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Identification of horseshoes chaos in a cable-stayed bridge subjected to randomly moving loads

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ABSTRACT

In this paper, the dynamic response of cable-stayed bridge loaded by a train of moving forces with stochastic velocity is investigated. The cable-stayed bridge is modelled by Rayleigh beam with linear elastic supports. The stochastic Melnikov method is derived and the mean-square criterion is used to determine the effects of stochastic velocity and cables number on the threshold condition for the inhibition of smale horseshoes chaos in the system. The results indicate that the intensity of the random component of the loads velocity can be contributed to the enlargement of the possible chaotic domain of the system, and/or increases the chances to have a regular behavior of the system. On the other hand, the presence of cables in cable-stayed bridges system increases it degree of safety and paradoxically can be contributed to its destabilization. Numerical simulations of the governing equations are carried out to confirm the analytical prediction. The effect of loads number on the system response is also investigated.

1. Introduction

Cable-stayed bridges have become very popular over the last three decades because of their aesthetic appeal, structural efficiency, enhanced stiffness compared with suspension bridges, ease of construction and comparatively small size of structures. Response prediction of this type of bridges subjected to randomly moving excitations is important for engineering practice [1,2].

The vibrations of a suspension bridge under a random train of moving loads are discussed in detail by Bryja and Śniady [3–5]. Generally, a very important parameter in the study of the vibration of bridges caused by moving loads is the velocity. Although there is scarcity of publications on this subject, one can mention the work of Zibdeh [6] who included the effect of random velocities on the dynamic response of a bridge traversed by a concentrated load. Chang et al. [7] investigated the dynamic response of a fixed–fixed beam with an internal hinge on an elastic foundation, which is subjected to a moving mass oscillator with uncertain parameters such as random mass, stiffness, damping, velocity and acceleration. In the same impetus, Śniady et al. [8,9] and Rystwej et al. [10] investigated on the problem of a dynamic response of a beam and a plate to the passage of a train of random forces. In this study they assumed that the random train of forces idealizes the flow of vehicles having random weights and travelling at the stochastic velocity. They show the effect of these stochastic quantities on the mean deflection of the beam.

On one hand, in all of the above-mentioned research, only the effect of stochastic parameters of the moving loads on the probabilistic features of the beam response namely the mean square amplitude and the probability density function is carried out. To the best knowledge of the authors, the effects of stochastic fluctuations of the load velocity and the number of cables on the possible appearance of horseshoes chaos in the cable-stayed bridge system have not been explored by the researchers yet. Thus in this paper, based on the Melnikov approach, which is widely used by most researchers [11–15], all these effects on the appearance of transverse intersection of perturbed and unperturbed heteroclinic orbits and the route to chaos are investigated.

Following this introduction, the effective model of cable-stayed bridge is presented in Section 2. Also, the random Melnikov analysis for the examination of the effect of a noisy part of velocity of moving loads and cables effects on the threshold condition for the inhibition of chaos is extended. Section 3 presents some numerical simulations to validate the theoretical predictions. Finally, Section 4 is devoted to the conclusion.

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2. The bridge model

This section is devoted to the presentation of the system (Section 2.1), the corresponding reduced modal equations (Section 2.2). The last Section 2.3 is devoted to the theoretical analysis of the random Melnikov analysis applied to the proposed model.

2.1. Mathematical modelling

The dynamic model of a cable-stayed bridge system investigated in this paper and shown in Fig. 1(a) is the semi-harp type with two symmetrical spans. The cable-stayed bridge is modelled by using a Rayleigh beam theory [16] (in order to take into account the high frequency motion of the beam) of finite length L with geometric nonlinearities on elastic supports with linear stiffness K_i^c subjected to an axial compressive loads $T_h{}^c$ due to the total contribution of the horizontal component of the tensile cables and a series of lumped loads p moving along the beam in the same direction with the same stochastic velocity v_k (see Fig. 1 (b)). We assume that the mass of the cables is negligible and they are regularly spaced on the beam. Since all the stay cable anchorage sections are fixed to move both horizontally and vertically, the whole pylon is assumed to be fixed.

The deformed beam can be described by the transverse deflection $W = W(X, t)$ and the rotation of the cross section of the beam $\theta = \theta(X, t)$. By Considering the classical damping force model for the viscosity materials and Newton's second law of motion for an infinitesimal element of the beam, the equation of motion for the small deformations $\left(\theta\left(X, t\right) \simeq \frac{\partial W(X,t)}{\partial X}\right)$ for this system is then given by

$$m_b \frac{\partial^2 W}{\partial t^2} - R_a \frac{\partial^4 W}{\partial X^2 \partial t^2} + c\frac{\partial W}{\partial t} + T_h^c \frac{\partial^2 W}{\partial X^2} + \sum_{i=0}^{N_c} K_i^c \delta\left[X - i\frac{L}{N_c}\right] W + EI\frac{\partial^2}{\partial X^2}\left[\frac{\partial^2 W}{\partial X^2}\left(1-\frac{3}{2}\left(\frac{\partial W}{\partial X}\right)^2\right)\right] = P\sum_{k=1}^{N_v}\varepsilon_k \delta\left[X - X_k(t-t_k)\right] \quad (1)$$

In which m_b, EI, ρ, R_a, c, $W(X, t)$ are the beam mass per unit length, the flexural rigidity of the beam, beam material density, the transverse Rayleigh beam coefficient, the damping coefficient and

