







$$\frac{\partial}{\partial t} \underline{p} = \nabla \cdot \underline{\underline{T}}$$
$$\frac{\partial}{\partial t} \underline{\underline{S}} = \operatorname{sym}\{\nabla \underline{v}\}$$

Exercises for Electromagnetic Field Theory I (EFT I) SS 2002

University of Kassel Department of EE/CS Electromagnetic Theory

Sheet 3

Exercise 6

A dielectric sphere with the relative permittivity ε_{in} and the radius R_K is brought into a homogenous electrostatic field with $\underline{\mathbf{E}}^{hom} = E_0 \underline{\mathbf{e}}_z$. The sphere is surrounded by a homogenous medium with the relative permittivity ε_{out} .

The electric potential outside of the sphere is then given by

$$\Phi_{out}(\underline{\mathbf{R}}) = -E_0 R \cos \vartheta \left\{ 1 - \frac{\varepsilon_{in} - \varepsilon_{out}}{\varepsilon_{in} + 2\varepsilon_{out}} \frac{R_K^3}{R^3} \right\} \quad R > R_K$$

and inside the sphere by

$$\Phi_{in}(\underline{\mathbf{R}}) = -E_0 \frac{3\varepsilon_{out}}{\varepsilon_{in} + 2\varepsilon_{out}} R\cos\vartheta \qquad 0 < R \le R_K$$

- a) Calculate the electric field strengths $\underline{\mathbf{E}}_{out}$ and $\underline{\mathbf{E}}_{in}$ outside and inside of the dielectric sphere.
- **b)** Show that $\underline{\mathbf{E}}_{out}$ and $\underline{\mathbf{E}}_{in}$ are source free.
- c) Calculate the closed surface integral

$$\oint_{S_K} \underline{\mathbf{E}} \cdot \underline{\mathbf{dS}}$$

where S_K is the surface of a sphere with the radius $R = \frac{1}{2}R_K$.

d) Calculate the closed surface integral

$$\oint_{S_K} \underline{\mathbf{E}} \cdot \underline{\mathbf{dS}}$$

where S_K is the surface of a sphere with the radius $R = 2R_K$.

Hint: Use the spherical coordinate system and the substitutions:

$$\alpha = \frac{\varepsilon_{in} - \varepsilon_{out}}{\varepsilon_{in} + 2\varepsilon_{out}} R_K^3, \quad \text{and} \quad \beta = \frac{3\varepsilon_{out}}{\varepsilon_{in} + 2\varepsilon_{out}}$$