$\frac{\partial}{\partial t}\underline{\boldsymbol{B}} = -\nabla \times \underline{\boldsymbol{E}}$

$\frac{\partial}{\partial t}\underline{\boldsymbol{D}} = \nabla \times \underline{\boldsymbol{H}}$

GhK
TET

$\frac{\partial}{\partial t}\underline{\boldsymbol{p}} = \nabla \cdot \underline{\underline{\boldsymbol{T}}}$

$\frac{\partial}{\partial t}\underline{\underline{\boldsymbol{S}}} = \mathrm{sym}\{\nabla \underline{\boldsymbol{v}}\}$

Exercises for Electromagnetic Field Theory I
(EFT I)
SS 2002

University of Kassel
Department of EE/CS
Electromagnetic Theory

Sheet 2

Exercise 3 (Gauss' Electric Law)

A sphere with the radius R_0 centered at the coordinate origin is charged with an inhomogeneous electric charge density given by

$$\varrho_{\mathrm{e}}(R) = \begin{cases} \varrho_{\mathrm{e}0}\frac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases} .$$

Here ϱ_{e0} and R_0 are given constants. The sphere is filled with vacuum and is embedded in vacuum.

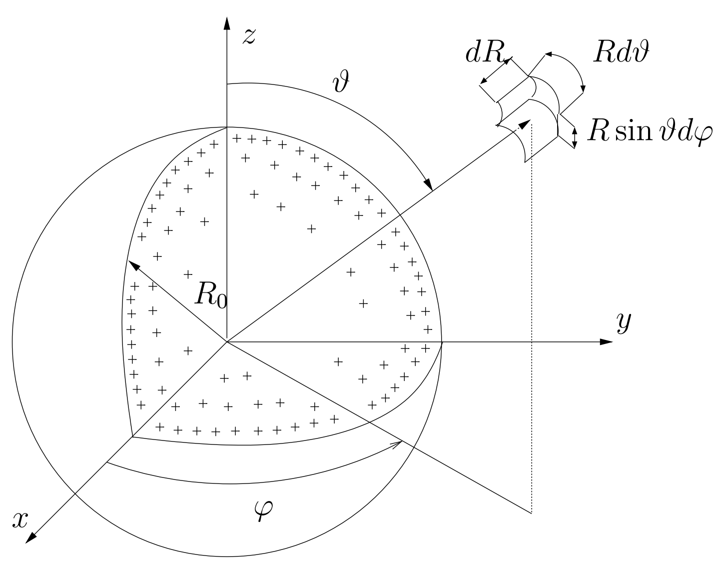

Figure 1: Spherically symmetric electric charge density distribution

a) Determine the electric flux density $\underline{\mathbf{D}}(\underline{\mathbf{R}})$.

b) Determine the electric field strength $\underline{\mathbf{E}}(\underline{\mathbf{R}})$.

c) Sketch the charge density, $(\varrho_e(R))$, the electric flux density, $(\underline{\mathbf{D}}(R))$ and the electric field strength, $(\underline{\mathbf{E}}(R))$, for $0 \leq R \leq 2R_0$.

Exercise 4

Given is a circular cylinder with the height $2Z_0$ and the Radius R_0 around the z-axis (compare figure 2):

$$Z = \{(r, \varphi, z)| 0 \leq r \leq R_0, 0 \leq \varphi \leq 2\pi, -Z_0 \leq z \leq Z_0\}.$$

In this cylinder there is a time harmonic current density distribution with:

$$\underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t) = \varrho_{e0} \frac{1}{3} \frac{r^2}{R_0} \cos\left(\frac{\pi}{2} \frac{z}{Z_0}\right) \omega \cos(\omega t)\, \underline{\mathbf{e}}_r, \qquad -Z_0 \leq z \leq Z_0, \quad 0 \leq r \leq R_0.$$

