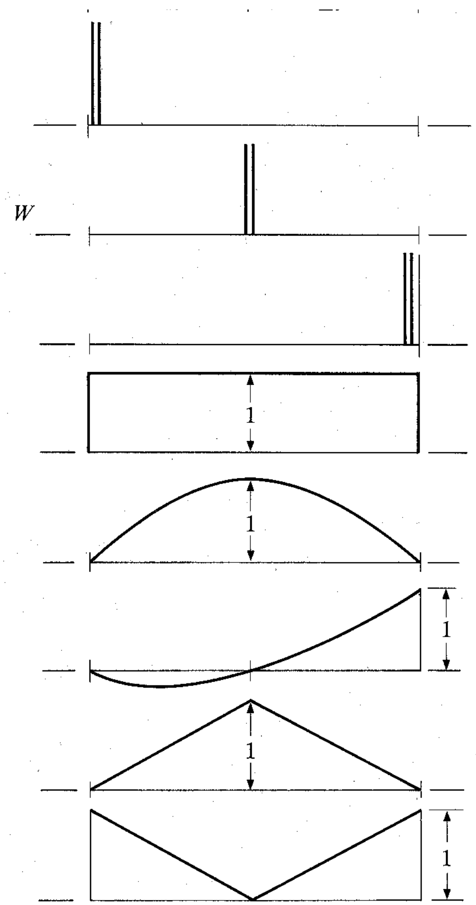
$$\int_{-1}^{1} W \begin{bmatrix} M \cdot \left(u^{n-1} \frac{d^2}{dt^2} N_{n-1} + u^n \frac{d^2}{dt^2} N_n + u^{n+1} \frac{d^2}{dt^2} N_{n+1} \right) \\ +C \cdot \left(u^{n-1} \frac{d}{dt} N_{n-1} + u^n \frac{d}{dt} N_n + u^{n+1} \frac{d}{dt} N_{n+1} \right) \\ +K \cdot \left(u^{n-1} N_{n-1} + u^n N_n + u^{n+1} N_{n+1} \right) \\ +f_*^{n-1} N_{n-1} + f_*^{n} N_n + f_*^{n+1} N_{n+1} \end{bmatrix} d\xi$$

$$f_* = f_r - p$$

$$\gamma = \frac{\int_{-1}^{1} W \left(\xi + \frac{1}{2} \right) d\xi}{\int_{-1}^{1} W \, d\xi}$$

$$\beta = \frac{\int_{-1}^{1} W \frac{1}{2} (1 + \xi) \xi \, d\xi}{\int_{-1}^{1} W \, d\xi}$$

Weighting functions W:



γ	β	
$-\frac{1}{2}$	0	
$\frac{1}{2}$	0	Central explicit
$\frac{3}{2}$	1	Backward
$\frac{1}{2}$	$\frac{1}{6}$	
$\frac{1}{2}$	$\frac{1}{10}$	Linear acceleration
$\frac{3}{2}$	$\frac{4}{5}$	Galerkin
$\frac{1}{2}$	$\frac{1}{12}$	Fox Goodwin
$\frac{1}{2}$	$\frac{1}{4}$	Average acceleration

3-point recurrence scheme

$$u^{n+1} = [M + \gamma\Delta t C + \beta\Delta t^2 K]^{-1} \cdot \left\{ \begin{aligned} &\left[2M - (1-2\gamma)\Delta t C - \left(\frac{1}{2} - 2\beta + \gamma\right)\Delta t^2 K\right] \cdot u^n \\ &-\left[M - (1-\gamma)\Delta t C + \left(\frac{1}{2} + \beta - \gamma\right)\Delta t^2 K\right] \cdot u^{n-1} \\ &+\beta\Delta t^2 f_*^{n+1} + \left(\frac{1}{2} - 2\beta + \gamma\right)\Delta t^2 f_*^n \\ &+\left(\frac{1}{2} + \beta - \gamma\right)\Delta t^2 f_*^{n-1} \end{aligned} \right\}$$

$$u^{n+1} = u_0 + G(f_r^{n+1})$$

with:

$$u_0 = [M + \gamma\Delta t C + \beta\Delta t^2 K]^{-1} \cdot \left\{ \begin{aligned} &\left[2M - (1-2\gamma)\Delta t C - \left(\frac{1}{2} - 2\beta + \gamma\right)\Delta t^2 K\right] \cdot u^n \\ &-\left[M - (1-\gamma)\Delta t C + \left(\frac{1}{2} + \beta - \gamma\right)\Delta t^2 K\right] \cdot u^{n-1} \\ &-\beta\Delta t^2 p^{n+1} + \left(\frac{1}{2} - 2\beta + \gamma\right)\Delta t^2 f_*^n \\ &+\left(\frac{1}{2} + \beta - \gamma\right)\Delta t^2 f_*^{n-1} \end{aligned} \right\}$$

$$G = \beta\Delta t^2 [M + \gamma\Delta t C + \beta\Delta t^2 K]^{-1}$$

Stability and accuracy

Solution for linear SDOF-system: $y(t) = Ye^{vt}$

and its recurrent form: $y_{n+1} = Ye^{v(t+\Delta t)} = \left(e^{v\Delta t}\right)Ye^{vt} = \lambda y_n$

yields a characteristic equation:

$$\lambda^2[m + \gamma\Delta tc + \beta\Delta t^2 k] + \lambda\left[-2m + (1-2\gamma)\Delta tc + \left(\frac{1}{2} - 2\beta + \gamma\right)\Delta t^2 k\right] + \left[m - (1-\gamma)\Delta tc + \left(\frac{1}{2} + \beta - \gamma\right)\Delta t^2 k\right] = 0$$

