Introduction to Chaos theory

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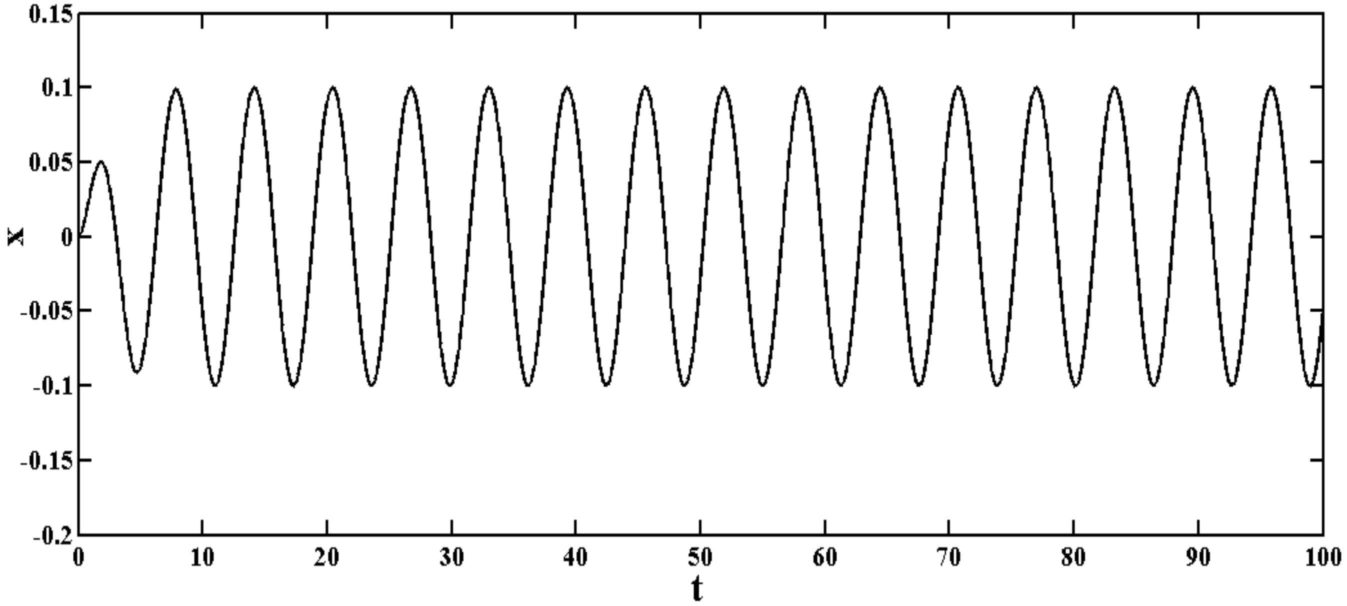
Outline

Chaos in dynamical systems

Linear system

- Model: $$M\ddot{x}+c\dot{x}+kx=f\left(t\right)$$

Response always periodic.

