Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

10th Lecture / 10. Vorlesung

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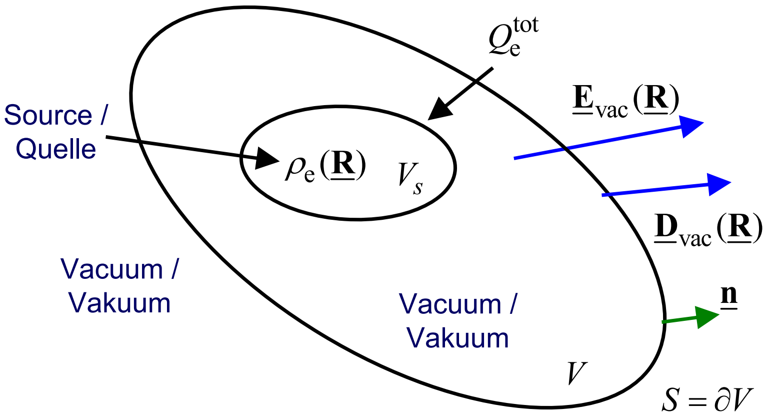
ES Fields – Electric Polarization of Materials / ES Felder – Elektrische Polarisation von Materialien



$$\oiint_{S_s=\partial V_s} \underline{\mathbf{D}}_{vac}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}\, dS = Q_e^{tot}$$

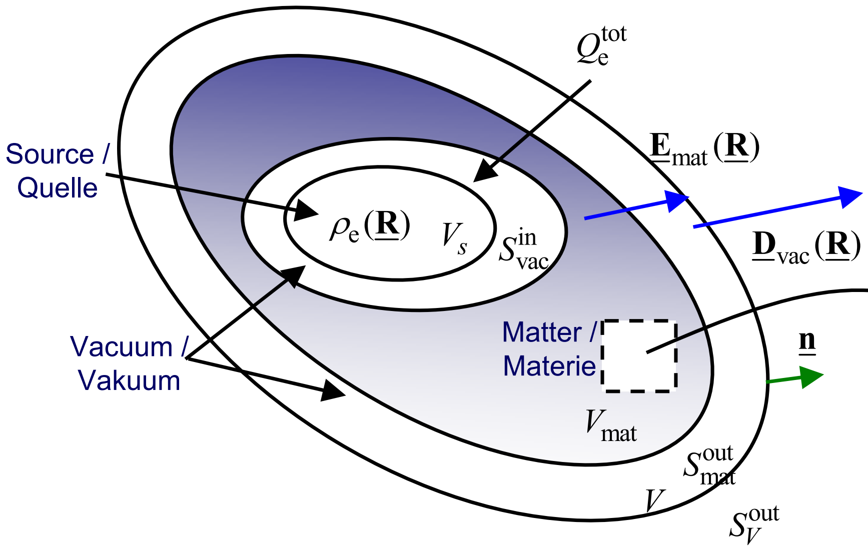
$$= \oiint_{S=\partial V} \underline{\mathbf{D}}_{vac}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}\, dS$$

$$\underline{\mathbf{D}}_{vac}(\underline{\mathbf{R}}) = \varepsilon_0 \underline{\mathbf{E}}_{vac}(\underline{\mathbf{R}})$$

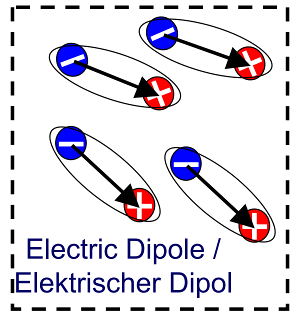
$$= \varepsilon_0 \underbrace{\varepsilon_{r,vac}}_{=1} \underline{\mathbf{E}}_{vac}(\underline{\mathbf{R}})$$

Vacuum / Vakuum

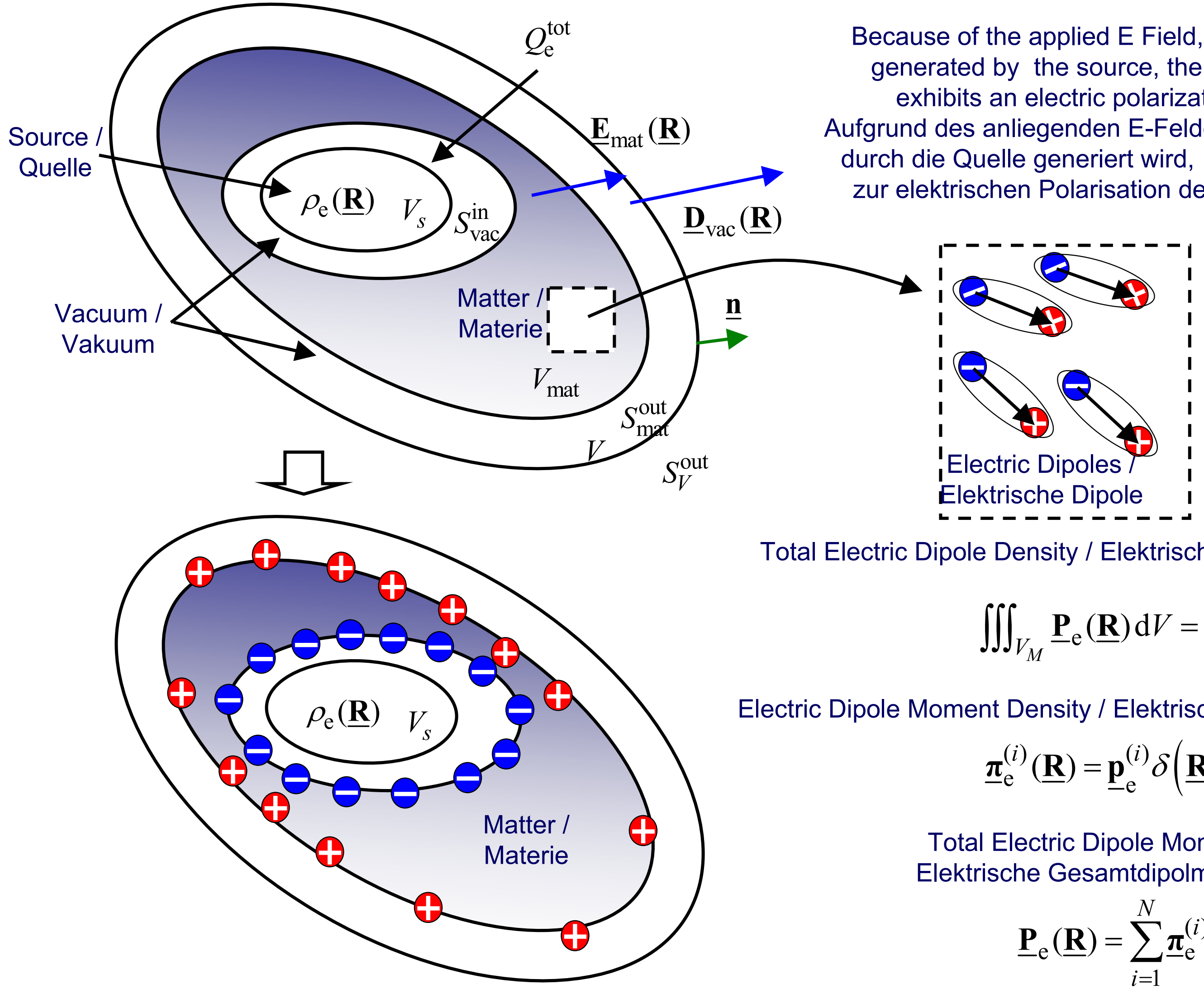
$$\varepsilon_{r,vac} = 1$$



Because of the applied E Field, which is generated by the source, the matter exhibits an electric polarization. /
Aufgrund des anliegenden E-Feldes, welches durch die Quelle generiert wird, kommt es zur elektrischen Polarisation der Materie



ES Fields – Electric Polarization of Materials / ES Felder – Elektrische Polarisation von Materialien



Because of the applied E Field, which is generated by the source, the matter exhibits an electric polarization. / Aufgrund des anliegenden E-Feldes, welches durch die Quelle generiert wird, kommt es zur elektrischen Polarisation der Materie

$\underline{\mathbf{p}}_e^{(i)}, i = 1,2,3,4$

Total Electric Dipole Density / Elektrisches Gesamtdipolmoment

$$\iiint_{V_M} \underline{\mathbf{P}}_e(\underline{\mathbf{R}})\,dV = \sum_{i=1}^{N} \underline{\mathbf{p}}_e^{(i)}$$

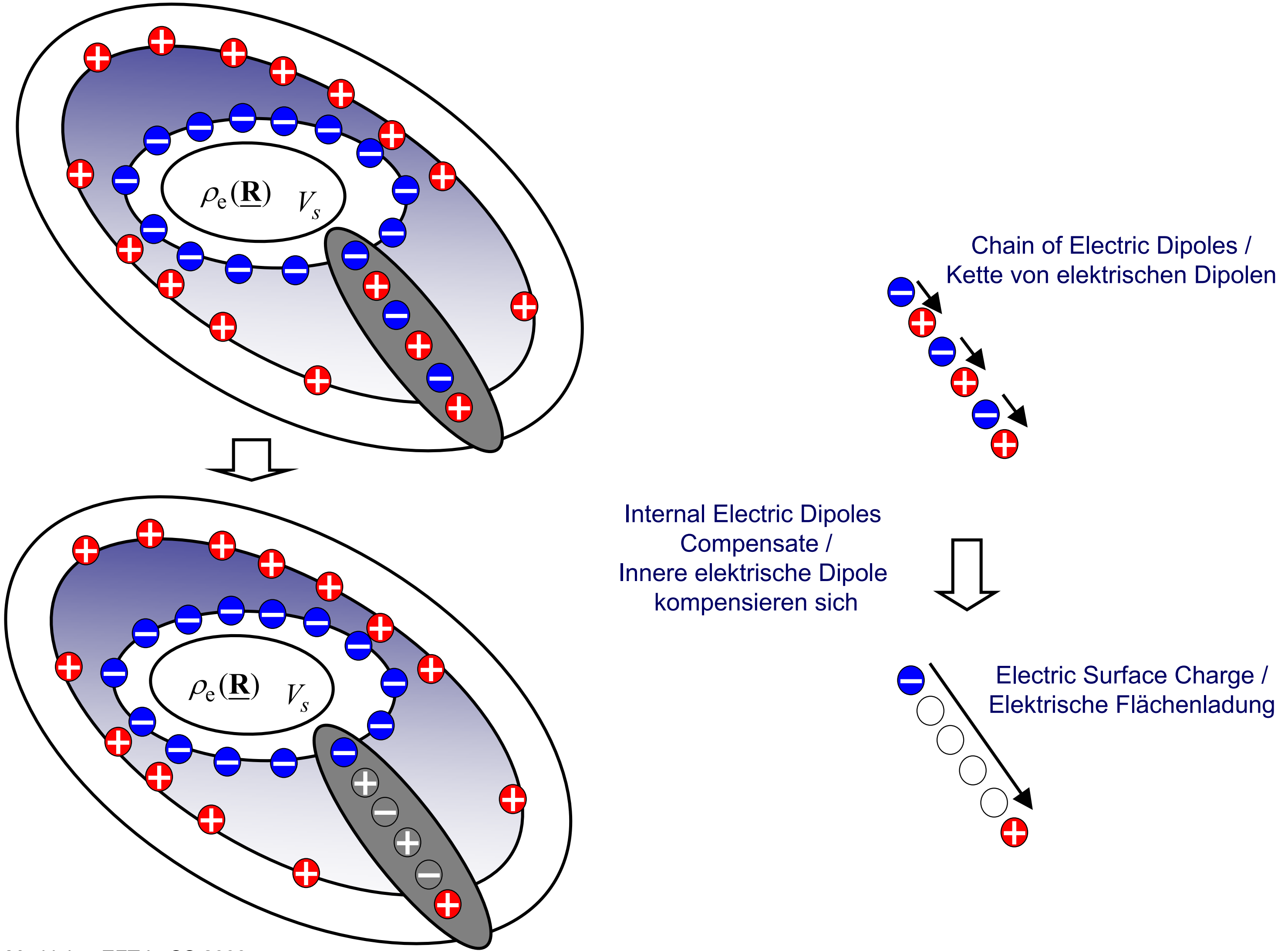
Electric Dipole Moment Density / Elektrische Dipolmomentendichte

$$\underline{\boldsymbol{\pi}}_e^{(i)}(\underline{\mathbf{R}}) = \underline{\mathbf{p}}_e^{(i)}\delta\left(\underline{\mathbf{R}} - \underline{\mathbf{R}}^{(i)}\right)$$