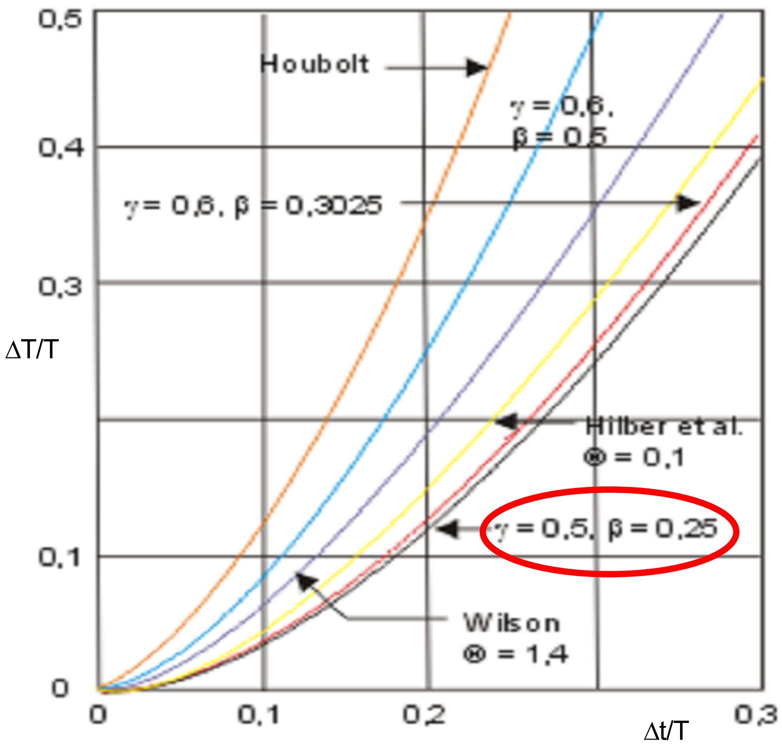
exact solution: $|\lambda| = 1$

stable solution with numerical damping: $|\lambda| < 1$

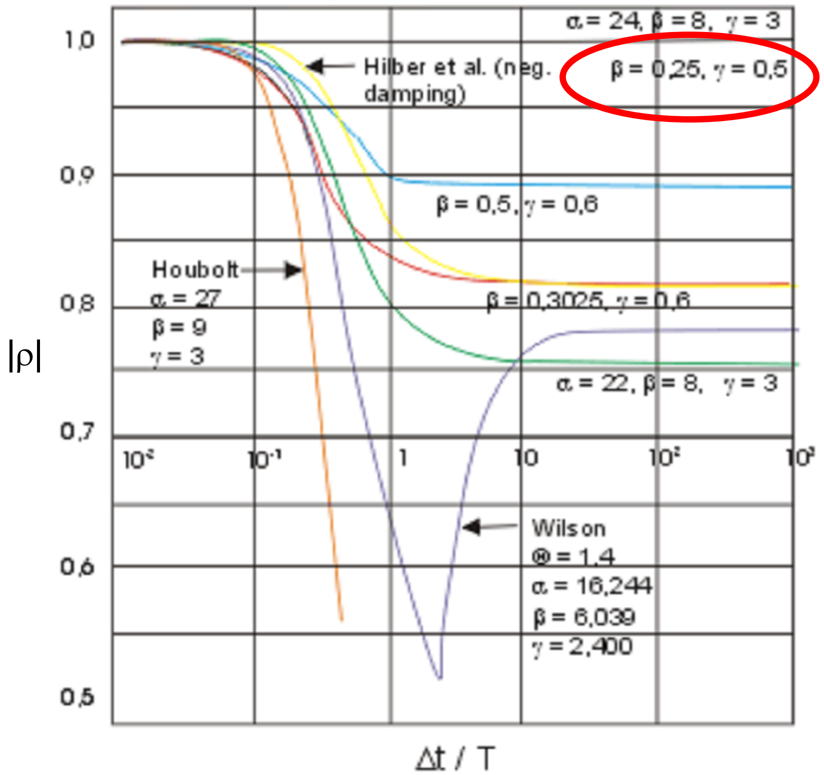
unstable solution: $|\lambda| > 1$

Stability and accuracy

period elongation



numerical damping



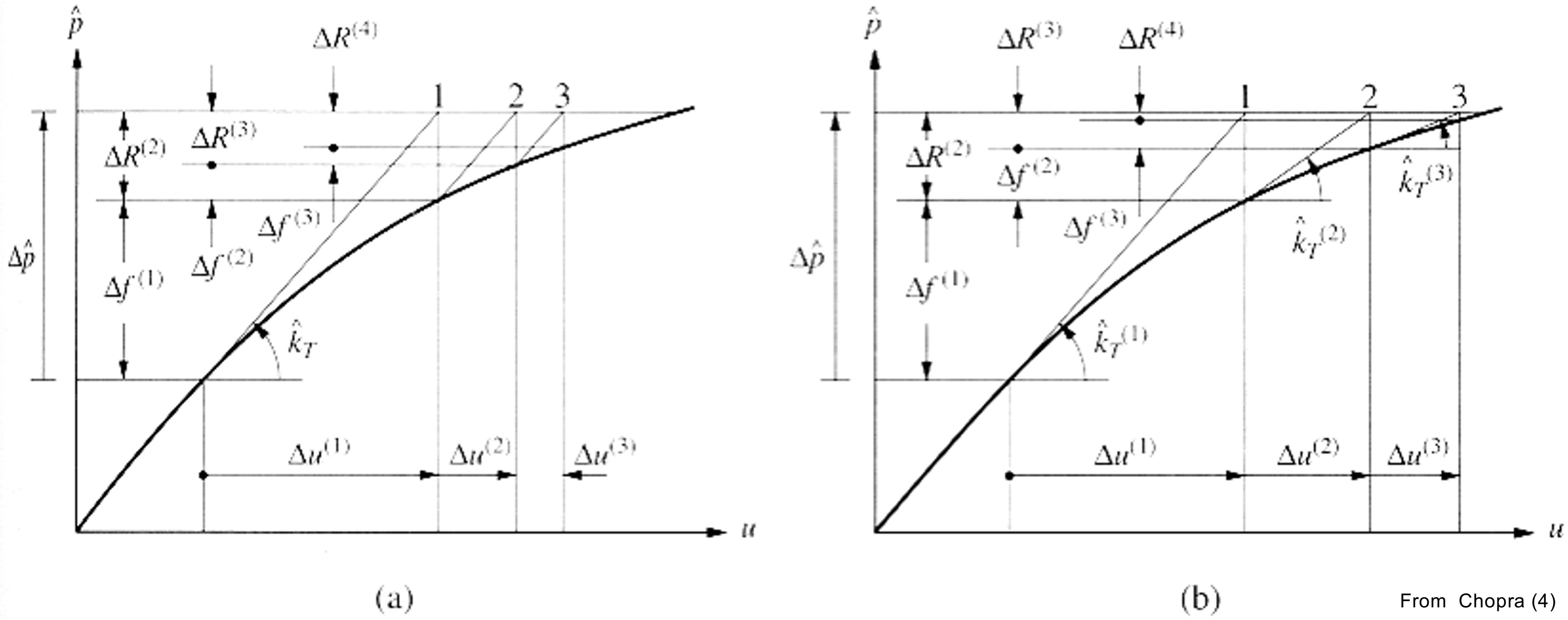
$\gamma = 0.5 \qquad \beta = 0.25$

unconditionally stable Newmark scheme without numerical damping

Newton-Raphson iteration within a time step

$$u^{n+1} = u_0 + G(f_r^{n+1})$$

Since the response f_r at time *n+1* is not known, an iteration within each time step is required to solve this equation:



From Chopra (4)

Figure 5.7.3 Iteration within a time step for nonlinear systems: (a) modified Newton–Raphson iteration; (b) Newton–Raphson iteration.

