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Exercises for EFT 1, Sheet 1
Exercises for Math. Foundations of EFT, Sheet 1
Solutions

Exercise 1. (a)

$$\begin{aligned}
\underline{\mathbf{A}} - \underline{\mathbf{B}} &= -\underline{\mathbf{e}}_x + 3\underline{\mathbf{e}}_y - 2\underline{\mathbf{e}}_z \\
\underline{\mathbf{A}} + \underline{\mathbf{B}} &= 3\underline{\mathbf{e}}_x - \underline{\mathbf{e}}_y + 4\underline{\mathbf{e}}_z \\
\hat{\underline{\mathbf{A}}} &= \frac{\underline{\mathbf{e}}_x + 2\underline{\mathbf{e}}_y + 3\underline{\mathbf{e}}_z}{\sqrt{14}} \\
\hat{\underline{\mathbf{B}}} &= \frac{2\underline{\mathbf{e}}_x - \underline{\mathbf{e}}_y + \underline{\mathbf{e}}_z}{\sqrt{6}} \\
\underline{\mathbf{A}} \cdot \underline{\mathbf{B}} &= 3 \\
\underline{\mathbf{A}} \times \underline{\mathbf{B}} &= 5\underline{\mathbf{e}}_x + 5\underline{\mathbf{e}}_y - 5\underline{\mathbf{e}}_z
\end{aligned}$$

(b) $\alpha = 70°$

(c) $\alpha = 74,5°$

Exercise 2. (a) $\alpha = 1$

(b) $\alpha = -2$

(c) $\alpha_{1,2} = -\frac{1}{2} \pm \frac{1}{2}\sqrt{5}$

Exercise 3. (a) $R = |\underline{\mathbf{R}}| = \sqrt{x^2 + y^2 + z^2}$, $\quad \hat{\underline{\mathbf{R}}} = \frac{\underline{\mathbf{R}}}{R}$,

(b) $\frac{\partial}{\partial x}\underline{\mathbf{R}} = \underline{\mathbf{e}}_x$, $\quad \frac{\partial}{\partial y}\underline{\mathbf{R}} = \underline{\mathbf{e}}_y$, $\quad \frac{\partial}{\partial z}\underline{\mathbf{R}} = \underline{\mathbf{e}}_z$,

(c) $\nabla R = \hat{\underline{\mathbf{R}}}$, $\quad \nabla \cdot \underline{\mathbf{R}} = 3$, $\quad \nabla \times \underline{\mathbf{R}} = \underline{0}$.

Exercise 4. (a) $|\underline{\mathbf{R}} - \underline{\mathbf{R}}'| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$

$\widehat{\underline{\mathbf{R}} - \underline{\mathbf{R}}'} = \frac{(x-x')\underline{\mathbf{e}}_x + (y-y')\underline{\mathbf{e}}_y + (z-z')\underline{\mathbf{e}}_z}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}$,

(b) $\frac{\partial}{\partial x}(\underline{\mathbf{R}} - \underline{\mathbf{R}}') = \underline{\mathbf{e}}_x$, $\quad \frac{\partial}{\partial y}(\underline{\mathbf{R}} - \underline{\mathbf{R}}') = \underline{\mathbf{e}}_y$, $\quad \frac{\partial}{\partial z}(\underline{\mathbf{R}} - \underline{\mathbf{R}}') = \underline{\mathbf{e}}_z$,

(c) $\nabla|\underline{\mathbf{R}} - \underline{\mathbf{R}}'| = \frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}'}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|} = \widehat{\underline{\mathbf{R}} - \underline{\mathbf{R}}'}$, $\quad \nabla\frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|} = \frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}'}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$.