Total Electric Dipole Moment Density / Elektrische Gesamtdipolmomentendichte

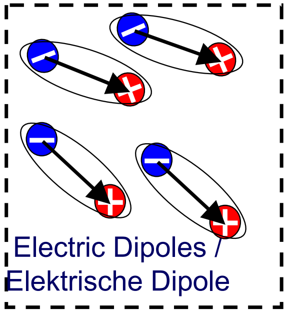
$$\underline{\mathbf{P}}_e(\underline{\mathbf{R}}) = \sum_{i=1}^{N} \underline{\boldsymbol{\pi}}_e^{(i)}(\underline{\mathbf{R}})$$

ES Fields – Electric Polarization of Materials / ES Felder – Elektrische Polarisation von Materialien



ES Fields – Electric Polarization of Materials / ES Felder – Elektrische Polarisation von Materialien

$$\iiint_{V_M} \underline{\mathbf{P}}_{\mathrm{e}}(\underline{\mathbf{R}})\,\mathrm{d}V = \sum_{i=1}^{N} \underline{\mathbf{p}}_{\mathrm{e}}^{(i)}$$



$\underline{\mathbf{p}}_{\mathrm{e}}^{(i)}, i = 1,2,3,4$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \begin{cases} \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}) + \underline{\mathbf{P}}_{\mathrm{e}}(\underline{\mathbf{R}}) & \underline{\mathbf{R}} \in V_{\mathrm{mat}} \\ \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}) & \underline{\mathbf{R}} \in V_{\mathrm{vac}} \end{cases}$$

$$\begin{aligned} \underline{\mathbf{D}}(\underline{\mathbf{R}}) &= \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}) + \underline{\mathbf{P}}_{\mathrm{e}}(\underline{\mathbf{R}}) \\ &= \varepsilon_0 \varepsilon_{\mathrm{r}}(\underline{\mathbf{R}}) \underline{\mathbf{E}}(\underline{\mathbf{R}}) \\ &= \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}) + \underbrace{\varepsilon_0 \left[\varepsilon_{\mathrm{r}}(\underline{\mathbf{R}}) - 1\right] \underline{\mathbf{E}}(\underline{\mathbf{R}})}_{= \underline{\mathbf{P}}_{\mathrm{e}}(\underline{\mathbf{R}})} \end{aligned}$$

$\varepsilon_{\mathrm{r}}(\underline{\mathbf{R}})$ Relative Permittivity / Relative Permittivität

$$\begin{aligned} \underline{\mathbf{P}}_{\mathrm{e}}(\underline{\mathbf{R}}) &= \varepsilon_0 \underbrace{\left[\varepsilon_{\mathrm{r}}(\underline{\mathbf{R}}) - 1\right]}_{= \chi_{\mathrm{e}}(\underline{\mathbf{R}})} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \\ &= \varepsilon_0 \chi_{\mathrm{e}}(\underline{\mathbf{R}}) \underline{\mathbf{E}}(\underline{\mathbf{R}}) \end{aligned}$$

$\chi_{\mathrm{e}}(\underline{\mathbf{R}})$ Electric Susceptibility / Elektrische Suszeptibilität

ES Fields – Electric Polarization of Materials / ES Felder – Elektrische Polarisation von Materialien

General Case / Allgemeiner Fall

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}) + \underline{\mathbf{P}}_\mathrm{e}(\underline{\mathbf{R}})$$

Isotropic Case / Isotroper Fall

$$\underline{\mathbf{P}}_\mathrm{e}(\underline{\mathbf{R}}) = \varepsilon_0 \chi_\mathrm{e}(\underline{\mathbf{R}}) \underline{\mathbf{E}}(\underline{\mathbf{R}})$$

$$\begin{aligned}\underline{\mathbf{D}}(\underline{\mathbf{R}}) &= \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}) + \underline{\mathbf{P}}_\mathrm{e}(\underline{\mathbf{R}}) \\ &= \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}) + \varepsilon_0 \chi_\mathrm{e}(\underline{\mathbf{R}}) \underline{\mathbf{E}}(\underline{\mathbf{R}}) \\ &= \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}) + \varepsilon_0 \left[\varepsilon_\mathrm{r}(\underline{\mathbf{R}}) - 1\right] \underline{\mathbf{E}}(\underline{\mathbf{R}})\end{aligned}$$

$$\chi_\mathrm{e}(\underline{\mathbf{R}}) = \varepsilon_\mathrm{r}(\underline{\mathbf{R}}) - 1$$

$$\varepsilon_\mathrm{r}(\underline{\mathbf{R}}) = \chi_\mathrm{e}(\underline{\mathbf{R}}) + 1$$

Anisotropic Case / Anisotroper Fall

$$\underline{\mathbf{P}}_\mathrm{e}(\underline{\mathbf{R}}) = \varepsilon_0 \underline{\underline{\boldsymbol{\chi}}}_\mathrm{e}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{E}}(\underline{\mathbf{R}})$$

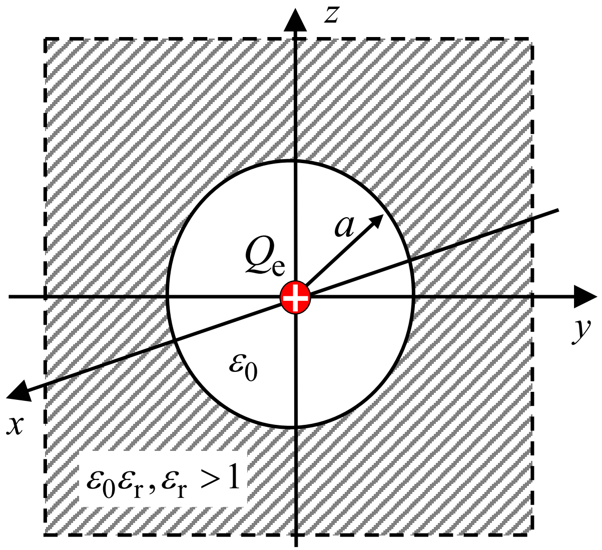
$$\begin{aligned}\underline{\mathbf{D}}(\underline{\mathbf{R}}) &= \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}) + \underline{\mathbf{P}}_\mathrm{e}(\underline{\mathbf{R}}) \\ &= \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}) + \varepsilon_0 \underline{\underline{\boldsymbol{\chi}}}_\mathrm{e}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{E}}(\underline{\mathbf{R}}) \\ &= \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}) + \varepsilon_0 \left[\underline{\underline{\boldsymbol{\varepsilon}}}_\mathrm{r}(\underline{\mathbf{R}}) - \underline{\underline{\mathbf{I}}}\right] \bullet \underline{\mathbf{E}}(\underline{\mathbf{R}})\end{aligned}$$

$$\underline{\underline{\boldsymbol{\chi}}}_\mathrm{e}(\underline{\mathbf{R}}) = \underline{\underline{\boldsymbol{\varepsilon}}}_\mathrm{r}(\underline{\mathbf{R}}) - \underline{\underline{\mathbf{I}}}$$

$$\underline{\underline{\boldsymbol{\varepsilon}}}_\mathrm{r}(\underline{\mathbf{R}}) = \underline{\underline{\boldsymbol{\chi}}}_\mathrm{e}(\underline{\mathbf{R}}) + \underline{\underline{\mathbf{I}}}$$

ES Fields – Electric Polarization of Materials – Example / ES Felder – Elektrische Polarisation von Materialien – Beispiel

Electric Point Charge Embedded in a Sphere Filled with Vacuum, which is Embedded in a Dielectric Material / Elektrische Punktladung eingebettet in einer mit Vakuum gefüllter Kugel, die in ein dielektrisches Material eingebettet ist.



$$\varepsilon(R) = \begin{cases} \varepsilon_0 & R < a \\ \varepsilon_0\varepsilon_r & R > a \end{cases}$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi}\frac{1}{R^2}\hat{\underline{\mathbf{R}}} = \frac{Q_e}{4\pi}\frac{1}{R^2}\underline{\mathbf{e}}_R$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \begin{cases} \dfrac{\underline{\mathbf{D}}(\underline{\mathbf{R}})}{\varepsilon_0} & R < a \\ \dfrac{\underline{\mathbf{D}}(\underline{\mathbf{R}})}{\varepsilon_0\varepsilon_r} & R > a \end{cases}$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \begin{cases} \varepsilon_0\underline{\mathbf{E}}(\underline{\mathbf{R}}) & R < a \\ \varepsilon_0\underline{\mathbf{E}}(\underline{\mathbf{R}}) + \underline{\mathbf{P}}_e(\underline{\mathbf{R}}) & R > a \end{cases}$$

$$\underline{\mathbf{P}}_e(\underline{\mathbf{R}}) = \begin{cases} \underline{\mathbf{0}} & R < a \\ \varepsilon_0(\varepsilon_r - 1)\underline{\mathbf{E}}(\underline{\mathbf{R}}) & R > a \end{cases}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \begin{cases} \dfrac{\underline{\mathbf{D}}(\underline{\mathbf{R}})}{\varepsilon_0} & R < a \\ \dfrac{\underline{\mathbf{D}}(\underline{\mathbf{R}})}{\varepsilon_0} + \dfrac{\underline{\mathbf{P}}_e(\underline{\mathbf{R}})}{\varepsilon_0} & R > a \end{cases}$$

$$= \begin{cases} \dfrac{Q_e}{4\pi\varepsilon_0}\dfrac{1}{R^2}\underline{\mathbf{e}}_R & R < a \\ \dfrac{Q_e}{4\pi\varepsilon_0}\dfrac{1}{R^2}\underline{\mathbf{e}}_R - \dfrac{\underline{\mathbf{P}}_e(\underline{\mathbf{R}})}{\varepsilon_0} & R > a \end{cases}$$