Newton-Raphson iteration within a time step

TABLE 5.7.1 MODIFIED NEWTON–RAPHSON ITERATION

1.0 *Initialize data.*

$$u_{i+1}^{(0)} = u_i \qquad f_S^{(0)} = (f_S)_i \qquad \Delta R^{(1)} = \Delta \hat{p}_i \qquad \hat{k}_T = \hat{k}_i$$

2.0 *Calculations for each iteration,* $j = 1, 2, 3, \ldots$

2.1 Solve: $\hat{k}_T \, \Delta u^{(j)} = \Delta R^{(j)} \Rightarrow \Delta u^{(j)}$.

2.2 $u_{i+1}^{(j)} = u_{i+1}^{(j-1)} + \Delta u^{(j)}$.

2.3 $\Delta f^{(j)} = f_S^{(j)} - f_S^{(j-1)} + (\hat{k}_T - k_T) \, \Delta u^{(j)}$.

2.4 $\Delta R^{(j+1)} = \Delta R^{(j)} - \Delta f^{(j)}$.

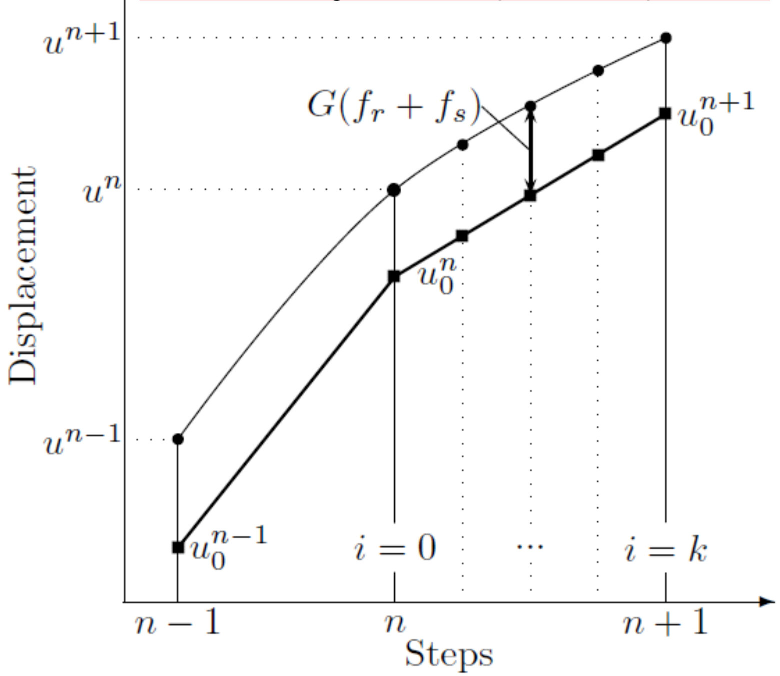
3.0 *Repetition for next iteration.* Replace j by $j + 1$ and repeat calculation steps 2.1 to 2.4.

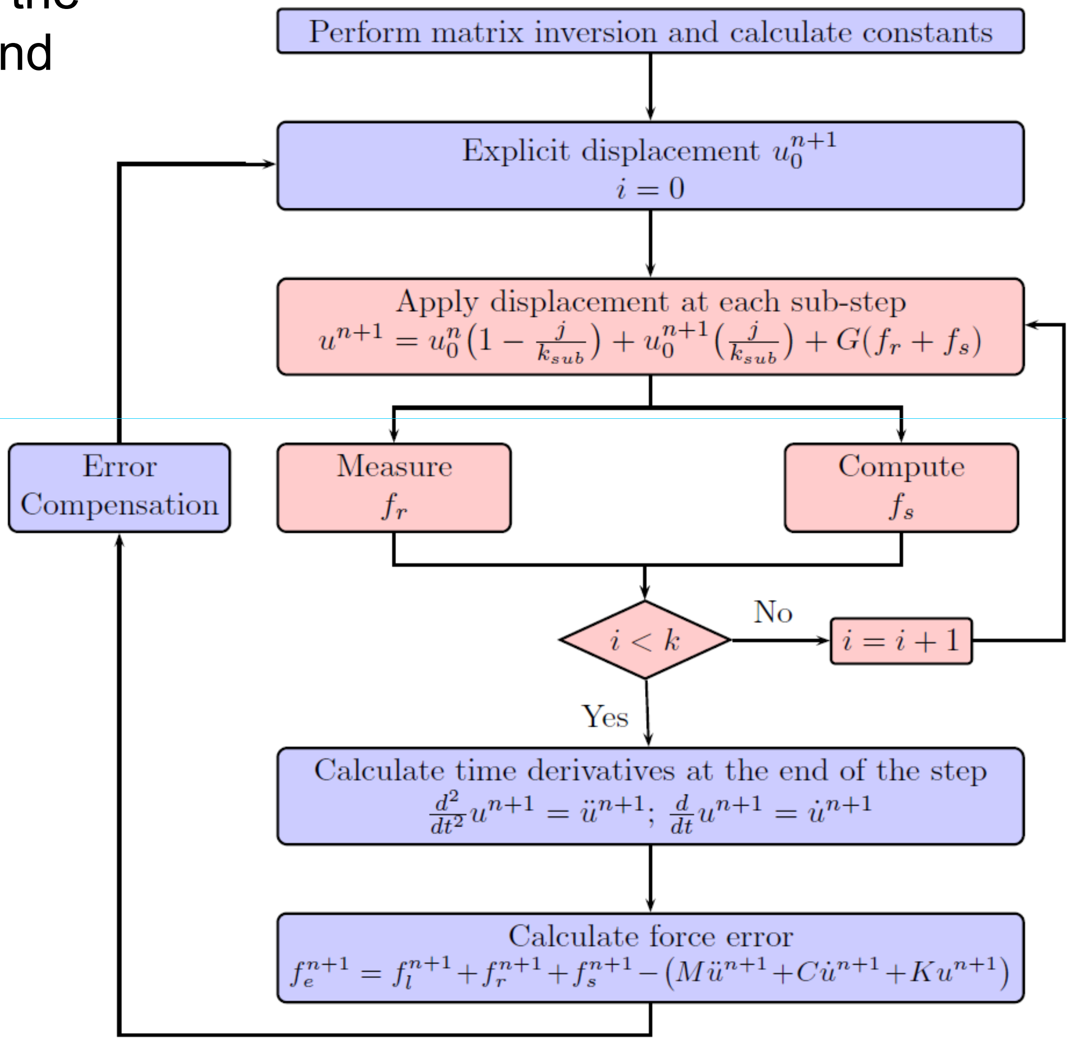
From Chopra (4)