ϱ_{e0} is a constant charge density with the unit $\frac{As}{m^3}$.
The excitation needs to be causal, that yields:

$$\begin{aligned} \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t) &= \underline{\mathbf{0}}, \qquad t \leq 0, \\ \varrho_{e0}(\underline{\mathbf{R}}, t) &= 0, \qquad t \leq 0. \end{aligned}$$

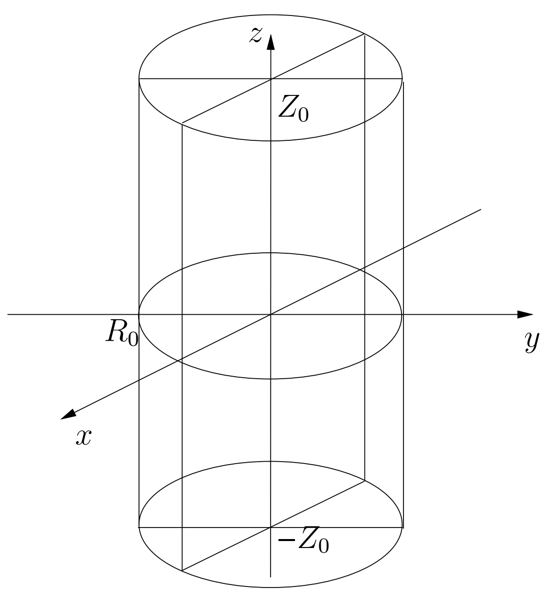

Figure 2: Circular cylinder with height $2Z_0$ and radius R_0.

a) Determine the electric current $i_e(t)$ through the closed surface of the cylinder Z.

b) Sketch the electric current $i_\mathrm{e}(t)$ over the circular frequency ω, for $\omega \geq 0$ and $t = 1s$.

c) Determine the charge density distribution $\varrho_\mathrm{e}(\underline{\mathbf{R}}, t)$ inside the cylinder Z for $t \geq 0$ using the continuity equation:

$$\nabla \cdot \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}}, t) - \frac{\partial}{\partial t}\varrho_\mathrm{e}(\underline{\mathbf{R}}, t) = 0.$$

d) Evaluate the volume integral

$$\iiint\limits_Z \frac{\partial}{\partial t}\varrho_\mathrm{e}(\underline{\mathbf{R}}, t) dV.$$

Which physical value did you just calculate ?

Exercise 5

A positive point charge is given at the origin. The resulting electric field is then given by

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{\underline{\mathbf{R}}}{R^3}$$

A positive point charge is to be moved from the position $P_1(a, 0, 0)$ to position $P_2(-a, b, c)$. Compute the integral

$$\int\limits_C \underline{\mathbf{E}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dR}}$$

for both paths C_1 and C_2 in the picture.
To do this, parametrize the path (if nescessary in parts) into

$$\underline{\mathbf{R}}(t) = x(t)\underline{\mathbf{e}}_x + y(t)\underline{\mathbf{e}}_y + z(t)\underline{\mathbf{e}}_z,$$

using t as contour parameter, form $\underline{\mathbf{dR}}$ and evaluate the integral.

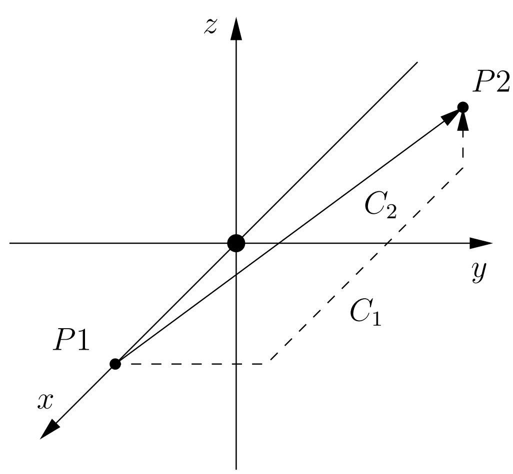

Figure 3: Movement through the field of a point charge on two different paths