ES Fields – Electric Polarization of Materials – Example / ES Felder – Elektrische Polarisation von Materialien – Beispiel (...)

$$\underline{\mathbf{P}}_\mathrm{e}(\underline{\mathbf{R}}) = \begin{cases} \underline{\mathbf{0}} & R < a \\ \varepsilon_0(\varepsilon_\mathrm{r} - 1)\underline{\mathbf{E}}(\underline{\mathbf{R}}) & R > a \end{cases}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \begin{cases} \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{1}{R^2}\underline{\mathbf{e}}_R & R < a \\ \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{1}{R^2}\underline{\mathbf{e}}_R - \frac{\underline{\mathbf{P}}_\mathrm{e}(\underline{\mathbf{R}})}{\varepsilon_0} & R > a \end{cases}$$

$$= \begin{cases} \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{1}{R^2}\underline{\mathbf{e}}_R & R < a \\ \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{1}{R^2}\underline{\mathbf{e}}_R - \frac{\varepsilon_0(\varepsilon_\mathrm{r} - 1)\underline{\mathbf{E}}(\underline{\mathbf{R}})}{\varepsilon_0} & R > a \end{cases}$$

$$= \begin{cases} \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{1}{R^2}\underline{\mathbf{e}}_R & R < a \\ \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{1}{R^2}\underline{\mathbf{e}}_R - (\varepsilon_\mathrm{r} - 1)\underline{\mathbf{E}}(\underline{\mathbf{R}}) & R > a \end{cases}$$

$R > a$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{1}{R^2}\underline{\mathbf{e}}_R - (\varepsilon_\mathrm{r} - 1)\underline{\mathbf{E}}(\underline{\mathbf{R}})$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) + (\varepsilon_\mathrm{r} - 1)\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{1}{R^2}\underline{\mathbf{e}}_R$$

$$\varepsilon_\mathrm{r}\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{1}{R^2}\underline{\mathbf{e}}_R$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{Q_\mathrm{e}}{4\pi\varepsilon_0\varepsilon_\mathrm{r}}\frac{1}{R^2}\underline{\mathbf{e}}_R$$

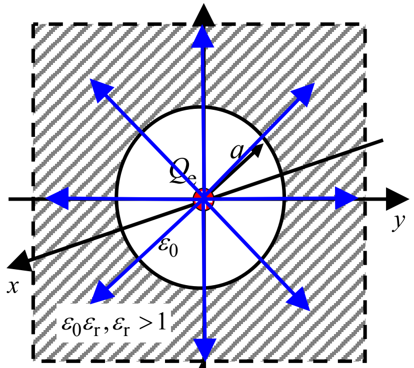
$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \begin{cases} \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{1}{R^2}\underline{\mathbf{e}}_R & R < a \\ \frac{Q_\mathrm{e}}{4\pi\varepsilon_0\varepsilon_\mathrm{r}}\frac{1}{R^2}\underline{\mathbf{e}}_R & R > a \end{cases}$$

ES Fields – Electric Polarization of Materials – Example / ES Felder – Elektrische Polarisation von Materialien – Beispiel (...)

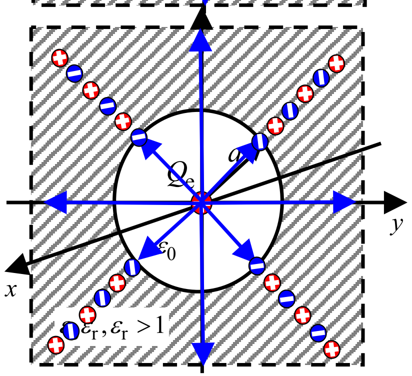
Permittivity Profile / Permittivitätsprofil

$$\varepsilon(R) = \begin{cases} \varepsilon_0 & R < a \\ \varepsilon_0 \varepsilon_\mathrm{r} & R > a \end{cases}$$

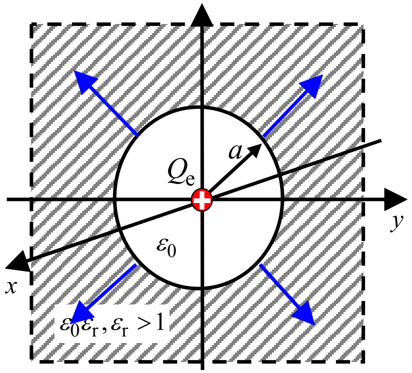
$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \frac{Q_\mathrm{e}}{4\pi} \frac{1}{R^2} \underline{\mathbf{e}}_R$$



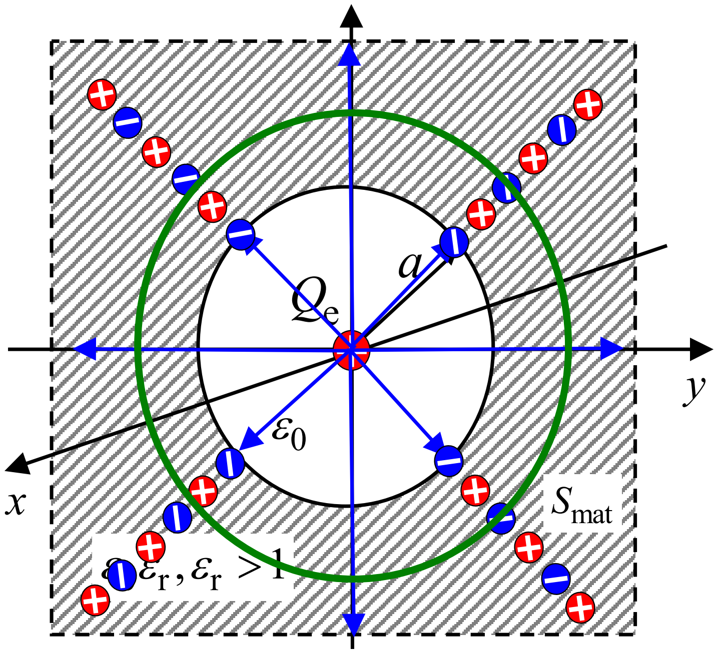
$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \begin{cases} \dfrac{Q_\mathrm{e}}{4\pi\varepsilon_0} \dfrac{1}{R^2} \underline{\mathbf{e}}_R & R < a \\ \dfrac{Q_\mathrm{e}}{4\pi\varepsilon_0\varepsilon_\mathrm{r}} \dfrac{1}{R^2} \underline{\mathbf{e}}_R & R > a \end{cases}$$



$$\underline{\mathbf{P}}_\mathrm{e}(\underline{\mathbf{R}}) = \begin{cases} \underline{\mathbf{0}} & R < a \\ \dfrac{Q_\mathrm{e}}{4\pi} \dfrac{\varepsilon_\mathrm{r} - 1}{\varepsilon_\mathrm{r}} \dfrac{1}{R^2} \underline{\mathbf{e}}_R & R > a \end{cases}$$



ES Fields – Electric Polarization of Materials – Example / ES Felder – Elektrische Polarisation von Materialien – Beispiel (...)



$$\oiint_{S_{\mathrm{mat}}} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \varepsilon_0 \oiint_{S_{\mathrm{mat}}} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} + \oiint_{S_{\mathrm{mat}}} \underline{\mathbf{P}}_{\mathrm{e}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}}$$

$$= Q_{\mathrm{e}}$$

$$\varepsilon_0 \oiint_{S_{\mathrm{mat}}} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = Q_{\mathrm{e}} - \underbrace{\oiint_{S_{\mathrm{mat}}} \underline{\mathbf{P}}_{\mathrm{e}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}}}_{=-Q_{\mathrm{e}}^{\mathrm{pol}}}$$

$$= Q_{\mathrm{e}} + Q_{\mathrm{e}}^{\mathrm{pol}}$$

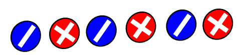
$$\oiint_{S_{\mathrm{mat}}} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = Q_{\mathrm{e}}$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}) + \underline{\mathbf{P}}_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$Q_{\mathrm{e}} = Q_{\mathrm{e}}^{\mathrm{unpaired}}$$

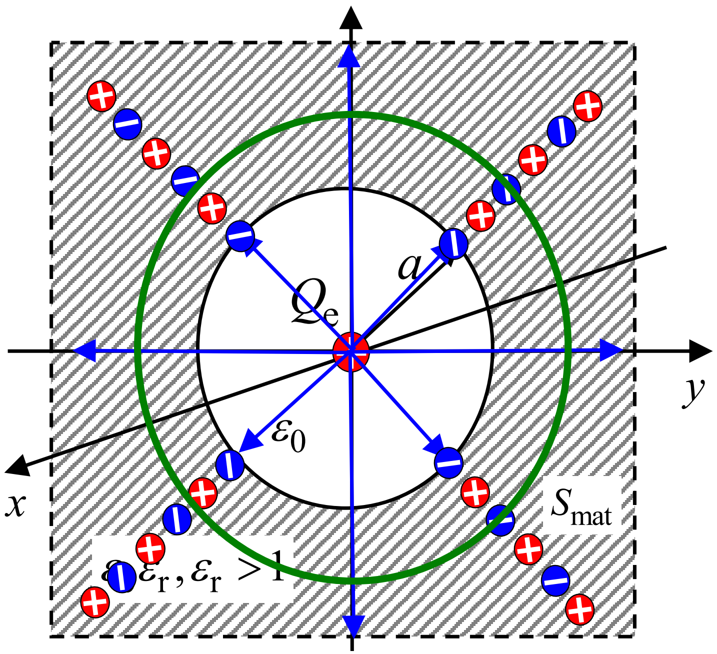
Unpaired Electric Charge / Ungepaarte elektrische Ladungen

$$Q_{\mathrm{e}}^{\mathrm{pol}} = Q_{\mathrm{e}}^{\mathrm{paired}}$$



Paired Electric Charge / Gepaarte elektrische Ladungen

ES Fields – Electric Polarization of Materials – Example / ES Felder – Elektrische Polarisation von Materialien – Beispiel (...)