The *Subfeed* algorithm

- Sub-stepping avoids iteration within the time step. This is 100-times faster and more robust.
- It can be used in so-called hybrid simulations where specimen are connected to numerical structures

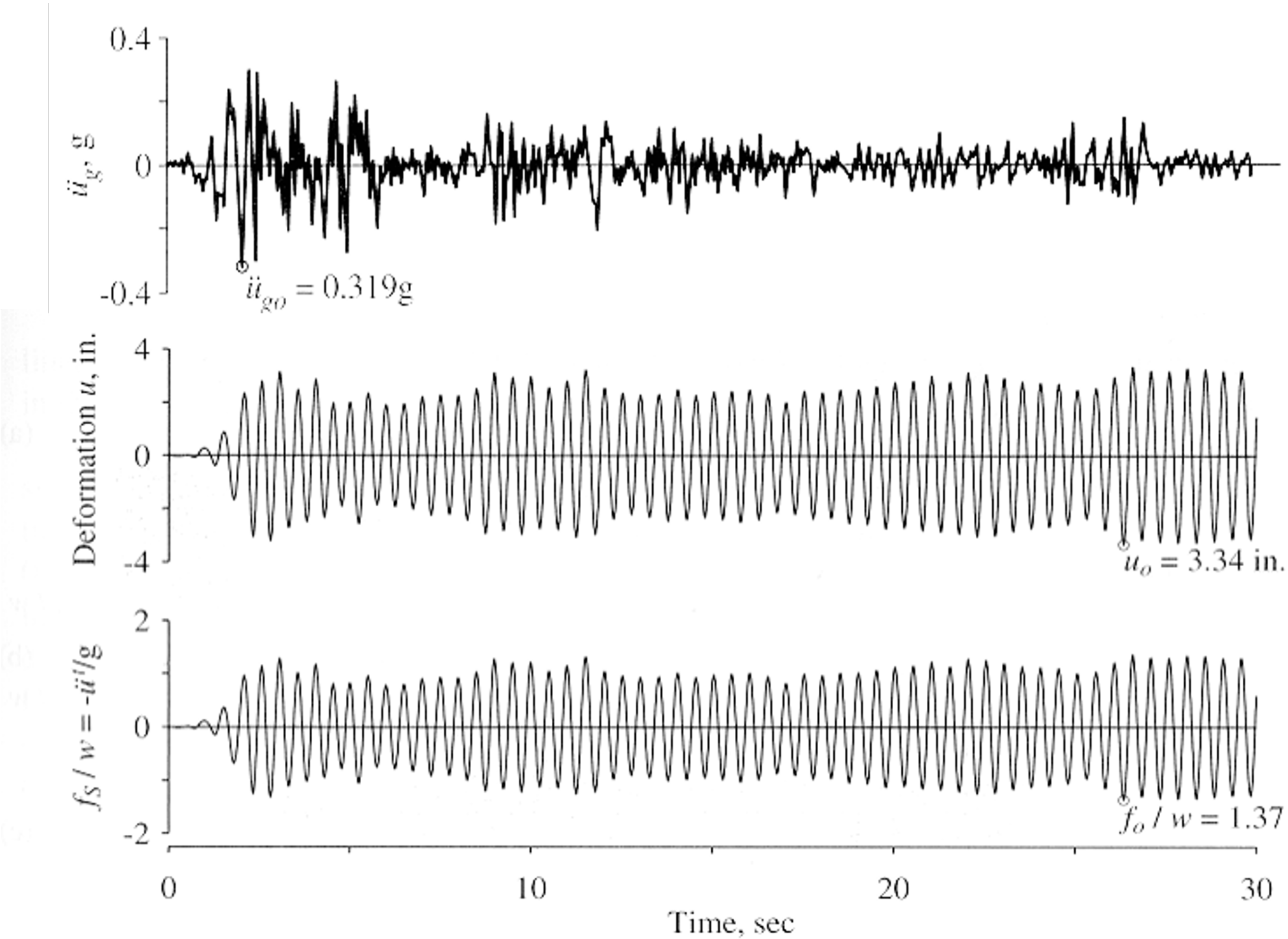
$$u^{n+1} = u_0^{n+1} + G(f_r^{n+1} + f_s^{n+1})$$





Examples of computed time histories

Response of linear SDOF-system with T=0,5 s and ζ=0 to El Centro ground motion



f_s *elastic resisting force*

$\ddot{u}^t$ *total acceleration of the mass*

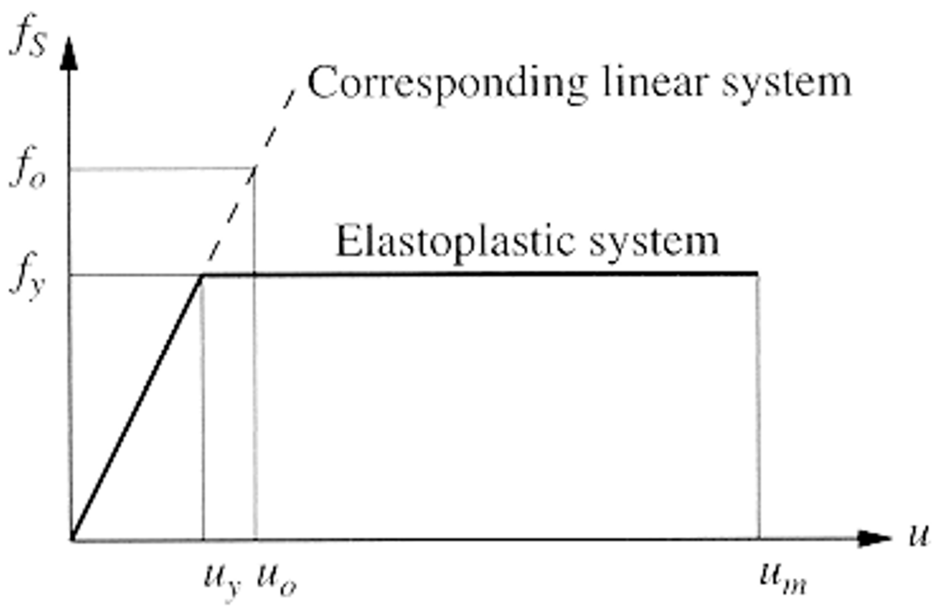
f_0 *peak value of the elastic resisting force*

u_0 *peak value of the total acceleration*

w *weight of the mass*

From Chopra (4)

