Aufgabe 1

$$\frac{\partial}{\partial t}\underline{\boldsymbol{B}} = -\nabla \times \underline{\boldsymbol{E}} \qquad \frac{\partial}{\partial t}\underline{\boldsymbol{D}} = \nabla \times \underline{\boldsymbol{H}}$$

GhK
TET

$$\frac{\partial}{\partial t}\underline{\boldsymbol{p}} = \nabla \cdot \underline{\underline{\boldsymbol{T}}} \qquad \frac{\partial}{\partial t}\underline{\underline{\boldsymbol{S}}} = \mathrm{sym}\{\nabla \underline{\boldsymbol{v}}\}$$

EFT I - KLAUSUR

„Einführung in die Theoretische Elektrotechnik“

vom 13.09.00

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a)

$$\Delta\Phi(R) = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial\Phi}{\partial R}\right) \overset{!}{=} 0$$

$$R^2\frac{\partial\Phi}{\partial R} = C_1$$

$$\Phi = \int \frac{C_1}{R^2}\,\mathrm{d}R + C_2$$

$$\Phi = -\frac{C_1}{R} + C_2$$

$$\Phi(1) = 0 \quad \to \quad -C_1 + C_2 = 0$$

$$\Phi(2) = 1 \quad \to \quad -\frac{C_1}{2} + C_2 = 1$$

$$\Rightarrow \quad C_1 = 2, \quad C_2 = 1$$

$$\Phi(R) = 2\left(1 - \frac{1}{R}\right)$$

b)

$$\underline{\mathbf{E}} = -\nabla\Phi = -\left[\frac{\partial}{\partial R}2\left(1 - \frac{1}{R}\right)\right]\underline{\mathbf{e}}_R$$

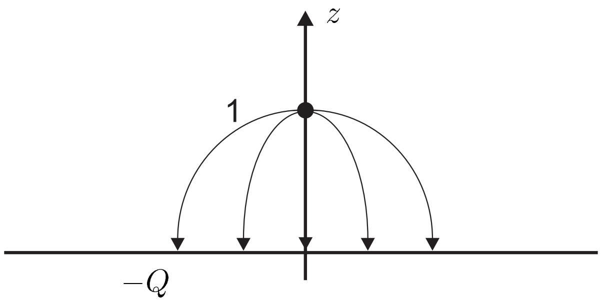
$$\underline{\mathbf{E}} = -\frac{2}{R^2}\,\underline{\mathbf{e}}_R$$

c)

$$\begin{aligned} U &= \Phi(2) - \Phi(1) = 1 \\ \text{oder} \quad U &= \int \underline{\mathbf{E}} \cdot \mathrm{d}\underline{\mathbf{R}} \\ &= \int_2^1 \left(-\frac{2}{R^2}\, \underline{\mathbf{e}}_R \right) \cdot \underline{\mathbf{e}}_R \, \mathrm{d}R \\ &= 1 \end{aligned}$$

Aufgabe 2

a) Anordnung und Feldlinien des elektrischen Feldes:



b) Spiegelladung bei $\underline{\mathbf{R}}_2 = -\underline{\mathbf{e}}_z$

$$\Phi = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0} \left(\dfrac{1}{|\,\underline{\mathbf{R}} - \underline{\mathbf{e}}_z\,|} - \dfrac{1}{|\,\underline{\mathbf{R}} + \underline{\mathbf{e}}_z\,|} \right) & \text{für} \quad z > 0 \\ \\ 0 & \text{sonst} \end{cases}$$

c)

$$\underline{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0} \left(\frac{\underline{\mathbf{R}} - \underline{\mathbf{e}}_z}{|\,\underline{\mathbf{R}} - \underline{\mathbf{e}}_z\,|^3} - \frac{\underline{\mathbf{R}} + \underline{\mathbf{e}}_z}{|\,\underline{\mathbf{R}} + \underline{\mathbf{e}}_z\,|^3} \right)$$

d) Für $z = 0$ ist

$$\begin{aligned} |\,\underline{\mathbf{R}} - \underline{\mathbf{e}}_z\,|^3 &= (x^2 + y^2 + 1)^{3/2} \\ &= |\,\underline{\mathbf{R}} + \underline{\mathbf{e}}_z\,|^3 \end{aligned}$$

Also:

$$\underline{\mathbf{E}}(z = 0) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{-2\,\underline{\mathbf{e}}_z}{(x^2 + y^2 + 1)^{3/2}}$$

Aufgabe 3

a)

$$\begin{aligned} I &= \int_0^{2\pi}\int_0^{r_0} \frac{I_0}{\pi r_0^2}\,\underline{\mathbf{e}}_z \cdot \underline{\mathbf{e}}_z\, r\,\mathrm{d}r\,\mathrm{d}\varphi \\ &= \frac{I_0}{\pi r_0^2}\,2\pi\,\frac{r_0^2}{2} = I_0 \end{aligned}$$