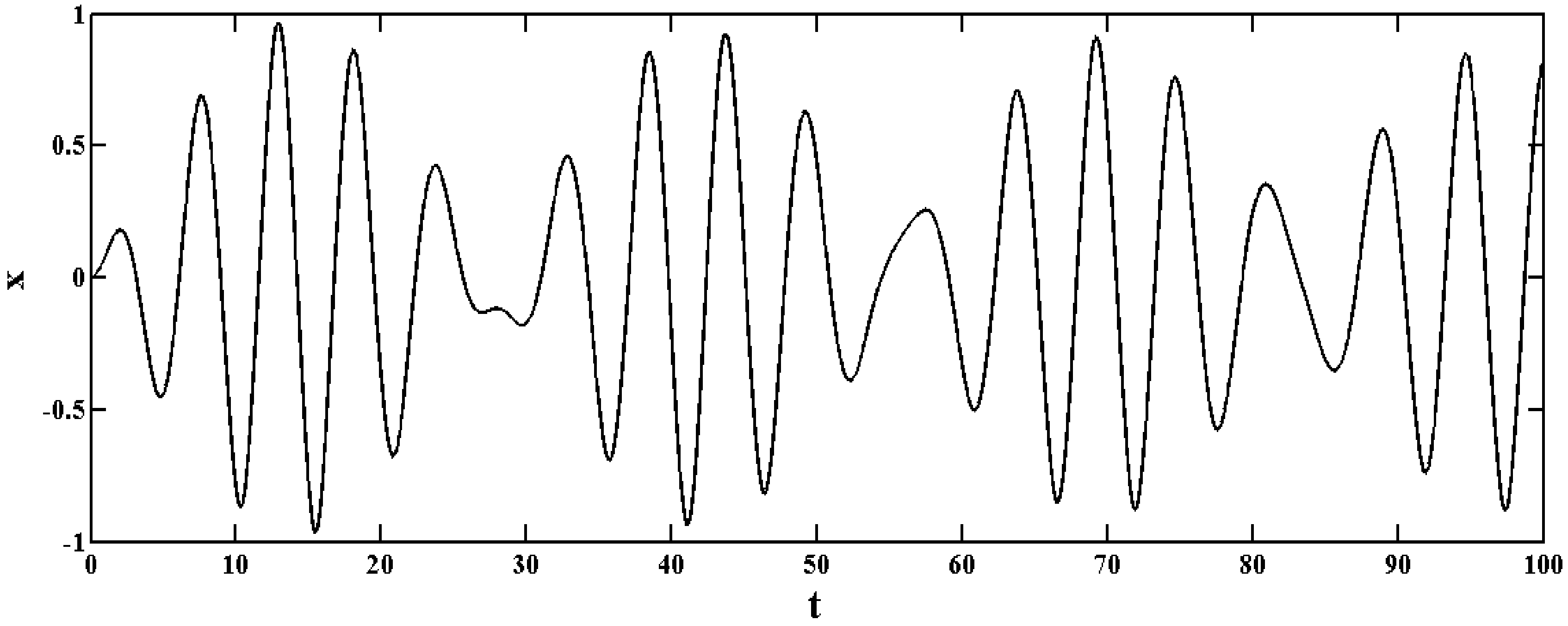


Chaos in dynamical systems

Non-linear system

- Model: $M\ddot{x}+cf\left(x,\dot{x}\right)+kg\left(x\right)=f\left(t\right)$

Response can be periodic or not depending of the parameter of the system.



Chaos in dynamical systems

- The presence of the nonlinearity on the system give more precision on their dynamics future

- Analytically it become quite impossible to obtain a complet solution of the system

- There exist some analytical methods to obtain some approximated solution called, pertubed methods, Averaging Methods, Harmonic balance methods and so on

- For some system with strong noninearity sometimes is only possible to obtain the solution numerically using some well known algorithm

Chaos in dynamical systems

Stochastic system:

- Unpredictable

- Probabilistic

Chaotic system:

- Predictable

- Deterministic

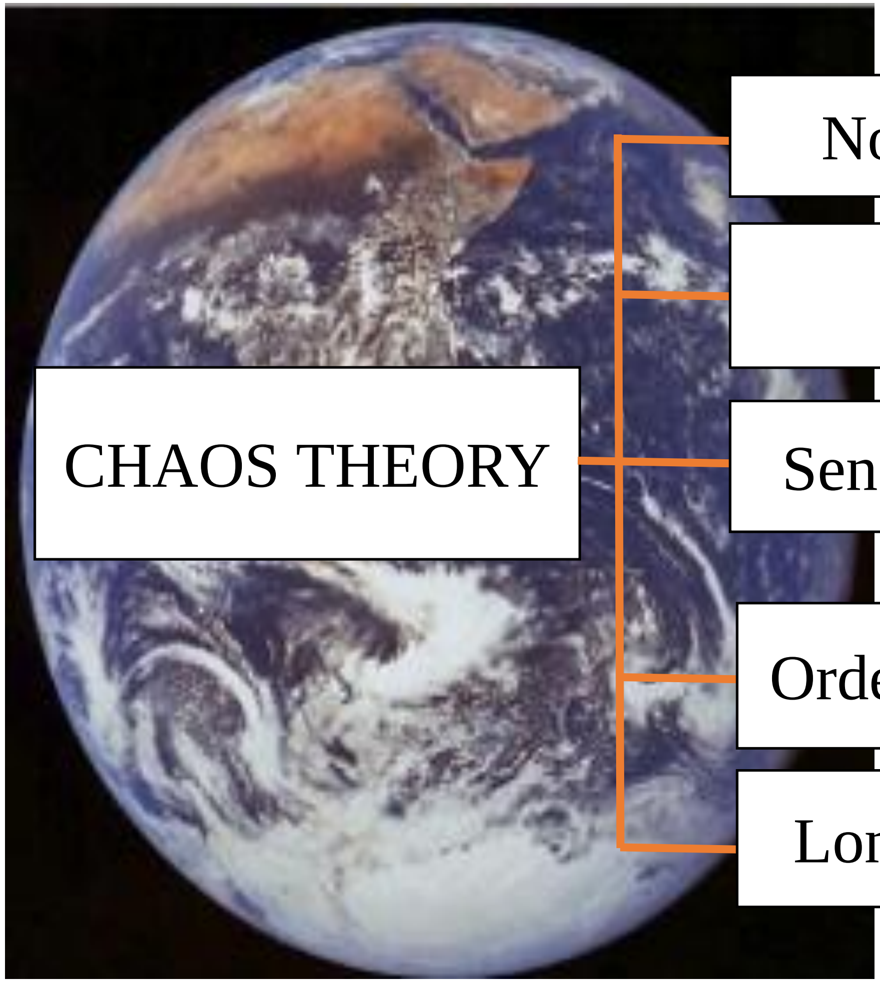
Chaos in dynamical systems

Chaotic systems are unstable since they tend not resist any outside disturbances but instead react in significant ways

- Dynamics system : simplified model for the time-varying behavior of and actual system. The system are described using differential equations specifying the rates of change for each variable.

- Deterministic system : system is which no randomness is involed in the development of future states of the system. This property implies that two trajectories emerging from two different close-by initial conditions separate exponentially in the course of time.

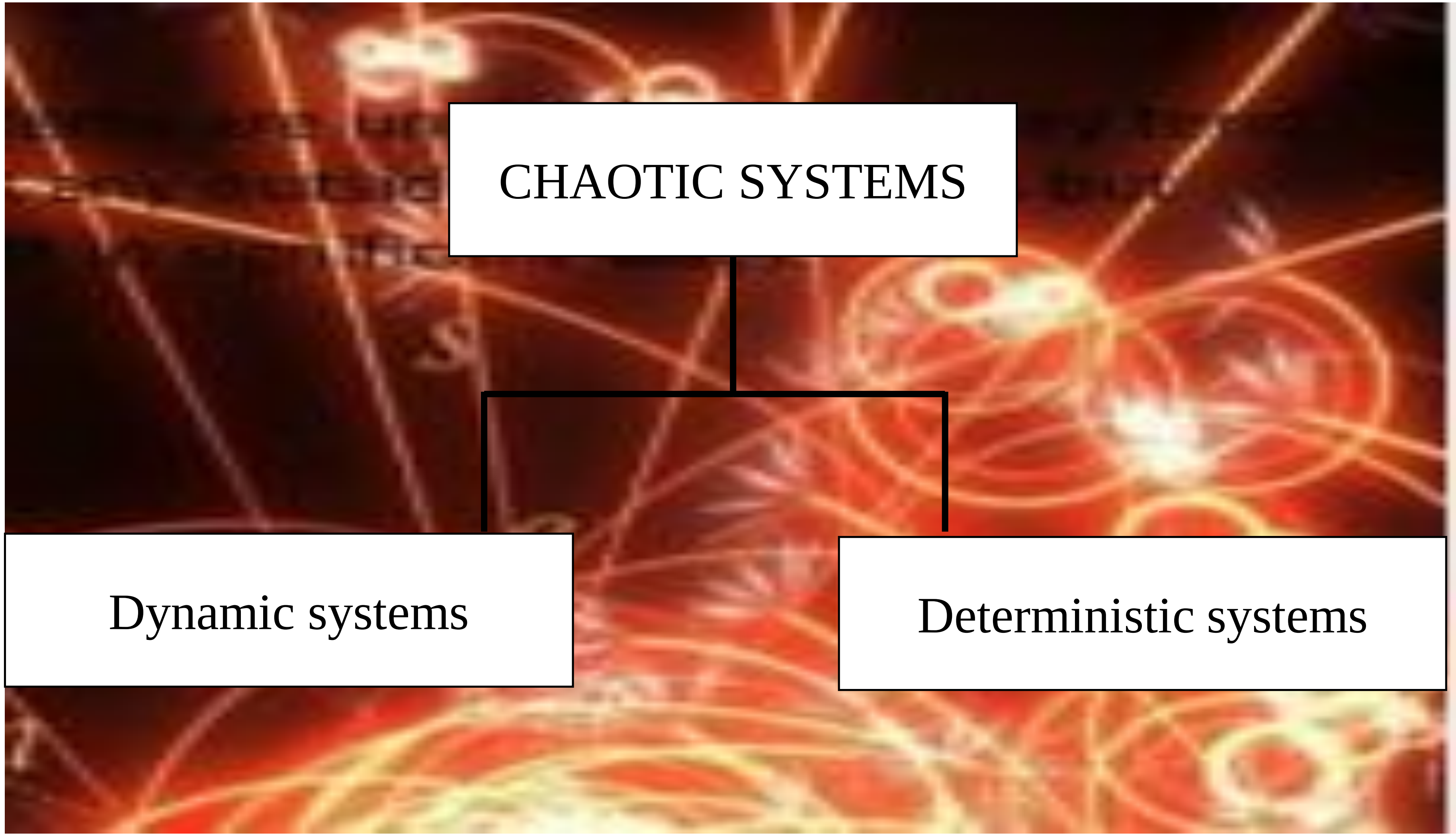
GENERALIZATION



CHAOS THEORY

- Nonlinearity
- Determinism
- Sensitivity to initial conditions
- Order in disorder
- Long-term prediction is mostly impossible