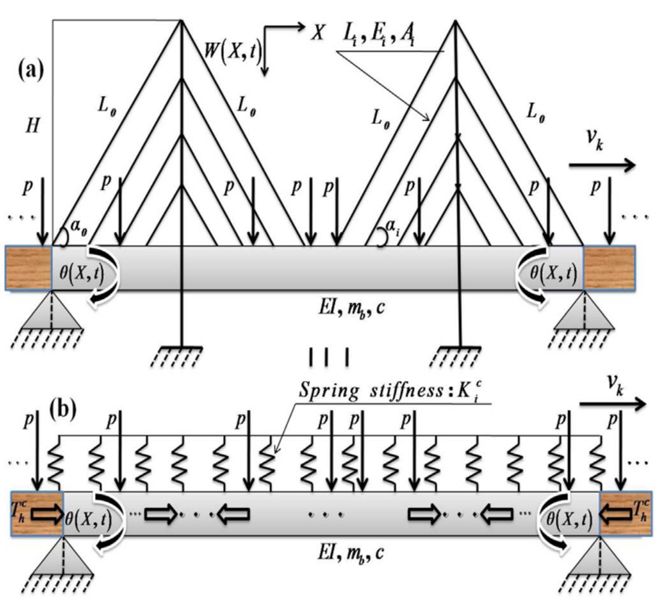

Fig. 1. Sketch of (a) the cable-stayed bridge system, (b) equivalent model under stochastic moving loads. The gravitational forces are represented by arrows p, whose separations are not uniform, for the speeds v_k are not identical.

the transverse defection of the beam at point X and time t respectively. $T_h{}^c$ is the axial compressive loads due to the total contribution of the horizontal component of the tensile cables. In Eq. (1), $m_b \frac{\partial^2 W}{\partial t^2}$ represents the inertia force of the beam per unit length, $R_a \frac{\partial^4 W}{\partial X^2 \partial t^2}$ is the rotary inertia force of the beam element (per unit length), $c\frac{\partial W}{\partial t}$ is the damping force of the beam per unit length, $T_h^c \frac{\partial^2 W}{\partial X^2}$ is the axial compressive load (per unit length) due to the horizontal component of the stay cables, $\sum_{i=0}^{N_c} K_i^c \delta\left[X - i\frac{L}{N_c}\right] W$ is the tension of cables per unit length, $EI\frac{\partial^2}{\partial X^2}\left[\frac{\partial^2 W}{\partial X^2}\left(1-\frac{3}{2}\left(\frac{\partial W}{\partial X}\right)^2\right)\right]$ is the nonlinear rigidity of beam essentially due to the Euler law which states that the bending moment of the beam is proportional to the change in the curvature produced by the action of the load [17,18]. This nonlinear term is obtained by using the Taylor expansion of the exact formulation of the curvature up to the second order. The term on the right-hand side of Eq. (1) is used to describe the series of random moving loads over the beam. $X_k(t - t_k)$ is the distance covered by the kth force to the time t. $t_k = (k-1)d/v_0$ = deterministic arriving time of the kth load at the beam. d is the spacing loads, $\delta(\cdot)$ denotes the Dirac delta function, N_v is the total number of moving loads. To facilitate a compact representation of the equations, a window function ε_k is defined: $\varepsilon_k = 0$ when the load has left the beam and $\varepsilon_k = 1$ while the load is crossing the beam [19]. N_c is the number of cables acting on the bridge and $\delta[X - i\frac{L}{N_c}]$ give the position of each action. $K_i{}^c$ is the linear stiffness of the cables. Their expression according to the particular characteristics of the stay cables is given by [22]

$$K_i^c = \frac{E_i A_i}{L_i}\sin^2(\alpha_i) \quad (2)$$

where α_i is the angle between the ith cable and the bridge deck, E_i, A_i, L_i are Young's Modulus, the cross section and the length of the ith cable respectively. For a finite, simply supported beam, the boundary and initial conditions have the forms

$$W(X,t)\big|_{X=0,L} = 0, \quad \left.\frac{\partial^2 W(X,t)}{\partial X^2}\right|_{X=0,L} = 0.$$
$$W(X,t)\big|_{t=0} = 0, \quad \left.\frac{\partial W(X,t)}{\partial t}\right|_{t=0} = 0 \quad (3)$$

It is well known that a more realistic and practical model of highway traffic loads takes into account the features of the Poisson process [20], or the ones of renewal counting process [21] to represent the vehicular traffic. So to derive the proposed model of external forces, we take into account the randomness of the velocity and assume the similar form of loads studied by Nikkhoo et al. [19]. The random velocities are assumed to be Gaussian distributed, i.e. that the loads travel with velocities v_k Gaussian distributed around the average speed v_0 [8]

$$\frac{dX_k(t-t_k)}{dt} = v_k(t-t_k) = v_0 + \sigma_v \xi_k(t-t_k)$$
$$0 \le X_k(t-t_k) \le L \quad (4)$$

Here $v_k(t - t_k)$ is the stochastic velocity of the kth force, v_0 the mean value of velocity, σ_v its standard deviation and $\xi_k(t - t_k)$ the velocity disturbances which we assume to be independent and stationary white noise random processes; i.e.

$$\langle v_k(t-t_k)\rangle = v_0, \quad \langle \xi_k(t-t_k)\rangle = 0$$

$$\langle \xi_k(t-t_k)\xi_j(t-t_k)\rangle = 0 \quad \text{for } k \neq j,$$

$$\langle \xi_k(t-t_k)\xi_k(t-t_k+\zeta)\rangle = \sigma_v^2\delta(\zeta) \tag{5}$$

The brackets $\langle\cdots\rangle$ denote the time average.