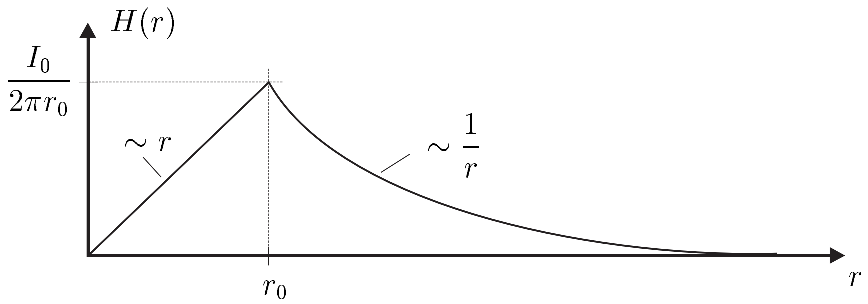
b)

$$\int_{\partial S} \underline{\mathbf{H}} \cdot \mathrm{d}\underline{\mathbf{R}} = \iint_S \underline{\mathbf{J}} \cdot \mathrm{d}\underline{\mathbf{S}}$$

Zylindersymmetrie $\quad\rightarrow\quad \underline{\mathbf{H}} = H(r)\,\underline{\mathbf{e}}_\varphi$

$$\begin{aligned} \int_{\partial S} \underline{\mathbf{H}} \cdot \mathrm{d}\underline{\mathbf{R}} &= \int_0^{2\pi} H(r)\,\underline{\mathbf{e}}_\varphi \cdot r\,\underline{\mathbf{e}}_\varphi \mathrm{d}\varphi = H(r)2\pi r \\ \iint_S \underline{\mathbf{J}} \cdot \mathrm{d}\underline{\mathbf{S}} &= \begin{cases} I_0 \left(\dfrac{r}{r_0}\right)^2 & \text{für} \quad 0 \le r \le r_0 \\ I_0 & \text{für} \quad r > r_0 \end{cases} \end{aligned}$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \begin{cases} \dfrac{I_0}{2\pi r_0^2}\,r\,\underline{\mathbf{e}}_\varphi & \text{für} \quad 0 \le r \le r_0 \\ \dfrac{I_0}{2\pi r}\underline{\mathbf{e}}_\varphi & \text{für} \quad r > r_0 \end{cases}$$



c)

Aufgabe 4

a)

$$\varrho(\underline{\mathbf{R}}) = Q\,\delta(\underline{\mathbf{R}}),$$
$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \frac{Q}{4\pi R^2}\,\underline{\mathbf{e}}_R$$

b)

$$\begin{aligned}\iint\limits_S \underline{\mathbf{D}} \cdot \mathrm{d}\underline{\mathbf{S}} &= \int\limits_0^{2\pi}\int\limits_0^{\pi} \frac{Q}{4\pi R_0^2}\,\underline{\mathbf{e}}_R \cdot \underline{\mathbf{e}}_R R_0^2 \sin\vartheta\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \\ &= Q\end{aligned}$$

$$\begin{aligned}\iiint\limits_B \nabla \cdot \underline{\mathbf{D}}\,\mathrm{d}V &= \iiint\limits_B Q\,\delta(\underline{\mathbf{R}})\,\mathrm{d}V \\ &= Q \iiint\limits_B \delta(\underline{\mathbf{R}})\,\mathrm{d}V \\ &= Q\end{aligned}$$

$$\iint\limits_S \underline{\mathbf{D}} \cdot \mathrm{d}\underline{\mathbf{S}} = \iiint\limits_B \nabla \cdot \underline{\mathbf{D}} \cdot \mathrm{d}V = Q$$

Aufgabe 5

$$Q = \iiint\limits_{-\infty}^{\infty} \varrho(\underline{\mathbf{R}})\,\mathrm{d}V$$

$$\begin{aligned}
Q &= \int\limits_0^{2\pi}\int\limits_0^{\pi}\int\limits_{R_1}^{R_2} \varrho_0 \frac{R_0^2}{R^2} \sin\left(\pi\frac{R-R_1}{R_2-R_1}\right) R^2 \sin\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \\
&= \varrho_0\, R_0^2\, 2\pi \cdot 2 \int\limits_{R_1}^{R_2} \sin\left(\pi\frac{R-R_1}{R_2-R_1}\right)\mathrm{d}R
\end{aligned}$$

<u>Substitution:</u>

$$x = \pi\frac{R-R_1}{R_2-R_1} \quad \curvearrowright \quad \frac{\mathrm{d}x}{\mathrm{d}R} = \frac{\pi}{R_2-R_1} \quad \curvearrowright \quad \mathrm{d}R = \frac{R_2-R_1}{\pi}\,\mathrm{d}x$$

<u>Also:</u>

$$\begin{aligned}
Q &= \varrho_0\, R_0^2\, 4\pi \int\limits_0^{\pi} \sin x\, \frac{R_2-R_1}{\pi}\,\mathrm{d}x \\
&= 4\varrho_0\, R_0^2\,(R_2-R_1) \int\limits_0^{\pi} \sin x\,\mathrm{d}x \\
&= 8\varrho_0\, R_0^2\,(R_2-R_1)
\end{aligned}$$