Chaos in dynamical systems



Chaos in dynamical systems

Types of dynamical systems:

- Differential equations: time is continuous (called flow)

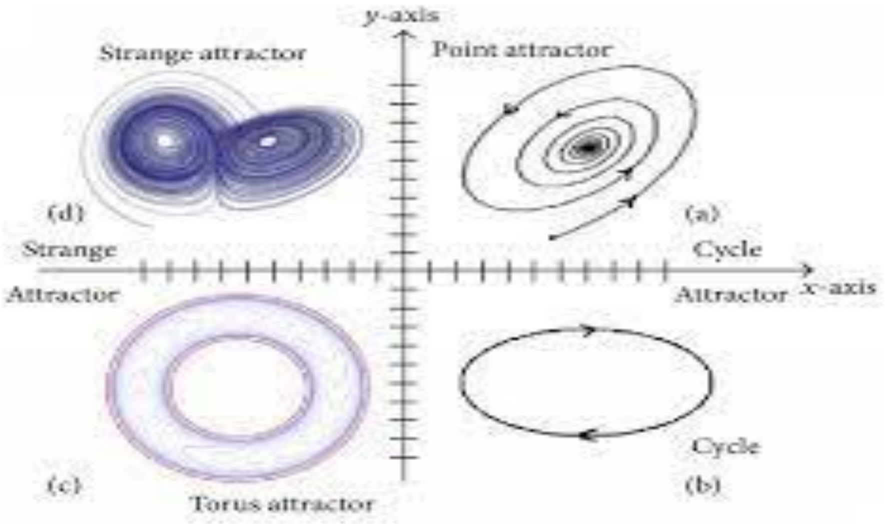
$$\frac{dx}{dt} = f(x), \qquad t \in \mathrm{R}^{\mathrm{N}}$$

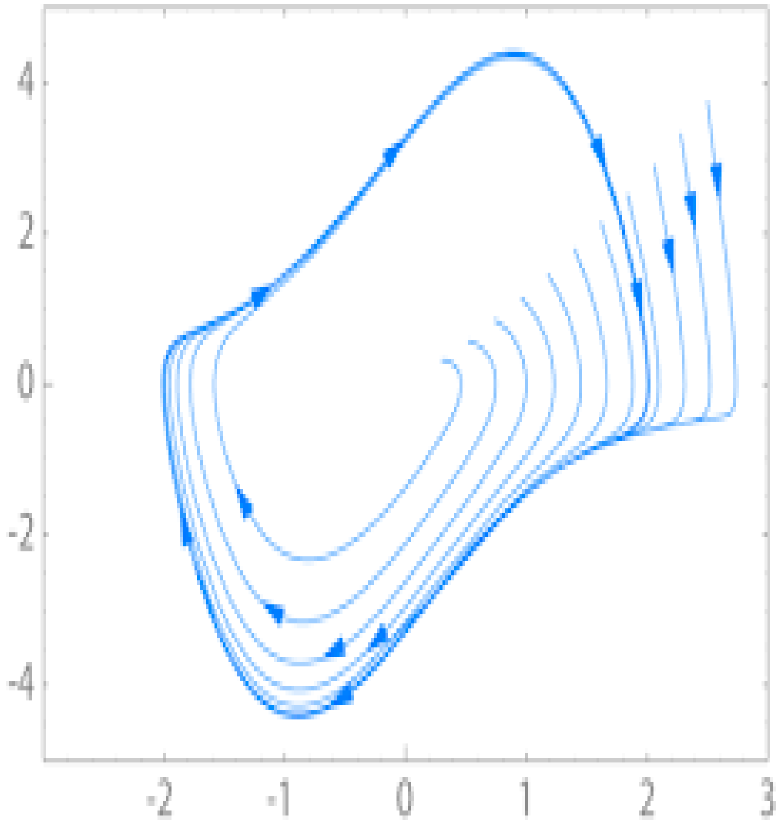
- Difference equations (iterated maps): time is discrete (called cascade)