2.2. Modal equations

If one takes into account the boundary conditions, the transversal deflection $W(X,t)$ for the simply-supported beam can be represented in a series form as

$$W(X,t) = \sum_{n=1}^{\infty} q_n(t)\sin\left(\frac{n\pi X}{L}\right) \tag{6}$$

where $q_n(t)$ is the amplitude of the nth mode, and $\sin(n\pi X/L)$ is the solution of the eigenvalue problem which depends on the boundary conditions of the free oscillations of the beam. It is convenient to adopt the following dimensionless variables:

$$\vartheta_n = \frac{q_n}{l_r}, \quad \tau = \omega_0 t \tag{7}$$

The equivalent stochastic dimensionless modal equation is obtained by substituting Eqs. (2) and (4) into Eq. (1) and considering the first mode of vibration in which almost the energy is concentrated:

$$\ddot{\vartheta}(\tau) + \lambda\dot{\vartheta}(\tau) + \left[1 - \frac{T_h^c}{T_{hcr}^c} + \chi\sum_{i=0}^{N_c}\frac{E_iA_i}{L_i}\sin^2(\alpha_i)\sin^2\left(\frac{\pi i}{N_c}\right)\right]\vartheta(\tau) + \beta\vartheta^3(\tau) = \Gamma\sum_{k=1}^{N_v}\varepsilon_k\sin\left[\Omega\tau + \gamma W(\tau-\tau_k)\right] \tag{8}$$

with

$$\Omega = \frac{\pi v_0}{L\omega_0}, \quad \Gamma = \frac{2PL^3}{l_rEI\pi^4}, \quad \lambda = \frac{cL^3}{\pi^2\sqrt{EI\left[L^2m_b + R_a\pi^2\right]}}, \quad \tau_k = \omega_0 t_k$$

$$\beta = -\frac{3}{8}\left(\frac{\pi l_r}{L}\right)^2, \quad \gamma = \frac{\pi\sigma_v}{L}, \quad \chi = \frac{2L^3}{EI\pi^4}, \quad T_{hcr}^c = \frac{EI\pi^2}{L^2}, \tag{9}$$

and

$$\omega_0 = \frac{\pi^2}{L}\sqrt{\frac{EI}{L^2m_b + R_a\pi^2}}, \quad R_a = m_br^2, \quad r = \sqrt{\frac{I}{S}}, \quad l_r = \frac{L}{2}. \tag{10}$$

Here l_r is a reference length of the beam and $W(\tau-\tau_k)$ is a unit Wiener stochastic process.

Eq. (8) amounts to a stochastic Duffing oscillator which describes the unbounded or catastrophic motion of the beam for:

$$T_h^c < \left(1 + \chi\sum_{i=0}^{N_c}\frac{E_iA_i}{L_i}\sin^2(\alpha_i)\sin^2\left(\frac{\pi i}{N_c}\right)\right)T_{hcr}^c \tag{11}$$

The catastrophic behavior of the beam is related to the configuration of the potential of the system, as described in detail in Ref. [23].

For the analytical purpose, let us consider the simplest case when the beam is subjected to the passage of a single moving load ($N_v = 1$). Also, we assume in the first case ($\varepsilon_k = 0$) that the dynamic response function is equal to zero ($\vartheta(\tau) = 0$) and for the second case ($\varepsilon_k = 1$) the response function is calculated from the equation

$$\ddot{\vartheta}(\tau) + \lambda\dot{\vartheta}(\tau) + (1+\alpha)\vartheta(\tau) + \beta\vartheta^3(\tau) = \eta(\tau) \tag{12}$$

where

$$\sigma = \chi\sum_{i=0}^{N_c}\frac{E_iA_i}{L_i}\sin^2(\alpha_i)\sin^2\left(\frac{\pi i}{N_c}\right), \quad \alpha = \sigma - \frac{T_h^c}{T_{hcr}^c},$$
$$\eta(\tau) = \Gamma\sin\left[\Omega\tau + \gamma W(\tau)\right]$$

$\alpha$is the total contribution of the stay cables structures on the dynamics of the cable-stayed bridges system. This is the main result of this part: The sudden appearance on the right-hand side of Eq. (12) of a harmonic function with constant amplitude and random phases (mathematically equivalent to frequency fluctuations of a nonmonochromatic drive [24]), which amounts to a bounded or sine-Wiener noise $\eta(\tau)$ [25], whose covariance is given by

$$C_\eta(\tau,\tau') = \langle\eta(\tau)\eta(\tau')\rangle = \frac{\Gamma^2}{2}\exp\left(-\frac{\gamma^2|\tau-\tau'|}{2}\right)\cos\Omega(\tau-\tau'). \tag{13}$$

In the following, the prediction of chaotic behavior for Eq. (12) will be investigated.