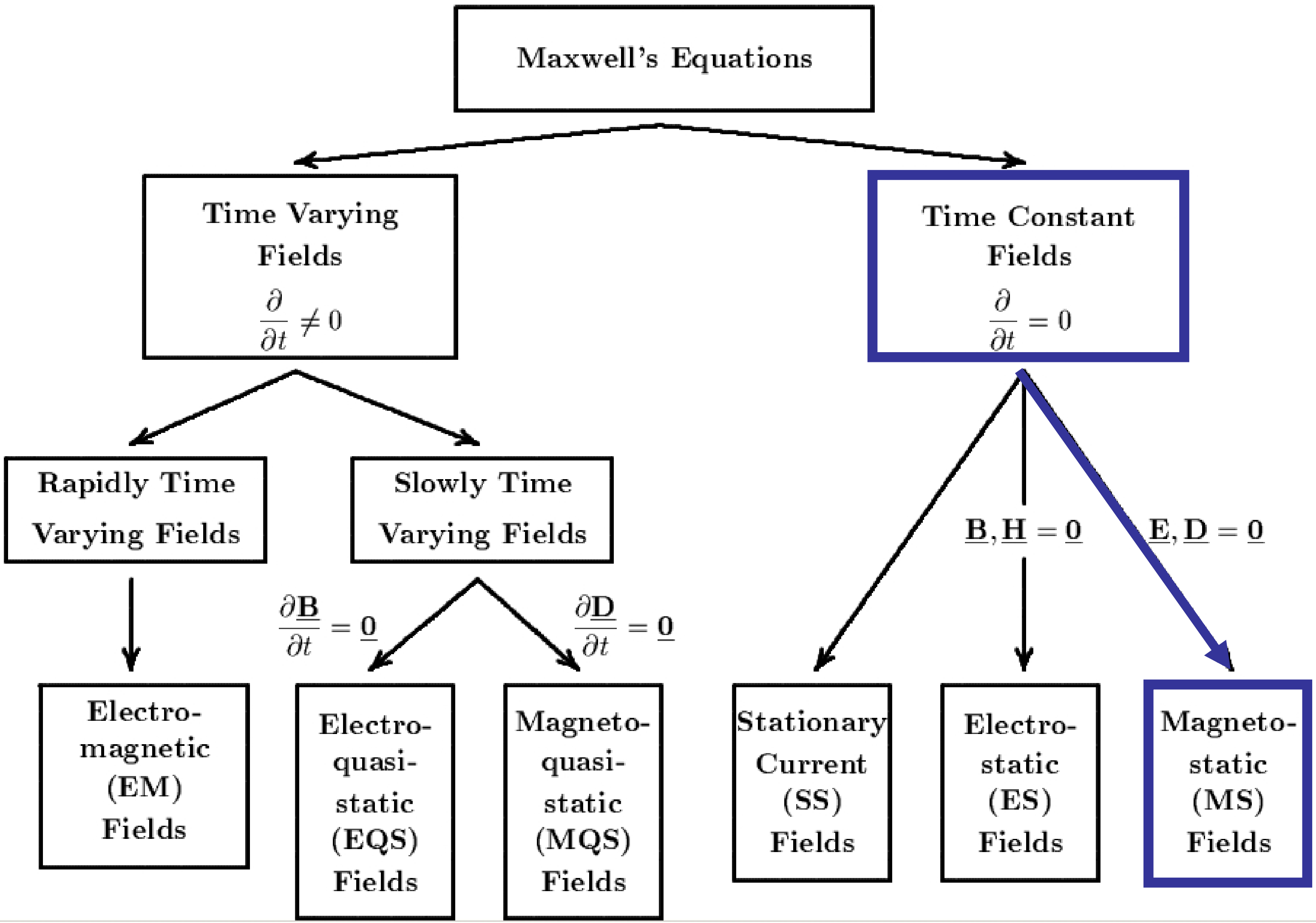
$$Q_e = \oiint_{S_{mat}} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \iiint_{V_{mat}} \nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}})\, \mathrm{d}V$$

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_e(\underline{\mathbf{R}}) = \rho_e^{\text{unpaired}}(\underline{\mathbf{R}})$$

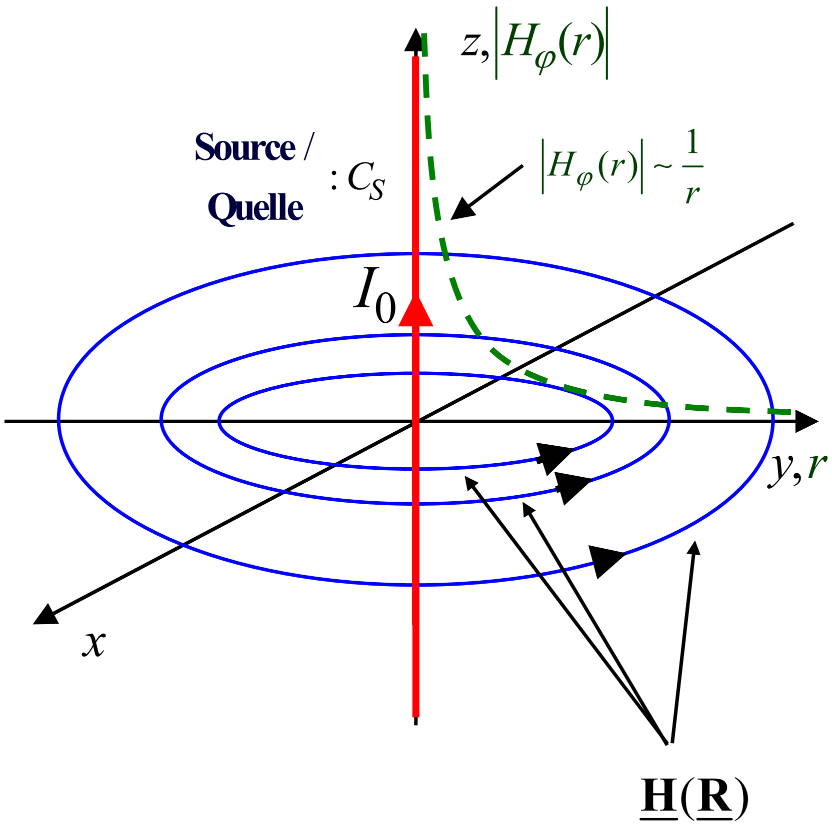
$$Q_e^{\text{pol}} = -\oiint_{S_{mat}} \underline{\mathbf{P}}_e(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = -\iiint_{V_{mat}} \nabla \bullet \underline{\mathbf{P}}_e(\underline{\mathbf{R}})\, \mathrm{d}V$$

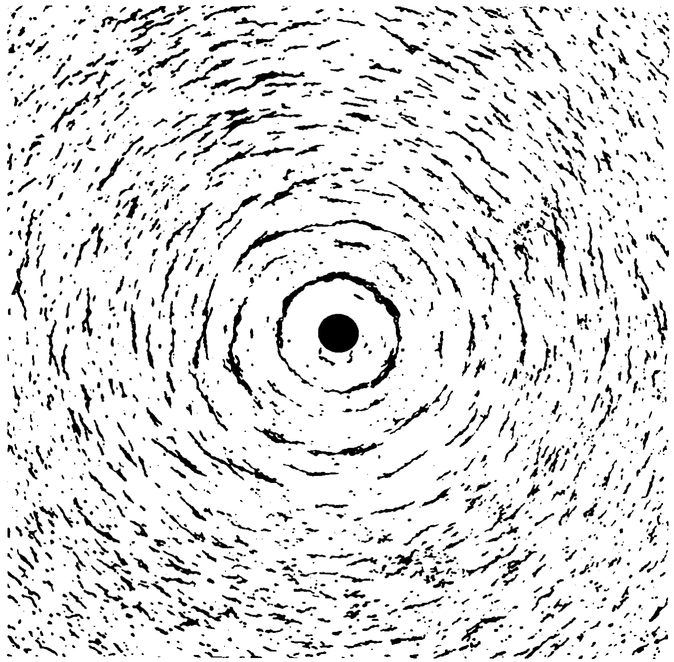
$$\nabla \bullet \underline{\mathbf{P}}_e(\underline{\mathbf{R}}) = -\rho_e^{\text{pol}}(\underline{\mathbf{R}}) = -\rho_e^{\text{paired}}(\underline{\mathbf{R}})$$

Magnetostatic (MS) Fields – Classification / Magnetostatische (MS) Felder - Klassifikation

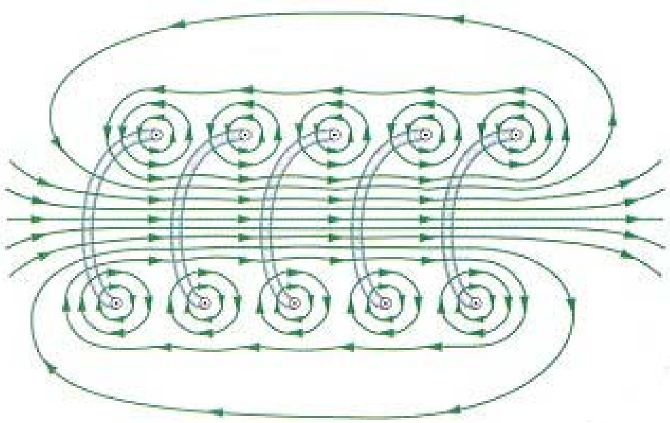


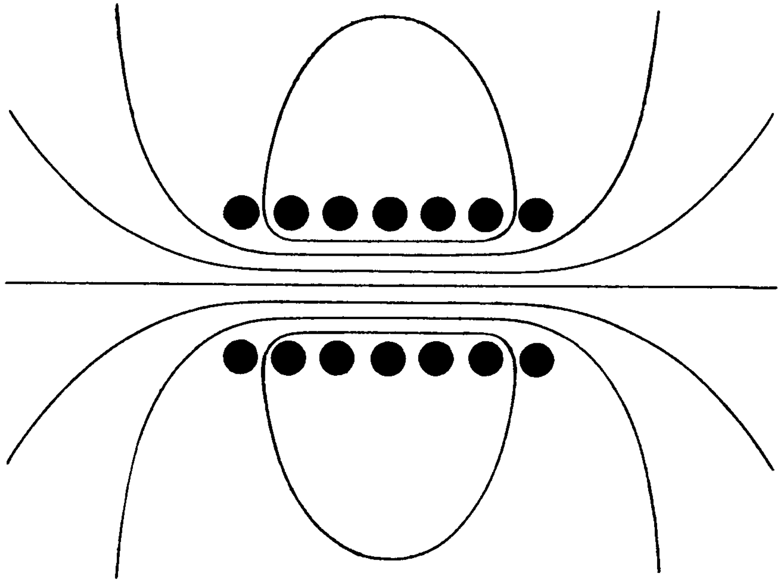
MS Fields – Magnetic Field of a Line Current / MS-Felder – Magnetfeld eines Linienstromes