Examples of computed time histories



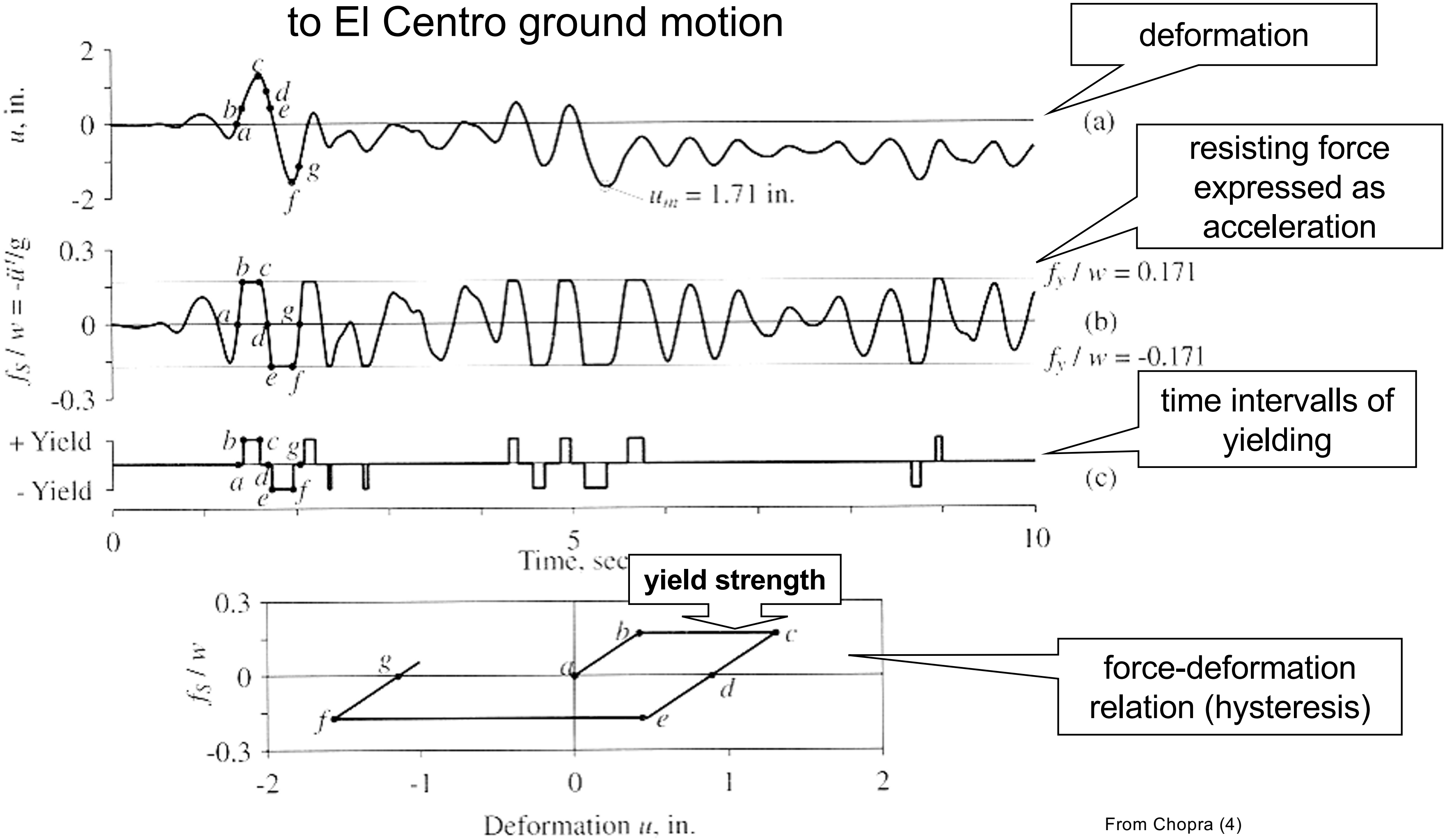
From Chopra (4)