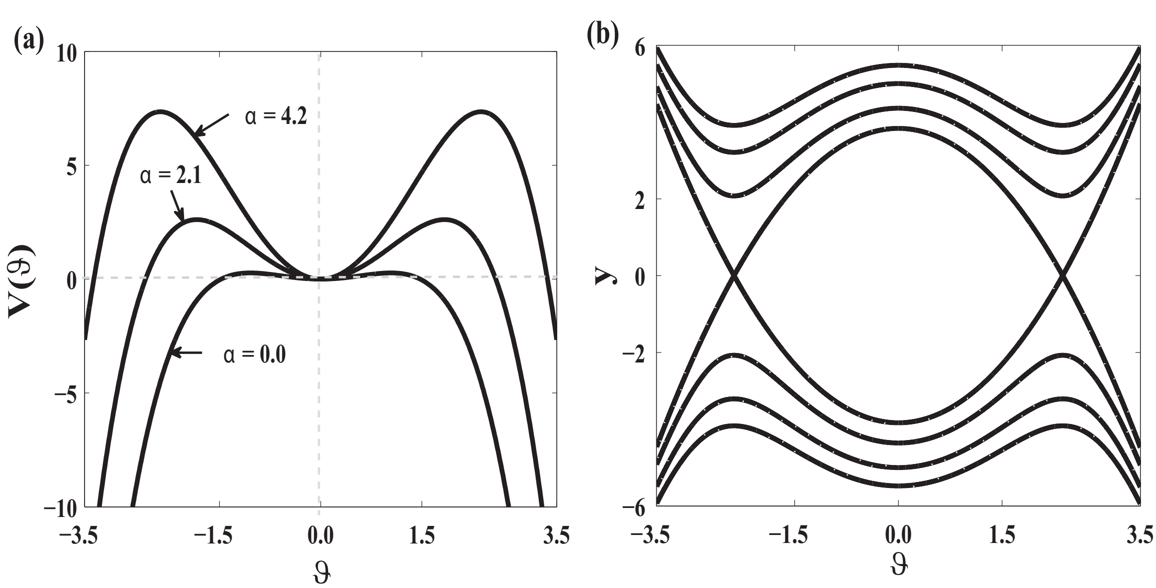

Fig. 2. Potential for different values of α (a), separatrix (closed curve) and phase space portrait (open lines) of the Catastrophic system Eq. (12) for $\alpha = 4.2$ (b).

2.3. Melnikov analysis and random chaos prediction

The aim of this subsection is to show the effect of stochastic velocity of the moving loads and the cables contribution on the basin boundaries. This is done by one of a few methods allowing analytical prediction of chaos occurrence: Melnikov method [11–15].

This method was extended to study stochastic dynamical system by Frey and Simiu [26]. To apply this method, we introduce a small parameter μ in Eq. (12) and rewrite the governing system as the following set of first order differential equation:

$$\begin{cases} \dot{\vartheta}(\tau) = y(\tau) \\ \dot{y}(\tau) = -\left(1+\alpha\right)\vartheta(\tau) - \beta\,\vartheta^3(\tau) + \mu\left[-\lambda\, y(\tau) + \eta(\tau)\right] \end{cases} \tag{14}$$

For $\mu = 0$, and after assuming that $\vartheta = \vartheta(\tau)$; $y = y(\tau)$, the system of Eq. (14) is the Hamiltonian system with Hamiltonian function

$$H(\vartheta, y) = \frac{1}{2}y^2 + \frac{1}{2}(1+\alpha)\vartheta^2 + \frac{\beta}{4}\vartheta^4 \tag{15}$$

and the potential function

$$V(\vartheta) = \frac{1}{2}(1+\alpha)\vartheta^2 + \frac{\beta}{4}\vartheta^4 \tag{16}$$

Fig. 2(a) shows an increases of an energy barrier of our system when the contribution of cables α varies. As $\beta < 0$, the system has three equilibrium points: a center point $\vartheta_{e0} = (0, 0)$ and two saddles $\vartheta_{e1} = \left(-\sqrt{-(1+\alpha)/\beta}, 0\right)$ and $\vartheta_{e2} = \left(\sqrt{-(1+\alpha)/\beta}, 0\right)$, as shown in Fig. 2(b). The saddle points are connected by heteroclinic orbits that satisfy the following equation:

$$\begin{aligned} \vartheta_{het}(\tau) &= \pm\sqrt{-\frac{1+\alpha}{\beta}}\tanh\left[\sqrt{\frac{1+\alpha}{2}}\cdot\tau\right] \\ y_{het}(\tau) &= \pm(1+\alpha)\cdot\sqrt{-\frac{1}{2\beta}}\ \sec\ h^2\left[\sqrt{\frac{1+\alpha}{2}}\cdot\tau\right] \end{aligned} \tag{17}$$

Melnikov theory defines the condition for the appearance of the so-called transverse intersection points between the perturbed and the unperturbed separatrix or the appearance of the fractality or erosion on the basin of attraction. This theory can be applied to Eq. (14) by using formula given by Wiggins in [12] as follows:

$$\begin{aligned} M_R(\tau_0) &= -\lambda\int_{-\infty}^{+\infty} y_{het}^2(\tau)\,d\tau + \int_{-\infty}^{+\infty} y_{het}(\tau)\,\eta(\tau+\tau_0)\,d\tau \\ &= I \pm Z(\tau_0) \end{aligned} \tag{18}$$

where

$$I = \frac{\lambda\sqrt{8(1+\alpha)^3}}{3\beta},$$

$$Z(\tau_0) = (1+\alpha)\cdot\sqrt{-\frac{1}{2\beta}}\int_{-\infty}^{+\infty}\sec\ h^2\left[\sqrt{\frac{1+\alpha}{2}}\cdot\tau\right]\eta(\tau+\tau_0)\,d\tau$$

In the following, we consider the simple zeros of the mean-square of the output analyzed through the random Melnikov function. The impulse response function of the system (14) is

$$h(\tau) = \vartheta_{het}(\tau)\cdot y_{het}(\tau) = \sqrt{\frac{(1+\alpha)^3}{2\beta^2}}\tanh\left[\sqrt{\frac{1+\alpha}{2}}\cdot\tau\right]\sec\ h^2\left[\sqrt{\frac{1+\alpha}{2}}\cdot\tau\right] \tag{19}$$

The associated frequency response can be expressed as follows:

$$\mathrm{H}(\omega) = \int_{-\infty}^{+\infty} h(\tau)e^{-j\omega\tau}\,d\tau = -\frac{j\pi\omega^2\csc h\left[\frac{\pi\omega}{2\sqrt{1+\alpha}}\right]}{2\sqrt{2\beta^2}} \tag{20}$$