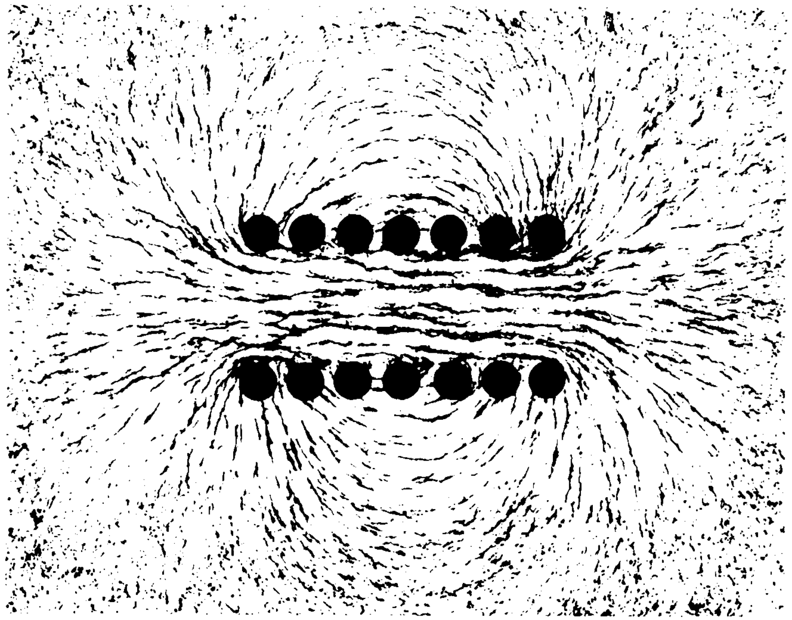




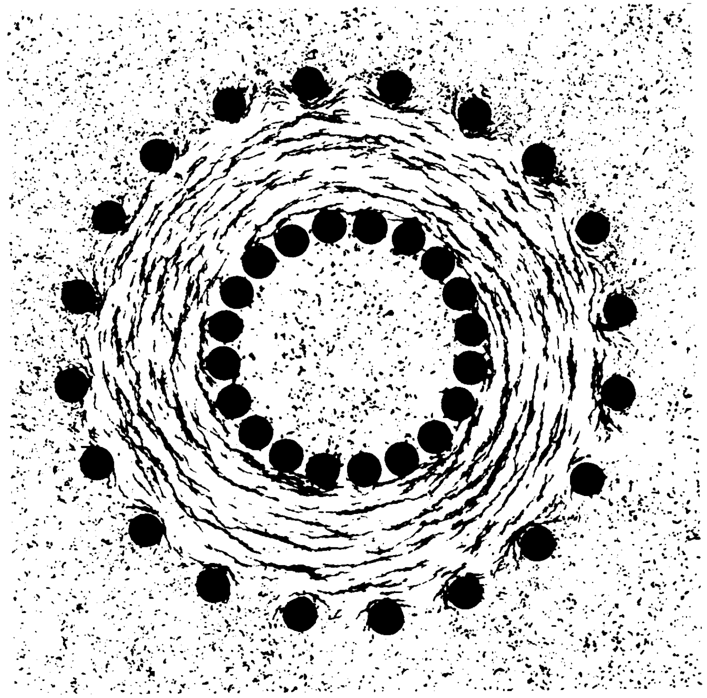
MS Fields – Magnetic Field of a Solenoid / MS-Felder – Magnetfeld eines Spule



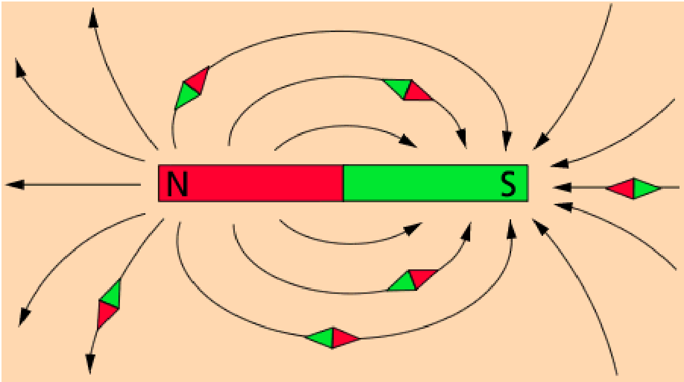


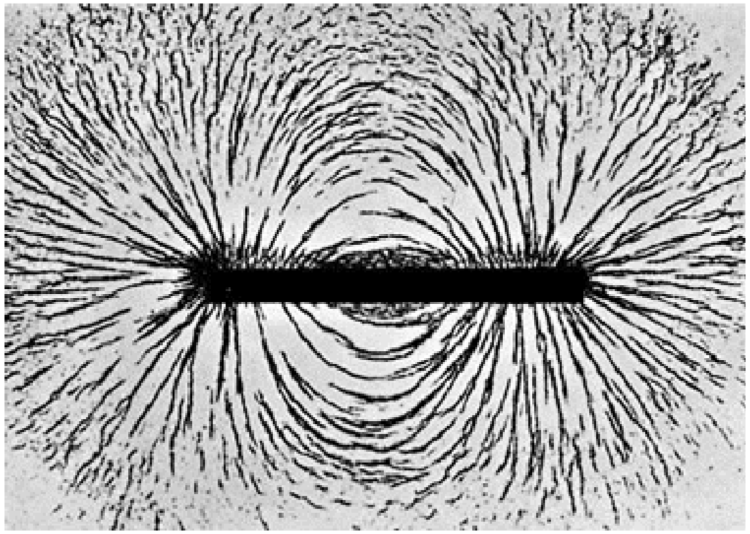


MS Fields – Magnetic Field of a Toroidal Coil / MS-Felder – Magnetisches Feld einer Ringspule



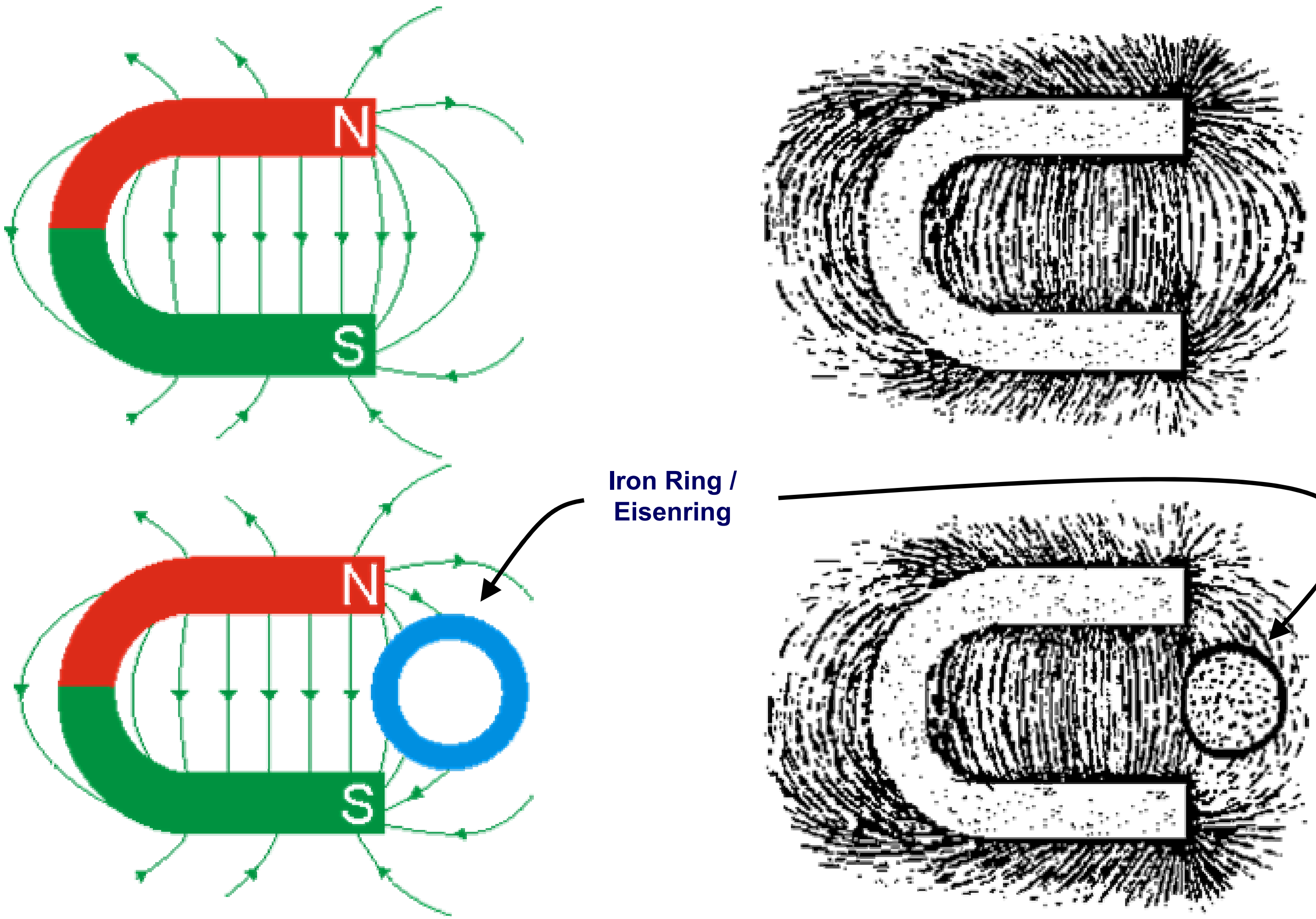
MS Fields – Magnetic Field of a Bar Magnet / MS-Felder – Magnetfeld eines Stabmagneten



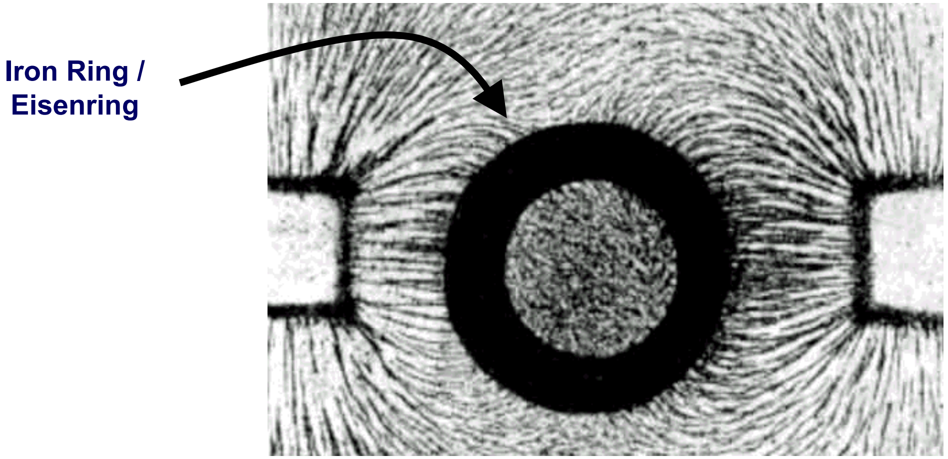


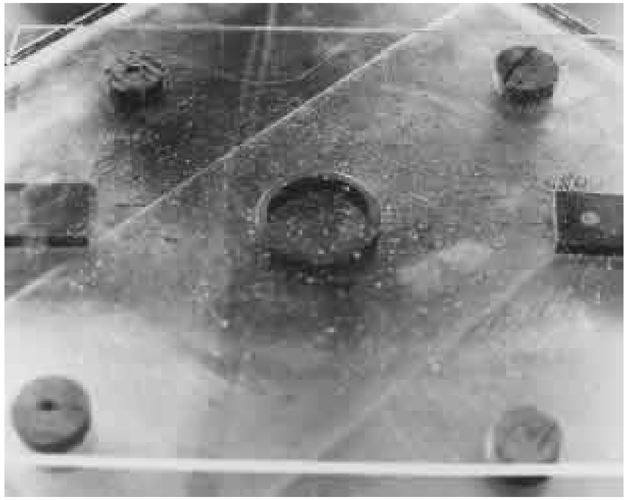
Magnetic Field Lines of a Bar Magnet – Visualized with Iron Filings /
Magnetfeldlinien eines Stabmagneten – Visualisiert mit Eisenfeilspäne

MS Fields – Magnetic Fields of a Horseshoe Magnet (Permanent Magnet) / MS-Felder – Magnetfeld eines Hufeinsenmagneten (Permanentmagnet)