$$\bar{f}_y = \frac{f_y}{f_o} = \frac{u_y}{u_o}$$ *normalized yield strength*

$$\mu = \frac{u_m}{u_y}$$ *ductility factor*

Examples of computed time histories

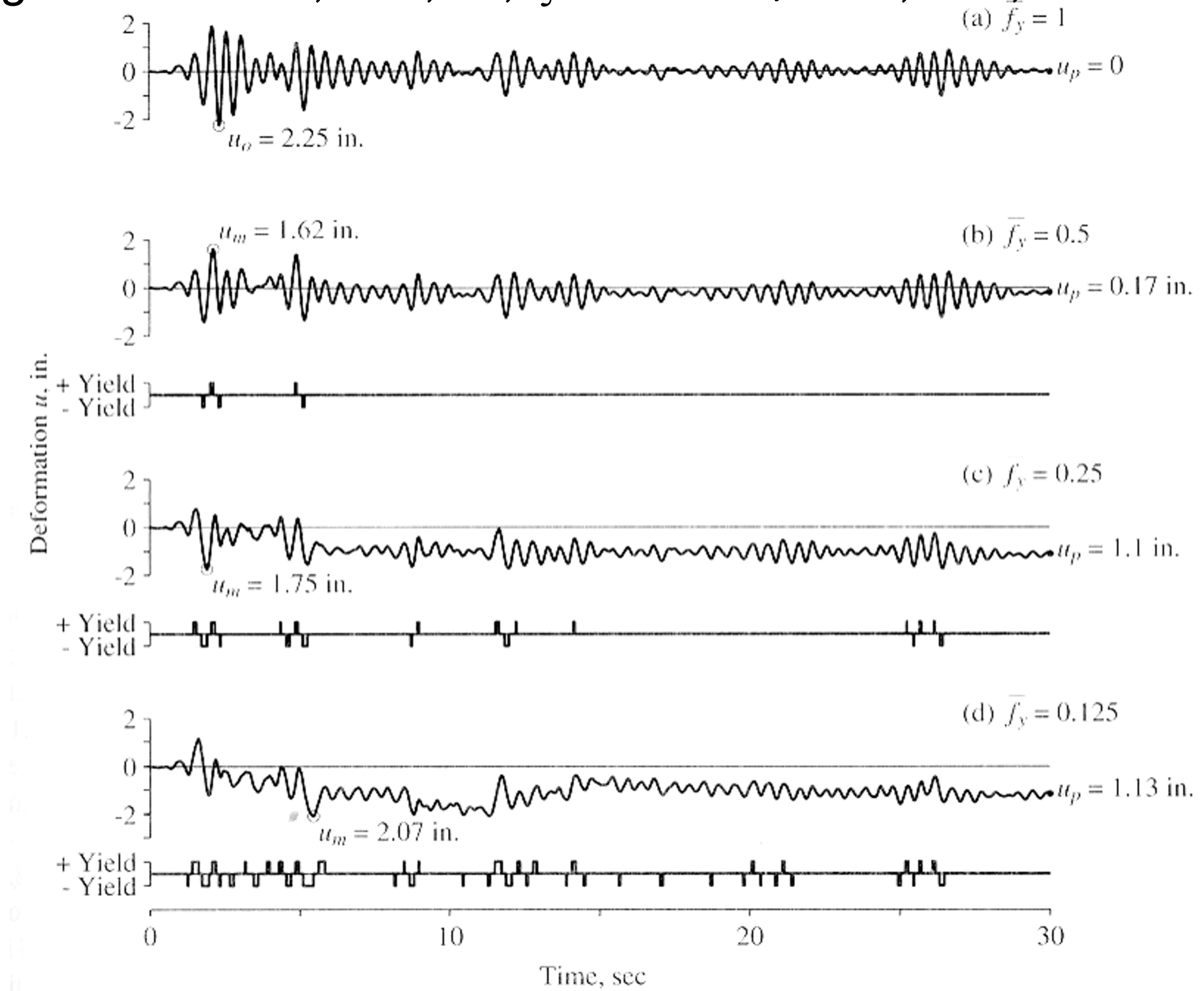
Response of elasto-plastic system with T=0,5 s, z=0 and $\bar{f}_y$=0,125 to El Centro ground motion



From Chopra (4)

Examples of computed time histories

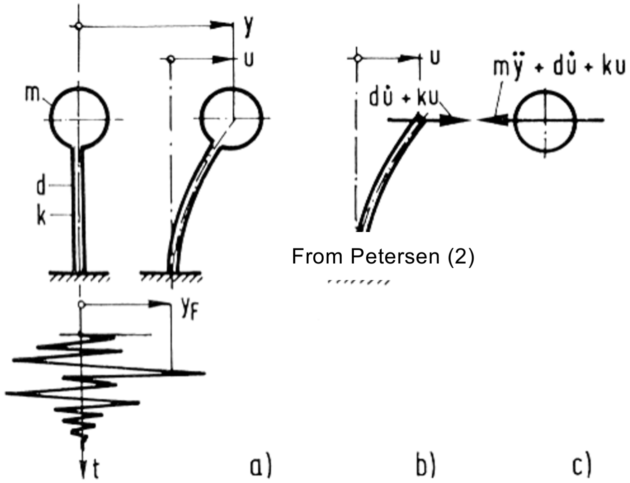
Deformation response and yielding of four systems due to El Centro ground motion; T=0,5 s, ζ=5% and $\bar{f}_y$=1- 0,5 - 0,25 and 0,125



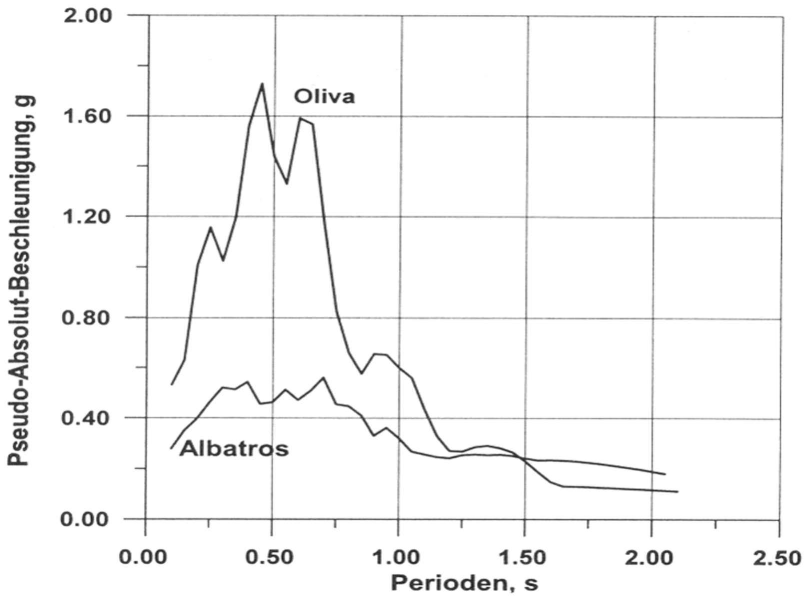
From Chopra (4)

Response spectra

viscously damped SDOF oscillator



From Petersen (2)



From Meskouris (5)

$$m \cdot \ddot{u} + d \cdot \dot{u} + k \cdot u = -m \cdot \ddot{y}_F(t) \quad \Big| \cdot \frac{1}{m}$$

$$\ddot{u} + 2 \cdot \omega \cdot \varsigma \cdot \dot{u} + \omega^2 \cdot u = -\ddot{y}_F(t)$$

solving this equation for various ω and ζ, but only for one ground motion gives the maximum absolute acceleration of every SDOF system subjected to this ground motion

where: Eigenfrequency: $\omega = \sqrt{\frac{k}{m}}$

Damping ratio: $\varsigma = \frac{d}{2 \cdot m \cdot \omega}$

Elastic design spectra given in codes

The acceleration response spectra in EC 8 are given with respect to the subsoil classes