Consequently, the variance of $Z(\tau_0)$, considered as the output of the system, is

$$\begin{aligned} \sigma_z^2 &= \int_{-\infty}^{+\infty}|\mathrm{H}(\omega)|^2 S_\eta(\omega) \\ d\omega &= \frac{\pi}{2}\left(\frac{\Gamma\gamma}{4\beta}\right)^2\int_{-\infty}^{+\infty}\omega^4\csc\ h^2 \\ &\left[\frac{\pi\omega}{2\sqrt{1+\alpha}}\right]\left\{\frac{\Omega^2+\omega^2+\frac{\gamma^4}{4}}{\left(\omega^2-\Omega^2+\frac{\gamma^4}{4}\right)^2+\Omega^2\gamma^4}\right\}d\omega \end{aligned} \tag{21}$$

Here $S_\eta(\omega)$ is the spectral density of the noise $\eta(\tau)$ defined by

$$\begin{aligned} S_\eta(\omega) &= \frac{(\Gamma\gamma)^2}{2\pi}\left[\frac{1}{4(\omega-\Omega)^2+\gamma^4} + \frac{1}{4(\omega+\Omega)^2+\gamma^4}\right] \\ &= \frac{(\Gamma\gamma)^2}{4\pi}\frac{\Omega^2+\omega^2+\frac{\gamma^4}{4}}{\left(\omega^2-\Omega^2+\frac{\gamma^4}{4}\right)^2+\Omega^2\gamma^4} \end{aligned} \tag{22}$$

Therefore, smale chaos appears (in mean-square response) when condition $I^2 \le \sigma_z^2$ is satisfied, i.e.

$$\left(\frac{\lambda\sqrt{8(1+\alpha)^3}}{3\beta}\right)^2 \le \frac{\pi}{2}\left(\frac{\Gamma\gamma}{4\beta}\right)^2 \times \int_{-\infty}^{+\infty}\omega^4\csc\ h^2 \left[\frac{\pi\omega}{2\sqrt{1+\alpha}}\right]\left\{\frac{\Omega^2+\omega^2+\frac{\gamma^4}{4}}{\left(\omega^2-\Omega^2+\frac{\gamma^4}{4}\right)^2+\Omega^2\gamma^4}\right\}d\omega \tag{23}$$

Or simply,

$$\frac{\lambda}{\Gamma} \le \frac{3\gamma}{16}\sqrt{\frac{\pi\int_{-\infty}^{+\infty}\omega^4\csc h^2\left[\frac{\pi\omega}{2\sqrt{1+\alpha}}\right]\left\{\frac{\Omega^2+\omega^2+\frac{\gamma^4}{4}}{\left(\omega^2-\Omega^2+\frac{\gamma^4}{4}\right)^2+\Omega^2\gamma^4}\right\}d\omega}{(1+\alpha)^3}} = \left(\frac{\lambda}{\Gamma}\right)_{cr} \tag{24}$$

where $(\lambda/\Gamma)_{cr}$ is the critical parameter for the chaotic motion of the nonlinear system. The integral in Eq. (24) can be computed numerically. One can thus get the threshold of bounded excitation amplitude versus the standard deviation of the stochastic velocity for different values of the mean driving frequency Ω, for $\alpha = 0.0$ (no cables) (as shown in Fig. 3(a)) and for $\alpha = 4.2$ (with 18 cables) (as shown in Fig. 3(b)). From Fig. 3, we can see that the threshold curve is a continuous line in the space $(\gamma, \lambda/\Gamma) \in R^2$. We observe that the area above the curves indicates the domain where the system goes from periodic to random as γ increases progressively, while below them the motion of system goes from chaos to random chaos as γ increases from zero and becomes more and more random and less chaotic as γ further increases. This domain is especially sensitive to initial conditions and fractal basin boundaries. Likewise, for certain values of the intensity of stochastic velocity ($\gamma \in [0, \gamma_{limit}]$), the increasing of mean driving frequency Ω still increases the chaotic field of the system. This effect does not appear any more for $\gamma > \gamma_{limit}$ (see Fig. 3(a)), the case is opposite. Fig. 3(b) illustrates the same effect while showing the contribution of the stay cables on the results obtained previously.

On the other hand, the effect of cable connections on the threshold amplitude of sine-Wiener noise excitation for the onset

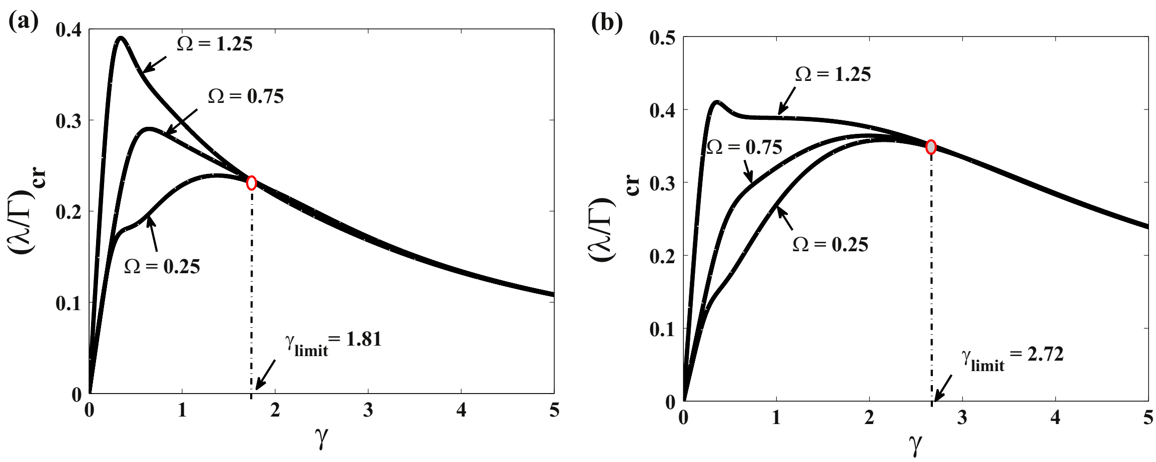

Fig. 3. Effects of the mean driving frequency Ω on the threshold curve of horseshoes chaos: (a) for $\alpha = 0.0$ (no cables) and (b) for $\alpha = 4.2$ (with 18 cables).