$$x_{n+1} = f(x_n), \qquad n = 0, 1, 2, \ldots$$

Attractors or fixed points

- .Attractors and limit cycle





- Video on fixed point

COMPLEXITY
EXPLORER
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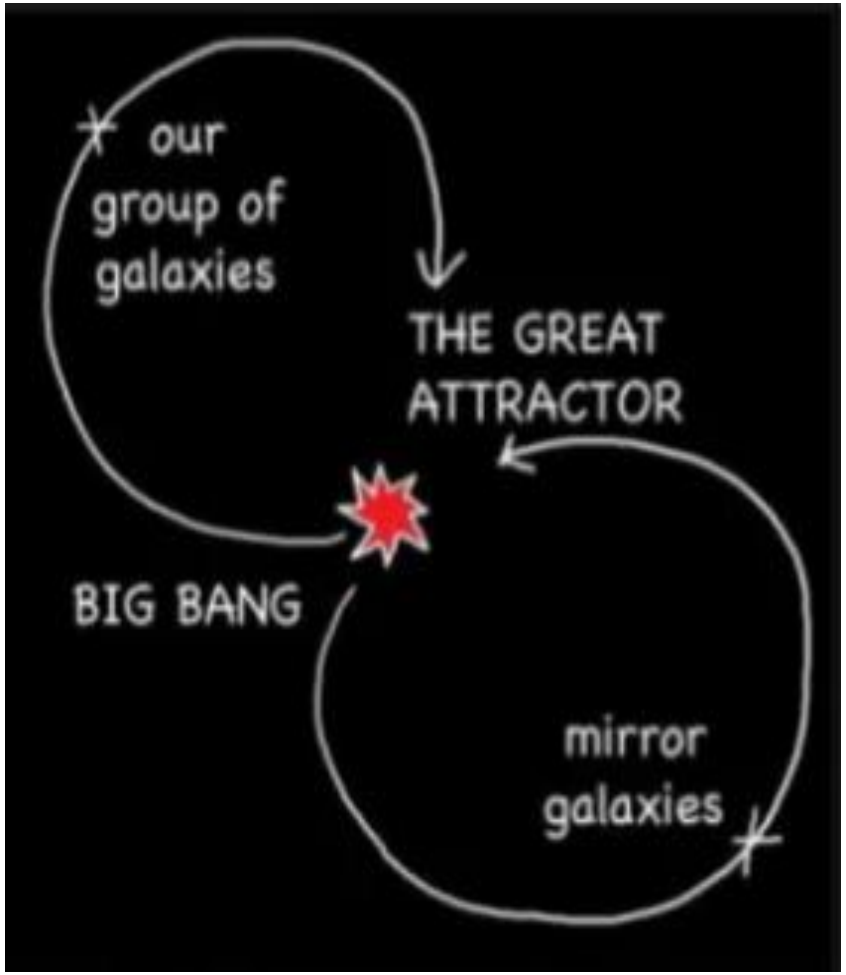
Chaos in dynamical system

Phase portrait and Poincare

- In chaos theory, systems evolve towards states called attractors. The evolution towards a specifie state is governed by a set of initial condition. An attractor is generated within the system itself.

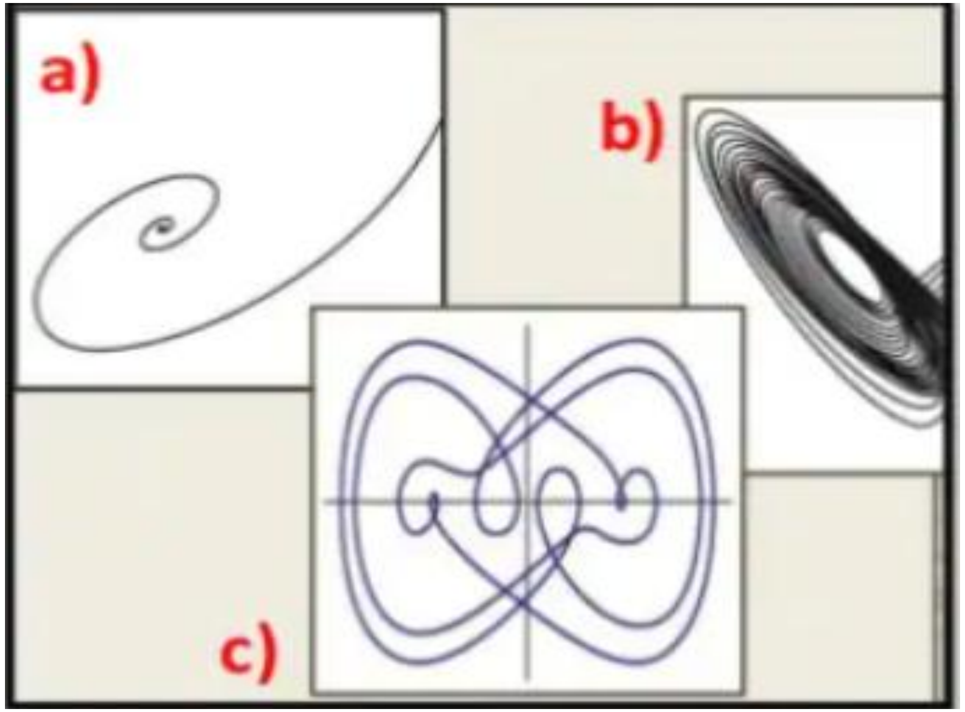
- Attractor: smallest unit which cannot itself be decomposed into two or more attractors with distinct basins of attraction.



