Dipl.-Ing. R. Hannemann SS 2001

Exercises for EFT 1, Sheet 4

Exercise 1.

a) $\int_{-\infty}^{\infty} \delta(x)\Phi(x)\,dx$

b) $\int_{-\infty}^{\infty} \delta(x)\Phi(x-x')\,dx$

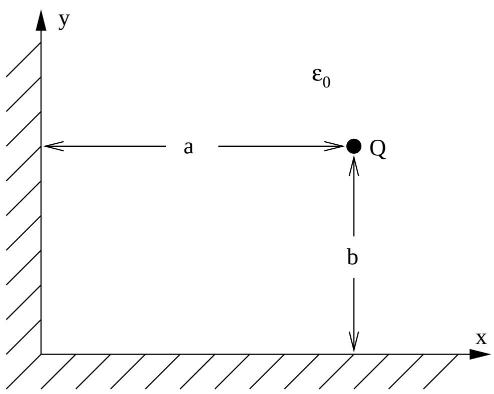
c) $\int_{-\infty}^{\infty} \delta(x-x_0)\Phi(x)\,dx$

d) $\int_{-\infty}^{\infty} \delta(x-x_0)\Phi(x-x')\,dx$

e) $\int_{-\infty}^{\infty} \delta(x-x_0)(f_1(x)+f_2(x))\,dx$

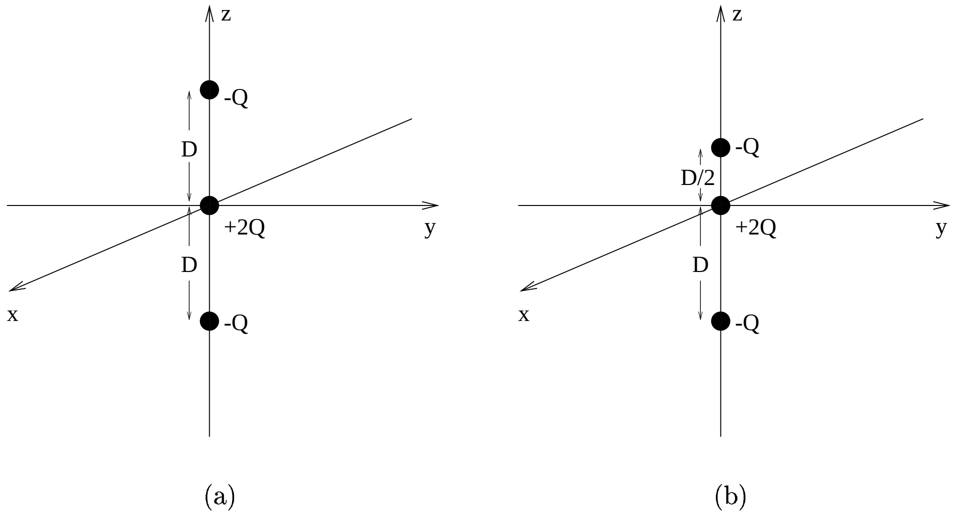
Exercise 2.
Find the potential Φ and the electrical field $\underline{\mathbf{E}}$ of a point charge, which is placed in the origin of a coordinate system.
Display the result in cartesian, cylindrical, and sherical coordinates.

Exercise 3.



A charge Q is located near two grounded planes ($\Phi = 0$), intersecting at right angles, as shown in the figure. Find the potential distribution of this configuration by using the method of images. Scetch the E–field lines and the potantial lines.

Exercise 4.



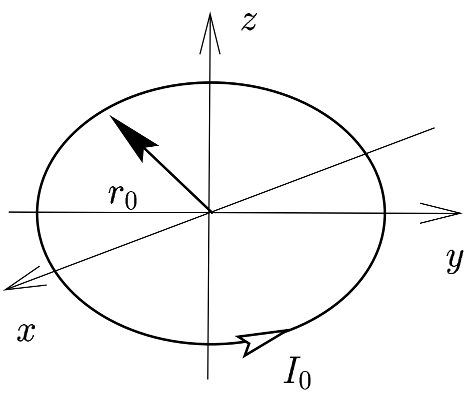
The figure shows two different distributions of three point charges.

1) Write down the volume charge density $\varrho(\underline{\mathbf{R}})$ for the cases (a) and (b) in a mathematical formula by using the δ–Distribution.

2) What is the dipol moment $\underline{\mathbf{p}}_e$ of the charge distributions in the figure (a) and figure (b)?

3) What is the quadrupole moment $\underline{\underline{\mathbf{q}}}_e$ of the charge distributions in the figure (a) and figure (b)?

Exercise 5.



A constant direct current is flowing through a circular wire loop with radius r_0 and of infinitesimal thickness. The wire loop is situated in the xy–plane. Calculate the magnetic flux density $\underline{\mathbf{B}}(\underline{\mathbf{R}})$ on the z–axis by solving the law of Biot–Savart

$$\underline{\mathbf{B}}(\underline{\mathbf{R}}) = \frac{\mu_0 I_0}{4\pi} \int_C \frac{\mathbf{d}\underline{\mathbf{R}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3} \quad .$$

Exercise 6.
A plane wave is given by

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}_0(\omega)e^{jk_0\hat{\underline{\mathbf{k}}}\cdot\underline{\mathbf{R}}} \quad .$$

In this formula $\hat{\underline{\mathbf{k}}}$ is the propagation direction and $\underline{\mathbf{E}}_0(\omega)$ the frequency dependent amplitudeand polarization of the field. k_0 is the wave number of free space

$$\begin{aligned} k_0 &= \frac{\omega}{c_0} \\ \text{with} & \\ c_0 &= \frac{1}{\sqrt{\varepsilon_0\mu_0}} \end{aligned}$$

where c_0 is the propagation velocity of the wave in free space.
Show that the plane wave is a solution of the homogeneous wave equation

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) + k_0^2\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = 0 \quad .$$

Hint: Use the cartesian coordinate system with

$$\begin{aligned} \underline{\mathbf{R}} &= x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z \\ \hat{\underline{\mathbf{k}}} &= \hat{k}_x\underline{\mathbf{e}}_x + \hat{k}_y\underline{\mathbf{e}}_y + \hat{k}_z\underline{\mathbf{e}}_z \quad . \end{aligned}$$

Hint:

$$\nabla\cdot(\Phi\underline{\mathbf{A}}) = \Phi\nabla\cdot\underline{\mathbf{A}} + \nabla\Phi\cdot\underline{\mathbf{A}}$$