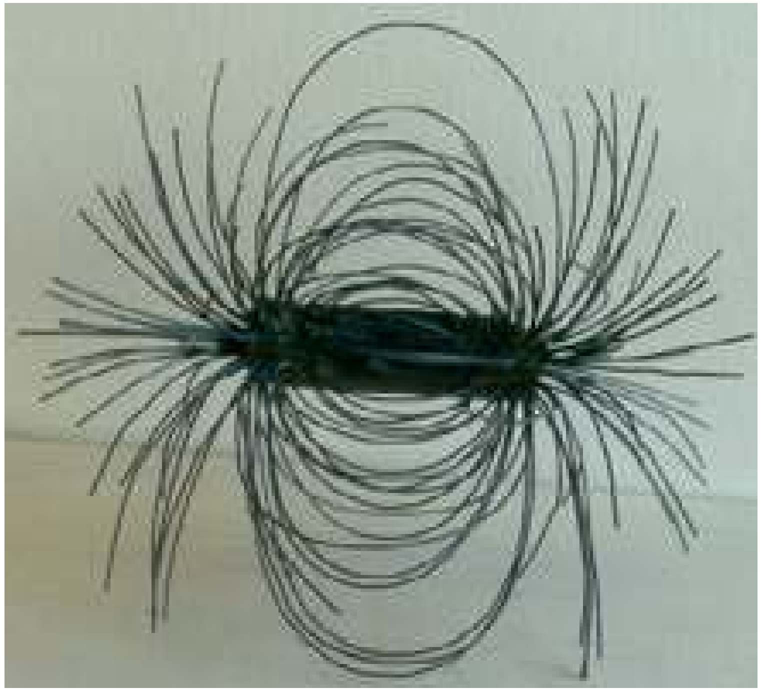


MS Fields – (…) Magnetic Shielding / MS-Felder – (...) Magnetische Abschirmung

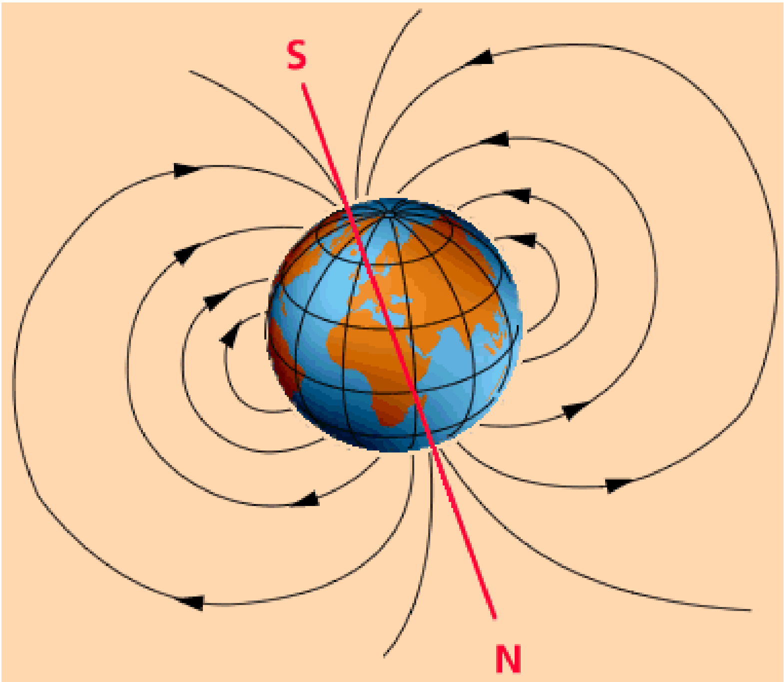


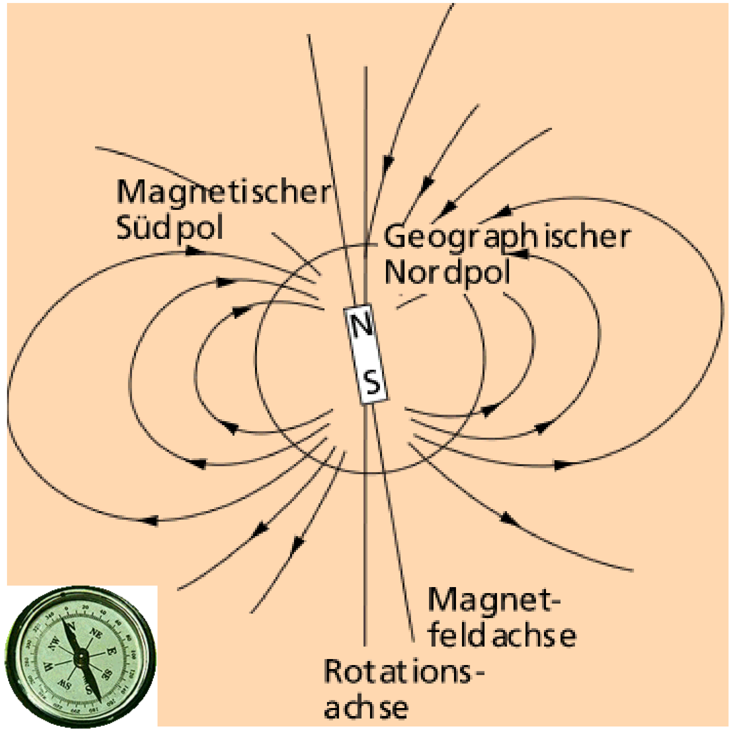


MS Fields – Magnetic Fields of Permanent Magnets / MS-Felder – Magnetfelder von Permanentmagneten