Phase portrait: Chaos indicator

a) Point attractor: There is only one outcome for the system. Death is a point attractor for living things.

b) Limit cycle or periodic attractor instead moving to a single state as in a point attractor, the system settles into a cycle.



c) Strange attractor or a chaotic attractor: double spiral which never repeats itself. Strange attractors are shapes with fractional dimension; the are fractals

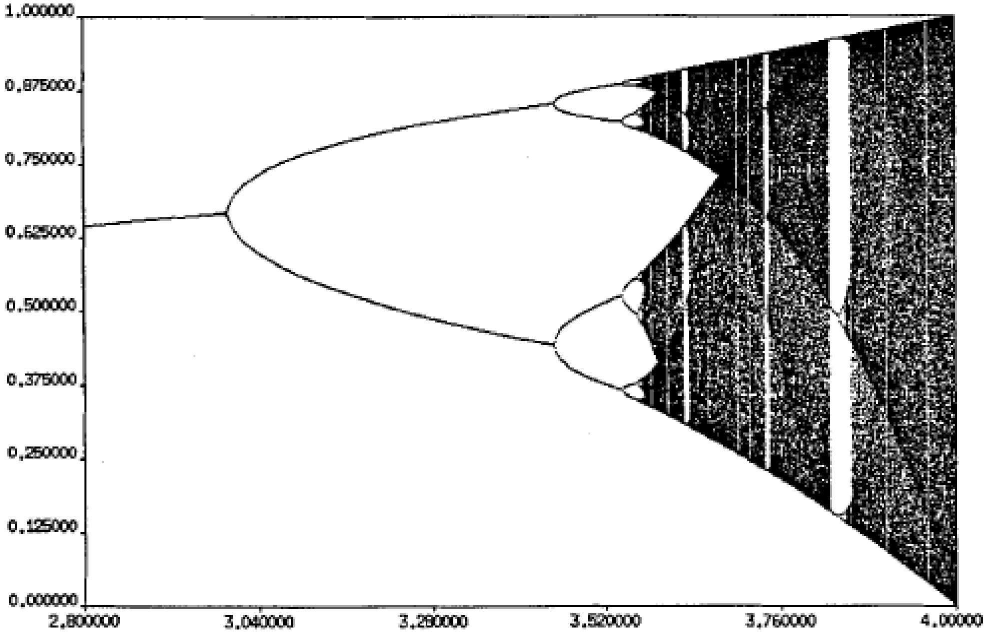
Bifurcation diagram: indicator of Chaos

To answer the question, we need discrete dynamical systems given by *one-dimensional maps*

- Bifurcation diagram for one-dimensional logistic map. Regular and chaotic dynamics