of chaos in the model is investigated as shown in Fig. 4. It is clear that the increase of number of stay cables first increases the threshold, and then decreases it. It is also shown from this figure that the lowest number of cables is dangerous for the stability of the structure, while the highest number contributed to increase the degree of safety of the bridge. The intensity of the random component of the loads velocity γ influences considerably this previous results as shown in Fig. 4.

Fig. 5 shows the correspondence between the parameter σ and the number of connections N_c. This result is obtained after a rigorous dimensioning of the model (noticed that the dimensioning of the model only takes into account the case of eighteen cables). We observe that by increasing the number of cables, their contribution on the dynamic of the bridge also increase and then starting from the 16th cable, saturates.

To detect the effect of loads number on the system response, Eq. (8) is solved numerically using stochastic fourth-order Runge Kutta method [27] and Fig. 6 is plotted. It is shown how an increase of the number of moving loads affects the mean square amplitude of the beam. In fact, when the value of N_v increases the

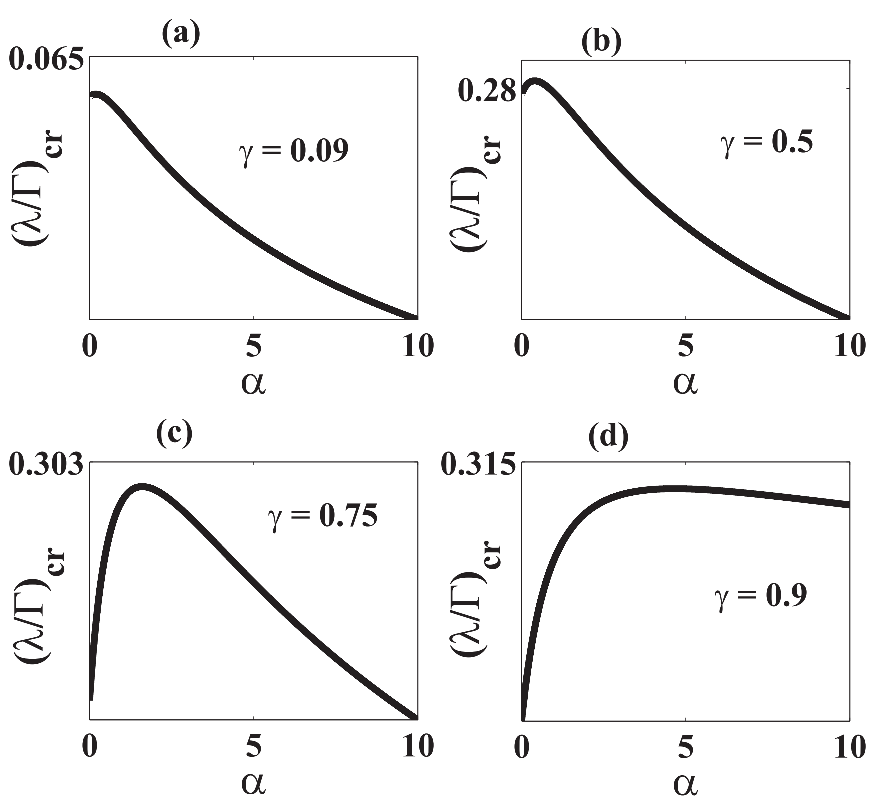

Fig. 4. Effects of stay cables contributions α on the threshold amplitude of sine-Wiener noise excitation for $\Omega = 0.75$.

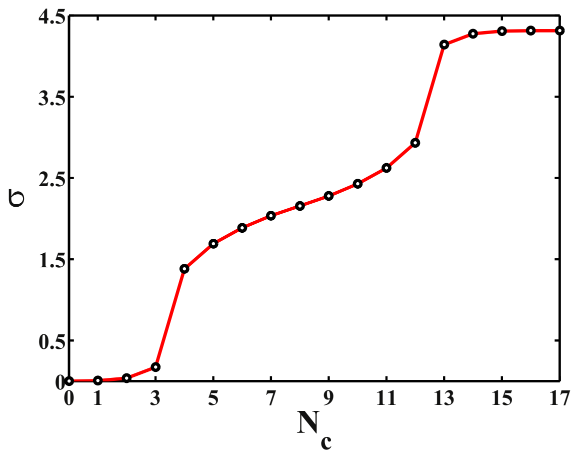

Fig. 5. Transversal contribution of cable connections σ as a function of their number N_c.