Table 3.1: Classification of subsoil classes

Subsoil class	Description of stratigraphic profile	Parameters		
		$V_{s,30}$ (m/s)	N_{SPT} (bl/30cm)	c_u (kPa)
A	Rock or other rock-like geological formation, including at most 5 m of weaker material at the surface	> 800	_	_
B	Deposits of very dense sand, gravel, or very stiff clay, at least several tens of m in thickness, characterised by a gradual increase of mechanical properties with depth	360 – 800	> 50	> 250
C	Deep deposits of dense or medium-dense sand, gravel or stiff clay with thickness from several tens to many hundreds of m	180 – 360	15 - 50	70 - 250
D	Deposits of loose-to-medium cohesionless soil (with or without some soft cohesive layers), or of predominantly soft-to-firm cohesive soil	< 180	< 15	< 70
E	A soil profile consisting of a surface alluvium layer with $V_{s,30}$ values of class C or D and thickness varying between about 5 m and 20 m, underlain by stiffer material with $V_{s,30}$ > 800 m/s			
S_1	Deposits consisting – or containing a layer at least 10 m thick – of soft clays/silts with high plasticity index (PI > 40) and high water content	< 100 (indicative)	_	10 - 20
S_2	Deposits of liquefiable soils, of sensitive clays, or any other soil profile not included in classes A –E or S_1			

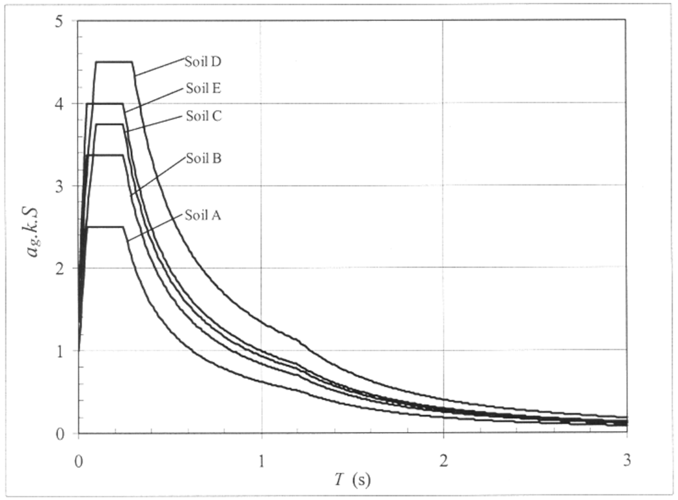


Figure 3.2: Elastic response spectrum, Type 2

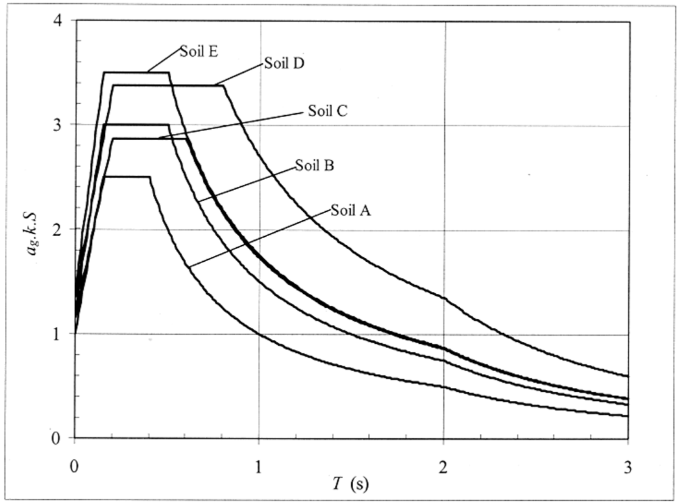
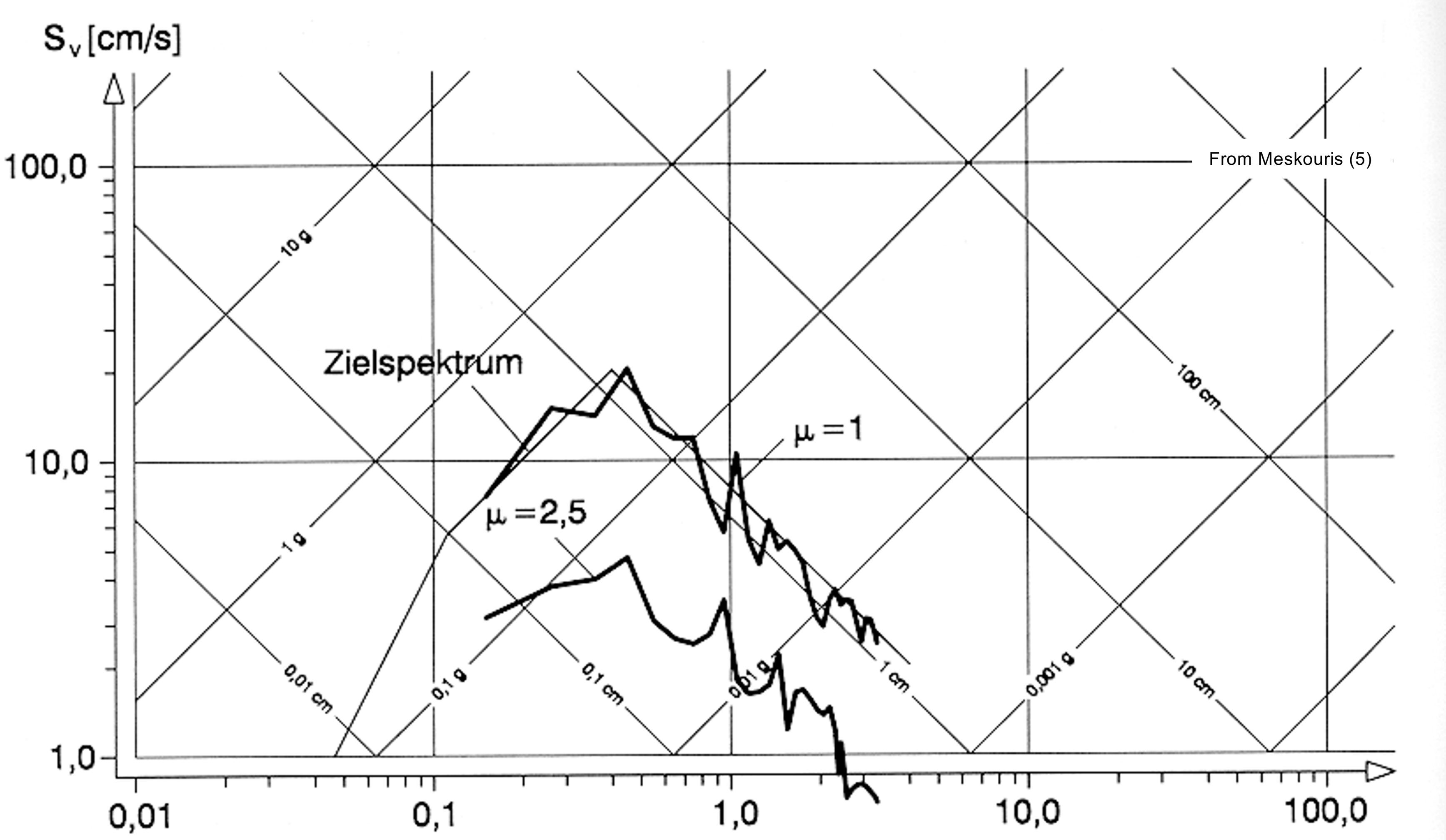


Figure 3.1: Elastic response spectrum, Type 1.