$$f_a : x \to ax(1-x), \quad x \in R^1, \quad a \in [0,4]$$

x



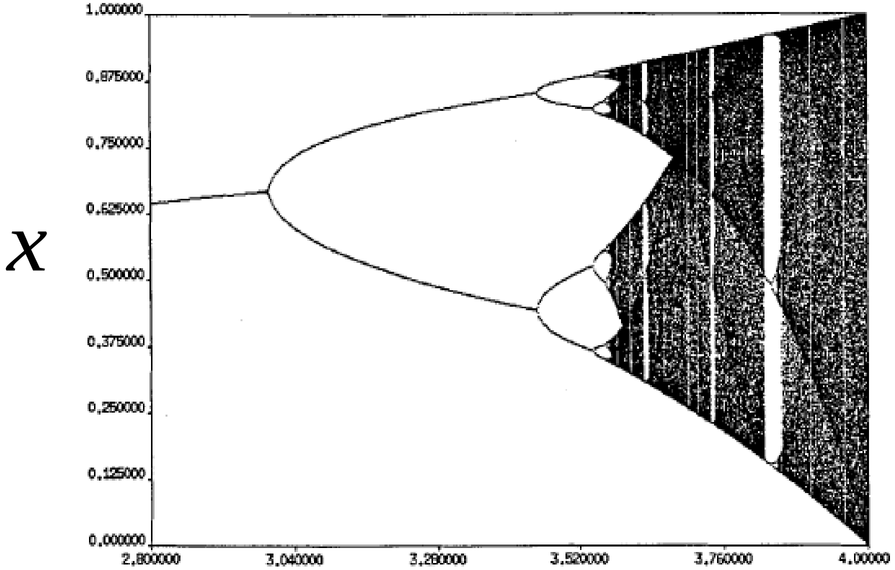
system parameter a

Lyapunoy exponent: indicator of Chaos

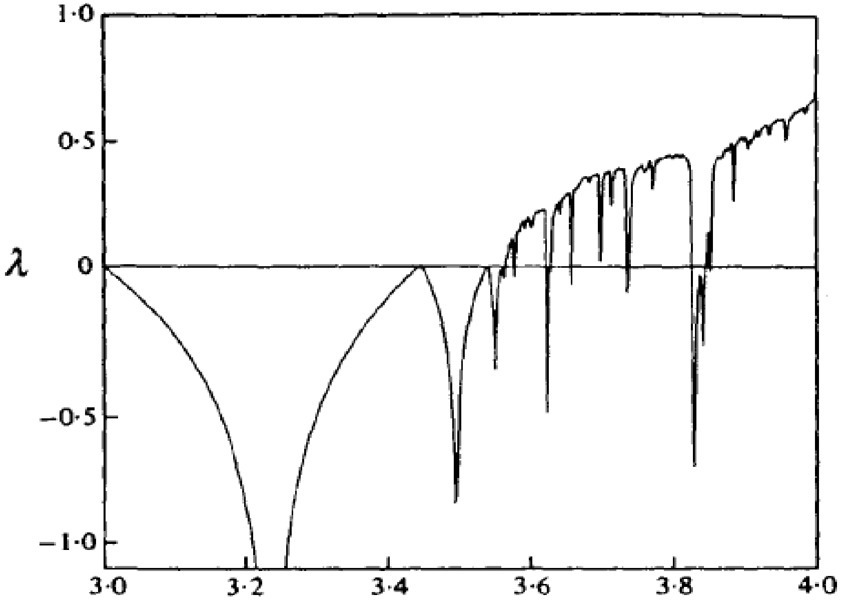
- Lyapunov exponent for logistic map.

$$f_a : x \rightarrow ax(1-x), \quad x \in R^1, \quad a \in [0,4]$$

Bifurcation diagram



Lyapunov exponent λ



parameter a

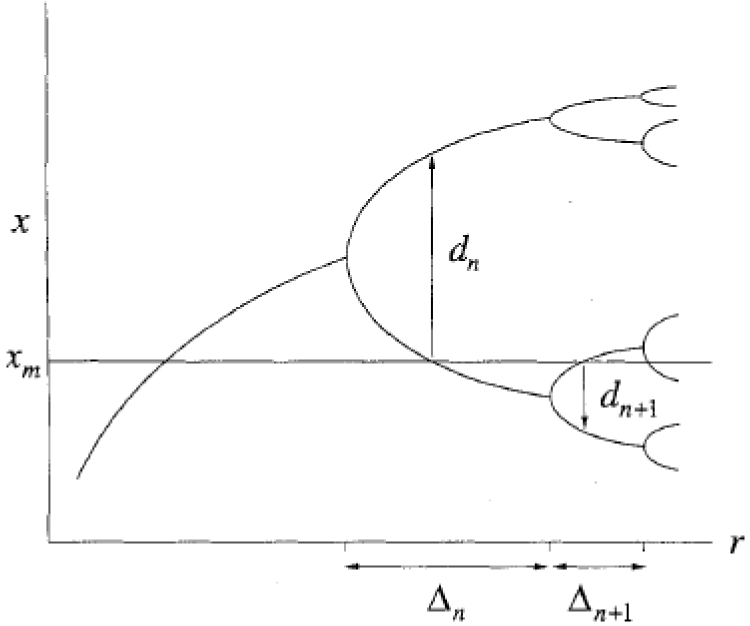
is positive on a nowhere dense, Cantor-like set of parameter *a*

How common is chaos in dynamical systems?