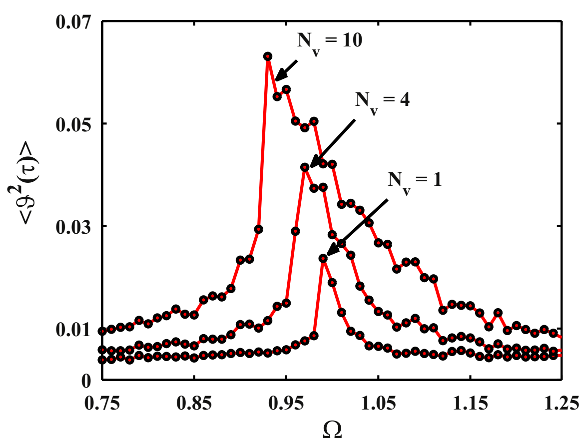

Fig. 6. Effects of loads number N_v on the mean square amplitude of the beam for $\gamma = 0.001$, $\alpha = 0.0$, $\Gamma = 0.002$.

Table 1
Values of the physical parameters of a cable-stayed bridge model, Fig. 1(a).

Physical parameters	Symbols	Values
Length of the bridge (m)	L	628.1
Young modulus of the bridge deck (MPa)	E	200.0
Cross-sectional area of the bridge deck (m^2)	S	4.8
Moment of inertia of the bridge deck (m^4)	I	12.0
Mass per unit length of the bridge deck (kg/m)	m_b	37680.0
Damping coefficient of the bridge deck (N s/m)	c	68.0
High length of the stay cables (m)	L_0	158.13
Young modulus of the cables (MPa)	E_i	131.0
Cross-sectional area of the cables (m^2)	A_i	5.48×10^{-4}
Horizontal tension of the cables (N/m)	T_h^c	5.3×10^6
Length of the pylon (m)	H	45.7

mean square vibration amplitude at the resonant state merely increases.

3. Numerical results

To validate the accuracy of the proposed analytical predictions, we solve numerically Eq. (12) using stochastic fourth-order Runge Kutta method [27] to display the shape of the basin of attraction. In calculation, the structural and material properties of a cable-stayed bridge model are given in Table 1 as (see Refs. [28,29]).

The dimensionless parameters are $\beta = -0.92$, $\lambda = 0.009$ (all numerical results shown in this paper will use these parameters). The other lengths of the cables are calculated by using the theorem of Thalès, by assuming of course that the cables on both sides of the towers which support them are parallel between them. Moreover, the various angles ranging between the cables and the bridge deck are evaluated by using the relation: $\alpha_i = \cos^{-1}\left(\frac{l_c}{2L_i}\right)$ where l_c is the distance separating the two impacts points on the bridge deck of the two symmetrical cables. Fig. 7 shows the sequence of the safe basin of system (14) plotted in order to verify the results provided by the Melnikov analysis. We first observe that for $\Omega = 0.75$ and $(\lambda/\Gamma) = 0.9$, the shape of the basins boundaries is regular. This reliability can be periodic for lower values of noise (Fig. 7(a), $\gamma = 0.003$) or random for higher values of noise (Fig. 7(b), $\gamma = 1.0$). These observations had already been predicted by the analytical developments presented in Section 2. Second, we take $\Omega = 0.75$ and $(\lambda/\Gamma) = 0.15$, the fractal boundaries of the safe basins have turned up, especially when the intensity of stochastic velocity is low (Fig. 7(c), $\gamma = 0.003$). By considering the highest values of this intensity, another rich motion occurred in our system: “Random chaos motion” (see Fig. 7(d), $\gamma = 1.0$), as predicted by the frontier of Fig. 3(a). Fig. 8 reveals the interesting role of cables stayed on the bridge safety. In fact, by increasing the number of cables (increasing ofα) on the bridge that enlarges the basin of attraction area and the fractality disappears progressively (for the chosen parameters here, the system goes from random chaos motion to random motion as α increases), increasing consequently the degree of predictibility of the system. This leads us to the conclusion that the numerical range is closed to the analytical one.

4. Conclusion

We have considered a stochastic dynamical system to inhibition of the chaotic responses on cable-stayed bridge subjected to train of forces moving with stochastic velocity. To do so, we have first modelled the cable-stayed bridge system by the full partial differential equation that we have thereafter reduced to an effective a single nonlinear one-dimensional equation. Second, the random terms have been modelled as equal weights moving with disordered velocity, thus neglecting other effects as the spread of the weights. Thereafter, the random Melnikov analysis has been used to seek the effects of velocity (mean value and stochastic value) and cable contribution on the structure failure, on the structure unpredictibility, and on the possible appearance of horseshoes chaos. We have found that the intensity of the random component of the loads velocity causes an increases of the threshold $(\lambda/\Gamma)_{cr}$ and then increases the chances to have a regular behavior of the bridge; after a maximum a further increase of the noise causes a decrease of the threshold and then enlarges the possible chaotic domain in parameter space. We have also sought the effect of cable contribution on the beam safety and found that the lowest number of cables is dangerous for the stability of the structure, while the highest number contributed to increase the degree of safety of the bridge. The identification of some rich dynamical behavior such as periodic, random, chaos and random chaos in the proposed model has been investigated analytically and validated numerically. Finally, we have shown how an increase of the number of moving loads affects the mean square amplitude of the beam.