Elastic and inelastic response spectra



Modal analysis

linear equations of motion for a MDOF, homogeneous, un-damped case

$$m \cdot \ddot{u} + k \cdot u = 0$$

solving the eigenvalue problem gives us the modal matrix Φ

$$\Phi = [\Phi_1 \; \Phi_2 \; \Phi_3 \ldots \Phi_n]$$

with this modal matrix, the equations of motion can be transform into modal coordinates

$$(\Phi^T \cdot m \cdot \Phi) \cdot \ddot{u} + (\Phi^T \cdot k \cdot \Phi) \cdot u = 0$$
$$M \cdot \ddot{\eta} + K \cdot \eta = 0$$

or for a damped case with ground acceleration:

$$(\Phi^T \cdot m \cdot \Phi) \cdot \ddot{u} + (\Phi^T \cdot c \cdot \Phi) \cdot \dot{u} + (\Phi^T \cdot k \cdot \Phi) \cdot u = -\Phi^T \cdot m \cdot \{j\} \cdot \ddot{u}_F(t)$$
$$M \cdot \ddot{\eta} + C \cdot \dot{\eta} + K \cdot \eta = 0$$

now we have uncoupled equations of motion for n SDOF-systems in modal coordinates

Modal analysis

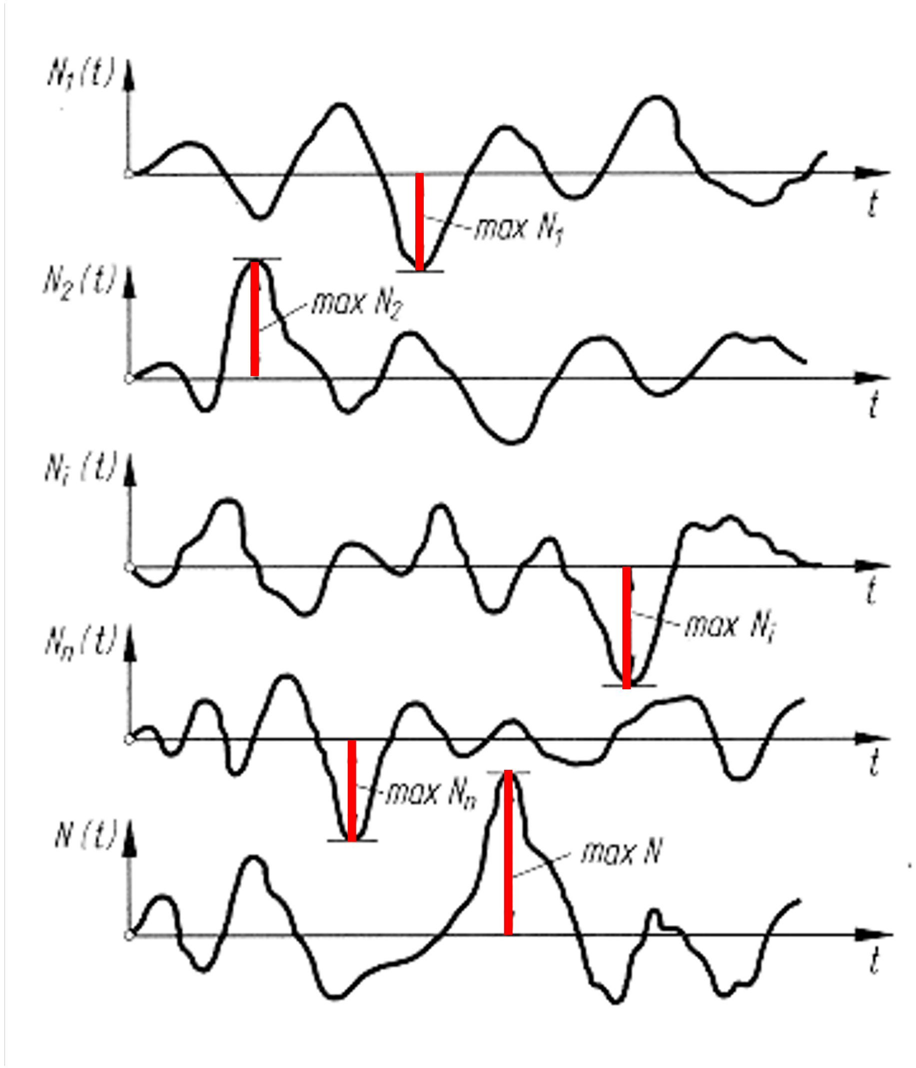
These n equations could be solved in known ways, so we get n solutions in modal coordinates in the time domain

$$\eta(t) = \begin{Bmatrix} \eta_1(t) \\ \eta_2(t) \\ \vdots \\ \eta_j(t) \\ \vdots \\ \eta_n(t) \end{Bmatrix}$$

With the modal matrix we are able to transform these solutions back into local coordinates

$$u(t) = \Phi \cdot \eta(t) = \begin{Bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_j(t) \\ \vdots \\ u_n(t) \end{Bmatrix}$$

Modal analysis using response spectra



The modal responses attain their peaks at different time instants. How do we have to combine these peak values?

$$S = \sum_{j=1}^{n} |S_j| \qquad Superposition\ of\ absolute\ values$$

$$S = \sqrt{\sum_{j=1}^{n} S_j^2}$$

$Square - Root - Sum - of - Square\ (SRSS)$

$$S = \sqrt{\sum_{j=1}^{n} \sum_{k=1}^{n} S_j \cdot \rho_{jk} \cdot S_k}$$

$\rho_{jk} = correlation\ coefficient$

$Complete\ Quadratic\ Combination\ (CQC)$

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