

Exercises for EFT 1, Sheet 4 Solutions

Exercise 1.

a) $\Phi(0)$

b) $\Phi(-x')$

c) $\Phi(x_0)$

d) $\Phi(x_0 - x')$

e) $f_1(x_0) + f_2(x_0)$

Exercise 2.

$$\Phi = \frac{1}{4\pi\epsilon_0} \frac{Q}{|\underline{\mathbf{R}}|}$$

$$\underline{\mathbf{E}} = \frac{R}{|\underline{\mathbf{R}}|} \frac{Q}{4\pi\epsilon_0}$$

Exercise 3.

Point source

$$+Q \text{ in } \underline{\mathbf{R}}_1 = -a\underline{\mathbf{e}}_x + b\underline{\mathbf{e}}_y$$

mirror sources

$$-Q \text{ in } \underline{\mathbf{R}}_2 = -a\underline{\mathbf{e}}_x + b\underline{\mathbf{e}}_y$$

$$-Q \text{ in } \underline{\mathbf{R}}_3 = +a\underline{\mathbf{e}}_x - b\underline{\mathbf{e}}_y$$

$$+Q \text{ in } \underline{\mathbf{R}}_4 = -a\underline{\mathbf{e}}_x - b\underline{\mathbf{e}}_y$$

$$\Phi(\underline{\mathbf{R}}) = \frac{1}{4\pi\epsilon_0} \left\{ \frac{+Q}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_1|} + \frac{-Q}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_2|} + \frac{-Q}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_3|} + \frac{+Q}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_4|} \right\}$$

Exercise 4.

$$\begin{aligned}\varrho(\underline{\mathbf{R}})_a &= -Q\delta(\underline{\mathbf{R}} - D\underline{\mathbf{e}}_z) + 2Q\delta(\underline{\mathbf{R}}) - Q\delta(\underline{\mathbf{R}} + D\underline{\mathbf{e}}_z) \\ \varrho(\underline{\mathbf{R}})_b &= -Q\delta(\underline{\mathbf{R}} - \frac{D}{2}\underline{\mathbf{e}}_z) + 2Q\delta(\underline{\mathbf{R}}) - Q\delta(\underline{\mathbf{R}} + D\underline{\mathbf{e}}_z)\end{aligned}$$

$$\begin{aligned}\underline{\mathbf{p}}_{ea} &= \mathbf{0} \\ \underline{\mathbf{p}}_{eb} &= Q\frac{D}{2}\underline{\mathbf{e}}_z\end{aligned}$$

$$\begin{aligned}\underline{\mathbf{q}}_{ea} &= -2QD^2\underline{\mathbf{e}}_z\underline{\mathbf{e}}_z \\ \underline{\mathbf{q}}_{eb} &= -Q\frac{5}{4}D^2\underline{\mathbf{e}}_z\underline{\mathbf{e}}_z\end{aligned}$$

Exercise 5.

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0\mu_0}{2} \frac{r_0^2}{(z^2 + r_0^2)^{\frac{3}{2}}} \underline{\mathbf{e}}_z$$