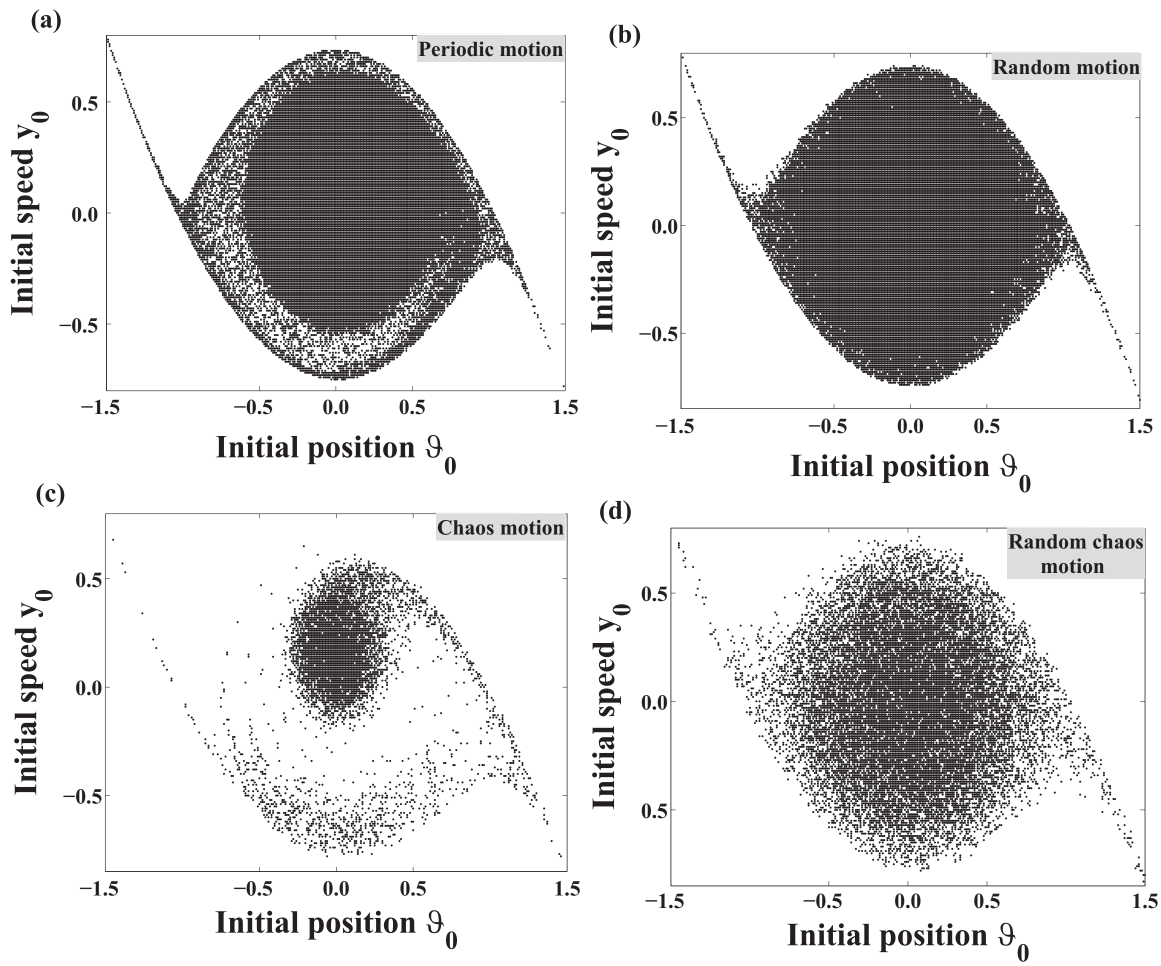

Fig. 7. Basins of attraction showing the confirmation of the analytical prediction for $\alpha = 0.0$, $\Omega = 0.75$: $(\lambda/\Gamma) = 0.9$; (a) $\gamma = 0.003$, (b) $\gamma = 1.0$ and $(\lambda/\Gamma) = 0.15$; (c) $\gamma = 0.003$, (d) $\gamma = 1.0$.

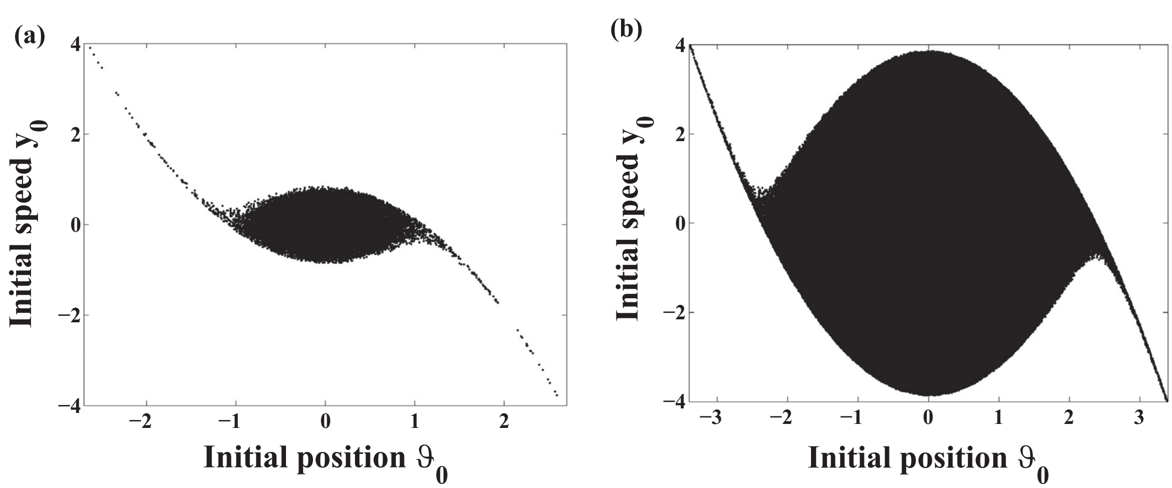

Fig. 8. Effects of cable contribution α on the basins of attraction: (a) $\alpha = 0.1$, (b) $\alpha = 4.2$. For $(\lambda/\Gamma) = 0.15$, $\Omega = 0.75$ and $\gamma = 1.0$.

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