$\frac{\partial}{\partial t}\underline{\boldsymbol{B}} = -\nabla \times \underline{\boldsymbol{E}}$

$\frac{\partial}{\partial t}\underline{\boldsymbol{D}} = \nabla \times \underline{\boldsymbol{H}}$

GhK
TET

$\frac{\partial}{\partial t}\underline{\boldsymbol{p}} = \nabla \cdot \underline{\underline{\boldsymbol{T}}}$

$\frac{\partial}{\partial t}\underline{\underline{\boldsymbol{S}}} = \mathrm{sym}\{\nabla \underline{\boldsymbol{v}}\}$

University of Kassel
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Department of Electrical Engineering
Electromagnetic Theory

Kassel, 19 September 2001
Time: 14.00 - 16.00

Electromagnetic Field Theory I (EFT I)
EXAM

Last Name: .... Registration No.: ....

First Name: .... Middle Name: ....

Signature: ________________________

Problem	1	2	3	4	$\sum_{i=1}^{4}$
Points:					

Mark: ______

Problem 1 [18 points]

Two circular, infinitely thin wire loops C_1 and C_2 with radii $R_1 = a$ and $R_2 = 3a$ ($a > 0$) are oriented in the xy–plane at $z = 0$ (see Figure 1.1). A constant direct current I_1 is flowing through the smaller loop C_1.

a) Determine the current I_2 in the second loop in such a way that the resulting magnetic flux density is vanishing at the origin of the coordinate system. Note the indicated direction of the current I_2 in Figure 1.1.

b) Determine the total electric current density $\underline{\mathbf{J}}(\underline{\mathbf{R}})$, which is a singular vector function.

c) Prove then for the calculated electric current density the following equation

$$\iint_S \underline{\mathbf{J}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}} \quad = \quad I_1 + I_2 \qquad (1.1)$$

where the surface S is the half–plane with $\underline{\mathbf{R}} = \{0 \leq x < \infty; y = 0; -\infty < z < \infty\}$ and $\underline{\mathbf{n}} = \underline{\mathbf{e}}_y$.

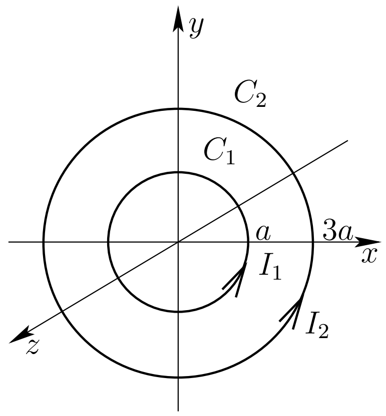

Figure 1.1: Infinitely thin, circular wire loops

Problem 2 [25 points]

A sphere of radius R_0 centered at the coordinate origin is charged with an inhomogeneous electric (volume) charge density

$$\varrho(R) \quad = \quad \varrho_0 \frac{R}{R_0} \qquad 0 \leq R \leq R_0 \quad .$$

There is no electric (volume) charge density outside of the sphere.
The sphere is filled with an inhomogeneous dielectric medium according to the inhomogeneous relative permittivity

$$\varepsilon_r(R) \quad = \quad 5 \frac{R}{R_0} \qquad 0 \leq R \leq R_0 \quad .$$

The embedding medium of the sphere is vacuum.

a) Draw a sketch of the problem.

b) Determine the electric flux density $\underline{\mathbf{D}}(\underline{\mathbf{R}})$.

c) Determine the electric field strength $\underline{\mathbf{E}}(\underline{\mathbf{R}})$.

d) Determine the electric polarization $\underline{\mathbf{P}}(\underline{\mathbf{R}})$.

e) Sketch for $0 \leq R \leq 2R_0$: 1. $\varrho(R)$, 2. $\varepsilon_{\mathrm{r}}(R)$, 3. $D(R)$, 4. $E(R)$, and 5. $P(R)$.

Problem 3 [15 points]

Given is the distribution of five electrostatic point charges as shown in Figure 3.1.

a) Determine the electric (volume) charge density mathematically.

b) Calculate the dipole moment of the charge distribution.

c) Calculate the quadrupole moment of the charge distribution.

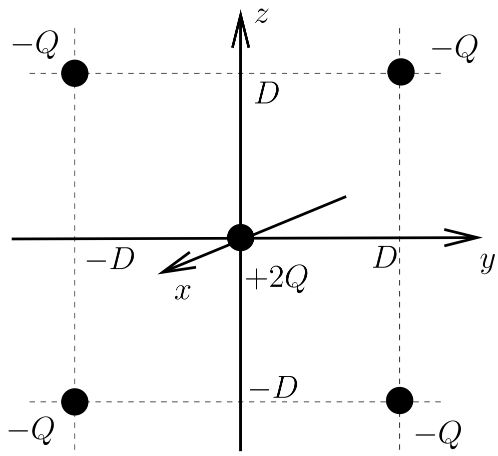

Figure 3.1: Electrostatic point charge distribution

Problem 4 [20 points]

Consider the electrostatic point charge distribution in Figure 4.1. The xy–plane is perfectly electrically conducting and grounded, i. e., $\sigma_\mathrm{e} \to \infty$ and $\Phi(\underline{\mathbf{R}}) = 0$ for $\underline{\mathbf{R}} \in xy$–plane.

a) Find the electrostatic potential $\Phi(\underline{\mathbf{R}})$ using the method of images.

b) Determine the electric surface charge density $\eta\,(x, y)$ along the xy–plane at $z = 0$.

c) At which positions does the electric surface charge density $\eta\,(x, y)$ reach an extremum?

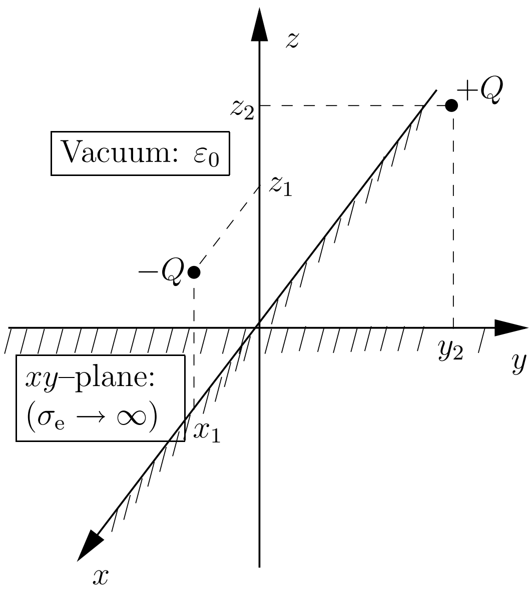

Figure 4.1: Electrostatic point charge distribution over a perfectly electrically conducting plane