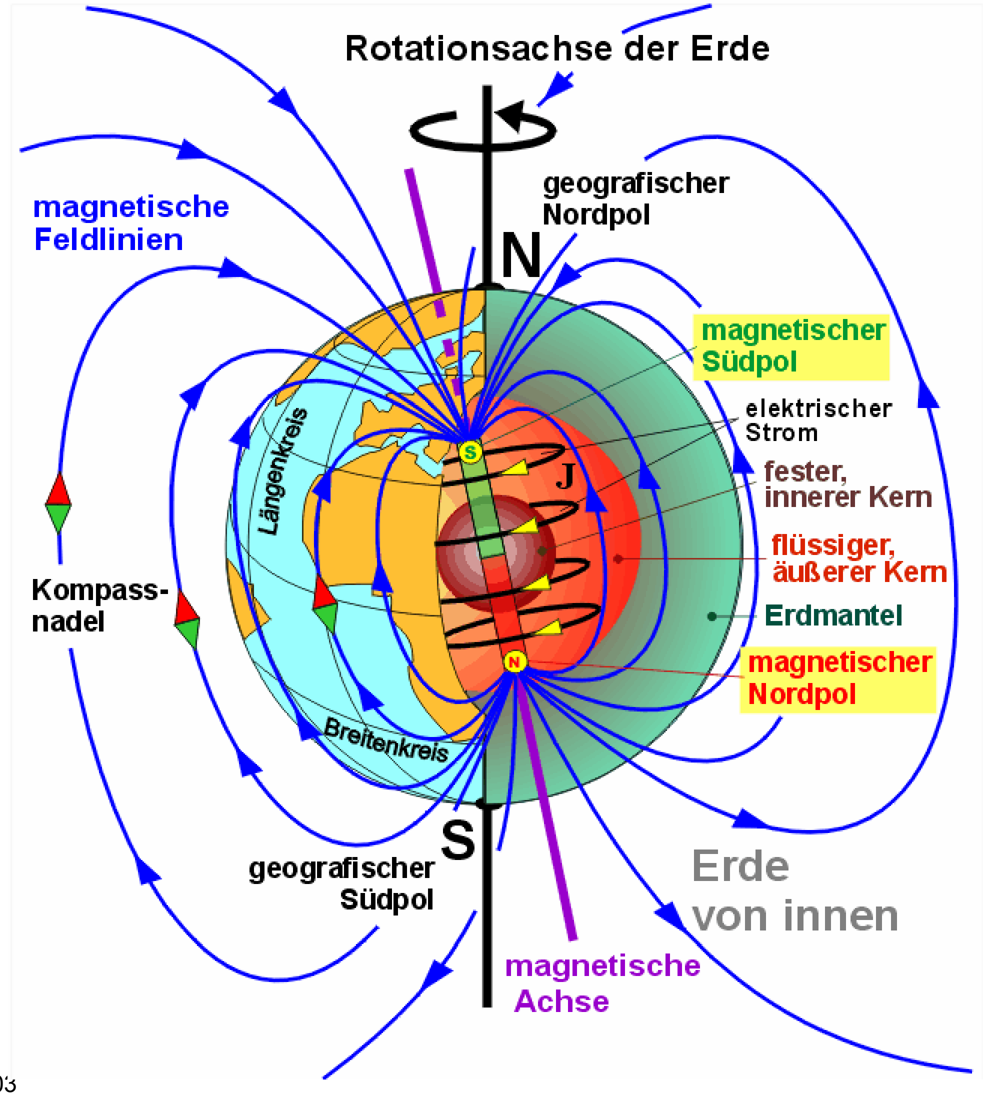


MS Fields – Magnetic Field of the Earth / MS-Felder – Magnetfeld der Erde

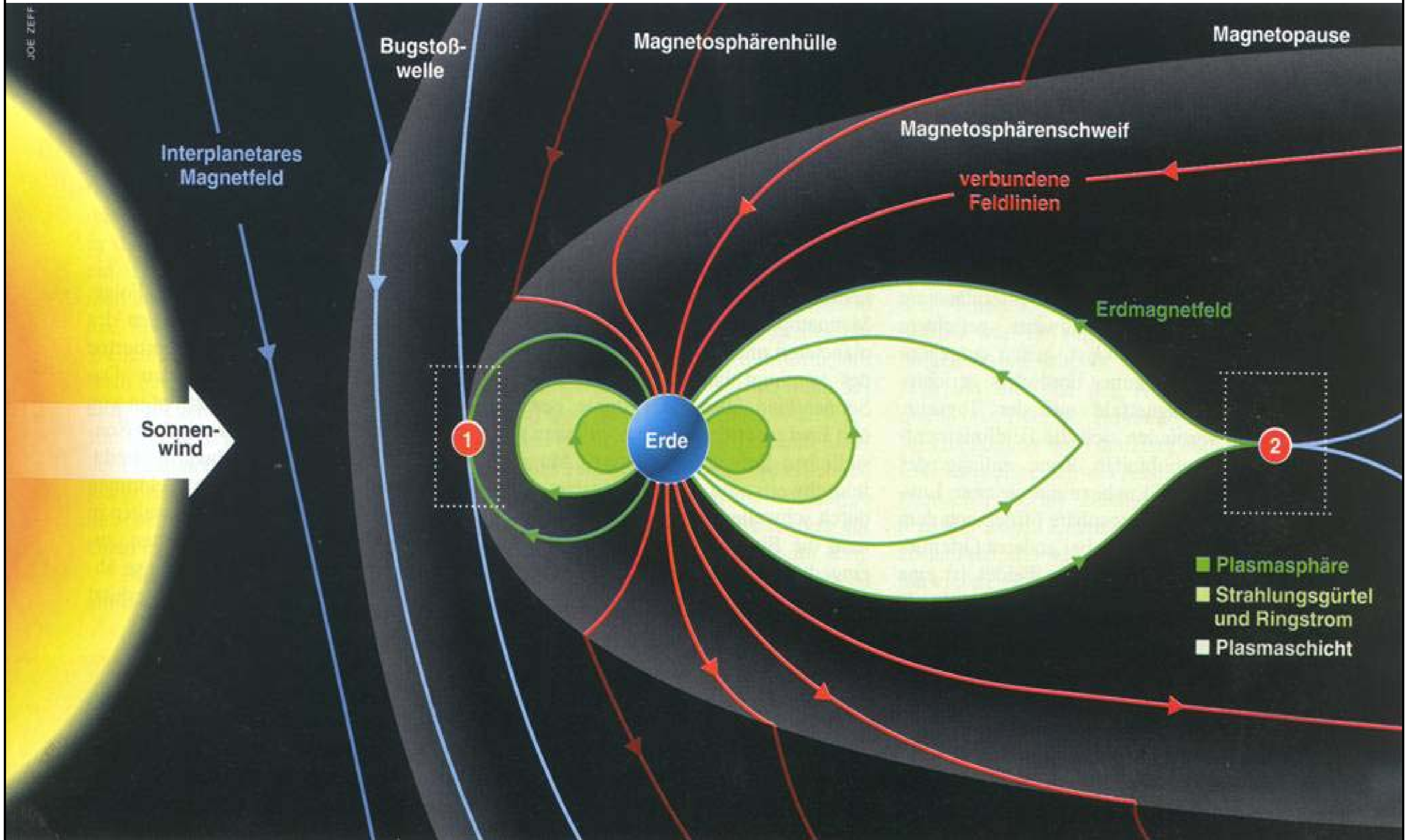




MS Fields – Magnetic Field of the Earth / MS-Felder – Magnetfeld der Erde



MS Fields – Magnetic Field of the Earth – Sun Wind / MS-Felder - Magnetfeld der Erde - Sonnenwind



MS Fields – Governing Equations / MS-Felder - Grundgleichungen

Integral Form / Integralform

$$\oint_{C=\partial S} \underline{\mathbf{H}}(\underline{\mathbf{R}}) \bullet d\underline{\mathbf{R}} = \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) \bullet d\underline{\mathbf{S}}$$

$$\oiint_S \underline{\mathbf{B}}(\underline{\mathbf{R}}) \bullet d\underline{\mathbf{S}} = 0$$

Differential Form / Differentialform

$$\nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}) = \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$\nabla \bullet \underline{\mathbf{B}}(\underline{\mathbf{R}}) = 0$$

$\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})$

is a Known Prescribed Electric Current Density: For Example a Electric Current Density in a Wire / ist eine bekannte vorgegebene elektrische Stromdichte: Zum Beispiel eine elektrische Stromdichte in einem Draht

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Because of / Weil $\nabla \bullet \underline{\mathbf{B}}(\underline{\mathbf{R}}) = 0$ $\underline{\mathbf{B}}(\underline{\mathbf{R}})$ **Can be Represented by / kann dargestellt werden über** $\underline{\mathbf{B}}(\underline{\mathbf{R}}) = \nabla \times \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}})$

In Vacuum we have / Im Vakuum gilt

$$\underline{\mathbf{B}}(\underline{\mathbf{R}}) = \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}})$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{1}{\mu_0} \underline{\mathbf{B}}(\underline{\mathbf{R}}) = \frac{1}{\mu_0} \nabla \times \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}})$$

MS Fields – Vector Potential / MS Felder – Vektorpotential

$$\nabla \bullet \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) = 0$$

Coulomb Gauge / Coulomb-Eichung

It follows / Es folgt

$$\mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}}) = \nabla \times \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}})$$

Applying the Curl Operator Gives / Die Anwendung des Rotationsoperators ergibt

$$\mu_0 \nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}) = \nabla \times \nabla \times \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) = \mu_0 \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$\begin{aligned}\nabla \times \nabla \times \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) &= \nabla \underbrace{\nabla \bullet \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}})}_{=0} - \nabla \bullet \nabla \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) \\ &= -\nabla \bullet \nabla \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) \\ &= -\Delta \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}})\end{aligned}$$

MS Fields – Vector Potential – Poisson and Laplace Equation / MS Felder – Vektorpotential – Poisson- und Laplace-Gleichung

$$\Delta\,\underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) = \begin{cases} -\mu_0\,\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) & \text{for / für} \quad \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) \neq \underline{\mathbf{0}} \quad \text{Vectorial Poisson Equation / Vektorielle Poisson-Gleichung} \\ 0 & \text{for / für} \quad \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}} \quad \text{Vectorial Laplace Equation / Vektorielle Laplace-Gleichung} \end{cases}$$

MS Fields – Vector Potential – (…) Special Solution / MS Felder – Vector Potential – (...) Spezielle Lösung

$$\Delta\, \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) = -\mu_0 \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})$$

Vector Possion’s Equation / Vektorielle Possion-Gleichung

Special Solution / Spezielle Lösung

$$\underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) = \mu_0 \iiint_{V_S} G(\underline{\mathbf{R}} - \underline{\mathbf{R}}')\, \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\, \mathrm{d}^3\underline{\mathbf{R}}'$$

With the Three-Dimensional Static Green’s Function / Mit der dreidimensionalen statischen Greenschen Funktion

$$G(\underline{\mathbf{R}} - \underline{\mathbf{R}}') = \frac{1}{4\pi} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|}$$

$$\begin{aligned}
\underline{\mathbf{H}}(\underline{\mathbf{R}}) &= \frac{1}{\mu_0} \nabla \times \underline{\mathbf{A}}_{\mathrm{e}}(\underline{\mathbf{R}}) \\
&= \frac{1}{\mu_0} \nabla \times \left[\mu_0 \iiint_{V_S} G(\underline{\mathbf{R}} - \underline{\mathbf{R}}')\, \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\, \mathrm{d}^3\underline{\mathbf{R}}' \right] \\
&= \iiint_{V_S} \nabla \times \left[G(\underline{\mathbf{R}} - \underline{\mathbf{R}}')\, \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}') \right] \mathrm{d}^3\underline{\mathbf{R}}'
\end{aligned}$$

$$\nabla \times (\Phi \underline{\mathbf{A}}) = \Phi \nabla \times \underline{\mathbf{A}} + \nabla\Phi \times \underline{\mathbf{A}}$$

$$\nabla \times [G(\underline{\mathbf{R}} - \underline{\mathbf{R}}')\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')] = G(\underline{\mathbf{R}} - \underline{\mathbf{R}}') \underbrace{\nabla \times \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')}_{=\underline{\mathbf{0}}} + \nabla G(\underline{\mathbf{R}} - \underline{\mathbf{R}}') \times \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')$$

MS Fields – Vector Potential – Biot-Savart’s Law / MS Felder – Vector Potential – Biot-Savartsche Gesetz

$$\nabla G(\underline{\mathbf{R}}-\underline{\mathbf{R}}') = \nabla \frac{1}{4\pi}\frac{1}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|} = \frac{1}{4\pi}\nabla\frac{1}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|} = -\frac{1}{4\pi}\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}'}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = -\iiint_{V_S}\frac{1}{4\pi}\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}'}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\mathrm{d}^3\underline{\mathbf{R}}'$$

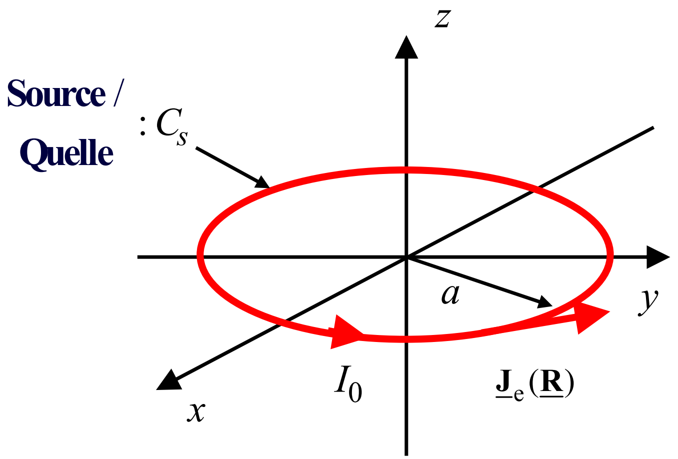
$$= \iiint_{V_S}\frac{1}{4\pi}\frac{\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\times(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\mathrm{d}^3\underline{\mathbf{R}}'$$

$$= \frac{1}{4\pi}\iiint_{V_S}\frac{\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\times(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\mathrm{d}^3\underline{\mathbf{R}}'$$

Biot-Savart’s Law (for a Given Volume Source) /
Biot-Savartsches Gesetz (für eine Volumenquelle)

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{1}{4\pi}\iiint_{V_S}\frac{\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}')\times(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\mathrm{d}^3\underline{\mathbf{R}}'$$

MS Fields – Biot-Savart's Law for a Line Source / MS Felder – Biot-Savartsches Gesetz für eine Linienquelle



$$\underline{\mathbf{J}}_{\rm e}(\underline{\mathbf{R}}) = I_0\,\delta(r - a)\,\delta(z)\,\underline{\mathbf{e}}_{\varphi}, \qquad a > 0$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{1}{4\pi}\iiint_{V_s} \frac{\underline{\mathbf{J}}_{\rm e}(\underline{\mathbf{R}}')\times(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\,{\rm d}^3\underline{\mathbf{R}}'$$

$$= \frac{1}{4\pi}\int\limits_{z'-\infty}^{\infty}\int\limits_{\varphi'=0}^{2\pi}\int\limits_{r'=0}^{\infty} \frac{I_0\delta(r'-a)\delta(z')\underline{\mathbf{e}}_{\varphi'}(\varphi')\times(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\,r'{\rm d}r'{\rm d}\varphi'{\rm d}z'$$

$$= \frac{I_0}{4\pi}\int\limits_{\varphi'=0}^{2\pi}\int\limits_{r'=0}^{\infty} \frac{\delta(r'-a)\underline{\mathbf{e}}_{\varphi'}(\varphi')\times(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|^3}\,r'{\rm d}r'{\rm d}\varphi'\Bigg|_{\underline{\mathbf{R}}':z'=0}$$

MS Fields – Biot-Savart's Law for a Line Source / MS-Felder – Biot-Savartsche Gesetz für eine Linienquelle

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0}{4\pi} \int\limits_{\varphi'=0}^{2\pi} \frac{\overbrace{\underline{\mathbf{e}}_{\varphi'}(\varphi')\, a\, \mathrm{d}\varphi'}^{=\underline{\underline{\mathbf{dR}}}'} \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3} \Bigg|_{\underline{\mathbf{R}}':r'=a;z'=0}$$

$$= \frac{I_0}{4\pi} \int\limits_{\varphi'=0}^{2\pi} \frac{\underline{\underline{\mathbf{dR}}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3} \Bigg|_{\underline{\mathbf{R}}':r'=a;z'=0}$$

$$= \frac{I_0}{4\pi} \int\limits_{C_s} \frac{\underline{\underline{\mathbf{dR}}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3} \Bigg|_{C_s:\underline{\mathbf{R}}':r'=a;0\le\varphi'<2\pi;z'=0}$$