- Cascade of period-doubling bifurcation Feigenbaum (1978).

$$\delta = \lim_{n\to\infty} \frac{r_n - r_{n-1}}{r_{n+1} - r_n} = 4.669\ldots$$

$$\frac{d_n}{d_{n+1}} \to \alpha = -2.5029\ldots$$



Sharkovsky ordering (1964)

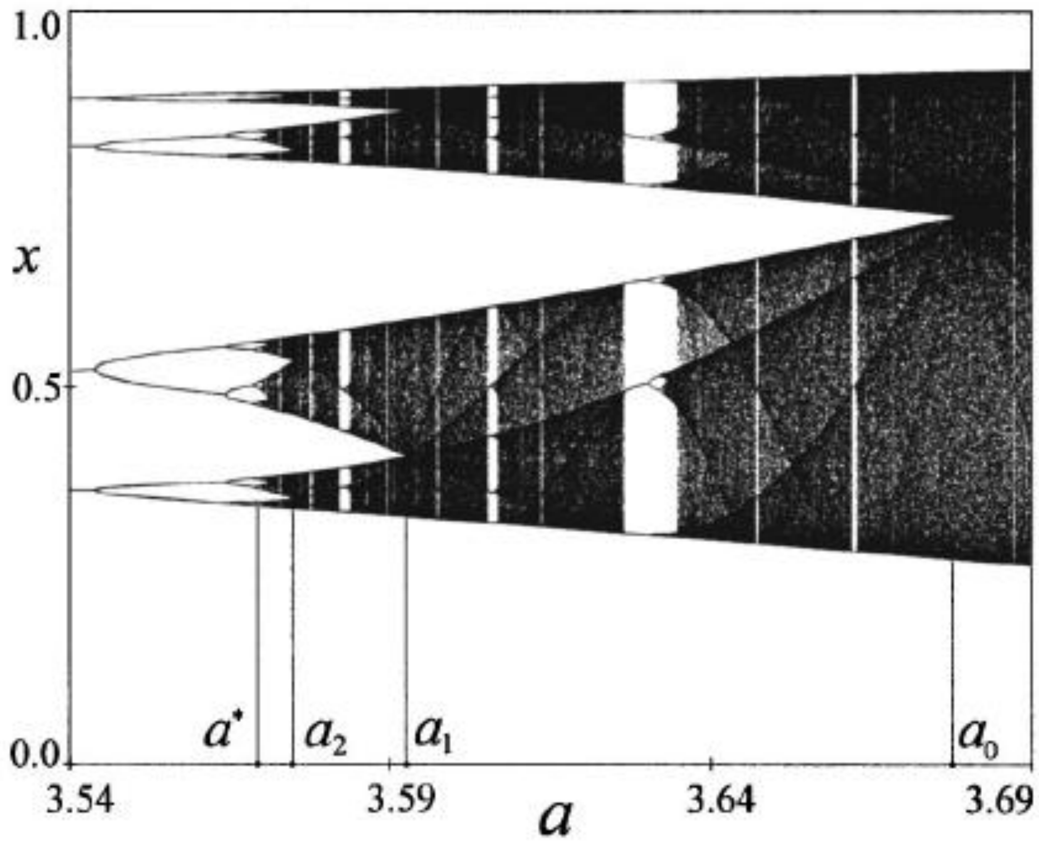
❑ Cascade of homoclinic bifurcations

$a_0 = 3.678\,573\,510\,428\,32\ldots,$

$a_1 = 3.592\,572\,184\,106\,97\ldots,$

$a_2 = 3.574\,804\,938\,759\,20\ldots,$

$a_3 = 3.570\,985\,940\,341\,61\ldots.$



Analytical technique

In generall analyticall is difficult to predict Chaos.

For engineering purpose, it should be nice to be able to derived in the space paraemeters of the system the conditions for wich chaos appears on the system.

Knowing one should be able to control the appearance of chaotic situation

for that it exist two technics : the Shilnikov and the Melnikov theorem.

In the next course we will focus on Melnikov techniques witch enable to detect the

Conclusions

- Eveything in the universe is under control of chaos or product of chaos.

- Irregularity leads to complex systems.

- Chaotic systems are very sensitive to the initial condition, this makes the system fairly unpredictable. They never repeat but they always have some order. That is the reason why chaos theory has been seen as potentially. ***"one of the three greatest triumphs of the 21 st century.'*** In 1991, James Marti speculated that ***"chaos might be the new world order"***

- It gives us a new concept of measurements and scales. It offers a fresh way to proceed with observational data.