

Exercises for EFT 1, Sheet 2
Exercises for Math. Foundations of EFT, Sheet 2
Solutions

Exercise 1.

- (a) $\int x \, dx = \frac{1}{2}x^2$
- (b) $\int x^2 \, dx = \frac{1}{3}x^3$
- (c) $\int \frac{1}{x} \, dx = \ln|x|$
- (d) $\int e^x \, dx = e^x$
- (e) $\int (x + y) \, dx = \frac{1}{2}x^2 + xy$
- (f) $\int (x + y) \, dy = xy + \frac{1}{2}y^2$
- (g) $\int \sin x \, dx = -\cos x$
- (h) $\int \cos x \, dx = \sin x$

Exercise 2.

- (a) $\frac{d}{dx}x = 1$
- (b) $\frac{d}{dx}x^2 = 2x$
- (c) $\frac{d}{dx}cx^5 = 5cx^4$ with c constant
- (d) $\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$
- (e) $\frac{d}{dx}c = 0$ with c constant
- (f) $\frac{d}{dx}\sin x = \cos x$
- (g) $\frac{d}{dx}\cos x = -\sin x$
- (h) $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$
- (i) $\frac{d}{dx}e^x = e^x$

Exercise 3.

(a) $\underline{\mathbf{A}} = R^2 \cos \varphi \underline{\mathbf{e}}_R$

(b) $\underline{\mathbf{A}} = z(r\underline{\mathbf{e}}_r + z\underline{\mathbf{e}}_z)$

Exercise 4.

(a) $\underline{\mathbf{A}} = R \cos^2 \vartheta \underline{\mathbf{e}}_R - R \cos \vartheta \sin \varphi \underline{\mathbf{e}}_\vartheta - R \sin \vartheta \underline{\mathbf{e}}_\varphi$

(b) $\underline{\mathbf{A}} = -r\underline{\mathbf{e}}_\varphi + z\underline{\mathbf{e}}_z$

Exercise 5.

(a) $\underline{\mathbf{A}} = (x^2 + y^2 + z^2)\underline{\mathbf{e}}_x + (x^2 + y^2)\underline{\mathbf{e}}_y$

(b) $\underline{\mathbf{A}} = R^2[(\sin \vartheta \cos \varphi + \sin^3 \vartheta \sin \varphi)\underline{\mathbf{e}}_R + (\cos \vartheta \cos \varphi + \sin^2 \vartheta \cos \vartheta \sin \varphi)\underline{\mathbf{e}}_\vartheta + (-\sin \varphi + \sin^2 \vartheta \cos \varphi)\underline{\mathbf{e}}_\varphi]$

(c) $\underline{\mathbf{A}} = [(r^2 + z^2) \cos \varphi + r^2 \sin \varphi]\underline{\mathbf{e}}_r - [(r^2 + z^2) \sin \varphi - r^2 \cos \varphi]\underline{\mathbf{e}}_\varphi$

(d) $\nabla \cdot \underline{\mathbf{A}} = 2x + 2y$

(e) $\nabla \cdot \underline{\mathbf{A}} = 2R \sin \vartheta \cos \varphi + 2R \sin \vartheta \sin \varphi$

(f) $\nabla \cdot \underline{\mathbf{A}} = 2r \cos \varphi + 2r \sin \varphi$

(g) $\nabla(\underline{\mathbf{e}}_x \cdot \underline{\mathbf{A}}) = 2x\underline{\mathbf{e}}_x + 2y\underline{\mathbf{e}}_y + 2z\underline{\mathbf{e}}_z$

(h) $\nabla(\underline{\mathbf{e}}_x \cdot \underline{\mathbf{A}}) = 2R\underline{\mathbf{e}}_R$

(i) $\nabla(\underline{\mathbf{e}}_x \cdot \underline{\mathbf{A}}) = 2r\underline{\mathbf{e}}_r + 2z\underline{\mathbf{e}}_z$

(j) $\nabla \times \underline{\mathbf{A}} = 2z\underline{\mathbf{e}}_y + (2x - 2y)\underline{\mathbf{e}}_z$

(k) $\nabla \times \underline{\mathbf{A}} = 2R[\sin \vartheta \cos \vartheta \cos \varphi \underline{\mathbf{e}}_R + (\sin \varphi - \sin^2 \vartheta \cos \varphi)\underline{\mathbf{e}}_\vartheta + \cos \vartheta \cos \varphi \underline{\mathbf{e}}_\varphi]$

(l) $\nabla \times \underline{\mathbf{A}} = 2z \sin \varphi \underline{\mathbf{e}}_r + 2z \cos \varphi \underline{\mathbf{e}}_\varphi + (2r \cos \varphi - 2r \sin \varphi)\underline{\mathbf{e}}_z$