Line Source with Contour C_S /
Linienquelle mit der Kontur C_S

$$C_S : \underline{\mathbf{R}}' : r' = a; 0 \le \varphi' < 2\pi; z' = 0$$

Biot-Savart Law for a Line Source with Contour C_S /
Biot-Savartsches Gesetz für eine Linienquelle mit der Kontur C_S

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0}{4\pi} \int\limits_{C_s} \frac{\underline{\underline{\mathbf{dR}}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

⇨ Arbitrary Contour C_S /
Beliebige Kontur C_S

MS Fields – Biot-Savart's Law for a Line Source / MS-Felder – Biot-Savartsche Gesetz für eine Linienquelle

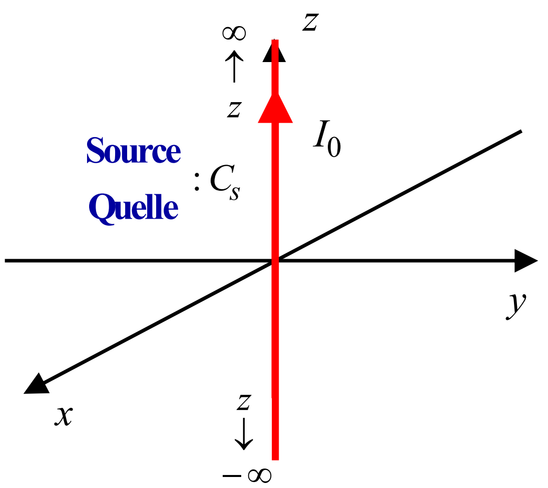
Biot-Savart's Law for a Volume Source V_s /
Biot-Savartsches Gesetz für eine Volumenquelle V_s

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{1}{4\pi} \iiint_{V_s} \frac{\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}}') \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3} \mathrm{d}^3 \underline{\mathbf{R}}'$$

Biot-Savart Law for a Line Source with Contour C_S /
Biot-Savartsches Gesetz für eine Linienquelle mit der Kontur C_S

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0}{4\pi} \int_{C_s} \frac{\underline{\mathbf{dR}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

MS Fields – Biot-Savart – Example: Infinite Thin and Infinite Long Wire Carrying a Constant Electric Current / MS-Felder – Biot-Savart – Beispiel: unendlich dünner, unendlich langer Draht, der einen konstanten elektrischen Strom führt



Biot-Savart Law for a
Line Source with Contour C_S /
Biot-Savartsches Gesetz
für eine Linienquelle mit der Kontur C_S

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0}{4\pi} \int_{C_S} \frac{\mathbf{d}\underline{\mathbf{R}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|^3}$$

$$\underline{\mathbf{R}} = r\underline{\mathbf{e}}_r(\varphi) + z\underline{\mathbf{e}}_z$$

$$\underline{\mathbf{R}}' = r'\underline{\mathbf{e}}_{r'}(\varphi') + z'\underline{\mathbf{e}}_z \Big|_{r'=0}$$

$$= z'\underline{\mathbf{e}}_z, \qquad -\infty \le z' \le \infty$$

$$\underline{\mathbf{R}} - \underline{\mathbf{R}}' = r\underline{\mathbf{e}}_r + (z - z')\underline{\mathbf{e}}_z$$

$$\mathbf{d}\underline{\mathbf{R}}' = \frac{\mathrm{d}}{\mathrm{d}z'}\underline{\mathbf{R}}' dz'$$

$$= \underline{\mathbf{e}}_z \mathrm{d}z'$$

$$\mathbf{d}\underline{\mathbf{R}}' \times (\underline{\mathbf{R}} - \underline{\mathbf{R}}') = \begin{vmatrix} \underline{\mathbf{e}}_r & \underline{\mathbf{e}}_\varphi(\varphi) & \underline{\mathbf{e}}_z \\ 0 & 0 & dz' \\ r & 0 & z - z' \end{vmatrix}$$

$$= r\,\underline{\mathbf{e}}_\varphi\, dz'$$

$$\left|\underline{\mathbf{R}} - \underline{\mathbf{R}}'\right|^3 = \left[r^2 + (z - z')^2\right]^{3/2}$$

MS Fields – Biot-Savart – Example: Infinite Thin and Infinite Long Wire Carrying a Constant Electric Current / MS-Felder – Biot-Savart – Beispiel: unendlich dünner, unendlich langer Draht, der einen konstanten elektrischen Strom führt (...)

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0}{4\pi} \int_{z'=-\infty}^{\infty} \frac{r\underline{\mathbf{e}}_\varphi(\varphi)}{\left[r^2 + (z-z')^2\right]^{3/2}} \mathrm{d}z'$$

$$= -\frac{I_0}{4\pi} r\underline{\mathbf{e}}_\varphi(\varphi) \int_{\alpha=\infty}^{-\infty} \frac{\mathrm{d}\alpha}{\left[r^2 + \alpha^2\right]^{3/2}}$$

$$= \frac{I_0}{4\pi} r\underline{\mathbf{e}}_\varphi(\varphi) \int_{\alpha=-\infty}^{\infty} \frac{\mathrm{d}\alpha}{\left[r^2 + \alpha^2\right]^{3/2}}$$

$$= \frac{I_0}{4\pi} r\underline{\mathbf{e}}_\varphi(\varphi) \int_{\alpha=-\infty}^{\infty} \frac{\mathrm{d}\alpha}{\left[r^2 + \alpha^2\right]^{3/2}}$$

$$= \frac{I_0}{4\pi} r\underline{\mathbf{e}}_\varphi(\varphi) \left. \frac{\alpha}{r^2\sqrt{r^2 + \alpha^2}} \right|_{\alpha=-\infty}^{\infty}$$

With the Substitution / Mit der Substitution

$$\alpha = z - z'$$

$$\mathrm{d}z' = -\mathrm{d}\alpha$$

$$z' = -\infty : \quad \alpha = z + \infty = \infty$$

$$z' = \infty : \quad \alpha = z - \infty = -\infty$$

$$\int \frac{\mathrm{d}x}{\left[ax^2 + b\right]^{3/2}} = \frac{x}{b\sqrt{ax^2 + b}}$$

with
mit $a = 1, \; b = r^2$

MS Fields – Biot-Savart – Example: Infinite Thin and Infinite Long Wire Carrying a Constant Electric Current / MS-Felder – Biot-Savart – Beispiel: unendlich dünner, unendlich langer Draht, der einen konstanten elektrischen Strom führt (...)

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \frac{I_0}{4\pi}\frac{1}{r}\frac{\operatorname{sgn}(\alpha)\,|\alpha|}{\sqrt{r^2+\alpha^2}}\Bigg|_{\alpha=-\infty}^{\infty}\underline{\mathbf{e}}_\varphi(\varphi)$$

$$= \frac{I_0}{4\pi}\frac{1}{r}\frac{\operatorname{sgn}(\alpha)\,|\alpha|}{|\alpha|\sqrt{1+r^2/\alpha^2}}\Bigg|_{\alpha=-\infty}^{\infty}\underline{\mathbf{e}}_\varphi(\varphi)$$

$$= \frac{I_0}{4\pi}\frac{1}{r}\left[\underbrace{\lim_{\alpha\to\infty}\frac{\operatorname{sgn}(\alpha)}{\sqrt{1+r^2/\alpha^2}}}_{=1} - \underbrace{\lim_{\alpha\to-\infty}\frac{\operatorname{sgn}(\alpha)}{\sqrt{1+r^2/\alpha^2}}}_{=-1}\right]\underline{\mathbf{e}}_\varphi(\varphi)$$

$$= \frac{I_0}{4\pi}\frac{1}{r}\underbrace{\left[1-(-1)\right]}_{2}\underline{\mathbf{e}}_\varphi(\varphi)$$

$$= \frac{I_0}{2\pi}\frac{1}{r}\underline{\mathbf{e}}_\varphi(\varphi)$$

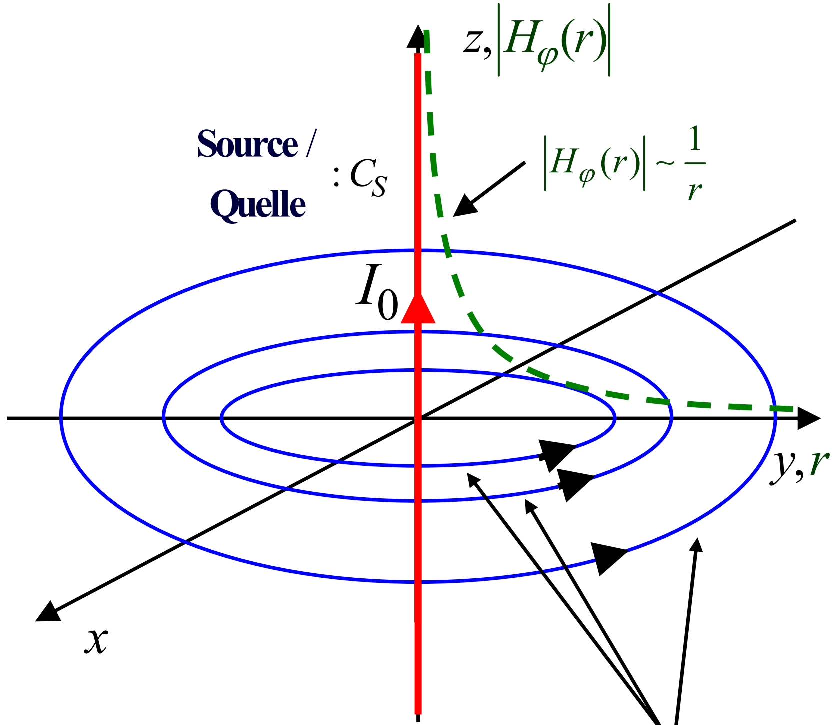
With the Signum Function / Mit der Signum-Funktion

$$\operatorname{sgn}(\alpha) = \begin{cases} -1 & \alpha < 0 \\ 1 & \alpha > 0 \end{cases}$$

$$\alpha = \operatorname{sgn}(\alpha)\ |\alpha|$$

With / Mit $\sqrt{\alpha^2} = |\alpha|$

MS Fields – Biot-Savart – Example: Infinite Thin and Infinite Long Wire Carrying a Constant Electric Current / MS-Felder – Biot-Savart – Beispiel: unendlich dünner, unendlich langer Draht, der einen konstanten elektrischen Strom führt (...)



Field Lines of the Magnetic Field Strength / Feldlinien der magnetischen Feldstärke $\underline{\mathbf{H}}(\underline{\mathbf{R}})$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}}) = \underbrace{\frac{I_0}{2\pi}\frac{1}{r}}_{=H_\varphi(r)} \underline{\mathbf{e}}_\varphi(\varphi)$$

$$= H_\varphi(r)\underline{\mathbf{e}}_\varphi(\varphi)$$

$$|\underline{\mathbf{H}}(\underline{\mathbf{R}})| = |H_\varphi(r)| \sim \frac{1}{r}$$

End of the 10th Lecture / Ende der 10. Vorlesung